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Official Statistics and its Impact on Good Governance and Economy of Pakistan

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Dr. Munir Ahmad
Editor

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I, on my behalf and on behalf of National college of Business Administration and Economics and the Islamic Countries Society of Statistical Sciences take this opportunity to thank you for sparing your precious time.

I thank Mian Shamim Haider, Chairman, NCBA&E and former Federal Minister for Railway, who for the love of education, has dedicated himself to the spreading of knowledge. I deeply appreciate his efforts for NCBA&E programs by creating alliances amongst statisticians and encouraging us in organizing such conferences for exchange of knowledge and experiences.

It is our cherished desire that we make intensive and sustained struggle to expand the horizons of research in general and particularly in Statistics. Statistical scientists of academia by sharing their scientific potential and expanding their databases will help Pakistan by bringing technology to their door-steps which, of course, could be utilized for the betterment of the Muslim Ummah.

The society, which would be celebrating its 25 years journey next year, was established way back in 1988, during the first Islamic Countries Conference on Statistical Sciences held at Lahore.

I feel really proud of its existence, as the Society had held eleven Islamic Countries Conferences on Statistical Sciences at Lahore, Morocco, Malaysia, and Indonesia, and numerous national conferences, seminars and workshops in Pakistan. The 12th Conference will be held in Doha, Qatar on December 21-24, 2012.

We need to look up to our ancestral scientific legacy. We cannot revive our scientific heritage by putting education outside the framework of national activities and seeking assistance from outside Islam. We need program with faith and follow the footsteps of our administrative giants. Hazrat Omar (RS); bravery and intellect of Hazrat Ali (AS). We had created and established many great educational institutions in the past but we could not sustain those great universities. We let others destroy our scientific and academic institutions and libraries which were rich and well established in Baghdad, Cordoba, Seville, Cairo, Toledo, and Fez. The calamity of oblivion of Islamic thoughts still persists. In the past our Ulemas were scientists and our scientists were renowned Ulemas. They were Imams, scientists and poets, who produced un-precedent inventions, discoveries, and philosophy. Now, unfortunately our Ulemas are considered as terrorists and terrorists are considered Ulemas. Those are still the bases of current scientific

advancements in the world. We were no doubt proud of Muslim scientists and their philosophic thoughts. Even now we have noble laureates in the Muslim world which show that we have the potential of becoming great scientists, social scientists and philosophers. We should not forget our past and current achievements but unfortunately, we still seem to live in Mohanjo Daro and pre-Egyptian civilization. It is still a common sight of wooden bullock driven carts on broken roads alongside Pajero and BMW running on Motor Ways in Pakistan. We are still proud of our ancestors and take pride in what they had achieved in the fields of science and technology. They excelled others in their achievements in many aspects of contemporary sciences. They kindled the lights of knowledge around the globe and shared unreservedly the fruit of their learning with others. It paints of very dismal and gloomy picture when we look at our present status and the directions of which we are heading today. I am not talking about the Muslim scientists; I am talking about the overall status of science and technology in the Muslim World. In the past, other nations learnt many things from us. Now, we will have to make advancements in the current state of our knowledge at an accelerated pace as a religious obligation. With committed and devoted applications, we can recreate, under the new system of coordination and interaction among Muslim scientists and scientific centers of Fez, Azhar, Maragha, Iran, Cordova, Grenada, Baghdad, Basra, Tashkent, Bukhara and Samarkand. These were the places where scientists from both East and West were attracted to. At this point, in our history, the Muslim communities have produced Scientists who were capable of excelling in their areas of specializations and were able to contribute significantly. However, this is very unfortunate that Muslim Ummah is unable to utilize the potential of their capable scholars and scientists. Unfortunately these persons have been serving the Western societies. It is an economic reality that wealth migrates to safe abodes. The same is true for the scientists and technologists who are attracted towards seats of learning and places of appreciation. They migrate to places where they are secure, welcomed, honoured and nurtured. I am sure you will agree with me that this is an opportune time to accept the challenges of rediscovering the glory of past and leading scientists to enter the 21st Century with prestige, honour and respect. It is a common question, "how long will it take us to revive our scientific and cultural supremacy?" I believe, if we take faith like our ancestors and act with their dedication and commitment, certainly, it will not take us too long to achieve the same heights of the past glory.

There are 57 Muslim countries with more than one billion population, which account for more than 25% of the World population. Everywhere the physical and human resources remain under-utilized. We need collective self-reliance in the Islamic Community, mutual cooperation and extensive exchange of scientists. The current state of Muslim world is due to decline in the knowledge of science and technology where Muslims had played a leading role in the past.

The theme of this conference is "Official Statistical and its impact on Good Governance and economy of Pakistan"

Official Statistical provide a true picture of a country and is an important element of a democratic society. People have been collecting official statistics since ancient times. Hindu Veda, old testaments, and Holy Quran and other divine scriptures had been talking

about people, its structure, economy and administration. Some scientific surveys had been conducted for good governance during Khalifat period.

In Pakistan, we did not get our share of statistical organization set up at the time of partition and consequently Pakistan had established Central Statistical Office in 1950. The first census was held in 1951. On the academic side, departments of statistics were established one in Dhaka and one in Lahore under the able leaders of Dr. Q.M. Hussain and Dr. M. Ziauddin. The discipline of statistics took its proper status now. Pakistan Statistical Association was also established in 1950 under able leadership of Dr. M. Ziauddin.

Prof. Dr. R.A. Fisher and P.C. Mahalanobis and many internationally renowned statisticians visited Pakistan and attended the Conferences. But unfortunately, PSA had become defunct.

We in 1988 held first Islamic Countries Conference on Statistical Sciences in Lahore and constituted Islamic Countries Society of Statistical Sciences with HQ at Lahore. We held Conferences and published its proceedings regularly.

We also published Pakistan Journal of Statistics in 1985 and since then we had been publishing it regularly without break. It has now entered 27th year of publication. The 28th volume is in press. It is enlisted in ISI Thomson and is one of 116 statistical journals of the world. Its impact factor has been doubled this year and is one of the two journals in statistics from Islamic Countries.

ISOSS and PJS should provide platform for the Pakistani Statisticians and should give a lead to Pakistan to be one of the statistically advanced countries.

Official Statisticians should play their role as Bureau of Census. Washington D.C. had played its role in developing statistics. Many techniques were developed and discovered by statisticians working in Bureau of Census. Similarly, I wish our Pakistan Bureau of Statistics should be able to develop and discover new statistical technologies. There are many areas where they can work and undertake research. Training Wing and R&D Sections of Pakistan Bureau of Statistics should work together and try solutions to many unsolved problems of official statistics. There are many new areas where they can work.

I am sure, if they call for academicians in the area, when there are now more than 100 Ph.Ds in Statistics in Pakistan and are very capable. Ph.Ds who can help Pakistan Bureau of Statistics in their endeavor of finding new technologies for improving Pakistan economy.

I strongly believe that planning has to be knowledge based and to be monitored by a strong team of statisticians. At present, there is no interaction between academicians and Statistics Officials at Federal as well as provincial levels. If Official Statistics is to be improved, academia must be involved very effectively and if statistical agencies need qualified incumbents, statistical agencies must be effectively involved in colleges and universities so that statisticians produced by colleges and universities are directly inducted in statistical organizations.

I will be talking on this aspect after this ceremony.

As I mentioned earlier, statistics and information are the basic ingredients of any development plan. I believe that the basic information, accurately collected and scientifically analyzed are the imperatives of planning in the countries.

I foresee ISOSS would develop into as a World Forum that can be managed on a collective vision of its active members. I invite all participants to become active member of the society. Dignity of top statisticians working both in public and private sectors is a pre-requisite to Society's strategy and action plans.

In the past, I proposed to institute a number of awards for young statisticians which could not be implemented for want of responses from statisticians, students, teachers and professionals. I am going to propose ISOSS Awards and hope that this time support will be received from all over.

I am also proposing to Government of Pakistan to create a new cadre of Para-Statisticians who have specific qualifications for the positions. A detailed report is being prepared. I will be talking about this cadre in my next talk. I once again thank you all for attending the conference and making it a great success.

Thank you.

INSTRUCTIONAL TECHNOLOGY: A TOOL OF EFFECTIVE LEARNING

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ABSTRACT

The objective of the study was to investigate the effects of instructional technology on students' learning. Five heads and head mistresses of Government Boys and Girls Secondary Schools of Wah Cantonment area were selected for the study. These schools were selected on two bases: Masters and Graduate Teachers, and schools having both arts and science streams. A questionnaire was statistically prepared for heads and head mistresses to take their opinion about the use of instructional material for effective learning. Percentage method was applied for data analysis by using Statistical Package for Social Sciences (SPSS). Major findings indicated that instructional technology is helpful to increase the efficiency of students' learning; improve the standard and quality of education, and teachers' behavior and classroom instruction; support learning activity; realize desirable teaching-learning objectives; bring accuracy in learning and take first-hand experience. Instructional technology is integral part of teaching-learning process. Instructional technology is the backbone of teaching process. Instructional technology acts as a tool for effective learning.

KEYWORDS

Students; Teachers; Teaching Process; Learning; Teaching-learning Process.

1. INTRODUCTION

Goal of education is attained when teaching acts as an art as well as science. When the imaginative and artistic abilities of the teachers are shown for creating worthwhile situation in class, it acts as an art and when teachers use logical, mechanical and procedural steps to attain goals, teaching becomes science (Joshi, 2005).

According to Khan (2011), just transferring of information to students is not teaching. Teaching is a planned activity about how to teach and guide the students for maximum learning. Teaching is a dynamic process in which all effort is put to maximize the learning experiences. Teaching is a way to improve thinking, understanding and learning process by utilizing several methods, techniques and instructional aids to improve the quality of instruction.

Quality of instruction is directly linked with the way the teachers teach in the classrooms. Teachers utilize several methods, techniques, strategies and instructional aids to make their teaching effective. They know that without using all these, they would not

be able to achieve their desired objectives. To achieve these objectives, they utilize technology in the classroom in a proper way.

Meaning of technology is different in the context of education. Before defining instructional technology, it is desirable to clear the concept of technology.

2. DEFINITION OF TECHNOLOGY

Naughton (1986) (as cited in Aggarwal, 1995) argued that technology can be considered as things as well as a social process. When we apply scientific and systematic knowledge to the practical work by involving 2 M's – man and machines, it is termed as technology of things. When we apply scientific and systematic knowledge to the practical work by involving hierarchical order, it is called as technology of social process. So it is not only the "tool" for the development of science but also the "change" in the social process.

Hiera (1976) (as cited in Aggarwal 1995) said that when scientific knowledge is applied to the practical purpose, then it is called technology.

All definitions show that the practical application of technology is to attain the specific purpose. In this way it plays two important roles, i.e. application of scientific knowledge and attainment of pre-determined objectives as it is a man made device to produce a reproducible effect.

So there is an ample reason to use technology in the classroom. When technology applies in the educational setting, it is termed as educational technology. Instructional Technology, Educational Technology, Audio-visual aids, Educational communication Technology, Audio-Visual Media, Learning Resources, Instructional or Educational Media, all have approximately the same meaning due to their same purpose, i.e. achievement of objects, goals or purpose. Hence instructional technology is the part and section of educational technology.

3. DEFINITION OF EDUCATIONAL TECHNOLOGY:

Unwin (1969) (as cited in Aggarwal 1995) argued that practical application of new and innovative skills and technology to impart knowledge and training by using media (print and electronic), new teaching method and provision of congenial atmosphere in which students are free to grow is educational technology.

Hadden (as cited in Aggarwal 1995) defined it as educational technology is concerned with theory and practice applicable to education by applying design and use of messages to control environment.

Leith (1967) (as cited in Aggarwal 1995) said that use of scientific, practical, procedural, and systematic knowledge about learning and its conditions to enhance the teaching-learning process is educational technology.

4. SCOPE OF EDUCATIONAL TECHNOLOGY

Aggarwal (1995) stated that scope of educational technology is as wide as education itself. Its scope ranges from the concrete educational process to the most abstract one that

includes the use of hardware, software and system analysis. There are three major areas of education in which technology has direct linkage:

1. Technology related to general education, administration and management
2. Technology related to general educational testing
3. Technology related to general instructional process

Rashid (2007) stated that research on educational technology always have an ambiguous agenda. Its agenda aim is to increase the efficiency of current practices, pedagogical change, design the science, address to the basic issues of teaching-learning process and social organizational structure. Because of its broad agenda, it utilizes all methodologies applied in social and life sciences.

Rashid (2007) viewed that educational technology promotes constructive and productive relation among new facilities and other factors that affect theory and practice of education. Educational technology embraces the utilization of new apparatus, equipments, methods, techniques, and their selections, adoption and coordination for effective learning. The shift is from predominantly intuition to critical, procedural, systematic and analytical approach. This shift includes adequate objectives; and proper use of appropriate technology for the effective assessment and modification of the students' learning. So the importance of technology in education settings is quite evident.

Learning does not occur in a vacuum. It takes place in an environment where instructional practices are appropriate to previous knowledge, cognition, aptitude, attitude, styles and strategies of thinking and so on.

Marshall, (2002) said that it is necessary to find out the ways to successful application of these technologies. Now the shift is from its applicability to its impact on student's effective learning.

New technology is immensely used in instruction. Instruction is no more without technology. Instructional Technology is used for teaching any subject assessing students' achievement and behavior; checking the attainment of instructions, objective; and modifying students' behavior etc.

5. INSTRUCTIONAL TECHNOLOGY:

Richart (2002) defined that instructional materials mean written and published textbooks and other supplementary materials (used by teachers in teaching) required in school.

Aggarwal (1995) stated that instructional technology is a system of 5 M's i.e. (machines, materials, media, men, and methods) which are inter-related with each other and work for the common cause i.e. fulfillment of specific educational objectives.

The description of 5 M's is:

Machines: Electronic or non-electronic
Materials: Teaching aids, text books or any supplementary material
Media: Print and Electronic
Men: Personnel involved in education process i.e. teachers, students, advisors etc.
Methods: Teaching methods to impart education

According to Aggarwal (1995), instructional technology is an applied or practical study where aim is to maximize the educational effects on student's learning, controlling educational purposes, educational content, teaching aids, teaching methods and materials, educational environment, conduct of students, behavior of teachers or instructors; and interrelation of students with teachers/instructors, teaching methods with objectives and content, educational content with teaching aids, and so on and so forth.

Instructional technology plays two roles – firstly introducing technological innovations in the field of education and secondly to optimize students' learning.

Mckown and Roberts (as cited in Aggarwal 1995) argued that instructional technology is supplementary devices by which the teacher, through the utilization of more than one sensory channel, is able to clarify the concepts of students.

Instructional technology is concerned with determining and providing appropriate stimuli to the learner to produce certain types of responses for making learning more effective.

Print, non-print or combination of both is instructional technology. Instructional aids are divided into two categories: basic and non-basic material.

Basic Material

Curriculum is considered as basic materials and adopted as a primary means to help students for the attainment of program outcomes. Subject-matter of Textbook and the educational approach also need to be considered as the basic learning material for students' proper learning. The construction of the content of instructional materials is an ongoing process. As the new technology is added, its contents will progress with the speed of its development.

Textbooks, supplementary reading materials, apparatus, tools, charts, maps etc. even pen pencils, chalk, notebooks used by the teachers and students, all are referred as basic learning materials. Today, supporting teaching materials cover both projected and non-projected aids. Some of these aids are costly or require advanced technology and handling perfection so these expensive aids have little attention of the teachers.

Instructional techniques are important as they influence students' academic achievement, behavior modification and other outcomes. Instructional materials provide the physical material to optimize the students' learning.

Instructional materials are helpful to direct the teachers how to teach and instruct, how to impart scientific knowledge, how to develop them professionally etc. Instructional materials are acting as a tool to promote standardized science education. Such materials are undoubtedly useful for the improvement of curricula and leave a deep impact on everyday teaching.

Non-Basic Material

Specially designed materials according to the need of circumstances, resources for individualized learning, library books, pamphlets etc. are non-basic materials. These aids are used by the subject specialists, teachers and administrators. For the selection of non-basic material, it is required to select them on the basis of predetermined objectives and program outputs.

6. IMPORTANCE OF INSTRUCTIONAL TECHNOLOGY

Vicarious experience can be gained from still pictures, films, filmstrips, resource persons, simulations, mockups, television and the like. The more concrete and realistic the vicarious experience, the more nearly it approaches the learning effectiveness of the first level. Of course, unless the learner realizes that he is dealing with a substitute; his learning may not be comparable to that of real-life learning.

According to Aggarwal (1995), instructional technology are those devices that help in clarification, establishment and correlation of different and complex concepts and enable the teachers to make their teaching concrete, effective, efficient, meaningful etc. They are helpful in promoting the learning process i.e. motivation-clarification-stimulation. The purpose of using instructional media is to clear the channel between learner and supportive materials. The basic assumption underlying audio-visual aids is that learning clear the understanding of the students from sense experience. The teacher must "show as well as tell". Audio-visual aids provides significant gains in informational learning, retention and recall, thinking and reasoning, activity interest imagination, better assimilation and individual's development. These aids are considered as the best stimuli for learning these areas i.e. why, what, how, when and where and answer the natural curiosity of the child by answering these questions. The most complex concepts become clearer by intelligently and skillfully designed teaching aids.

The audio-visual aids are the best motivators. The students reduce verbalism by taking clear ideas and bringing accuracy in learning. When our senses are involved, formation of clear images is confirmed. It is beyond doubt that the first-hand experience is the best type of educative experience. But it is neither practicable nor desirable to provide such experience to pupils. Substituted experiences may be provided under such conditions. There are many inaccessible objects and phenomena. For example, it is not possible for an average man to climb the Mount Everest. There are innumerable such things to which it is not possible to have direct access so, in all such cases, these aids help us.

"Mere chalk and talk" do not fulfill the teaching requirements. Provision of variety of tools for classroom teaching enhances students' learning. When audio-visual aids are employed, the chances of freedom for children will increase as they are free to move, walk, talk, comment etc. In such congenial atmosphere of classroom, students start work because they want to work not because of their teachers' willingness. Many teaching aids invite students to handle them so they will become more confident as compare to earlier. Audio-visual aids contribute to increase receptivity. The maxims of teaching are properly utilized with the help of instructional technology.

Teaching-learning process retains attention. Instructional technology is that helping aid that capture and sustain the students' attention and interest throughout the studies. The use of audio-visual aids provides a touch of reality to the learning situation.

Gillani (2005) stated that the use of a variety of audio-visual aids helps in meeting the needs of different type of students. Use of audio-visual aids stir the imagination, thinking process and reasoning power of the students and calls for creativity, and inventiveness and other higher mental activities of students and thus helps the development of higher faculties among the students. Use of audio-visual aids helps in the learning of other

concepts, principles and solving the real problems of life by making appropriate positive transfer of learning and training in the classroom. A balanced, rational and scientific use of these aids develops motivation, attracts the attention and interest of the students and provides a variety of creative outlets for the utilization of their tremendous energy and thus keeps them busy in the class work in this way, the overall classroom environment becomes conducive to create discipline.

There is no substitute of first-hand experience in the educational settings. There are many things on which it is not possible for the teacher to involve students in such experiences. So in all such conditions, it is preferable to use them.

What is gained in terms of learning, need to be fixed and imprint on the minds of the students. Instructional technology helps in achieving this objective by providing several activities, experiences and stimuli to the learner.

Due to the importance of these aids, it is desirable to find out the heads views about the use of instructional technology for better and effective learning. The literature shows the importance of these aids, whether heads give importance for utilizing it and how much importance they are given to these aids; this is the concerned area of this study.

7. RESEARCH METHODOLOGY

7.1 Sample

The sample was consisting of five head masters and head mistress of Government Secondary Schools having both arts and science stream in Wah Cantonment area. These schools were selected on the basis of the qualification of head masters and head mistress as Masters.

7.2 Research Instrument

A self-developed questionnaire on three point rating scale was administered to check their views about the impact of instructional technology on students' learning and its importance in teaching-learning process. The questionnaire was thoroughly examined and validated by experts to check the appropriateness of items.

7.3 Collection of Data

Data was collected for study through questionnaire which was approved by the supervisor and validated by the experts. The researcher herself visited the schools. The questionnaire was given to the heads that was completed in the stipulated period of time.

7.4 Analysis of Data

The data was analyzed by using Percentage. According to Garrett and Woodworth (2011), it is feasible to apply it when sample exhibits a certain behavior or possess a definite attitude or other characteristics when it is impossible to measure these attributes directly.

Table 1: Motivation to Teachers from Heads

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
Heads motivate the teaching staff of their school to use audio-visual aids.	3	60%	2	40%	00	0%	5	100%

Table 1 show that the majority (60%) of the heads agreed that teachers gain encouragement from heads to use audio-visual aids to make their teaching attractive. Some (40%) of the teachers said that effective learning is possible if heads boast-up teachers to use audio-visual aids in the classroom instructions but heads do not motivate the teachers to use them in classroom.

The graphical representation is shown in figure I.
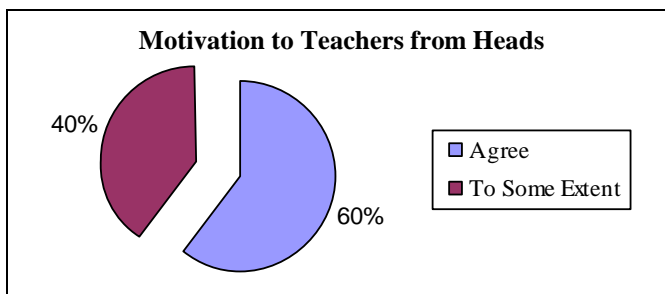
Figure I: Motivation to Teachers from Heads

Table 2: Guidance from Heads and Teachers to Use Instructional Technology

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
Heads and teachers guide the students to respond actively to the audio-visual aids.	3	60%	1	20%	1	20%	5	100%

Table 2 shows that the majority (60%) of the heads said that students show positive results if heads and teachers guide and motivate them. Some (20%) of the heads gave less value to the guidance about the proper use of instructional material. Some (20%) of the heads disagreed with the statement and said that heads and teachers do not provide guidance to the students and heads do not appreciate the teachers to use them for effective learning.

The graphical representation is shown in figure II.
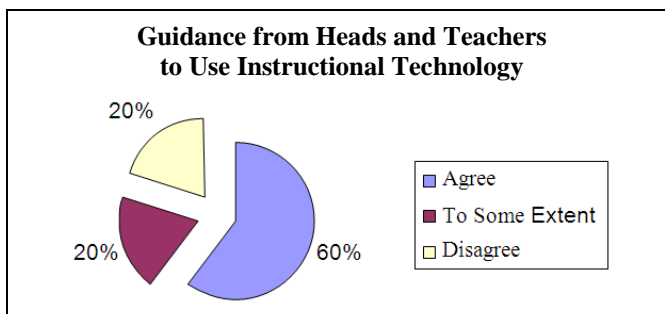
Figure II: Guidance from Heads and Teachers to Use Instructional Technology

Table 3: Instructional Technology Increase Learning Efficiency

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
Instructional technology can increase the efficiency of learning of students.	4	80%	1	20%	00	0%	5	100%

Table 3 shows that the majority (80%) of the heads agreed that effectiveness of students' learning can be improved with the help of instructional technology. Some (20%) of the heads gave less importance to the use of technology for the development of understanding of students' concepts.

The graphical representation is shown in figure III.
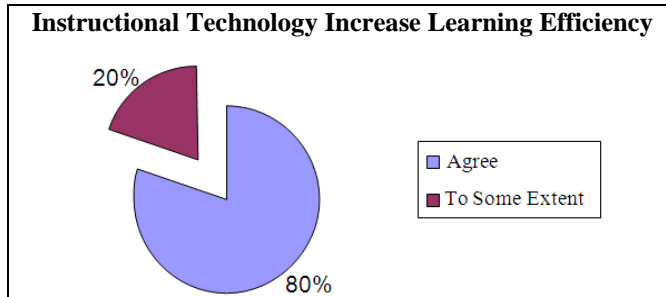
Figure III: Instructional Technology Increases Learning Efficiency

Table 4: Instructional Technology Improve Quality of Education

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
We can improve the quality of education with the use of educational technology.	4	80%	1	20%	00	0%	5	100%

Table 4 shows that the majority (80%) of the heads was of the view that the standard of education will be automatically improved if teachers apply instructional technology in their teaching while 20% gave less value to the role of instructional technology for the effective learning of students in educational process.

The graphical representation is shown in figure IV.
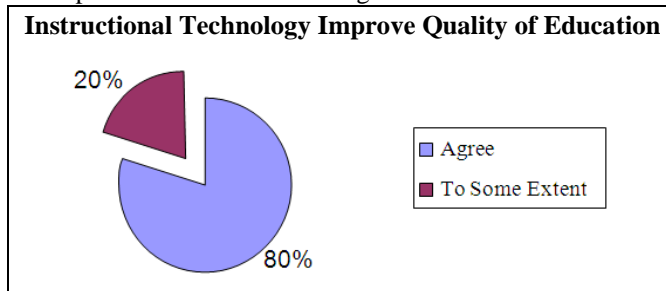
Figure IV: Instructional Technology Improve Quality of Education
Table 5: Improvements in Teachers' Behavior and Classroom Instruction

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
Instructional technology is helpful in the improvement of teacher's behavior and classroom instruction	5	100%	00	0%	00	0%	5	100%

Table 5 shows that all heads (100%) were agreed that teachers effectively communicate with their students when they teach them with the help of instructional technology. They were of the view that instructional technology is helpful for the improvement of teachers' behavior and classroom instruction.

The graphical representation is shown in figure V.
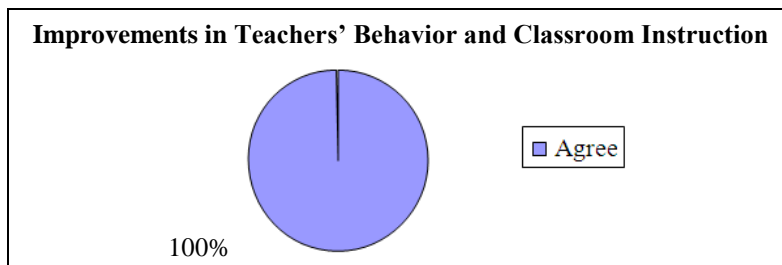
Figure V: Improvements in Teachers' Behavior and Classroom Instruction

Table 6: Use of Instructional Technology Support Learning Activity

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
The uses of technological resources support learning activity.	4	80%	1	20%	00	0%	5	100%

Table 6 shows that the majority (80%) of the heads was of the opinion that the achievement level of students will be improved by the use of technological resources while some (20%) heads said that sometimes technological resources support learning activities only when students are busy and show attention towards teachers.

The graphical representation is shown in figure VI.
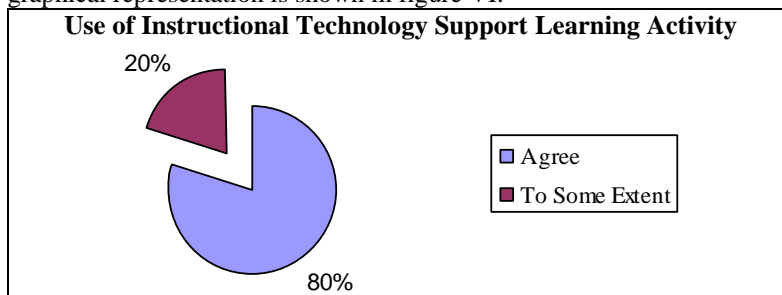
Figure VI: Use of Instructional Technology Support Learning Activity

Table 7: Realization of Desired Teaching-Learning Objectives

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
Audio-visual aids help in the realization of desired teaching-learning objectives.	4	80%	00	0%	1	20%	5	100%

Table 7 shows that the majority (80%) of the heads was of the view that teachers can attain their objectives when they teach in the classroom with the help of audio-visual aids. Some (20%) heads said that audio-visual aids play no role for the fulfillment of desired teaching-learning objectives.

The graphical representation is shown in figure VII.
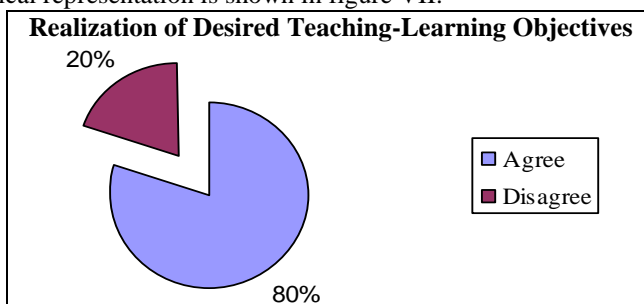
Figure VII: Realization of Desired Teaching-Learning Objectives

Table 8: Bring Accuracy in Learning

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
These aids help to bring accuracy in learning of students.	4	80%	00	0%	1	20%	5	100%

Table 8 shows that the majority (80%) of the heads agreed that students get accurate and perfect knowledge from those teachers who use audio-visual aids in the classroom instructions. Some (20%) heads were disagreed about the accuracy of learning by using instructional technology.

The graphical representation is shown in figure VIII.
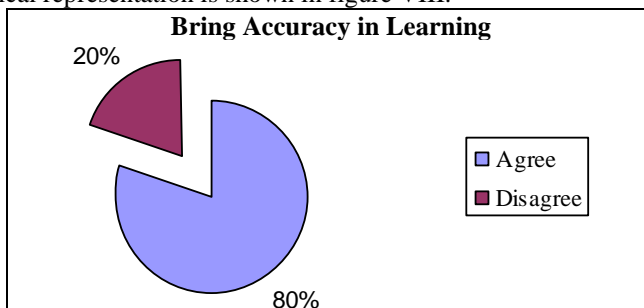
Figure VIII: Bring Accuracy in Learning
Table 9: Provision of Firsthand Knowledge

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
The audio-visual aids provide firsthand experience/knowledge to students.	4	80%	1	20%	00	0%	5	100%

Table 9 shows that the majority (80%) of the heads was in the favour that the students get firsthand knowledge and experience which is everlasting for them because they are able to use their senses properly. Some (20%) of the heads said that sometimes it is possible for the teachers to provide firsthand information to the students.

The graphical representation is shown in figure IX.
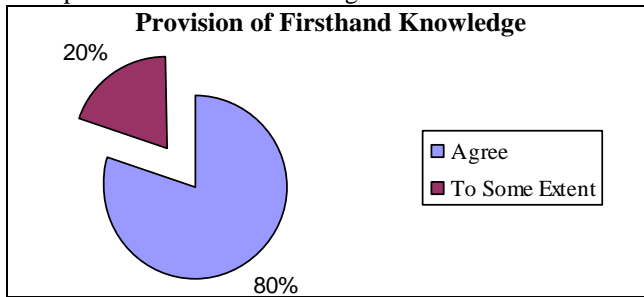
Figure IX: Provision of Firsthand Knowledge

Table 10: Provision of Reality to Learning Situation

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
The uses of audio-visual aids provide a touch of reality to the learning situation.	4	80%	1	20%	00	0%	5	100%

Table 10 shows that the majority (80%) of the heads agreed with this statement that there are more chances to learn complex concepts that are near to reality for the students if teachers teach them with the help of teaching aids. Some (20%) of the heads gave less importance to the role of teaching aids for the better learning of students in real life situation.

The graphical representation is shown in figure X.
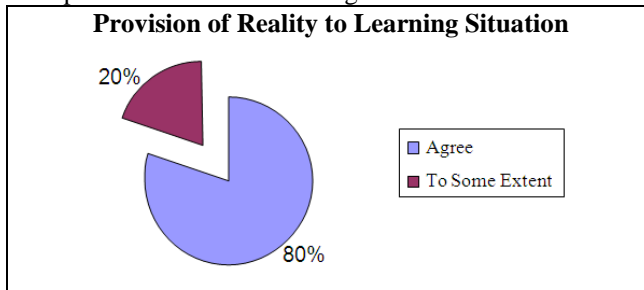
Figure X: Provision of Reality to Learning Situation
Table 11: Integral Part of Teaching-Learning Process

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
Instructional material is integral part of teaching- learning process.	4	80%	00	0%	1	20%	5	100%

Table 11 shows that the majority (80%) of the heads said that it is not possible to separate teaching-learning process from the use of instructional material in classroom. Some (20%) heads said that teaching-learning process can be done even they do not use instructional material.

The graphical representation is shown in figure XI.
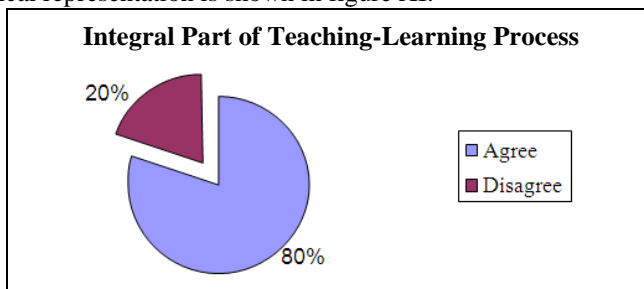
Figure XI: Integral Part of Teaching-Learning Process

Table 12: Possibility to Meet the Diverse Needs of Teachers and Students

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
Instructional material helps to meet the diverse needs of teachers and students.	4	80%	00	0%	1	20%	5	100%

Table 12 shows that the majority (80%) of the heads agreed that both teachers and students can be able to meet their diverse needs with the use of instructional material and some (20%) heads disagreed about the role of instructional material in meeting the diverse needs of teachers and students.

The graphical representation is shown in figure XII.
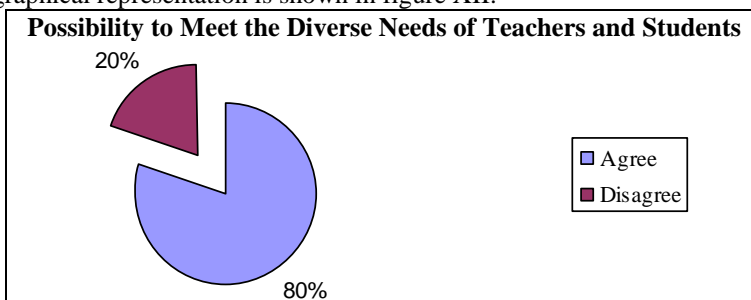
Figure XII: Possibility to Meet the Diverse Needs of Teachers and Students

Table 13: Provision of Feedback to Teachers and Students

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
Instructional material provides feedback both to teachers and students.	5	100%	00	0%	00	0%	5	100%

Table 13 shows that all heads (100%) agreed that instructional material are helpful to provide feedback to teachers and students to improve the efficiency of teaching and learning.

The graphical representation is shown in figure XIII.

Provision of Feedback to Teachers and Students
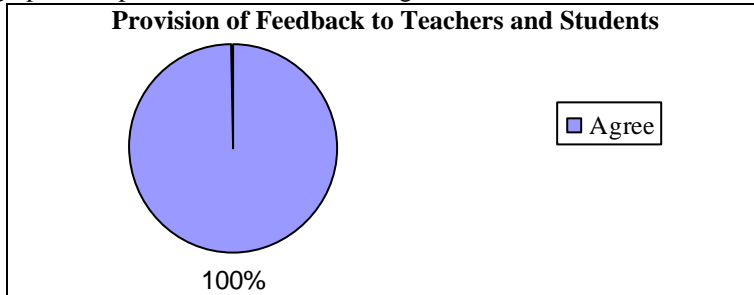
Figure XIII: Provision of Feedback to Teachers and Students

Table 14: Motivation from Teachers to Students for Using Instructional Material

Statement	Agree		To some extent		Disagree		Total	
	f	%	f	%	f	%	f	%
Teachers motivate students to consult the other books/ material beside the textbooks.	3	60%	2	40%	00	0%	5	100%

Table 14 shows that the majority (60%) of the heads agreed that teachers encourage students to visit the library and consult the other related material to show the outstanding performance in the class. Some (40%) of the heads gave less importance to the use of other instructional material in the academic achievement of student.

The graphical representation is shown in figure XIV.

Motivation from Teachers to Students for Using Instructional Material
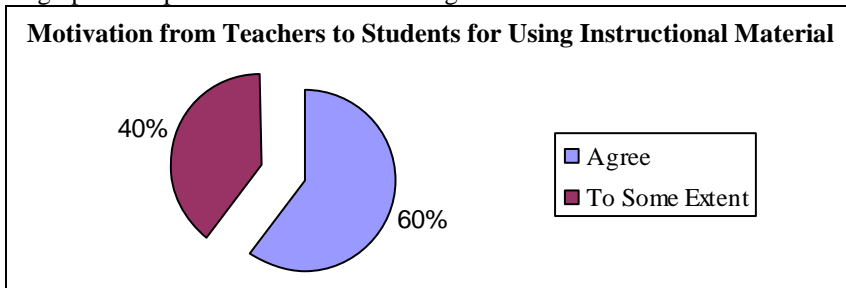
Figure XIV: Motivation from Teachers to Students for Using Instructional Material

**Graphical Representation of Importance of Instructional Materials
for Effective Learning**

Instructional Technology: A Tool of Effective Learning
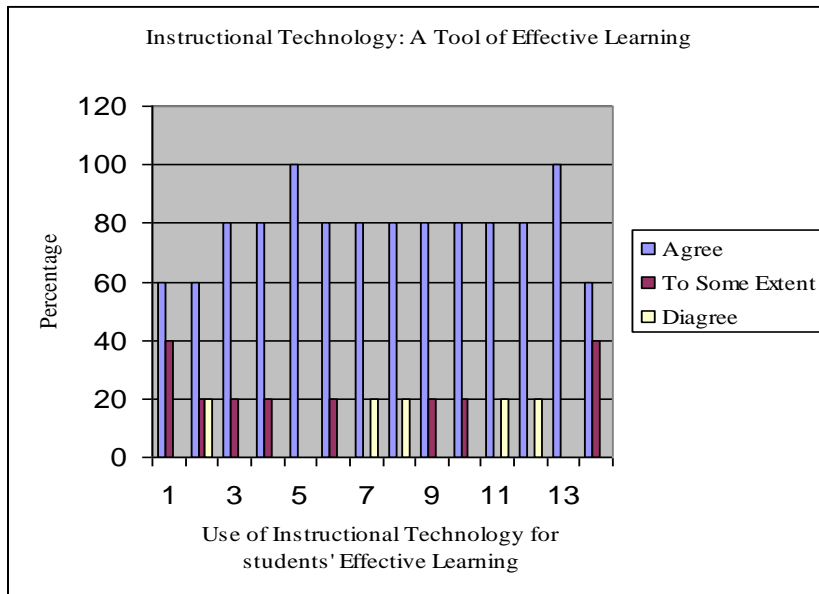
RESULTS AND DISCUSSION

Majority of the heads were of the view that instructional technology helps in effective learning. They provide firsthand experience, ever-lasting learning and clarification of concepts. Instructional material is necessary in education system; it provide basis for the development of understanding. Instructional material is selected according to the topic and age of the students. When topic is taught by using appropriate and relevant instructional aids, the concepts become clearer. Same findings were reported by Jones (2007), Beggs (2000), and Groves and Zemel (1999) who found that improvement in teaching and learning and students' achievement in the classroom is only possible by instructional aids.

The analysis showed that motivation from heads and teachers play a major role in active participation of students in the class which was supported by Ostman (2006) who stated that reinforcement for the students by the teachers, provide them little opportunity for developing working habit in the classroom.

Further, the study analysis showed that instructional technology is helpful in the improvement of students' learning and teachers' behaviour which was supported by Martin (2006) who identified that educational technology brings change in the behaviour of teachers, teachers use technology for the better learning of students.

   The analysis showed that objectives of teaching and learning can be achieved by the use of teaching aids which was supported by Levent (2006) who found that technology support the learning and helps in the attainment of goals.

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A STUDY OF THE EFFECT OF MANAGEMENT STYLES ON PERFORMANCE OF STUDENTS AT SECONDARY LEVEL IN SARGODHA

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ABSTRACT

Management focuses on the entire organization from both a short and a long-term perspective. The manager and management both are responsible for the betterment of both the organization and student, while, management is the process of forming a strategic vision, setting objectives, crafting a strategy and then implementing and executing the strategy. The major objective of the study was to find out the effect of school head's management styles on the performance of secondary class students in Sargodha. The research question which addressed was; whether there are effects of management style on student's performance or not? The instrument was developed to evaluate and appraise the performance of students to find out the relationship between management style and student's performance. Two separate structured questionnaires were administered by the researcher in four secondary schools of Sargodha. Forty students participated in the study and filled the questionnaire. A sample of 10 students from each school was selected out of total ensiled students in the secondary schools. Moreover four heads of the schools also participated in the study. The data collected was tabulated, analyzed and interpreted by using percentage formula. It was found that head teachers had adopted different style of decision making. One of them disclosed that they made pre-planning before decision making and some take decision under pressure. A few head teachers like self working. Head teachers were found self –centered only one of them quoted that sometimes he gave the opportunity to students for discussing the issues relevant to them. Students cannot point out difficulties. It was recommended that experienced managers and head teachers should be appointed to secondary schools. In-service training should also be given to the head teachers to improve their leadership styles.

INTRODUCTION

The concept of management has been introduced in from the works of business and trade and now educational management has followed commercial management. In England in 1988, Education Reforms Act introduced management styles and patterns with an attempt to make school more like commercial business. This trend has earlier roots. In 1983, the department of education and science in England issued circular recommending management training for the heads of educational institutions on the pattern adopted from commerce and industry. A school of thought is of the view that management of school should be run on the same style and pattern as that of banks, hotels and other commercial organizations.

DEFINITIONS

1. Wittgensteiner Kliniken describes that "Management is the managerial process of forming a strategic vision, setting objectives, crafting a strategy and then implementing and executing the strategy. A good management style is a blend of both efficiency and effectiveness. There is no point in acting efficiently if what you are doing will not have the desired effect. Management techniques can be viewed as either bottom-up, top-down or collaborative processes".

2. According to Stoner et al. (1997, p.7), Management is the process of planning, organizing, leading and controlling the work or member of an organization and using all available organizational resources. Although intelligence, initiative and self assurance are associated with high managerial performance, the single most important factor may be manner and style in which the manger can set the tone by influencing the attitude of the employees about their work.

Role of Manager

Manfred Davidmann (2008) provides a short summary of the role of managers under authoritarian and participative styles of management. Managers who initiate a quality improvement process must incorporate several basic principles into their management style:

- A firm commitment to and support for quality
- A concern for the satisfaction of staff and users of health services
- A focus on problem solving to improve quality
- Respect for staff and their abilities
- A willingness to collect and use data to determine the nature and size of problems and to improve processes.

ELEMENTS OF MANAGEMENT AND MANAGERIAL FUNCTIONS

There are various elements of management according to different authors. The elements are as follow: According to Weihbrick, Fayol (2004, p.64)

Fayol considered elements of management, i.e., planning, organizing, commanding and controlling as its functions. All the managerial work has to be organized in the framework of his functions. All the new ideas or techniques would readily fall in one of these classifications. Therefore, the primary functions of management are planning, organizing, staffing and controlling.

Bill, D. (2006, p.20) described the functions of educational management as follows:

1. Planning

Planning is an integral part of a sound decision making. It involves choosing a course of action that every enterprise and department is to follow. By planning we ponder on what to do, when to do, how to do, and who is to do it. It bridges the gap between our present position and the one we want to reach. It needs lot of innovation, as only a manager with a futuristic approach will be able to visualize and shape the future in his mind. Planning makes unthinkable possible.

2. Organizing

Organizing is that part of management that involves establishing an intentional structure of role for people in an enterprise to fill. It is intentional in the sense of making sure that all the tasks necessary to accomplish goals are assigned and it is hoped, assigned to people who can do those best.

Organizing involves the following activities:
i)   Marking those activities which are necessary to the achievement of goals.
ii) Bringing those activities into groups or sections.
iii) Assigning those groups of activities to the managers.
iv) Delegating authority to managers to perform these activities.
v)   Carries out those measures which ensure coordination at all levels between activities, sections and individuals.

The purpose of an organization structure is to help in creating an environment for human's performance. It is then a tool of managing and not and end in an end itself.

3. Staffing

The purpose of staffing is to ensure that all the positions in the organizing are filled with suitable persons; and to keep them filled in case of any vacancy or shortage. Firstly, it determines the duties to be performed by each person; then devices a method to search for those who possess the competencies and traits to fulfill them. Through staffing the newly inducted persons are guided and trained.

4. Leading

Leading is influencing people so that they will strive willingly and enthusiastically towards the achievement of organization and group goals. It has to do with the predominantly inter personnel aspects of managing. All managers would agree that their most important problems arise from people, their desires and attitudes, their behavior as individuals and in groups and the need for effective manager also to be effective's leaders. Since leadership implies followership and people tend to follow these in whom they see a means of satisfying their own needs, wishes and desires. It is understandable that leading involves motivation, leadership styles, approaches and communication.

5. Controlling

Controlling is the measuring and correcting of activities of subordinates to assure that events conform to plan, shows where negative deviations exists and by putting in notation action to correct deviation, helps ensure accomplishment of plans .Although planning must precedes controlling ,plans are not self–achieving. The plan guides managers in the use of resources to accomplish specifics goals. The then activities and checked to determine whether they conform to planned action.

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**NECESSARY MANAGERIAL SKILLS FOR
THE HEADS OF INSTITUTIONS**

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The job of educational institutional head is multi-dimensional including the tasks at various levels of management. In schools the tasks of heads are planning, strategic

planning, budgeting, resolving conflicts, disciplining, rewarding improving productivity, managing time, and managing change, managing ethics and leading. The Principal's job is complex in nature. Principal needs certain skills to perform his/her duties and activities associated with being a school head. The head needs skills that are technical, human and conceptual skills. Technical skills include knowledge of and proficiency in a certain specialized field, such as computers, financial and managerial accounting, or manufacturing. Human skills involve the ability to work well with other people both individually and in a group. Heads deal directly with people. Heads with good human skills are able to get the best out of their people. They should know how to communicate, motivate, lead, and inspire enthusiasm and trust. This is equally important at all levels of management. The conceptual skills those heads must have to think and conceptualize about abstract and complex situations. Using these skills heads must be able to see the organization as a whole, understand the relationship among various subunits, and visualize how the organization fits into its broader environment. Whetten and Cameron (2008, p. 43) provided an empirical derivation of effective leadership skills. They are:

    i)   Verbal communication (including listening),
    ii)   Managing time and stress,
    iii)   Managing individual decisions,
    iv)   Recognizing, defining, and solving problems,
    v)   Motivating and influencing others,
    vi)   Delegating,
    vii) Setting goals and articulating a vision,
    viii) Self awareness,
    ix)   Team building,
    x)   Managing conflict

The Head/Principal in educational institution has to perform different types of managerial tasks for the effective principal-ship and as the educational policy developer and the manger, coordinator of the educational institution. They need the following types of skills that encompass conceptual, communication, effectiveness, and interpersonal aspects.

According to Govinda (2002, pp.212-213) head of educational institution has to play multiple roles. The following are some of the main roles, which a heads have to play in their educational institution.

- **Academic Administration**: under the academic administration, the curriculum management activities hold an important place including implementing of curricular calendar, framing of timetable, timely and effective curriculum transaction continued and comprehensive evaluation of students through examination, organization of co-curricular activities.
- **General Administration**: it involves head giving admission to students, handling students and teacher union, work allocation. Checking absenteeism and alternate arrangements, seeking cooperating, supervision and quantity and quality assessment of teacher's work, need identifications for capacity building and making arrangement for the same.
- **Financial Management:** it included generation and mobilization of resources, allocation and utilization of resources, monitoring and evaluation of expenditures.

Heads has to spend considerable time in supervising the work accountants since they do not have specialized training in the field of financial management , it takes a lot to time to understand, follow and then supervise the related work. They still remain skeptical about spending the available funds, which affect the smooth flow of institutional activities. This generates demand for financial management training.

- **Material Resources Management:** heads have no power to incur expenditures as and when required. Thy have budgetary rigidity and for change they have to seek the permission of higher authority.
- **Community Linkage:** establishing linkage with community to seek teaching help from capable individuals of village. It is notable that most of the time institutional heads were dealing with admission pressure, discipline problems, political and administrative pressure and restoring institutional credibility in the eyes of public.

Therefore, heads should be trained in individual and group interaction, community linkage and relationship, financial affairs as well as routine administrative matters.

SIGNIFICANCE

The study will be helpful and useful for the head teacher and students as contains information about management style and performance of student. It cannot be claimed that that after knowing good management style, the head teacher try to adopt it, but the aim is to enable them to provide awareness about the impact of management style on student's performance and it could be rightly claimed that if head teacher adopt right style of management, they would improve the student's performance.

OBJECTIVES OF THE STUDY

Objectives of the study were as follows:
- To know about the administration of head teachers.
- To know whether there are effects of management style on students' performance.
- To find out relationship between management style and students' performance.
- To enable the head teachers to guide their students in realizing their full academic potential.

RESEARCH PROCEDURE

The procedure adopted for this research study was as follows:

Population:
All the students and headmasters of Govt. Boys High School of Sargodha city comprised population of the study. Two separate structured questionnaires were administered by the researcher in four secondary schools of Sargodha. Forty students participated in the study and filled the questionnaire. A sample of 10 students from each school was selected out of total ensiled students in the secondary schools. Moreover four heads of the schools also participated in the study.

Sampling:

Four secondary schools of Sargodha city were selected through random sampling were:

- Central Model High School Sargodha
- Govt. Khalqia High School Sargodha
- Govt. High School No 1 Sargodha
- Govt. Boys Comprehensive High School Sargodha.

Research Tool:

Two questionnaires developed and administrated for data collection. The questionnaire for the students was comprised of 15 items and the questionnaire for head teachers was also comprised of 15 items. All the items were prepared in the view of the requirement of the study. After thorough consultation, necessary changes were made and then the scale was administered. All the questionnaires were delivered by hand to the respondents. For each statement, there were two responses i.e. Yes and No. The respondents were to respond one of them.

1. **Collection of Data:**
   Questionnaires were collected from the students of secondary school, Sargodha and twenty questionnaires were collected from head teacher of secondary schools Sargodha.
2. **Data analysis:**
   The data collected was tabulated, analyzed and interpreted by using percentage formula, findings and conclusions were drawn and recommendations were made on the basis of data.

RESEARCH FINDINGS

The data collected through the two questionnaires was analyzed and results were tabulated and analyzed by using percentage formula.

Findings for the Questionnaire for the Student:

Questionnaire was distributed among 40 % students. All the students responded. The questionnaire was analyzed. Following were the findings of the questionnaire:

- 47% say that their head teacher is a good administrator.
- 33% students are of the view that they have opportunities to discuss class results with the head teacher.
- 76% students arc of the view that they are encouraged to participate in 10% co-curricular activities.
- 83% students say that their head teacher takes class.
- 22% students say that they complaint about the lack of facilities to the head teachers.
- 66% students are of the view that institution preparing them for the future.
- 27% students say that number of teachers is sufficient in their school.
- 87% students say that head teacher takes interest in the institutional programmes.
- 49% students say that head teachers appreciate and recognize good behavior of students.

- Students say that there is a lack of balance in provision of co-curricular activities.
- 49% students say that head teacher always provide learning facilities.
- 26% students are of the view that head teacher has constituted management committees to run the institution.
- 71% students say that head teacher has leadership qualities.
- 29% students say that head teacher likes the self working.
- 26% students are of the view that they point out difficulties before the head teacher.

Findings from the Questionnaire for the Teachers:

Questionnaire was distributed among 20 head teachers of 20 secondary of Sargodha. All of them responded.

- 27% head teachers say that their style of decision making is behavioral
- 51% head teachers say that their style of decision making is normative
- 89% head teachers say that their style of decision making is descriptive
- 13% head teachers say that their style of decision making is conceptual
- 23% head teachers say that their style of decision making is analytical
- 76% head teachers say that they remained composed and firm while decision making.
- 91% head teachers say that the process of decision making helps to lead good decisions.
- 24% head teachers say that their style of decision making is making directive.
- 54% head teachers say that their style of decision making is non-directive.
- 58% head teachers say that they co-ordinate others at the time of decision making.
- 93% head teachers say that they do some planning before decision making.
- 33% head teachers say that they take plenty of time in making decisions.
- 97% head teachers say that they feel pressure while taking decision.
- 7% head teachers accept external pressure in decision making.
- 62% head teachers say that they make decisions after conducting formal inquiry.

CONCLUSIONS

On the basis of findings the researchers concluded that most head teachers adopted different styles of decision making. Most head teachers do some planning before decision making. Head teachers feel pressure while taking decisions. Head teachers do not accept external pressure in decision making. A few students have opportunities to discuss class results. A few head teachers like self working. There is a lack of balance in provision of co-curricular activities. Students cannot point out difficulties of head teachers.

RECOMMENDATIONS

On the basis of conclusion the researchers recommended:

1. Well experienced and highly qualified head teachers may be appointed to very secondary school.
2. The number of teacher may be increase in secondary schools.
3. In service training should be given to the head teachers.

4. Head teachers should be appointed on permanent basis.
5. Special allowances should be given to the head teachers.
6. Head teachers should also be given chances to avail facilities like accommodation, medical and transport etc.

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NPL's OF PAKISTAN

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ABSTRACT

This study focuses the current trend of nonperforming loans (NPL's) which are bringing threats on conventional Banks of Pakistan and bringing highly impact on their performances. The study reflects that borrowers face difficulties in paying loans, although their nonpayment of loans or failure in payment or being default or disability to reach on contract or an agreement for 90 days or 3 month that brings highly impact on the performance of the conventional Banks. As well as the multiple regressions has also performed to know the final outcome. The interest/markup, discount rate, credit risk, liquidity risk, solvency risk & earning risk has found in a high variation and they are useful to make predictions and the final suggestion regarding nonperforming loans as a preventive measure to reduce, minimize & control NPL's is that the Efficient Loan Appraisal Techniques should be introduce & adopted which is based on risk measurements & conventional investment analysis and the rule of issuance of loan should be in control with the objectives of institution. This study must be review due to recent crisis of nonperforming loans (NPL's).

1. INTRODUCTION

A loan which has in default for 90 days or 3 month called Nonperforming loans (NPL's), these loans caused by nonpayment or failure in payment, although it relies on the agreement. Continuously increasing figures in nonperforming loans bringing threats in Banking Sector of Pakistan. The enormous figure of all Banks & DFI's of NPL's is over Rs.623,193 million, Net NPL's to Rs. 207,812 million & Net NPL's to Net Loans 6.14 million as on December 31ˢᵗ 2011. Banks are part of financial institutions and their function is to provide funds against collateral or non collateral security which is the conversion of assets from excess to shortage of amount in economy. It is risky in treating or dealing effectively to accomplish their task, which is task oriented. Although cash is a part of assets, when cash or loan could not be covered, then it effects on liquidity risk and credit growth, which puts the Bank into trouble. The high borrowers or long terms borrowers are expected to be defaulters, so the long term or big borrowers should be treated carefully. Banks must have strategy to reduce NPL's. Banks found with highest significance in Nonperforming loans are Public sector Bank, among all Banking industry. Nonperforming loans bring impact on operating risk, credit risk, monetary policy, market risk, liquidity risk, debt/equity risk, interest rate risk, reputation risk, earning risk, legal risk & solvency risk. Banks must have well structured strategy for the recovery of loan and the Bank must design or apply its own strategy for the recovery of loans.

2. LITERATURE REVIEW

Unsettlement of loans or a loan which has been in default for 90 days or 3 month will grow NPL's, these loans caused by nonpayment or failure in payment and unrealized markup will also be added to NPL's. Earlier than the financial crisis of the Asia, finance experts believed that the Bank should be treated as supplementary administration reserves and in order to manage growing liquidity problems & nonperforming loans in the Banking system. *(Victor Shih, 2004)*. Nonperforming loans contain principle payments, interest & additional financial data. Loan losses could be increased in future after the disclose of loan loss provision and income statement disclosed or revealed as accrued expense. All uncollectable loans supposed to be as loan charge offs which are assets write-off that must be managed separately in financial statement and it can be subsequent from income statement and balance sheet. *(Dr. Ishrat Husain, 2002)* Every organization builds on its assets and loans are mentioned as receivables in the balance sheet and the unrealized mark up would also be include when these receivables cannot be collected then they will considered as non performing loans. Nonperforming loans bring highly impact on conventional Banks.

Gross NPLs/gross advances and net NPLs/net advances necessarily monitored because they are reasonable and simple. Better quality of loans is issued due to carefulness, these ratios could be decline after a while and assets could be affected. The non-provisioning of NPL's could bring an impact or a threat on the performance of the entire Banking system. The higher the provision, the lower the systemic risk will be *(Dr. Ishrat Husain, 2002)*. The Banks must monitor the payment series of the borrowers and take some legal action against the defaulters less monitoring could be the reason of nonperforming loans. Nonperforming loans could not be removed from conventional Banks but it can be minimize by increasing security requirement, reducing unsecured loans and making restrictions on late payment, the strategy should be flexible and well structured.

The default risk of lenders evaluation depends on the loan arrangement, which is obviously depend on the employment, income and credit history of the borrower at issuing loan, household has connection or linked with marginal risk (Higgins, 1999). Although failure in adjusting the credit risk will increase the loan default risk, hence instant or immediate rising in the cost of financial institution causes decreases the further borrowing of retail credit.

3. RESEARCH METHODOLOGY

It is the basic structure of the study. In other word, I am going to make a participative Survey. It is basically a qualitative and causal research. This study has done in year 2011. The primary data has collected through questionnaire for which I have made 250 sample sizes but 50 of them have dropped out due to fake or incomplete information, so 200 sample size has chosen as a final sample size. 100 sample sizes have made to collect data from borrowers and 100 made to collect data from Bankers. Borrowers & Bankers would be the sample unit of the research, so the data has collected from 100 borrowers and 100 Bankers. The sample technique that I have used for study is snowball sampling and the secondary data has collected from State Bank of Pakistan.

4. HYPOTHESIS TESTING

The main part of the study where I have to test my hypotheses by analysis of T-test and F-test where significance level must be less than 0.05 ($P<0.05$) which will be the acceptance or rejection of hypotheses and this will be decide on the basis of the significance. I have made three main hypotheses in order to prove my research for which I have created, H_1 = Borrowers face difficulties in paying loans which creates nonperforming loans, H_2 = Nonperforming loans will bring impact on Conventional Banks and null hypotheses H_0 = Borrowers face difficulties in paying loans which does not create nonperforming loans, nonperforming loans will not bring impact on Conventional Banks. The primary data which I have collected through survey has Cronbach's Alpha (α) = 0.905 which means data is 90.5% reliable.

Table 4.1:
Results of Testing H_1

Reasons that create NPL's	Model 1							
	Constant α_0	Coefficient β_1	R	R^2	Adj. R^2	F (P-Value)	Coefficient's P-value	H_1 Results
Total Sample			0.970	0.941	0.935	160.007 (0.000)	(0.000)	Accept
Interest Rate/ Mark Up	0.350	0.938	0.951	0.903	0.902	917.135 (0.000)	(0.000)	Accept
Income	2.215	0.530	0.600	0.360	0.354	55.182 (0.000)	(0.000)	Accept
Inflation	2.268	0.524	0.591	0.349	0.342	52.476 (0.000)	(0.000)	Accept
Taxes	3.791	0.196	0.266	0.071	0.061	7.450 (0.004)	(0.004)	Accept
Investment	3.791	0.196	0.354	0.125	0.116	14.020 (0.000)	(0.000)	Accept
Discount Rate	0.696	0.855	0.915	0.837	0.835	503.268 (0.000)	(0.000)	Accept
Security Requirement	3.600	0.252	0.343	0.118	0.109	13.105 (0.000)	(0.000)	Accept
Unsecured Loans	4.068	0.137	0.189	0.036	0.026	3.614 (0.060)	(0.060)	Reject
Allowance to Late or Nonpayment	4.104	0.119	0.156	0.024	0.014	2.448 (0.121)	(0.121)	Reject

Table 4.2:
Results of Testing H$_2$

Impact of NPL's on Conventional Bank	Model 2							
	Constant α_0	Coefficient β_1	R	R^2	Adj. R^2	F (P-Value)	Coefficient's P-value	H$_2$ Results
Total Sample			0.915	0.837	0.825	67.548 (0.000)	(0.000)	Accept
Credit Risk	0.696	0.855	0.874	0.763	0.761	315.684 (0.000)	(0.000)	Accept
Liquidity Risk	1.873	0.616	0.743	0.552	0.548	120.896 (0.000)	(0.000)	Accept
Market Risk	2.098	0.570	0.711	0.505	0.500	100.165 (0.000)	(0.000)	Accept
Monetary Policy Rate	2.277	0.532	0.681	0.464	0.458	84.769 (0.000)	(0.000)	Accept
Debt/Equity Risk	1.249	0.741	0.778	0.605	0.601	149.843 (0.000)	(0.000)	Accept
Solvency Risk	1.465	0.695	0.810	0.656	0.652	186.601 (0.000)	(0.000)	Accept
Earning Risk	1.331	0.724	0.828	0.686	0.683	214.394 (0.000)	(0.000)	Accept

5. DATA ANALYSIS

The data I have collected are primary and secondary first I have to analyze secondary data, these data has provided by State Bank of Pakistan as you can see the tables. This secondary data which is mentioned below, only all Banks data of 2011 has analyzed because it's the compilation of the data. In year 2011 the NPL's Of All Banks have increased to Rs. 607,145, Net NPL's has also increased to Rs. 201,705 and Net NPL's to Net Loans has increased to 6.04%. The figures of year 2010 are from January to June 2010. Half year data of year 2010 has taken because of less availability of data.

Table 5.1:
Non Performing Loans

(End Period 2010-11: Million Rupees)

Banks	Nonperforming Loans		Net Non-Performing Loans		Net NPLs to Net Loans (%)	
	2010	2011	2010	2011	2010	2011
All Banks	459,840	607,145	123,094	201,705	3.81	6.04
Commercial Banks	431,352	572,548	113,563	187,913	3.62	5.79
Public Sector						
Commercial Banks	115,996	186,606	32,104	93,023	5.19	13.45
Local Private Banks	307,630	378,369	79,811	94,161	3.28	3.78
Foreign Banks	7,726	7,574	1,648	729	1.96	1.1
Specialized Banks	28,488	34,597	9,531	13,792	10.00	14.69

5.2 Reasons

The study has conducted to know the reasons that create the problem for the borrowers in paying loans that's why I have to collect primary data from the borrowers. The main objective is to the grasp the actual variable which creates the trouble for the borrowers in paying loans; loans are actual pay by the borrowers. First I have designed a model in which I have taken nonperforming loans as a dependent variable and interest/markup, income, inflation, taxes, investment, discount rate, security requirement, unsecured loans & allowance to late or nonpayment's have taken as an independent variable. The Pearson's correlation coefficient and regression equation has analyzed to covariances and relationship among the variables to know the variables that creates the problem for the borrowers in paying loans, their nonpayment of loans change into nonperforming loans.

$$NPL_{it} = \alpha_0 + \beta_1 * INT + \varepsilon_{it} \text{ ----- } eq. (1) \qquad \text{(Model 1)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * INC_{it} + \varepsilon_{it} \text{ ----- } eq. (2) \qquad \text{(Model 1)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * INF_{it} + \varepsilon_{it} \text{ ----- } eq. (3) \qquad \text{(Model 1)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * TX_{it} + \varepsilon_{it} \text{ ----- } eq. (4) \qquad \text{(Model 1)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * INV_{it} + \varepsilon_{it} \text{ ----- } eq. (5) \qquad \text{(Model 1)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * DR_{it} + \varepsilon_{it} \text{ ----- } eq. (6) \qquad \text{(Model 1)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * SR_{it} + \varepsilon_{it} \text{ ----- } eq. (7) \qquad \text{(Model 1)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * UL_{it} + \varepsilon_{it} \text{ ----- } eq. (8) \qquad \text{(Model 1)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * NP_{it} + \varepsilon_{it} \text{ ----- } eq. (9) \qquad \text{(Model 1)}$$

NPL is nonperforming loans, **INT** is interest, **INC** is income, **INF** inflation, **TX** is taxes, **INV** is investment, **DR** is discount rate, **SR** is security requirement, **UL** is unsecured loans & **NP** is allowance to late or nonpayment's.

5.3 Impacts

The task is to know the variable that brings impact on conventional Banks, and to design a model that could find out them. This study focuses on the performance of the Conventional Bank but also focuses on improving and making them better. First, I have designed second model in which I have taken nonperforming loans as a dependent variable and I have taken credit risk, liquidity risk, market risk, monetary policy rate, debt/equity risk, solvency risk & earning risk as an independent variable. The Pearson's correlation coefficient and regression equation has analyzed to covariances and relationship among the variables to know the variables that brings impact on Conventional Bank.

$$NPL_{it} = \alpha_0 + \beta_1 * CR + \varepsilon_{it} \text{ ----- } eq. (10) \qquad \text{(Model 2)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * LR_{it} + \varepsilon_{it} \text{ ----- } eq. (11) \qquad \text{(Model 2)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * MR_{it} + \varepsilon_{it} \text{ ----- } eq. (12) \qquad \text{(Model 2)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * MPR_{it} + \varepsilon_{it} \text{ ----- } eq. (13) \qquad \text{(Model 2)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * DER_{it} + \varepsilon_{it} \text{ ----- } eq. (14) \qquad \text{(Model 2)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * SR_{it} + \varepsilon_{it} \text{ ----- } eq. (15) \qquad \text{(Model 2)}$$

$$NPL_{it} = \alpha_0 + \beta_1 * ER_{it} + \varepsilon_{it} \text{ ----- } eq. (16) \qquad \text{(Model 2)}$$

NPL is nonperforming loans, **CR** is credit risk, **LR** is liquidity risk, **MR** is market risk, **MPR** is monetary policy rate, **DER** is debt/equity risk, **SR** is solvency risk & **ER** is earning risk.

6. COMMENTS AND CONCLUSION

To get final outcome I have to perform multiple regression which is use to know the combine effect in reasons of nonperforming loans which is the reflection of total sample as you can see in (Table 4.1) and the combination of equation (from eq.1 to eq.7) where data has collected from borrowers which has predictor i.e. interest/markup & discount rate. R = 0.970 which is highly correlated this predicts nonperforming loans very well and R represents multiple correlation coefficient and R square = 0.941 means 94.1% of the variance in nonperforming loans can be predicted by the combinations of interest/markup & discount rate. The adjusted R square = 0.935, $F_{9,90}$ = 160.007 with p = 0.000 means the significance level is less than 0.05 (p<0.05).

$$NPL = 0.186 + 0.601INT + 0.326DR \qquad \text{----- } eq. (17)$$

To obtain final outcome of impact of nonperforming loans on conventional Banks I have performed multiple regression which is the reflection of total sample as you can see

in (Table 4.2) and the combination of equation (from eq.10 to eq.16) where data has collected from Bankers which has predictor i.e. credit risk, liquidity risk, debt/equity risk, solvency risk & earning risk. R = 0.915 which is highly correlated this predicts nonperforming loans very well and R represents multiple correlation coefficient and R square = 0.837 means 83.7% of the variance in nonperforming loans can be predicted by the combinations of credit risk, liquidity risk, debt/equity risk, solvency risk & earning risk. The adjusted R square = 0.825, $F_{7,92} = 67.548$ with p = 0.000 means the significance level is less than 0.05 (p<0.05).

$$NPL's = 0.700+3.447CR-0.048LR+1.717DER-1.704SR-2.588ER \quad \text{- eq. (18)}$$

The Conventional Banks must contain following methods in their procedures (1) approximation of non-performing loans and assign it to the subsequent borrowers although always imagine regarding nonpayment of loans or uncovered loans, how their transaction are performed, in the way of increase principal outstanding, and (2) Approximate that the interest are received, except of receivable, and same with the interest payable, in this way nonperforming loans will not be affected. While the difficulty or obstacles that bring impact on non-performing loans is related with the procedures of the Banks, there must be destructive loan collection strategy. The absence of destructiveness is admired in the developing & developed countries and this is supposed as the Banks definite aspect and adds to the crisis of nonperforming loans. The competencies of the Banks should be maintained and increased until they improve their selves and cover a common procedure. Since the economic uncertainty, depression/ recession in the global surroundings must be handled to control the domestic circumstances of Pakistan. Because the economic uncertainties are not in a control of Banks, then they should permit the government to design its own strategy. The qualities and abilities of the managers to control the supply and the rising demand of the customers can be the trouble that could occur in the workforce. Thus, inconsistencies in policies & strategies should be present in case of failure in evaluating the qualities and abilities of the entities or individuals.

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TO UTILIZE TEACHING STAFF SKILLS ESPECIALLY OF GOVT. SCHOOLS

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ABSTRACT

As teaching as an honorable profession and teachers are the respectable part of any society. These are the personnel which may change the society and give a right direction to it. They are the build of any nation. In this paper an attempt will be made that how to improve and utilize skills of the teaching staff.

KEYWORDS

Utilize, Teaching, Skills, Nation.

1. INTRODUCTION

a Teacher

A teacher or schoolteacher is a person who provides education for pupils (children) and students (adults). The role of teacher is frequently formal and continuing, carried out at a school or other place of formal education. In many countries, a person who wishes to become a teacher must first obtain specified professional qualifications or credentials from a university or college. These professional qualifications may include the study of pedagogy, the science of teaching. Teachers, like other professionals, may have to continue their education after they qualify, a process known as continuing professional development. Teachers may use a lesson plan to facilitate student learning, providing a course of study which is called the curriculum.

A teacher's role may vary among cultures. Teachers may provide instruction in literacy and numeric, craftsmanship or vocational training, the arts, religion, civics, community roles, or life skills.

A teacher who facilitates education for an individual may also be described as a personal tutor, or, largely historically, a governess.

In some countries, formal education can take place through home schooling. Informal learning may be assisted by a teacher occupying a transient or ongoing role, such as a family member, or by anyone with knowledge or skills in the wider community setting.

Religious and spiritual teachers, such as gurus, mullahs, rabbis, pastors/youth pastors and lamas, may teach religious texts such as the Qumran, Torah or Bible.

In education, teachers facilitate student learning, often in a school or academy or perhaps in another environment such as outdoors. A teacher who teaches on an individual basis may be described as a tutor.

The objective is typically accomplished through either an informal or formal approach to learning, including a course of study and lesson plan that teaches skills, knowledge and/or thinking skills. Different ways to teach are often referred to as pedagogy. When deciding what teaching method to use teachers consider students' background knowledge, environment, and their learning goals as well as standardized curricula as determined by the relevant authority. Many times, teachers assist in learning outside of the classroom by accompanying students on field trips. The increasing use of technology, specifically the rise of the internet over the past decade, has begun to shape the way teachers approach their roles in the classroom.

Teaching using pedagogy also involve assessing the educational levels of the students on particular skills. Understanding the pedagogy of the students in a classroom involves using differentiated instruction as well as supervision to meet the needs of all students in the classroom. Pedagogy can be thought of in two manners. First, teaching itself can be taught in many different ways, hence, using a pedagogy of teaching styles. Second, the pedagogy of the learners comes into play when a teacher assesses the pedagogic diversity of his/her students and differentiates for the individual students accordingly.

Throughout the history of education the most common form of school discipline was corporal punishment. While a child was in school, a teacher was expected to act as a substitute parent, with all the normal forms of parental discipline open to them.

Co-teaching has also become a new trend amongst educational institutions.
Co-teaching is defined as two or more teachers working harmoniously to fulfill the needs of every student in the classroom. Co-teaching focuses the student on learning by providing a social networking support that allows them to reach their full cognitive potential. Co-teachers work in sync with one another to create a climate of learning.

In past times, corporal punishment (spanking or paddling or caning or strapping or birching the student in order to cause physical pain) was one of the most common forms of school discipline throughout much of the world. Most Western countries, and some others, have now banned it, but it remains lawful in the United States following a US Supreme Court decision in 1977 which held that paddling did not violate the US Constitution. [1] Sudbury model democratic schools claim that popularly based authority can maintain order more effectively than dictatorial authority for governments and schools alike. They also claim that in these schools the preservation of public order is easier and more efficient than anywhere else. Primarily because rules and regulations are made by the community as a whole, thence the school atmosphere is one of persuasion and negotiation, rather than confrontation since there is no one to confront. Sudbury model democratic schools' proponents argue that a school that has good, clear laws, fairly and democratically passed by the entire school community, and a good judicial system for enforcing these laws, is a school in which community discipline prevails, and in which an increasingly sophisticated concept of law and order develops, against other schools today, where rules are arbitrary, authority is absolute, punishment is capricious, and due process of law is unknown.[2][3] Since teachers can affect how students perceive the course materials, it has been found that teachers who showed enthusiasm towards the course materials and students can affect a positive learning experience towards the course materials. On teacher/course evaluations, it was found that teachers who have a positive

disposition towards the course content tend to transfer their passion to receptive students. [4] These teachers do not teach by rote but attempt to find new invigoration for the course materials on a daily basis.[5] Teachers that exhibit enthusiasm can lead to students who are more likely to be engaged, interested, energetic, and curious about learning the subject matter. Recent research has found a correlation between teacher enthusiasm and students' intrinsic motivation to learn and vitality in the classroom. [6] Research shows that student motivation and attitudes towards school are closely linked to student-teacher relationships. Enthusiastic teachers are particularly good at creating beneficial relations with their students. Their ability to create effective learning environments that foster student achievement depends on the kind of relationship they build with their students. [7][8][9][10]. Useful teacher-to-student interactions are crucial in linking academic success with personal achievement.[11] Here, personal success is a student's internal goal of improving himself, whereas academic success includes the goals he receives from his superior. A teacher must guide his student in aligning his personal goals with his academic goals. Students who receive this positive influence show stronger self-confidence and greater personal and academic success than those without these teacher interactions.[10][12][13] Misconduct by teachers, especially sexual misconduct, has been getting increased scrutiny from the media and the courts.[14] A study by the American Association of University Women reported that 9.6% of students in the United States claim to have received unwanted sexual attention from an adult associated with education; be they a volunteer, bus driver, teacher, administrator or other adult; sometime during their educational career.[15]

A study in England showed a 0.3% prevalence of sexual abuse by any professional, a group that included priests, religious leaders, and case workers as well as teachers. [16]

Chris Keats, the general secretary of National Association of Schoolmasters Union of Women Teachers, said that teachers who have sex with pupils over the age of consent should not be placed on the sex offenders register and that prosecution for statutory rape "is a real anomaly in the law that we are concerned about." This has led to outrage from child protection and parental rights groups. [17]

International schools generally follow an English-speaking, Western curriculum and are aimed at expatriate communities. [18]

Salaries for Nursery, Primary and Secondary School teachers ranged from £20,133 to £41,004 in September 2007, although some salaries can go much higher depending on experience and extra responsibilities. [19]

Salaries for primary teachers in the Republic of Ireland depend mainly on seniority (i.e. holding the position of principal, deputy principal or assistant principal), experience and qualifications. Extra pay is also given for teaching through the Irish language, in a Gaeltacht area or on an island. The basic pay for a starting teacher is €30,904 p.a., rising incrementally to €59,359 for a teacher with 25 years' service. A principal of a large school with many years' experience and several qualifications (M.A., H.Dip., etc.) could earn over €90,000.[20]

In Scotland, anyone wishing to teach must be registered with the General Teaching Council for Scotland (GTCS). Teaching in Scotland is an all graduate profession and the

normal route for graduates wishing to teach is to complete a programme of Initial Teacher Education (ITE) at one of the seven Scottish Universities who offer these courses. Once successfully completed, "Provisional Registration" is given by the GTCS which is raised to "Full Registration" status after a year if there is sufficient evidence to show that the "Standard for Full Registration" has been met.[21]

For the salary year beginning April 2008, unprompted teachers in Scotland earned from £20,427 for a Probationer, up to £32,583 after 6 years teaching, but could then go on to earn up to £39,942 as they complete the modules to earn Chartered Teacher Status (requiring at least 6 years at up to two modules per year.) Promotion to Principal Teacher positions attracts a salary of between £34,566 and £44,616; Deputy Head, and Head teachers earn from £40,290 to £78,642.[22]

In the United States, each state determines the requirements for getting a license to teach in public schools. Teaching certification generally lasts three years, but teachers can receive certificates that last as long as ten years.[23] Public school teachers are required to have a bachelor's degree and the majority must be certified by the state in which they teach. Many charter schools do not require that their teachers be certified, provided they meet the standards to be highly qualified as set by No Child Left Behind. Additionally, the requirements for substitute/temporary teachers are generally not as rigorous as those for full-time professionals. The Bureau of Labor Statistics estimates that there are 1.4 million elementary school teachers,[24] 674,000 middle school teachers,[25] and 1 million secondary school teachers employed in the U.S.[26]

In the past, teachers have been paid relatively low salaries. However, average teacher salaries have improved rapidly in recent years. US teachers are generally paid on graduated scales, with income depending on experience. Teachers with more experience and higher education earn more than those with a standard bachelor's degree and certificate. Salaries vary greatly depending on state, relative cost of living, and grade taught. Salaries also vary within states where wealthy suburban school districts generally have higher salary schedules than other districts. The median salary for all primary and secondary teachers was $46,000 in 2004, with the average entry salary for a teacher with a bachelor's degree being an estimated $32,000. Median salaries for preschool teachers, however, were less than half the national median for secondary teachers, clock in at an estimated $21,000 in 2004.[27] For high school teachers, median salaries in 2007 ranged from $35,000 in South Dakota to $71,000 in New York, with a national median of $52,000.[28] Some contracts may include long-term disability insurance, life insurance, emergency/personal leave and investment options.[29] The American Federation of Teachers' teacher salary survey for the 2006-07 school year found that the average teacher salary was $51,009.[30] In a salary survey report for K-12 teachers, elementary school teachers had the lowest median salary earning $39,259. High school teachers had the highest median salary earning $41,855.[31] Many teachers take advantage of the opportunity to increase their income by supervising after-school programs and other extracurricular activities. In addition to monetary compensation, public school teachers may also enjoy greater benefits (like health insurance) compared to other occupations. Merit pay systems are on the rise for teachers, paying teachers extra money based on excellent classroom evaluations, high test scores and for high success at their overall school. Also, with the advent of the internet, many teachers are now selling their lesson

plans to other teachers through the web in order to earn supplemental income, most notably on TeachersPayTeachers.com.[32]

Teachers in Wales can be registered members of trade unions such as NUT or NASUWT and reports in recent years suggest that the average age of teachers in Wales is falling with teachers being younger than in previous years.[33] A growing cause of concern are that attacks on teachers in Welsh schools which reached an all-time high between 2005 and 2010.[34]

b. Teaching

A process in which teacher thought and the learners (Students learn. Either on formal or informal way. Teaching is a very sensitive job. It builds the environment to live. Hence care must be taken while performing this job. A Teacher may be a person an environment or experience. Environment is the surrounding in which people live hence people create the environment. Hence human is the important character of any environment. And teacher is the key responsible . Because he teaches to the whole.

2. HOW TO UTILIZE THE SKILLS

Hence the skills may be utilized by providing good environment. The difficulties to perform the duty may be minimized. Its may be by providing some update sort of training. Also by improving facilities in schools.

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COMPUTER TO HELP IN SCHOOL SYSTEMS

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ABSTRACT

It is understood that schools are the nurseries of a nation future. These are schools where a child enrolled and start learning. So a nation future will be bright if the schools are good. Because these are the places for learning. In this paper it will be studied that how much computer is help full to enhance the teaching learning process. If the start is better than further its will be easy to proceed the teaching learning process.

KEYWORDS

School, Learning, Nurseries, Start.

1. INTRODUCTION

i Education

Education in its broadest, general sense is the means through which the aims and habits of a group of people lives on from one generation to the next.[1] Generally, it occurs through any experience that has a formative effect on the way one thinks, feels, or acts. In its narrow, technical sense, education is the formal process by which society deliberately transmits its accumulated knowledge, skills, customs and values from one generation to another, e.g., instruction in schools. Etymologically, the word education is derived from the Latin ēducātiō ("A breeding, a bringing up, a rearing) from ēdūcō ("I educate, I train") which is related to the homonym ēdūcō ("I lead forth, I take out; I raise up, I erect") from ē- ("from, out of") and dūcō ("I lead, I conduct").[2] Systems of schooling involve institutionalized teaching and learning in relation to a curriculum, which itself is established according to a predetermined purpose of the schools in the system. In formal education, a curriculum is the set of courses, and their content, offered at a school or university. As an idea, curriculum stems from the Latin word for race course, referring to the course of deeds and experiences through which children grow to become mature adults. A curriculum is prescriptive, and is based on a more general syllabus which merely specifies what topics must be understood and to what level to achieve a particular grade or standard.

An academic discipline is a branch of knowledge which is formally taught, either at the university, or via some other such method. Each discipline usually has several sub-disciplines or branches, and distinguishing lines are often both arbitrary and ambiguous. Examples of broad areas of academic disciplines include the natural sciences, mathematics, computer science, social sciences, humanities and applied sciences. [4]

ii Primary Education

Primary (or elementary) education consists of the first 5–7 years of formal, structured education. In general, primary education consists of six or eight years of schooling starting at the age of five or six, although this varies between, and sometimes within, countries. Globally, around 89% of primary-age children are enrolled in primary education, and this proportion is rising. [6] Under the Education For All programs driven by UNESCO, most countries have committed to achieving universal enrollment in primary education by 2015, and in many countries, it is compulsory for children to receive primary education. The division between primary and secondary education is somewhat arbitrary, but it generally occurs at about eleven or twelve years of age. Some education systems have separate middle schools, with the transition to the final stage of secondary education taking place at around the age of fourteen. Schools that provide primary education, are mostly referred to as primary schools. Primary schools in these countries are often subdivided into infant schools and junior school.

iii Secondary schools

In most contemporary educational systems of the world, secondary education comprises the formal education that occurs during adolescence. It is characterized by transition from the typically compulsory, comprehensive primary education for minors, to the optional, selective tertiary, "post-secondary", or "higher" education (e.g., university, vocational school for adults. Depending on the system, schools for this period, or a part of it, may be called secondary or high schools, gymnasiums, lyceums, middle schools, colleges, or vocational schools. The exact meaning of any of these terms varies from one system to another.

iv Indigenous Education

Indigenous education refers to the inclusion of indigenous knowledge, models, methods and content within formal and non-formal educational systems. Often in a post-colonial context, the growing recognition and use of indigenous education methods can be a response to the erosion and loss of indigenous knowledge and language through the processes of colonialism. Furthermore, it can enable indigenous communities to "reclaim and revalue their languages and cultures, and in so doing, improve the educational success of indigenous students."[7]

v Alternative Education

Alternative education, also known as non-traditional education or educational alternative, is a broad term that may be used to refer to all forms of education outside of traditional education (for all age groups and levels of education). This may include not only forms of education designed for students with special needs (ranging from teenage pregnancy to intellectual disability), but also forms of education designed for a general audience and employing alternative educational philosophies and methods.

Alternatives of the latter type are often the result of education reform and are rooted in various philosophies that are commonly fundamentally different from those of traditional compulsory education. While some have strong political, scholarly, or philosophical orientations, others are more informal associations of teachers and students

dissatisfied with certain aspects of traditional education. These alternatives, which include charter schools, alternative schools, independent schools, home schooling and auto didacticism vary widely, but often emphasize the value of small class size, close relationships between students and teachers, and a sense of community.

Alternative education may also allow for independent learning and engaging class activities. [8]

vi Higher Education

Higher education, also called tertiary, third stage, or post secondary education, is the non-compulsory educational level that follows the completion of a school providing a secondary education, such as a high school or secondary school.

Higher education generally involves work towards a degree-level or foundation degree qualification. In most developed countries a high proportion of the population (up to 50%) now enters higher education at some time in their lives. Higher education is therefore very important to national economies, both as a significant industry in its own right, and as a source of trained and educated personnel for the rest of the economy.

2. LEARNING MODALITIES

There has been work on learning styles over the last two decades. Dunn and Dunn [11] focused on identifying relevant stimuli that may influence learning and manipulating the school environment, at about the same time as Joseph Renzulli [12] recommended varying teaching strategies. Howard Gardner [13] identified individual talents or aptitudes in his Multiple Intelligences theories. Based on the works of Jung, the Myers-Briggs Type Indicator and Kersey Temperament Sorter[14] focused on understanding how people's personality affects the way they interact personally, and how this affects the way individuals respond to each other within the learning environment. The work of David Kolb and Anthony Gregory's Type Delineator [15] follows a similar but more simplified approach.

It is currently fashionable to divide education into different learning "modes". The learning modalities [16] are probably the most common:

  i   **Visual**: learning based on observation and seeing what is being learned.
  ii **Auditory:** learning based on listening to instructions/information.
  iii **Kinesthetic**: learning based on hands-on work and engaging in activities.

Although it is claimed that, depending on their preferred learning modality, different teaching techniques have different levels of effectiveness, [17] recent research has argued "there is no adequate evidence base to justify incorporating learning styles assessments into general educational practice."[18]

A consequence of this theory is that effective teaching should present a variety of teaching methods which cover all three learning modalities so that different students have equal opportunities to learn in a way that is effective for them. [19] Guy Claxton has

questioned the extent that learning styles such as VAK are helpful, particularly as they can have a tendency to label children and therefore restrict learning.[20][21]

3. INSTRUCTION

Instruction is the facilitation of another's learning. Instructors in primary and secondary institutions are often called teachers, and they direct the education of students and might draw on many subjects like reading, writing, mathematics, science and history. Instructors in post-secondary institutions might be called teachers, instructors, or professors, depending on the type of institution; and they primarily teach only their specific discipline. Studies from the United States suggest that the quality of teachers is the single most important factor affecting student performance, and that countries which score highly on international tests have multiple policies in place to ensure that the teachers they employ are as effective as possible. [22] With the passing of NCLB in the United States (No Child Left Behind), teachers must be highly qualified.

4. TECHNOLOGY

One of the most substantial uses in education is the use of technology. Also technology is an increasingly influential factor in education. Computers and mobile phones are used in developed countries both to complement established education practices and develop new ways of learning such as online education (a type of distance education). This gives students the opportunity to choose what they are interested in learning. The proliferation of computers also means the increase of programming and blogging. Technology offers powerful learning tools that demand new skills and understandings of students, including Multimedia, and provides new ways to engage students, such as Virtual learning environments. One such tool are virtual manipulative, which are an "interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge. In short, virtual manipulatives are dynamic visual/pictorial replicas of physical mathematical manipulatives, which have long been used to demonstrate and teach various mathematical concepts. Virtual manipulatives can be easily accessed on the Internet as stand-alone applets, allowing for easy access and use in a variety of educational settings. Emerging research into the effectiveness of virtual manipulatives as a teaching tool have yielded promising results, suggesting comparable, and in many cases superior overall concept-teaching effectiveness compared to standard teaching methods.[citation needed] Technology is being used more not only in administrative duties in education but also in the instruction of students. The use of technologies such as PowerPoint and interactive whiteboard is capturing the attention of students in the classroom. Technology is also being used in the assessment of students. One example is the Audience Response System (ARS), which allows immediate feedback tests and classroom discussions. [23]

Information and communication technologies (ICTs) are a "diverse set of tools and resources used to communicate, create, disseminate, store, and manage information."[24] These technologies include computers, the Internet, broadcasting technologies (radio and television), and telephony. There is increasing interest in how computers and the Internet can improve education at all levels, in both formal and non-formal settings.[25] Older

ICT technologies, such as radio and television, have for over forty years been used for open and distance learning, although print remains the cheapest, most accessible and therefore most dominant delivery mechanism in both developed and developing countries.[26] In addition to classroom application and growth of e-learning opportunities for knowledge attainment, educators involved in student affairs programming have recognized the increasing importance of computer usage with data generation for and about students. Motivation and retention counselors, along with faculty and administrators, can impact the potential academic success of students by provision of technology based experiences in the University setting. [27]

The use of computers and the Internet is in its infancy in developing countries, if these are used at all, due to limited infrastructure and the attendant high costs of access. Usually, various technologies are used in combination rather than as the sole delivery mechanism. For example, the Kothmale Community Radio Internet uses both radio broadcasts and computer and Internet technologies to facilitate the sharing of information and provide educational opportunities in a rural community in Sri Lanka.[28] The Open University of the United Kingdom (UKOU), established in 1969 as the first educational institution in the world wholly dedicated to open and distance learning, still relies heavily on print-based materials supplemented by radio, television and, in recent years, online programming.[29] Similarly, the Indira Gandhi National Open University in India combines the use of print, recorded audio and video, broadcast radio and television, and audio conferencing technologies.[30]

The term "computer-assisted learning" (CAL) has been increasingly used to describe the use of technology in teaching. Classrooms of the 21st century contain interactive white boards, tablets, mp3 players, laptops, etc. Wiki sites are another tool teachers can implement into CAL curriculums for students to understand communication and collaboration efforts of group work through electronic means.[citation needed] Teachers are encouraged to embed these technological devices and services in the curriculum in order to enhance students learning and meet the needs of various types of learners.

5. CONCLUSION AND FUTURE WORK

This was education and its categories. Type of education and instruction were also described. In future the process may be further enhances by using some their modern techniques. And it is clear that computer and technology may be used more efficiently. Just like to computerize the education system and make the record secure.

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PROJECT DESIGNING IN COMPUTER SCIENCE

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ABSTRACT

To design a project is a very important issue. Because a system is computerized from manual. So to cope with the problem the issues will be discussed. And with the help of practical projects it will be analyzed that how to improve and enhance the subject. Also it will be shown practically that how the front end is made more attractive and user friendly.

KEY WORDS

Project, Issue, System, Manual, End user

1. INTRODUCTION

The grading and judging of computer science project can often be a confusing job especially for those without a background in computer science.[1] In software engineering, a design pattern is a general reusable solution to a commonly occurring problem within a given context in software design. A design pattern is not a finished design that can be transformed directly into code. It is a description or template for how to solve a problem that can be used in many different situations. So patterns are formalized best practices that you must implement yourself in your application.[1] Object-oriented design patterns typically show relationships and interactions between classes or objects, without specifying the final application classes or objects that are involved. Many patterns imply object-orientation or more generally mutable state, and so may not be as applicable in functional programming languages, in which data is immutable or treated as such.

Design patterns reside in the domain of modules and interconnections. At a higher level there are architectural patterns that are larger in scope, usually describing an overall pattern followed by an entire system.[2]

Patterns originated as an architectural concept by Christopher Alexander (1977/79). In 1987, Kent Beck and Ward Cunningham began experimenting with the idea of applying patterns to programming and presented their results at the OOPSLA conference that year.[3][4] In the following years, Beck, Cunningham and others followed up on this work.

Design patterns gained popularity in computer science after the book *Design Patterns: Elements of Reusable Object-Oriented Software* was published in 1994 by the so-called "Gang of Four" (Gamma et al.). That same year, the first Pattern Languages of

Programming Conference was held and the following year, the Portland Pattern Repository was set up for documentation of design patterns. The scope of the term remains a matter of dispute. Notable books in the design pattern genre include:

- Gamma, Erich; Richard Helm, Ralph Johnson, and John Vlissides (1995). *Design Patterns: Elements of Reusable Object-Oriented Software*. Addison-Wesley. ISBN 0-201-63361-2.
- Buschmann, Frank; Regine Meunier, Hans Rohnert, Peter Sommerlad (1996). *Pattern-Oriented Software Architecture, Volume 1: A System of Patterns*. John Wiley & Sons. ISBN 0-471-95869-7.
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- Freeman, Eric T; Elisabeth Robson, Bert Bates, Kathy Sierra (2004). *Head First Design Patterns*. O'Reilly Media. ISBN 0-596-00712-4.

Although design patterns have been applied practically for a long time, formalization of the concept of design patterns languished for several years.[5]

In 2009 over 30 contributors collaborated with Thomas Erl on his book, *SOA Design Patterns*.[6] The goal of this book was to establish a de facto catalog of design patterns for SOA and service-orientation.[7] (Over 200+ IT professionals participated world-wide in reviewing Erl's book and patterns.) These patterns are also published and discussed on the community research site

Efforts have also been made to codify design patterns in particular domains, including use of existing design patterns as well as domain specific design patterns. Examples include user interface design patterns,[9] information visualization,[10] secure design,[11] "secure usability",[12] Web design [13] and business model design.[14]

The annual Pattern Languages of Programming Conference proceedings [15] include many examples of domain specific patterns.

The documentation for a design pattern describes the context in which the pattern is used, the forces within the context that the pattern seeks to resolve, and the suggested solution.[23] There is no single, standard format for documenting design patterns. Rather, a variety of different formats have been used by different pattern authors. However, according to Martin Fowler, certain pattern forms have become more well-known than others, and consequently become common starting points for new pattern-writing efforts.[24] One example of a commonly used documentation format is the one used by Erich Gamma, Richard Helm, Ralph Johnson and John Vlissides (collectively known as the "Gang of Four", or GoF for short) in their book *Design Patterns*. It contains the following sections:

- **Pattern Name and Classification:** A descriptive and unique name that helps in identifying and referring to the pattern.
- **Intent:** A description of the goal behind the pattern and the reason for using it.
- **Also Known As:** Other names for the pattern.
- **Motivation (Forces):** A scenario consisting of a problem and a context in which this pattern can be used.
- **Applicability:** Situations in which this pattern is usable; the context for the pattern.
- **Structure:** A graphical representation of the pattern. Class diagrams and Interaction diagrams may be used for this purpose.
- **Participants:** A listing of the classes and objects used in the pattern and their roles in the design.
- **Collaboration:** A description of how classes and objects used in the pattern interact with each other.
- **Consequences:** A description of the results, side effects, and tradeoffs caused by using the pattern.
- **Implementation:** A description of an implementation of the pattern; the solution part of the pattern.
- **Sample Code:** An illustration of how the pattern can be used in a programming language.
- **Known Uses:** Examples of real usages of the pattern.
- **Related Patterns:** Other patterns that have some relationship with the pattern; discussion of the differences between the pattern and similar patterns
- The concept of design patterns has been criticized in several ways.
- The design patterns may just be a sign of some missing features of a given programming language (Java or C++ for instance). Peter Norvig demonstrates that 16 out of the 23 patterns in the Design Patterns book (which is primarily focused on C++) are simplified or eliminated (via direct language support) in Lisp or Dylan.[25] See also Paul Graham's essay Revenge of the Nerds.[26]
- The idea may not be as new as suggested by the authors: for instance the Model-View-Controller paradigm is an example of a "pattern" which predates the concept of "design patterns" by several years.
- Moreover, inappropriate use of patterns may unnecessarily increase complexity.[27]

2. PROCESS EXPLANATİON

The process can be explain by the following improvements in different projects.
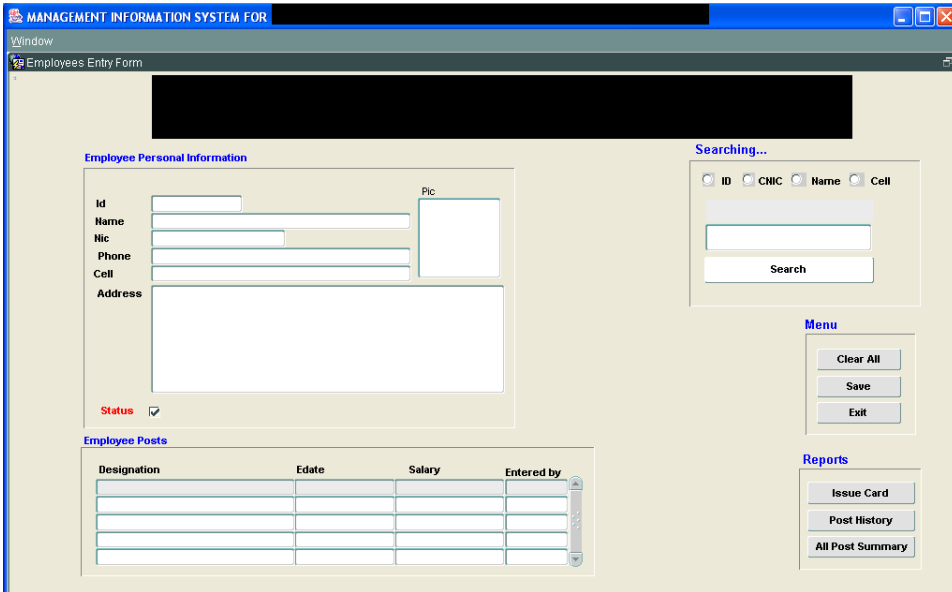
Fig. 1: This is the first figure of a software front end design.
The look is not so attractive
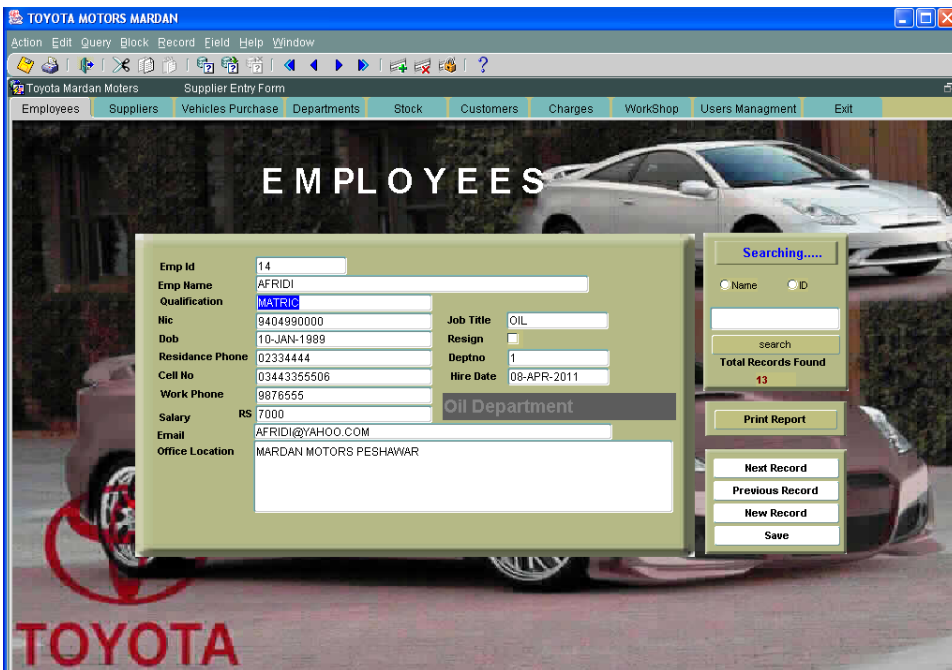
Fig.2: Now this the second design. Here a lot of area is free.
And the picture in the background is not necessary.
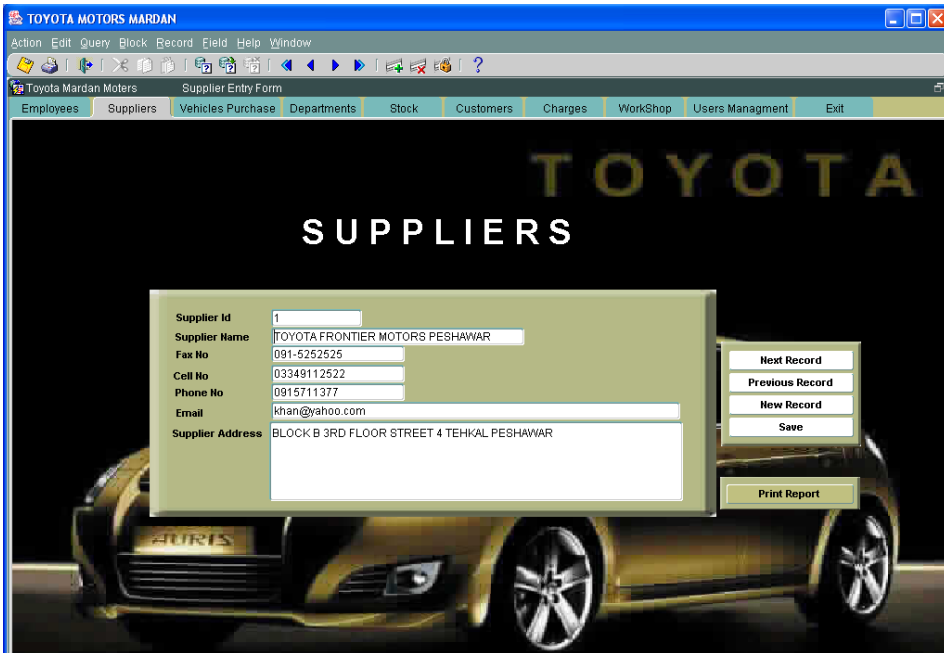
Fig.3: Now in the third design it enhanced upto some extent but still the area is free.
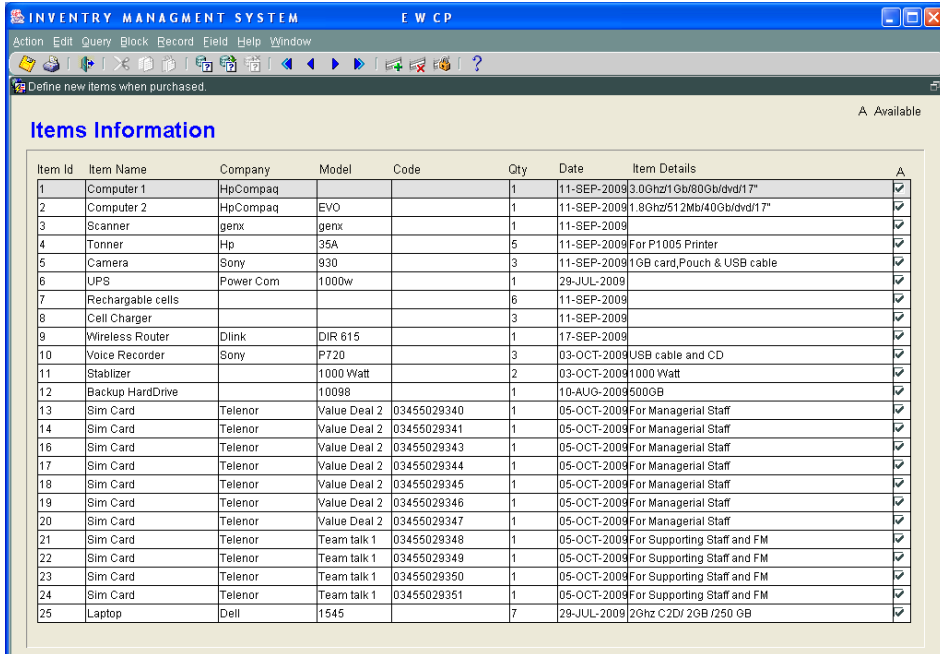
Fig.4: In this design the whole area is covered. And also most of the
information is available regarding the record.

Fig.5: In this design the whole information is displayed on a single screen. Hence the design is very improved.

3. CONCLUSION AND FUTURE WORK

In this research it is studied that how to improve the design of a project. Also it is studied to utilize the free space in a design. In future the its may be extended to make more improvements.

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RISK MANAGEMENT THROUGH COMPUTER SYSTEMS

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ABSTRACT

Risk management is very challenging job. Risk may be minimized by using computer systems instead of manual systems. Because these are run by the computer and computer neither get bored nor get tired. In this research it will be proved that how these systems are performing their jobs. Some examples from the real world will be taken for the practical point of view. Finally it will be discussed that how to improve these systems to make the work more systematic and consistent.

KEYWORDS

Risk, Management, Systems, Potential, Computer

1. INTRODUCTION

1.1 Risk

Risk is the potential that a chosen action or activity (including the choice of inaction) will lead to a loss (an undesirable outcome). The notion implies that a choice having an influence on the outcome exists (or existed). Potential losses themselves may also be called "risks". Almost any human endeavour carries some risk, but some are much more risky than others.

The Oxford English Dictionary cites the earliest use of the word in English (in the spelling of risque) as from 1621, and the spelling as risk from 1655. It defines risk as:

(Exposure to) the possibility of loss, injury, or other adverse or unwelcome circumstance; a chance or situation involving such a possibility [1] The ISO 31000 (2009) /ISO Guide 73 definition of risk is the 'effect of uncertainty on objectives'. In this definition, uncertainties include events (which may or not happen) and uncertainties caused by a lack of information or ambiguity. This definition also includes both negative and positive impacts on objectives.

Another definition is that risks are future problems that can be avoided or mitigated, rather than current ones that must be immediately addressed. [2]

1.2 Risk Management

Risk management is the identification, assessment, and prioritization of risks (defined in ISO 31000 as the effect of uncertainty on objectives, whether positive or negative) followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events[3]

2. USE OF COMPUTER IN RISK MANAGEMENT

2.1 Computer Software

Computer software is a collection of computer programs and related data that provide the instructions for telling a computer what to do and how to do it. In other words, software is a conceptual entity which is a set of computer programs, procedures, and associated documentation concerned with the operation of a data processing system. We can also say software refers to one or more computer programs and data held in the storage of the computer for some purposes. In other words software is a set of programs, procedures, algorithms and its documentation. Program software performs the function of the program it implements, either by directly providing instructions to the computer hardware or by serving as input to another piece of software. The term was coined to contrast to the old term hardware (meaning physical devices). In contrast to hardware, software is intangible, meaning it "cannot be touched".[4] Software is also sometimes used in a more narrow sense, meaning application software only. Sometimes the term includes data that has not traditionally been associated with computers, such as film, tapes, and records.[5]

Examples of computer software include:

2.1.1   Application software

Application Software Includes end-user applications of computers such as Microsoft office or video games, and ERP software for groups of users. Middleware controls and co-ordinates distributed systems.

2.1.2   Programming languages

It defines the syntax and semantics of computer programs. For example, many mature banking applications were written in the COBOL language, originally invented in 1959. Newer applications are often written in more modern programming languages. Nadra Software is in Visual Basic

2.1.3   System software

It includes operating systems (e.g. Windows, Linux), which govern computing resources. Today[when?] large[quantify] applications running on remote machines such as Websites are considered[by whom?] to be system software, because[citation needed] the end-user interface is generally through a graphical user interface, such as a web browser.

2.1.4   Test ware

It is software for testing hardware or a software package.

2.1.5   Firmware

It is low-level software often stored on electrically programmable memory devices. Firmware is given its name because it is treated like hardware and run ("executed") by other software programs.

2.1.6   Shrink ware

It is the older name given to consumer-purchased software, because it was often sold in retail stores in a shrink-wrapped box. Device drivers control parts of computers such as

disk drives, printers, CD drives, or computer monitors. Programming tools help conduct computing tasks in any category listed above. For programmers, these could be tools for debugging or reverse engineering older legacy systems in order to check source code compatibility.

Computer Software is programs to manage the system either Computer (Hardware) itself or the man made systems. Computer Software in the physical world means to computerize a system from manual. Software development is a process start with the problem definition and end with the maintenance of the system. Why organizations and management wants to computerize a system e.g. Nadra make the work of ID card more speedy and consistent. Its because of the computer software that chance to the loss of data is minimize. Also the works become speedy. And now all the facilities are available near about it one place.

These software minimize the risk just like in highways it manage the traffic and gather the data about the vehicle which may help the administration to manage it.

3. INTERNET AND COMMUNICATION

The Internet is a global system of interconnected computer networks that use the standard Internet Protocol Suite (TCP/IP) to serve billions of users worldwide. It is a network of networks that consists of millions of private, public, academic, business, and government networks, of local to global scope, that are linked by a broad array of electronic, wireless and optical networking technologies.[6] The Internet can also be defined as a worldwide interconnection of computers and computer networks that facilitate the sharing or exchange of information among users. The Internet carries a vast range of information resources and services, such as the inter-linked hypertext documents of the World Wide Web (WWW) and the infrastructure to support electronic mail.

Most traditional communications media including telephone, music, film, and television are reshaped or redefined by the Internet, giving birth to new services such as Voice over Internet Protocol (VoIP) and IPTV. Newspaper, book and other print publishing are adapting to Web site technology, or are reshaped into blogging and web feeds. The Internet has enabled or accelerated new forms of human interactions through instant messaging, Internet forums, and social networking. Online shopping has boomed both for major retail outlets and small artisans and traders. Business-to-business and financial services on the Internet affect supply chains across entire industries.

The origins of the Internet reach back to research of the 1960s, commissioned by the United States government in collaboration with private commercial interests to build robust, fault-tolerant, and distributed computer networks. The funding of a new U.S. backbone by the National Science Foundation in the 1980s, as well as private funding for other commercial backbones, led to worldwide participation in the development of new networking technologies, and the merger of many networks. The commercialization of what was by the 1990s an international network resulted in its popularization and incorporation into virtually every aspect of modern human life. As of 2009, an estimated one-quarter of Earth's population uses the services of the Internet.

The Internet has no centralized governance in either technological implementation or policies for access and usage; each constituent network sets its own standards. Only the overreaching definitions of the two principal name spaces in the Internet, the Internet Protocol address space and the Domain Name System, are directed by a maintainer organization, the Internet Corporation for Assigned Names and Numbers (ICANN). The technical underpinning and standardization of the core protocols (IPv4 and IPv6) is an activity of the Internet Engineering Task Force (IETF), a non-profit organization of loosely affiliated international participants that anyone may associate with by contributing technical expertise. So it minimize risk in the field of communication by giving confirmation to the sender and the receiver about the transmission.

4. SPEED, ACCURACY AND STORAGE OF DATA

Computer minimize the risk because it has the following characteristics

4.1 Speed:

Computers work at very high speed and are much faster than humans. A second is very large time period time for computer. A computer can perform billions of calculations in a second. The time used by a computer to perform an operation is called the processing speed. Computer speed is measured in Mega Hertz (MHz).

4.2 Storage:

A computer can store a large amount of data permanently. User can use this data at any time. We can store any type of data in a computer. Text, graphic, pictures, audio and video files can be stored easily. The storage capacity of the computer is increasing rapidly. Storage of computer is measured in (bit, Byte, Megabyte, Gigabyte, Terabyte).

Processing: A computer can process the given instructions. It can perform different types of processing like addition, subtraction, multiplication and division. It can also perform logical functions like comparing two numbers to decide which one is the bigger etc.

Accuracy: Accuracy means to provide results without any error. Computers can process large amount of data and generate error-free results. A modern computer performs millions of operations in nano second without any error.

5. IN THE FIELD OF MEDICAL AND ENGINEERING

Computer may be used in the field of medical for different purpose. Patient data may be collect easy for the analysis and future prediction. Experts systems may be developed to use in the remote areas where the access of specialist doctors are difficult. Computer may be used for online operation systems in special circumstances. Computer is use in medical labs to produce fast and accurate results during the treatment of different diseases. [7]

Use of computer in the field of engineering is very vast. Even there are many branches of computer Software tools that have been developed to support these activities are considered CAE tools. CAE tools are being used, for example, to analyze the strength and performance of components and assemblies. The term encompasses simulation,

justification, and optimization of products and manufacturing tools. In the future, CAE systems will be major providers of information to help support design teams in decision making. In regard to information networks, CAE systems are individually considered a single node on a total information network and each node may interrelate with other nodes on the network. CAE systems can provide support to businesses. This is achieved by the use of reference architectures and their ability to place information views on the business process. Reference architecture is the basis from which information model, especially product and manufacturing models.[9]

6. IN EDUCATION

Computer science or computing science (sometimes abbreviated CS) is the learning of the theoretical foundations of information and computation, and of practical techniques for their execution and application in computer systems. It is often described as the efficient study of algorithmic processes that produce, explain, and transform information. In this work, I use Data Mining algorithms from the field of computer science for the analysis process to prove experimentally and practically that how reliable, efficient and fast are these for the analysis of data in education? A solid mathematical threshold (0 to 1) is set to analyze the data.[8] By practical analysis the risk is minimized by giving the produced results to the experts.

7. IN DEFENSE

Wars fought prior to the advent of digital computing could take years to complete and resulted in tremendous loss of property on both sides. Weapons of mass destruction used to be the fastest way to achieve the objective. A discussion with Dan Carroll, Vietnam War veteran and current contract engineer for major aerospace companies revealed that in World War II and in Vietnam, American forces employed a technique known as carpet bombing to wipe out enemy troops and resources. Many large bombers would over fly an area and drop thousands of bombs in an effort to destroy the enemy. This proved to be very costly for both sides in terms of property and human life.

Assuming that war is inevitable and that enemies must sometimes be destroyed, small, powerful computers are useful. Weapons can now be made intelligent enough to know precisely where and what the target is. It is now possible to launch a projectile from hundreds of miles away and destroy one particular building in an enemy installation. This sort of precision warfare is much less expensive in the long run and makes much more efficient use of resources. When human beings must be killed, we have the ability to spare the vast majority of people in a city who are innocent bystanders and target only those directly involved in the war effort.

8. INFORMATION RETRIEVAL

Information retrieval is a sub-field of computer science that deals with the computerized storage and retrieval of documents [11].

9. CONCLUSION AND FUTURE WORK

Computer systems are very helpful in risk management process. Risk may be minimizing by using and enhancing the computer systems. In future computer systems may be making more advance and intelligent to overcome risk management more systematically.

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ROLE OF SOCIO-ECONOMIC AND DEMOGRAPHIC FACTORS IN DECIDING INTERNAL MIGRATION IN PAKISTAN

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ABSTRACT

The decision of migration has varied on the basis of socio-economic and demographic factors. In the context of internal migration people migrate from rural to urban areas as a rational human capital investment decision to reap economic rewards in the form of better economic opportunities and benefits. The objective of this paper is to investigate the role of socio-economic and demographic factors in decision of internal migration. The data was obtained from Pakistan Labour Force Survey 2009-10 which is conducted by Federal Bureau of Statistics. The dependent variable has dichotomous response; migrants and non migrants and independent variables are demographic; age, sex, marital status and in socioeconomics; education levels and training of employees. The Hierarchical Log-linear model has been used to identify model that best describing the relationship between these categorical variables. The findings of this study showed that the socioeconomics and demographic factors such as age, education, marital status and gender play a significant role in taking decision of internal migration in Pakistan.

KEYWORD

Migration, Socio-economic and Demographic Indicators, Pakistan Labour Force, Hierarchical Log-linear Model

1. INTRODUCTION

Migration means movement of people from one administrative district to another administrative district at any time of their lives. It excludes movement of population within the current district (FBS, 2009-10). The decision of migration has varied on the basis of socio-economic and demographic factors. The distribution of population in a country is influenced by the characteristics of the sending and receiving areas in terms of push and pulls factors, that resulting in rural-urban, urban-urban, rural-rural and urban-rural migration flows. In the context of internal migration people migrate from rural to urban areas as a rational human capital investment decision to reap economic rewards in the form of better economic opportunities and benefits.

Hoynes, et al. (2006) suggested, access to labor-market opportunities is an important determinant of both individuals' and families' poverty status, those are living in different location with different densities of migrant inflows may subject to different local labour market conditions and this leads them to have varying poverty rates across these set of location. In fact, such type of consequences of internal migration may become more

popular in developing countries, also emerging and transitional economics that have experienced profound economic and social changes during the last 30 years.

The native males who were living in provinces observed significantly increase in migrant inflows in their labor market outcomes. Higher the inflow of recent migrants lowered the native males' employment ratio play a significant role. The negative consequences of migrant inflows seem to be considered increase with both education and age i.e. the recent migrant ratio's estimated negative effects that are found to be highest pronounced for the older population with the highest educational qualifications and to be least pronounced for the younger population with the lowest educational qualifications [Berker (2010)].

Migration has always added to household earnings. The education increases the chances of migration which results in higher earnings. Low earning of household and lack of employment opportunities in labour market are main causes for internal migration in Pakistan. Migration is also provides employment opportunities at purpose which results in well being of household [Arif (2005)].

1.1 Objective of Study

The objective of this study is to investigate the role of socioeconomic and demographic factors in decision of internal migration in Pakistan.

2. LITERATURE REVIEW

Ahmed and Sirageldin (1993) carried a study on socio-economic determinants of labour mobility and developed the model for internal migration in Pakistan. The results of this study indicated that, in general migration in Pakistan were selective, especially in terms of age, education and occupation.

Nam (1997) conducted a study to examine factors which affect the employment situation of residents in Hanoi city (both migrants and non-migrants). The finding of this study showed that the employment between migrants and non-migrants in Hanoi was significant, and these differences was reduced, but not eliminated, when other factors (i.e., demographic and socioeconomic factors) were controlled.

Wolbers (1998) conducted a study to examine the effect of education on labour migration between employment and unemployment persons in the Netherlands. The findings of this study showed that unemployment among the individuals with the lower educational qualification was higher than among those with the higher educational level. So less educated have a greater chance to losing their jobs than for those with the higher educated.

Khan and Shehnaz (2000) conducted a study internal labour mobility pattern and determinants in Pakistan. In this study both the statistical and econometric analysis were used to measure the internal mobility and determinants. The statistical analysis showed that mostly male and female undertook the decision to migrant for non economic motives, this pattern is higher in urban-urban migration than to rural-urban migration. In econometric analysis the migration as a human capital investment, education, technical

and vocational training played a significantly positive effect on the probability of migration decision for both males and females.

Akram, et al. (2002) carried out study to examine the factors that has an effect on migration direction with reference to employment status of the migrants in case of male. The multinomial logistic regression model was used for analysis for relationship between migrants and their employment status. From this study found that person's decision for migration in Punjab are significantly affected by his age, education level, family type, marital and employment status.

Filiztekin (2010) conducted empirical study on the determinants of internal migration in Turkey. The result showed that both the economic factors such as income differentials and unemployment rates, and social factors like presence of social networks along with personal characteristics such as age and education levels play a significant role on migration.

Saarwar and Sial (2011) conducted a study to examine the effect of education on decision to migrate and on earnings of migrants within human capital framework. The result showed the probability to migrate increase: decision to migrate diminishes with age, human capital determinants like education and experience had a significant effect on earning of migrants. It implies that education and earning have a positive relation.

3. DATA SOURCE AND ANALYSIS TECHNIQUES

The data for this study is taken from the Labour Force Survey (LFS) 2009-10 which was conducted by Federal Bureau of Statistics (FBS), Pakistan. The sample of 36400 households consists of 263501 individuals is given in the said survey. There are some missing cases in Labour Force Survey data related to variables age, sex, marital status, education levels and training. So after excluding missing cases for any variable the ultimately sample size is 187732. We used Hierarchical Log-linear analysis for data analysis purpose.

4. RESULTS AND DISCUSSION

The log-linear analysis for model selection procedure is used to identify model that best describing the relationship between categorical variables and it analyzed in multi-way cross tabulations. Here we used to build the model by using backward elimination method and apply with saturated model with parameters estimates and test the partial association in terms of main effects, second order, third order and higher order. We have aim to investigate the association of these variables and also the strength of association by estimating the parameters values and odd ratio.

The valid sample size is 187732 and weighted valid cases according to given factors are 81089 that contains information on these variables. The variables description and their categories level presented in table A-1 age has form four categories in ordinal scale (10-35, 35-55, 56-75, >76), sex has two categories in nominal (1=male, 2=female), marital status has four categories in nominal scale (1= never marriage, 2= marriage, 3= widow, 4= divorce), training has three categories they are all in nominal scale (1= yes on job, 2= yes off job, 3= no), education level has five categories with ordinal scale (No

formal education, below matric, matric less inter, inter less degree, degree and above) and migration status has also two categories with nominal scale (1= migrants, 2= non migrants).

In table-2 contains information on k-way and higher order effects and shows the probability that provides the observed significance level for the test that k-way and higher order effects are zero. Second part of table shows just k-way effects it calculates by using upper portion of table and it also test that k-way effects are zero. The likelihood chi-square with no parameters and have only the mean is 478388.703 and first order effect is 53505.996 shown in upper part of table-2. The difference between mean and first order effect is 478388.703-53505.996 = 424882.706 which displayed on the first line of the second portion of this table. This difference is measure of knowing about how much the model improve when first order effects are included. The small sig. (.0000) value shows that the hypothesis of first order effects being zero is rejected and accept alternative hypothesis which means that first order effect is present.

Now this is same procedure apply to the second order effect the difference 53505.996 - 2245.051= 51260.945 displayed on the second line of second portion of same table-2. And the addition of second order effect improves the likelihood chi-square by 51260.945 this is also significant it means that there is second order effect is present in model. The difference between 2245.051 - 397.885 = 1847.166 is display on the third line of second portion of table-2 and also significant third order effect and similarly the difference between 397.885 - 81.477 = 316.408 displayed on forth line of second portion of table-2 also significant this indicates that this model support up to forth order effects and remaining five and six order does not help because p-value is not significant.

Goodness of fit test statistics shown in table-3 it represents the value of Likelihood Ratio Chi- Square and Pearson Chi-Square with its d.f and sig. values. Goodness of fit statistics provides the measure how well the model is fit for given data and for large sample size both statistics are equivalent. The small value of both chi-square statistics and the greater p value from significance value (.05) indicate a good model. According to given p-values against test statistics, the p value is greater than significance value (.05) and value of test statistics are small, this provides us that our final model after backward elimination is perfect fit for the given data.

The partial association given in table-4 which contains the partial chi-square with its sig. in term of the main effects, first order interaction term, second order, third order, forth order and fifth order interaction. This table shows the significance of partial association in all these terms. This table shows that all main effects age, sex, marital status, educational level and training are significant this indicate that all these variables are significantly related with migration and also all combinations of first order interaction parameters have significance partial association like age and migration, sex and migration, educational level and migration, training and migration, marital status and migration.

According to table-4 mostly 2nd order interactions terms are significant like Age * Edu_level * Migration, Sex * Edu_level * Migration, marital_status * edu_level * migration, Sex * Training * Migration, sex * training * migration, Age * Sex * Migration, and so on shown in table-4. There are single 3rd order partial association is

significant like Sex * Marital_Status * Edu_level * Migration and all others higher order interaction terms are not significant. This table also represents the k way and higher order decision as in that table we concluded that up to forth order interaction terms significantly have association between them.

The parameter estimated value for main effects, odd ratios and 95% confidence interval for odd ratio is presented in table-5. The positive value of estimate of parameters shows the positive relationship and increase in response category and negative value show negative relationship and decrease in response category. The odd ratio tells us both direction of relationship and also provides the magnitude or strength of association. In this table we calculated odd ratio by taking the antilogarithm of their estimated parameters. Age have four categories but the forth category deal as a reference category for others and provide three estimates for age variable and age provide significant effect for three categories. In case of sex variable has two categories and first category of male and 2nd is female and compare male to female so sex has significant effect in migration. Marital status has four categories and forth category used as a reference category and never married vs. divorce is not significant but other two categories have significant effect. Training has negatively significant impact on migration. The educational level has five categories and last category take as a reference category all others compare with it first two has positive impact on migration and other two have negative effect.

Table-6 represents the estimated parameter values of interaction effects, Odd Ratio and 95% Confidence Interval for Odd Ratio. Age has significant effect for three categories with migration decision. If we compare age group of 10-35 years of age with >76 then migration occurs in 10-35 years age of persons more than >76 of age. According to odd ratio we interpret as the odds ratio is 1.468 times higher (95% C-I 1.322 to 1.629) for those who have age is between 10-35 years than those who have age greater than 76 years. And for the second category of age group odd ratio indicates that, the odd ratio is 1.134 time higher (95% C-I 1.019 to 1.263) for those who have age is between 35-55 years than those who have age greater than 76 years and for the last odd ratio of age category indicates, the estimated odd of taking decision of migration for group of age 56-75 years are 0.831 times (95% C-I 0.735 to 0.939) the estimated odds for age group greater than 76 years. So these results shows that a mostly migration occurs in group of age 10-35 than all other three categories.

Interaction terms for Sex and migration is insignificant effect. Marital status and migration has significant effect for first category and not for next two categories. Training has negative impact on migration, the estimated odd of taking decision of migration for persons who have training on job 0.891 times (95% C-I 0.803 to 0.987) the estimated odds for those who have no training and the interaction terms for migration and the persons who have training off job. The interaction terms for 1st category for no formal education with migration have significant effect and interpret as the odd ratio is 1.240 times higher (95% C-I 1.790 to 2.270) for those who have no formal education than those who have higher degree and other three levels have insignificant effect. educational level has five categories and last category take as a reference category all others compare with it first two has positive impact on migration and other two have negative effect. The odd ratio is 1.240 times higher (95% C-I 1.1218 to 1.439) for those who have no formal education than those who have higher degree (see table-6). Table-7 represents the 2nd

order interaction effects with migration. Age, marital status has jointly effect on migration decision.

5. CONCLUSION

The findings of this study showed that the socioeconomics and demographic factors such as age, education, marital status and gender play a significant role in taking decision of migration. Furthermore the findings showed that age is an important determinant for migration and mostly migration occurs at the younger age 10-35 than older one. Education levels play a significant role and according to this finding mostly migration occurs in Pakistan at no formal education level. Marital status also has a significant effect with migration decision of respondents and mostly migration occurs in case of female marriage. Training has negative effect with migration decision. Gender plays a significant role and mostly migration occurs for male than female.

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Table 1:
Description of Variables with its Categories

Variables	No of categories	Names of categories
Age	4	Age 1 (10-35), age 2 (35-55), age 3 (56-75), age 4 (>76)
Sex	2	Sex 1 (1=male), sex 2 (2=female)
Marital status	4	Marital status (1= never marriage, 2= marriage, 3= widow, 4= divorce)
Migration status	2	Migration (1= migrants, 2= non migrants)
Training	3	Training (1= yes on job, 2= yes off job, 3= no)
Educational level	5	No formal education, below matric, matric less inter, inter less degree, degree and above

Table 2:
K-Way and Higher-Order Effects

	K	df	Likelihood Ratio		Pearson	
			Chi-Square	Sig.	Chi-Square	Sig.
K-way and Higher Order Effects	1	959	478388.703	.000	3000593.039	.000
	2	945	53505.996	.000	59712.769	.000
	3	867	2245.051	.000	2680.909	.000
	4	647	397.885	1.000	485.186	1.000
	5	318	81.477	1.000	71.409	1.000
	6	72	2.599	1.000	1.643	1.000
K-way Effects	1	14	424882.706	.000	2940880.271	.000
	2	78	51260.945	.000	57031.860	.000
	3	220	1847.166	.000	2195.723	.000
	4	329	316.408	.681	413.777	.001
	5	246	78.878	1.000	69.766	1.000
	6	72	2.599	1.000	1.643	1.000

Table 3:
Goodness-of-Fit Test Statistics

	Test Statistics	df	Sig.
Likelihood Ratio Chi- Square	147.256	444	1.000
Pearson Chi-Square	137.490	444	1.000

Table 4:
Partial Association with Migration only

Effects	df	Partial Chi-Square	p-value
Age*Sex*Marital_Status*Training*Migration	18	1.456	1.000
Age*Sex*Marital_Status*Edu_level*Migration	36	9.046	1.000
Age*Sex*Training*Edu_level*Migration	24	15.835	.894
Age*Marital_Status*Training*Edu_level*Migration	72	18.820	1.000
Sex*Marital_Status*Training*Edu_level*Migration	24	10.041	.994
Age*Sex*Marital_Status*Migration	9	7.347	.601
Age*Sex*Training*Migration	6	3.027	.805
Age*Marital_Status*Training*Migration	18	6.744	.992
Sex*Marital_Status*Training*Migration	6	3.422	.754
Age*Sex*Edu_level*Migration	12	6.890	.865
Age*Marital_Status*Edu_level*Migration	36	23.654	.943
Sex*Marital_Status*Edu_level*Migration	12	24.695	.016
Age*Training*Edu_level*Migration	24	18.092	.799
Sex*Training*Edu_level*Migration	8	9.239	.323
Marital_Status*Training*Edu_level*Migration	24	23.464	.493
Age*Sex*Migration	3	10.699	.013
Age*Marital_Status*Migration	9	2.339	.985
Sex*Marital_Status*Migration	3	145.969	.000
Age*Training*Migration	6	4.090	.665
Sex*Training*Migration	2	17.064	.000
Marital_Status*Training*Migration	6	6.471	.373
Age*Edu_level*Migration	12	39.689	.000
Sex*Edu_level*Migration	4	21.856	.000
Marital_Status*Edu_level*Migration	12	50.999	.000
Training*Edu_level*Migration	8	14.603	.067
Age*Migration	3	568.694	.000
Sex*Migration	1	69.799	.000
Marital_Status*Migration	3	294.451	.000
Training*Migration	2	536.227	.000
Edu_level*Migration	4	530.497	.000
Age	3	81117.369	.000
Sex	1	28206.523	.000
Marital_Status	3	104051.497	.000
Training	2	109359.823	.000
Edu_level	4	43099.148	.000
Migration	1	59048.350	.000

Table 5:
Estimated Parameter values of main effects, Odd Ratio (OR)
and 95% Confidence Interval for OR

Variables	Estimated Parameters	Sig.	OR	95% CI for OR
Age (years)				
10-35 vs. >76	1.150	.000	3.160	2.844 - 3.504
36-55 vs. >76	.568	.000	1.765	1.586 - 1.964
56-75 vs. >76	-.386	.000	.680	0.601 - 0.769
Sex				
Sex	0.380	.000	1.462	1.366 - 1.568
Marital Status				
Never married vs. divorce	.049	.425	1.050	.932 - 1.184
married vs. divorce	1.630	.000	5.104	4.627 - 5.635
widow vs. divorce	-.507	.000	.6022	0.534 - 0.681
Training				
Yes on job vs. no	-.683	.000	0.505	0.460 - .560
yes off job vs. no	-0.277	.000	0.758	0.687 - 1.194
Educational Level				
No Formal Education vs. Degree and Above	0.706	.000	2.026	1.790 - 2.270
Below Matric vs. Degree and Above	.278	.000	1.320	1.156 - 1.508
Matric Less Inter vs. Degree and Above	-.210	.004	0.811	0.702 - 0.935
Inter But Less Degree vs. Degree and Above	-.481	.000	0.618	0.535 - 0.715

Table 6:
Estimated Parameter values of interaction effects,
Odd Ratio and 95% Confidence Interval for Odd Ratio

Variables	Estimated Parameters	Sig.	OR	95% CI for OR
Age*Migration				
10-35 vs. >76	.384	.000	1.468	1.322 - 1.629
36-55 vs. >76	.126	.021	1.134	1.019 -1.263
56-75 vs. >76	-.185	.003	0.831	0.735 - 0.939
Sex*Migration				
Sex	.359	.026	1.040	0.970 - 1.114
Marital Status* Migration				
Never married vs. divorce	.167	.006	1.182	1.048 - 1.332
married vs. divorce	.074	.139	1.077	0.976 - 1.188
widow vs. divorce	-.067	.278	0.935	0.828 - 1.054
Training* Migration				
Yes on job vs. no	-.116	.027	0.891	0.803 - 0.987
yes off job vs. no	-.071	.164	0.932	0.844 - 1.029
Educational Level* Migration				
No formal edu vs. degree and above	.240	.000	1.240	1.1218 - 1.439
Below matric vs. degree and above	.073	.279	1.073	0.943 - 1.228
Matric less inter vs. degree and above	-.043	.559	0.958	0.186 - 1.106
Inter but less degree vs. degree and above	-.124	.094	0.883	0.763 - 1.021

Table 7:
Estimated Parameter values of 1st order interaction effects,
Odd Ratio and 95% Confidence Interval for Odd Ratio

Variables	Estimated Parameters	Sig.	OR	95% CI for OR
Age*marital Status*Migration	.358	.000	1.430	1.229 - 1.663

A GENERAL CLASS OF MEAN ESTIMATORS USING MIXTURE OF AUXILIARY VARIABLES IN TWO-PHASE SAMPLING IN THE PRESENCE OF NON-RESPONSE ON BOTH PHASES

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ABSTRACT

In this paper we have suggested a general class of estimators in two-phase sampling to estimate the population mean of study variable in the case when non-response occur on both phases and also on study and/or auxiliary variable (s). We are using several continuous and categorical auxiliary variables simultaneously while constructing the class. Also we are assuming that the information on all auxiliary variables is not available for population (no information case) as mostly the practical cases. The expressions mean square error of suggested class has been derived. Furthermore several special cases of proposed class have been identified.

KEY WORDS

Mixture of Multi-Auxiliary Variables; Non-response; Two Phase Sampling; Generalized Estimator; No Information Case.

1. INTRODUCTION

The estimation of the population mean is an important issue in sampling theory and several efforts have been made to improve the precision of the estimators. As the use of auxiliary variable can enhance the precision of estimator when the study variable is highly correlated with auxiliary variable (Singh et al. (2010)). The use of multi-auxiliary variables is based on increase in precision as well as based on affordable cost. There could be the following cases for the use of auxiliary information in two phase sampling (a) no information case (NIC) (b) partial information case (PIC) (c) full information case (FIC). If population information about none of the auxiliary variables is available then it is called NIC. If population information about some of the auxiliary variables is available then it is called PIC. If population information about all of the auxiliary variables is available then it is called FIC (Samiuddin and Hanif, (2007)).

Problem of non-response is very common in sample surveys which are conducted in the field of social sciences, agriculture and medical sciences. Almost all surveys suffer from the problem of non-response which can be either unit non-response on item non-response. There is a need to consider non-response as it decreases the precision of an estimate and produces biased estimates.

To address this problem the following sampling scheme is used. When sampling frame for study variable and/or population information about auxiliary variable is not

available then two phase sampling is used to decrease the cost. In two phase sampling, total population of N units is divided into two units N_1 shows responding units N_2 shows non-responding units a large first phase sample of size n_1 is selected and auxiliary information is recorded then, a subsample at second phase of size n_2 is selected from the first phase sample and information about study variable y as well as auxiliary variable x is collected such that $n_2 < n_1$. At second phase, let v_1 be the responding units and v_2 be those units which did not supply information. Then, a subsample of r_2 units is selected from the m_2 units. Following this sampling scheme, Hansen and Hurwitz (1946) considered the problem of non-response at second phase and suggested estimator is

$$t_{1(2)} = w_1 \bar{y}_{1\,2} + w_2 \bar{y}_{2r\,2}, \tag{1.1}$$

where $w_1 = \dfrac{n_1}{n}$ and $w_2 = \dfrac{n_2}{n}$, are the proportions for responding and non-responding units.

Variance of the estimator is given by

$$V(t_{1(2)}) = \left(\frac{1}{n_2} - \frac{1}{N} \right) S_y^2 + \frac{W_2(k_2 - 1)}{n_2} S_{y_2}^2, \tag{1.2}$$

Following the same strategy, Singh and Kumar (2008) suggested ratio, product and regression estimators in two phase sampling with non-response at second phase sample. The ratio estimator with bias and MSE is given by

$$t_{r(2)} = \bar{y}_2^* \left(\frac{\bar{x}_1}{\bar{x}_2^*} \right) \left(\frac{\bar{x}_1}{\bar{x}} \right), \tag{1.3}$$

$$Bias(t_{6(2)}) = \frac{1}{\bar{X}} \left[\left(\frac{1}{n_2} - \frac{1}{n_1} \right) S_x^2 (3R - 2\beta) + \frac{W_2(k_2 - 1)}{n_2} S_{x_2}^2 (R - \beta_{(2)}) \right],$$

and

$$MSE(t_{6(2)}) = \left(\frac{1}{n_2} - \frac{1}{n_1} \right) S_y^2 + 4RS_x^2(R - \beta)$$

$$+ \frac{W_2(k_2 - 1)}{n_2} S_{y_2}^2 + RS_{x_2}^2 (R - 2\beta_{(2)}) + \left(\frac{1}{n_1} - \frac{1}{N} \right) S_y^2,$$

The product estimator is

$$t_{p(2)} = \bar{y}_2^* \left(\frac{\bar{x}_2^*}{\bar{x}_1} \right) \left(\frac{\bar{x}}{\bar{x}_1} \right), \tag{1.4}$$

$$Bias(t_{7(2)}) = \frac{1}{\bar{X}} \left[\left(\frac{1}{n_2} - \frac{1}{n_1} \right) S_x^2 (3R + 2\beta) + \frac{W_2(k_2 - 1)}{n_2} S_{x_2}^2 \beta_{(2)} \right],$$

$$MSE(t_{7(2)}) = \left(\frac{1}{n_2} - \frac{1}{n_1}\right) S_y^2 + 4RS_x^2(R+\beta)$$

$$+ \frac{W_2(k_2-1)}{n_2} \ S_{y_2}^2 + RS_{x_2}^2(R+2\beta_{(2)}) \ + \left(\frac{1}{n_1} - \frac{1}{N}\right) S_y^2,$$

The regression estimator with *MSE* is

$$t_{reg(2)} = \bar{y}_2^* + d_1 \ \bar{x} - \bar{x}_2^* + d_2 \ \bar{x}_1 - \bar{x} \ , \tag{1.5}$$

and

$$MSE(t_{8(2)}) = \left(\frac{1}{n_2} - \frac{1}{n_1}\right) S_y^2 + d_2 S_x^2(d_2 - 2\beta)$$

$$+ \frac{W_2(k_2-1)}{n_2} \ S_{y_2}^2 + d_1 S_{x_2}^2(d_1 - 2\beta_{(2)}) \ + \left(\frac{1}{n_1} - \frac{1}{N}\right) S_y^2,$$

Singh et al. (2010) proposed exponential estimator when non-response is present at second phase and further consider non-response at both study variable y and auxiliary variable x and when only at auxiliary variable. Zakia (2011) proposed a generalized class for mean estimation using multi-auxiliary variables in two phase sampling considering non-response at both phases.

$$t_{rcre} = \left[\ \bar{y}_2^* + a\sum_{i=1}^{p} \alpha_i(\bar{x}_{(1)i}^* - \bar{x}_{(2)i}^*) \right] \left[b\prod_{i=1}^{q} \ \left(\ \bar{x}_{(1)i}^* \quad \bar{x}_{(2)i}^* \ \right)^{-1} \right)^{c\beta_i}$$

$$+ d \exp \left\{ \sum_{i=1}^{s} e\gamma_i \left(\ \bar{x}_{(1)i}^* \ ^{-1} \ \bar{x}_{(2)i}^* \ \right) \right\} \right], \tag{1.6}$$

with

$$MSE(t_{rcre}) = \lambda_2 S_y^2 + \theta_2 S_{y(2)}^2 - q_{1\times m}^t T_{m\times m}^{-1} q_{m\times 1}$$

Naik and Gupta (1996),Jhajj et al. (2006) and Shabbir and Gupta (2007) used information of single auxiliary attribute in two phase sampling.Hanif et al. (2009) extended the family of estimators proposed by Jhajj et al. (2006) using information on two auxiliary attributes in double sampling. Haq et al. (2011) suggested an estimator for full information case (an improved form of the estimator proposed by Shabbir and Gupta (2007)).

2. SUGGESTED CLASS OF ESTIMATORS

Consider the total population (denoted by U) of N units is divided into two sections: one is the section (denoted by U_1) of N_1 units, which would be available on the first attempt at the first stage and the other section (denoted by U_2) of N_2 units, which are not available on the first attempt at the first phase but will be available on the second attempt. From N units, a first phase sample (denoted by u_1) of n_1 units is drawn by simple random sampling without replacement (SRSWOR).A second phase sample (denoted by

u_2) of n_2 units (i.e. $n_2 < n_1$) is drawn from n_1 by simple random sampling without replacement (SRSWOR) and the variable of interest y is measured at second phases. At second phase let m_1 units supply information which is denoted by v_1 and m_2 units refuse to response which is denoted by v_2, where $v_1 = u_2 \cap U_1$ and $v_2 = u_2 \cap U_2$. A sub-sample (denoted by v_{2m}) of r_2 units is randomly taken from the m_2 non-respondents by applying the strategy defined of Hansen and Hurwitz (1946) and this sub sample is specified by

$$r_2 = \frac{m_2}{k_2},$$

$k_2 > 1$. Assume that no non response is observed in this sub sample. Let x_i, z_j, w_k denotes the set of multi-auxiliary quantitative variables for $i = 1,2,3,...,q_1$, $j = 1,2,3,...,q_3$ and

$k = 1,2,3,...,q_5$ respectively and population mean $\bar{X}_i = N^{-1} \sum_{t=1}^{N} x_t$, $\bar{X}_{(1)i} = N_1^{-1} \sum_{t=1}^{N_1} x_t$,

$\bar{X}_{(2)i} = N_2^{-1} \sum_{t=1}^{N_2} x_t$, $\bar{Z}_j = N^{-1} \sum_{t=1}^{N} z_t$, $\bar{Z}_{(1)j} = N_1^{-1} \sum_{t=1}^{N_1} z_t$, $\bar{Z}_{(2)j} = N_2^{-1} \sum_{t=1}^{N_2} z_t$, $\bar{W}_k = N^{-1} \sum_{t=1}^{N} w_t$,

$\bar{W}_{(1)k} = N_1^{-1} \sum_{t=1}^{N_1} w_t$ and $\bar{W}_{(2)k} = N_2^{-1} \sum_{t=1}^{N_2} w_t$ denote the population means of the responding

and non-responding units. Let $\bar{x}_{(1)i} = n_1^{-1} \sum_{t=1}^{n_1} x_t$, $\bar{z}_{(1)j} = n_1^{-1} \sum_{t=1}^{n_1} z_t$ and $\bar{w}_{(1)k} = n_1^{-1} \sum_{t=1}^{n_1} w_t$

denotes the sample mean of all n_1 units, and $\bar{x}'_{(1)i} = m_1^{-1} \sum_{t=1}^{m_1} x_t$, $\bar{x}'_{(2)i} = m_2^{-1} \sum_{t=1}^{m_2} x_t$,

$\bar{z}'_{(1)j} = m_1^{-1} \sum_{t=1}^{m_1} z_t$, $\bar{z}'_{(2)j} = m_2^{-1} \sum_{t=1}^{m_2} z_t$, $\bar{w}'_{(1)k} = m_1^{-1} \sum_{t=1}^{m_1} w_k$ and $\bar{w}'_{(2)k} = m_2^{-1} \sum_{t=1}^{m_2} w_t$ denote the sample

means of the m_1 responding and m_2 non-responding units. Further, let $\bar{x}_{(r_2)i} = \frac{1}{r_2} \sum_{t=1}^{r_2} x_t$,

$\bar{z}_{(r_2)j} = \frac{1}{r_2} \sum_{t=1}^{r_2} z_t$ and $\bar{w}_{(r_2)k} = \frac{1}{r_2} \sum_{t=1}^{r_2} w_t$ denotes the sample mean of the $r_2 = \frac{m_2}{k_2}$, $k_2 > 1$

sub-sampled units. Let $t_3 = e \left\{ 1 + \frac{f}{2} \alpha_5'{}_{1 \times q_5} Z_{q_5 \times q_5} d_{w_{q_5} \times 1} + \frac{l}{2} \alpha_6'{}_{1 \times q_6} E_{q_6 \times q_6} d_{\varepsilon_{q_6} \times 1} \right\}$

denotes another set of multi-auxiliary qualitative variables for $i' = 1,2,3,...,q_2$, $j' = 1,2,3,...,q_4$, $k' = 1,2,3,...,q_6$ with population proportions

$\Phi_{i'} = N^{-1} \sum_{t=1}^{N} \tau_t$, $\Phi_{(1)i'} = N_1^{-1} \sum_{t=1}^{N_1} \tau_t$, $\Phi_{(2)i'} = N_2^{-1} \sum_{t=1}^{N_2} \tau_t$, $\Psi_{j'} = N^{-1} \sum_{t=1}^{N} \omega_t$, $\Psi_{(1)j'} = N_1^{-1} \sum_{t=1}^{N_1} \omega_t$,

$\Psi_{(2)j'} = N_2^{-1} \sum_{t=1}^{N_2} \omega_t$, $E_{k'} = N^{-1} \sum_{t=1}^{N} \varepsilon_t$, $E_{(1)k'} = N_1^{-1} \sum_{t=1}^{N_1} \varepsilon_t$ and $E_{(2)k'} = N_2^{-1} \sum_{t=1}^{N_2} \varepsilon_t$ denote the

population proportions of the responding and non-responding groups. Let $\tau_{(1)i'} = n_1^{-1} \sum_{t=1}^{n_1} \tau_t$

$, \omega_{(1)j'} = n_1^{-1} \sum_{t=1}^{n_1} \omega_t$ and $\varepsilon_{(1)k'} = n_1^{-1} \sum_{t=1}^{n_1} \varepsilon_t$ denotes the sample proportions of all n_1 units, and

$\tau'_{(1)i'} = m_1^{-1} \sum_{t=1}^{m_1} \tau_t$, $\tau'_{(2)i} = m_2^{-1} \sum_{t=1}^{m_2} \tau_t$, $\omega'_{(1)j'} = m_1^{-1} \sum_{t=1}^{m_1} \omega_t$, $\omega'_{(2)j'} = m_2^{-1} \sum_{t=1}^{m_2} \omega_t$, $\varepsilon'_{(1)k'} = m_1^{-1} \sum_{t=1}^{m_1} \varepsilon_t$

and $\varepsilon'_{(2)k'} = m_2^{-1} \sum_{t=1}^{m_2} \varepsilon_t$ denote the sample proportions of the m_1 responding and m_2 non-

responding units. Further, let $\tau_{(r_2)i'} = \frac{1}{r_2} \sum_{t=1}^{r_2} \tau_t$, $\omega_{(r_2)j'} = \frac{1}{r_2} \sum_{t=1}^{r_2} \omega_t$ and $\varepsilon_{(r_2)k'} = \frac{1}{r_2} \sum_{t=1}^{r_2} \varepsilon_t$

denotes the sample proportions of the $r_2 = \frac{m_2}{k_2}$, $k_2 > 1$ sub-sampled units.

Let us define sampling errors for quantitative and qualitative auxiliary variables as

$\bar{e}^*_{y(2)} = \bar{y}_{(2)} - \bar{Y}$, $\bar{e}^*_{x(1)i} = \bar{x}_{(1)i} - \bar{X}_i$, $\bar{e}^*_{x(2)i} = \bar{x}_{(2)i} - \bar{X}_i$, $\bar{e}^*_{\tau(1)i'} = \tau^*_{(1)i'} - \Phi_{i'}$, $\bar{e}^*_{\tau(2)i'} = \tau^*_{(2)i'} - \Phi_{i'}$

$\bar{e}^*_{z(1)j} = \bar{z}_{(1)j} - \bar{Z}_j$, $\bar{e}^*_{z(2)j} = \bar{z}_{(2)j} - \bar{Z}_j$, $\bar{e}^*_{\omega(1)j'} = \bar{\omega}^*_{(1)j'} - \Psi_{j'}$, $\bar{e}^*_{\omega(2)j'} = \bar{\omega}^*_{(2)j'} - \Psi_{j'}$,

$\bar{e}^*_{w(1)k} = \bar{w}^*_{(1)k} - \bar{W}_k$ $\bar{e}^*_{w(2)k} = \bar{w}^*_{(2)k} - \bar{W}_k$, $\bar{e}^*_{\varepsilon(1)k'} = \bar{\varepsilon}^*_{(1)k'} - E_{k'}$, $\bar{e}^*_{\varepsilon(2)k'} = \bar{\varepsilon}^*_{(2)k'} - E_{k'}$

Some useful expectations are

$$E(\bar{e}^*_{y\,2}) = E(\bar{e}^*_{x\,2\,i}) = E(\bar{e}^*_{x\,1\,i}) = E(\bar{e}^*_{\tau\,2\,i'}) = E(\bar{e}^*_{\tau\,1\,i'}) = E(\bar{e}^*_{z\,2\,j}) = E(\bar{e}^*_{z\,1\,j})$$

$$= E(\bar{e}^*_{\omega\,2\,j'}) = E(\bar{e}^*_{\omega\,1\,j'}) = E(\bar{e}^*_{w\,2\,k}) = E(\bar{e}^*_{w\,1\,k}) = E(\bar{e}^*_{\varepsilon\,2\,k'}) = E(\bar{e}^*_{\varepsilon\,1\,k'}) = 0$$

Let $d_{x_{q_1 \times 1}} = \bar{e}^*_{x(2)i} - \bar{e}^*_{x(1)i}$, $d_{z_{q_3 \times 1}} = \bar{e}^*_{z(2)j} - \bar{e}^*_{z(1)j}$ and $d_{w_{q_5 \times 1}} = \bar{e}^*_{w(2)k} - \bar{e}^*_{w(1)k}$ are differences of sampling errors for quantitative variables.

$$d_{\tau_{q_2 \times 1}} = \bar{e}^*_{\tau(2)i'} - \bar{e}^*_{\tau(1)i'}, \quad d_{\omega_{q_4 \times 1}} = \bar{e}^*_{\omega(2)j'} - \bar{e}^*_{\omega(1)j'} \text{ and } d_{\varepsilon_{q_6 \times 1}} = \bar{e}^*_{\varepsilon(2)k'} - \bar{e}^*_{\varepsilon(1)k'}$$

are differences for qualitative variables. Also

$X_{q_{j} \times q_j} = \left[\bar{X}_i^{-1} \right]_{q_{j} \times q_j}$; $i = 1, 2, \ldots q_1$, $Z_{q_{j} \times q_j} = \left[\bar{Z}_i^{-1} \right]_{q_{j} \times q_j}$; $j = 1, 2, 3 \ldots q_3$ and $W_{q_{j} \times q_j} = \left[\bar{W}_i^{-1} \right]_{q_{j} \times q_j}$

; $k = 1, 2, 3 \ldots q_5$ are the diagonal matrices for quantitative variables and

$\Phi_{q_{j} \times q_j} = \left[\Phi_i^{-1} \right]_{q_{j} \times q_j}$; $i' = 1, 2, 3 \ldots q_2$, $\Psi_{q_{j} \times q_j} = \left[\Psi_{i'}^{-1} \right]_{q_{j} \times q_j}$; $j' = 1, 2, 3 \ldots q_4$ and $E_{q_{j} \times q_j} = \left[E_i^{-1} \right]_{q_{j} \times q_j}$;

$k' = 1, 2, 3 \ldots q_6$ are the diagonal matrices for qualitative variables.

The generalized class of regression-cum-ratio-exponential estimators for estimating population mean in the presence of non-response at both phases using mixture of multi-auxiliary variables can be suggested as

$$t_{mix} = t_1(t_2 + t_3),$$

(2.1)

where

$$t_1 = \left[\eta \bar{y}_2^* + a\sum_{i=1}^{q_1} \alpha_{1i} \; \bar{x}_{(1)i}^* - \bar{x}_{(2)i}^* + b\sum_{i'=1}^{q_2} \alpha_{2i'} \; \tau_{(1)i'}^* - \tau_{(2)i'}^* \right], \tag{2.2}$$

$$t_2 = c\prod_{j=1}^{q_3} \left(\frac{\bar{z}_{(1)j}^*}{\bar{z}_{(2)j}^*} \right)^{d\alpha_{3j}} \prod_{i=1}^{q_4} \left(\frac{\omega_{(1)j'}^*}{\omega_{(2)j'}^*} \right)^{h\alpha_j}, \tag{2.3}$$

and

$$t_3 = e\,\exp\left\{ \sum_{k=1}^{q_5} f\alpha_{5k}\left(\frac{\bar{w}_{(2)k}^* - \bar{w}_{(1)k}^*}{\bar{w}_{(2)k}^* + \bar{w}_{(1)k}^*} \right) + \sum_{k'=1}^{q_6} l\alpha_{6k'}\left(\frac{\varepsilon_{(2)k'}^* - \varepsilon_{(1)k'}^*}{\varepsilon_{(2)k'}^* + \varepsilon_{(1)k'}^*} \right) \right\}. \tag{2.4}$$

Now expressing t_1 in terms of sampling errors, we have

$$t_1 = \left[\eta \; \bar{e}_{y_{(2)}}^* + \bar{Y} + a\sum_{i=1}^{q_1} \alpha_{1i} \; \bar{e}_{x(1)i}^* - \bar{e}_{x(2)i}^* + b\sum_{i'=1}^{q_2} \alpha_{2i'} \; \bar{e}_{\tau(1)i'}^* - \bar{e}_{\tau(2)i'}^* \right].$$

Expressing t_2 in terms of sampling errors and using binomial expansion, ignoring second and higher order terms in (2.3) and (2.4) respectively, we get

$$t_2 = c\left[1 + d\sum_{j=1}^{q_3} \frac{\alpha_{3j}}{\bar{\bar{Z}}_j} \; \bar{e}_{z(1)j}^* - \bar{e}_{z(2)j}^* + h\sum_{j'=1}^{q_4} \frac{\alpha_{4j'}}{\Psi_{j'}} \; \bar{e}_{\omega(1)j'}^* - \bar{e}_{\omega(2)j'}^* \right],$$

and

$$t_3 = e\,\exp\left[\sum_{k=1}^{q_5} \frac{f\alpha_{5k}}{2\bar{W}_k} \; \bar{e}_{w(2)k}^* - \bar{e}_{w(1)k}^* - \bar{e}_{w(2)k}^* - \bar{e}_{w(1)k}^* \left(\frac{\bar{e}_{w(2)k}^* + \bar{e}_{w(1)k}^*}{2\bar{W}_k} \right) \right.$$

$$\left. + \sum_{k'=1}^{q_6} \frac{l\alpha_{6k'}}{2\mathrm{E}_{k'}} \; \bar{e}_{\varepsilon(2)k'}^* - \bar{e}_{\varepsilon(1)k'}^* - \bar{e}_{p(2)k'}^* - \bar{e}_{p(1)k'}^* \left(\frac{\bar{e}_{\varepsilon(2)k'}^* + \bar{e}_{\varepsilon(1)k'}^*}{2\mathrm{E}_{k'}} \right) \right]$$

Simplifying and expressing t_1, t_2 and t_3 in matrix notation, we get

$$t_1 = \left[m\,\bar{e}_{y_{(2)}}^* + \bar{Y} - a\alpha_1^t{}_{1\times q_1}\,d_{q_1\times 1} - b\alpha_2^t{}_{1\times q_2}\,d_{q_2\times 1} \right], \tag{2.5}$$

$$t_2 = c\left[1 - d\alpha_3^t{}_{1\times q_3}\,Z_{q_3\times q_3}d_{1\times q_3} - h\alpha_4^t{}_{1\times q_4}\,\Psi_{q_4\times q_4}d_{q_4\times 1} \right] \tag{2.6}$$

and

$$t_3 = e\left[1 + \frac{f}{2}\alpha_5^t{}_{1\times q_5}\,W_{q_5\times q_5}d_{q_5\times 1} + \frac{l}{2}\alpha_6^t{}_{1\times q_6}\,\mathrm{E}_{q_6\times q_6}d_{q_6\times 1} \right] \tag{2.7}$$

Substituting the value of t_1, t_2 and t_3 in (2.1), we get

$$t_{mix} = \left[\eta \; \overset{-*}{e}_{y_{(2)}} + \overline{Y} \; - a\alpha_1^{t}{}_{1\times q_1} \; d_{q_1\times 1} - b\alpha_2^{t}{}_{1\times q_2} \; d_{q_2\times 1} \right.$$

$$\left[1 - cd\alpha_3^{t}{}_{1\times q_3} \; Z_{q_3\times q_3} d_{1\times q_3} - ch\alpha_4^{t}{}_{1\times q_4} \; \Psi_{q_4\times q_4} d_{q_4\times 1} \right.$$

$$\left. + 2^{-1} ef\alpha_5^{t}{}_{1\times q_5} \; W_{q_5\times q_5} d_{q_5\times 1} + 2^{-1} el\alpha_6^{t}{}_{1\times q_6} \; E_{q_6\times q_6} d_{q_6\times 1} \right]$$

After simplification, we get

$$t_{mix} - \overline{Y} = \eta \overset{-*}{e}_{y_{(2)}} + (\eta-1)\overline{Y} - a\alpha_1^{t}{}_{1\times q_1} \; d_{q_1\times 1} - b\alpha_2^{t}{}_{1\times q_2} \; d_{q_2\times 1} - \eta\overline{Y}cd\alpha_3^{t}{}_{1\times q_3} \; Z_{q_3\times q_3} d_{q_3\times 1}$$

$$- \eta\overline{Y}ch\alpha_4^{t}{}_{1\times q_4} \; \Psi_{q_4\times q_4} d_{q_4\times 1} + 2^{-1}\eta\overline{Y}ef\alpha_5^{t}{}_{1\times q_5} \; W_{q_5\times q_5} d_{q_5\times 1} + 2^{-1}\eta\overline{Y}el\alpha_6^{t}{}_{1\times q_6} \; E_{q_6\times q_6} d_{q_6\times 1}$$

$$(2.8)$$

or

$$t_{mix} - \overline{Y} = \; m\overset{-*}{e}_{y_{(2)}} + (m-1)\overline{Y} - h^{t}{}_{1\times m} H_{m\times 1}$$

where

$$h^{t} = \left[\alpha_1^{t}{}_{1\times q_1} \quad \alpha_2^{t}{}_{1\times q_2} \quad \alpha_3^{t}{}_{1\times q_3} \quad \alpha_4^{t}{}_{1\times q_4} \quad \alpha_5^{t}{}_{1\times q_5} \quad \alpha_6^{t}{}_{1\times q_6} \right]_{1\times m}$$

and

$$H^{t} = \left[ad_{q_1\times 1} \quad bd_{q_2\times 1} \quad m\overline{Y}cdZ_{q_3\times q_3} d_{q_3\times 1} \quad m\overline{Y}ch\Psi_{q_4\times q_4} d_{q_4\times 1} \right.$$

$$\left. -2^{-1} m\overline{Y}efW_{q_5\times q_5} d_{q_5\times 1} - 2^{-1} m\overline{Y}el E_{q_6\times q_6} d_{q_6\times 1} \right]_{1\times m}$$

$$; \sum_{i=1}^{6} q_i = m$$

For obtaining the expression of *MSE*

$$E(t_{mix} - \overline{Y})^2 = E(\eta\overset{-*}{e}_{y_{(2)}} + (\eta-1)\overline{Y} - h^{t}{}_{1\times m} H_{m\times 1})^2 \; ,$$

$$(2.9)$$

To find the optimum value of unknown vector $h_{m\times 1}$ for which *MSE of* t_{mix} will be minimum, differentiating (2.9) w.r.t $h_{m\times 1}$ and equating it equal to zero.

$$E(m\overset{-*}{e}_{y_{(2)}} + (m-1)\overline{Y} - h^{t}{}_{1\times m} H_{m\times 1})H_{m\times 1} = 0, \qquad (2.10)$$

or

$$\eta E(H_{m\times 1}\overset{-*}{e}_{y_{(2)}}) + (\eta-1)\overline{Y}E(H_{m\times 1}) - h_{m\times 1}E(H_{m\times 1}H^{t}{}_{1\times m}) = 0,$$

or

$$h_{m\times 1} = \eta\Lambda^{-1}{}_{m\times m}\Omega_{m\times 1}, \qquad (2.11)$$

where $E(H_{m\times 1}\overset{-*}{e}_{y_{(2)}}) = \Omega_{m\times 1}$ and $E(H_{m\times 1}H^{t}{}_{1\times m}) = \Lambda_{m\times m} = \left[\Delta_{ij}\right]_{m\times m}$

where

$$\Delta_{11} = aE(d_{q_1 \times 1}d^t_{1 \times q_1})a^t = a^2 \Delta_{q_1 \times q_1},$$

$$\Delta_{12} = aE(d_{q_1 \times 1}d^t_{1 \times q_2})b = ab\Delta_{q_1 \times q_2},$$

$$\Delta_{13} = acd\bar{Y}E(d_{q_1 \times 1}d^t_{1 \times q_3})Z_{q_3 \times q_3} = acd\bar{Y}\Delta_{q_1 \times q_3}Z_{q_3 \times q_3},$$

$$\Delta_{14} = \bar{Y}achE(d_{q_1 \times 1}d^t_{1 \times q_4})\Psi_{q_4 \times q_4} = \bar{Y}ach\Delta_{q_1 \times q_4}\Psi_{q_4 \times q_4},$$

$$\Delta_{15} = -aef2^{-1}\bar{Y}E(d_{q_1 \times 1}d^t_{1 \times q_5})W_{q_5 \times q_5} = -aef2^{-1}\bar{Y}\Delta_{q_1 \times q_5}W_{q_5 \times q_5},$$

$$\Delta_{16} = -ael2^{-1}\bar{Y}E(d_{q_1 \times 1}d^t_{1 \times q_3})Z_{q_3 \times q_3} = -ael2^{-1}\bar{Y}_{q_1 \times q_3}Z_{q_3 \times q_3},$$

$$\Delta_{22} = bE(d_{q_2 \times 1}d^t_{1 \times q_2})b^t = b^2 \Delta_{q_2 \times q_2},$$

$$\Delta_{23} = bcd\bar{Y}E(d_{q_2 \times 1}d^t_{1 \times q_3})Z_{q_3 \times q_3} = bcd\bar{Y}\Delta_{q_2 \times q_3}Z_{q_3 \times q_3},$$

$$\Delta_{24} = \bar{Y}bchE(d_{q_2 \times 1}d^t_{1 \times q_4})\Psi_{q_4 \times q_4} = \bar{Y}bch\Delta_{q_2 \times q_4}\Psi_{q_4 \times q_4},$$

$$\Delta_{25} = -bef2^{-1}\bar{Y}E(d_{q_2 \times 1}d^t_{1 \times q_5})W_{q_5 \times q_5} = -bef2^{-1}\bar{Y}\Delta_{q_2 \times q_5}W_{q_5 \times q_5},$$

$$\Delta_{26} = -bel2^{-1}\bar{Y}E(d_{q_2 \times 1}d^t_{1 \times q_6})E_{q_6 \times q_6} = -bel2^{-1}\bar{Y}\Delta_{q_2 \times q_6}E_{q_6 \times q_6},$$

$$\Delta_{33} = (cd\bar{Y})^2 E(d_{q_3 \times 1}d^t_{1 \times q_3})Z^2_{q_3 \times q_3} = (cd\bar{Y})^2 \Delta_{q_3 \times q_3}Z^2_{q_3 \times q_3},$$

$$\Delta_{34} = (\bar{Y}c)^2 dhZ_{q_3 \times q_3}E(d_{q_3 \times 1}d^t_{1 \times q_4})\Psi_{q_4 \times q_4} = (\bar{Y}c)^2 dhZ_{q_3 \times q_3}\Delta_{q_3 \times q_4}\Psi_{q_4 \times q_4},$$

$$\Delta_{35} = -cdef2^{-1}\bar{Y}^2 Z_{q_3 \times q_3}E(d_{q_3 \times 1}d^t_{1 \times q_5})W_{q_5 \times q_5} = -cdef2^{-1}\bar{Y}^2 Z_{q_3 \times q_3}\Delta_{q_3 \times q_5}W_{q_5 \times q_5},$$

$$\Delta_{36} = -cdel2^{-1}\bar{Y}^2 Z_{q_3 \times q_3}E(d_{q_3 \times 1}d^t_{1 \times q_6})E_{q_6 \times q_6} = -cdel2^{-1}\bar{Y}^2 Z_{q_3 \times q_3}\Delta_{q_3 \times q_6}E_{q_6 \times q_6},$$

$$\Delta_{44} = (ch\bar{Y})^2 E(d_{q_4 \times 1}d^t_{1 \times q_4})\Psi^2_{q_4 \times q_4} = (ch\bar{Y})^2 \Delta_{q_4 \times q_4}\Psi^2_{q_4 \times q_4},$$

$$\Delta_{45} = -chef2^{-1}\bar{Y}^2 \Psi_{q_4 \times q_4}E(d_{q_4 \times 1}d^t_{1 \times q_5})W_{q_5 \times q_5} = -chef2^{-1}\bar{Y}^2 \Psi_{q_4 \times q_4}\Delta_{q_4 \times q_5}W_{q_5 \times q_5},$$

$$\Delta_{46} = -ehcl2^{-1}\bar{Y}^2 \Psi_{q_4 \times q_4}E(d_{q_4 \times 1}d^t_{1 \times q_6})E_{q_6 \times q_6} = -ehcl2^{-1}\bar{Y}^2 \Psi_{q_4 \times q_4}\Delta_{q_4 \times q_6}E_{q_6 \times q_6},$$

$$\Delta_{55} = \left(ef2^{-1}\right)^2 W_{q_5 \times q_5}\bar{Y}^2 E(d_{q_5 \times 1}d^t_{1 \times q_5}) = \left(ef2^{-1}\right)^2 W_{q_5 \times q_5}\bar{Y}^2 \Delta_{q_5 \times q_5},$$

$$\Delta_{56} = e^2 fl4^{-1}\bar{Y}^2 W_{q_5 \times q_5}E(d_{q_5 \times 1}d^t_{1 \times q_6})E_{q_6 \times q_6} = e^2 fl4^{-1}\bar{Y}^2 W_{q_5 \times q_5}\Delta_{q_5 \times q_6}E_{q_6 \times q_6},$$

$$\Delta_{66} = \left(efl2^{-1}\bar{Y}E_{q_6 \times q_6}\right)^2 E(d_{q_6 \times 1}d^t_{1 \times q_6}) = \left(efl2^{-1}\bar{Y}E_{q_6 \times q_6}\right)^2 \Delta_{q_6 \times q_6}.$$

From (2.9) we have,

$$MSE(t_{mix}) = E(\eta \bar{e}^*_{y(2)} + (\eta - 1)\bar{Y} - h^t_{1 \times m}H_{m \times 1})(\eta \bar{e}^*_{y(2)} + (\eta - 1)\bar{Y} - h^t_{1 \times m}H_{m \times 1}),$$

or

$$MSE(t_m) = E(m\bar{e}^*_{y(2)} + (m-1)\bar{Y} - h^t_{1 \times m}H_{m \times 1})(m\bar{e}^*_{y(2)} + (m-1)\bar{Y})$$

$$- h^t_{1 \times m}E(m\bar{e}^*_{y(2)} + (m-1)\bar{Y} - h^t_{1 \times m}H_{m \times 1})H_{m \times 1}, \tag{2.12}$$

Using the normal equation (2.7) in (2.12) , we have

$$MSE(t_{mix}) = \left[\eta \bar{e}^*_{y(2)} + (\eta-1)\bar{Y}\right]\left[\eta \bar{e}^*_{y(2)} + (\eta-1)\bar{Y} - h^t_{1\times m} H_{m\times 1}\right],$$

or

$$MSE(t_{mix}) = \eta^2 E\left[\bar{e}^*_{y(2)}\right]^2 + \left[(\eta-1)\bar{Y}\right]^2 - \eta h^t_{1\times m} E\left[H_{m\times 1}\bar{e}^*_{y(2)}\right],$$

or

$$MSE(t_{mix}) = \eta^2\left[\lambda_2 S_y^2 + \theta S_{y_2}^2\right] + \left[(\eta-1)\bar{Y}\right]^2 - \eta h^t_{1\times m}\Omega_{m\times 1},$$

or

$$MSE(t_{mix}) = \left[\eta^2 - 2\eta + 1\right]\bar{Y}^2 + \eta^2\Gamma, \qquad\qquad (2.13)$$

where

$$\Gamma = \left[\lambda_2 S_y^2 + \theta S_{y_2}^2\right] - \Omega_{1\times m}\Lambda^{-1}_{m\times m}\Omega_{m\times 1},$$

To find the optimum value of η for which *MSE* will be minimum, differentiating *MSE* with respect to *η* and equating to zero, we have

$$\eta_{opt} = \frac{\bar{Y}^2}{\bar{Y}^2 + \Gamma} = \left[1 + \Gamma\bar{Y}^{-2}\right]^{-1},$$

Finally minimum value of *MSE* is

$$MSE(t_{mix}) = \left[\left\{\left[1 + \bar{Y}^{-2}\Gamma\right]^{-1} - 1\right\}\bar{Y}\right] + \left[1 + \bar{Y}^{-2}\Gamma\right]^{-2}\Gamma, \qquad\qquad (2.14)$$

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3. SOME SPECIAL CASES

Special case of proposed class using quantitative variables

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
1	1	1	0	1	1	0	1	1	0	Generalized class using quantitative variables	$t_{quan} = \left[\bar{y}_2 + \sum\limits_{i=1}^{q_1} \alpha_{1i}\ \bar{x}^*_{(1)i} - \bar{x}_{(2)i}\right]\left[\prod\limits_{j=1}^{q_3}\left(\frac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}}\right)^{\alpha_{3j}} + \exp\left\{\sum\limits_{k=1}^{q_5}\alpha_{5k}\left(\frac{\bar{w}_{(2)k} - \bar{w}^*_{(1)k}}{\bar{w}_{(2)k} + \bar{w}^*_{(1)k}}\right)\right\}\right]$
2	1	1	0	1	0	0	0	0	0	Regression estimator	$t_{(reg)} = \left[\bar{y}_2 + \sum\limits_{i=1}^{q_1}\alpha_{1i}\ \bar{x}^*_{(1)i} - \bar{x}_{(2)i}\right]$
3	1	0	0	1	1	0	0	0	0	ratio estimator	$t_{(r)} = \bar{y}_2 \prod\limits_{j=1}^{q_3}\left(\frac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}}\right)^{\alpha_{3j}}$
4	1	0	0	0	1	0	1	1	0	exponential estimator	$t_{(\exp)} = \exp\left\{\sum\limits_{k=1}^{q_5}\alpha_{5k}\left(\frac{\bar{w}_{(2)k} - \bar{w}^*_{(1)k}}{\bar{w}_{(2)k} + \bar{w}^*_{(1)k}}\right)\right\}$
5	1	1	0	1	1	0	0	0	0	Regression-cum ratio estimator	$t_{(rcr)} = \left[\bar{y}_2 + \sum\limits_{i=1}^{q_1}\alpha_{1i}\ \bar{x}^*_{(1)i} - \bar{x}_{(2)i}\right]\left[\prod\limits_{j=1}^{q_3}\left(\frac{\bar{z}^*_{(1)j}}{\bar{z}_j}\right)^{\alpha_{3j}}\right]$
6	1	1	0	1	-1	0	0	0	0	Regression-cum product estimator	$t_{(rcp)} = \left[\bar{y}_2 + \sum\limits_{i=1}^{q_1}\alpha_{1i}\ \bar{x}^*_{(1)i} - \bar{x}_{(2)i}\right]\left[\prod\limits_{j=1}^{q_3}\left(\frac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}}\right)^{-\alpha_{3j}}\right]$
7	1	0	0	1	1	1	1	1	1	Ratio-cum expo Ratio estimator	$t_{(rcer)} = \left[\prod\limits_{j=1}^{q_3}\left(\frac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}}\right)^{d\alpha_{3j}} + \exp\left\{-\sum\limits_{k=1}^{q_5}\alpha_{5k}\left(\frac{\bar{w}_{(2)k} - \bar{w}^*_{(1)k}}{\bar{w}_{(2)k} + \bar{w}^*_{(1)k}}\right)\right\}\right]$
8	1	0	0	1	1	1	1	-1	-1	Ratio-cum expo product estimator	$t_{(rcep)} = \left[\prod\limits_{j=1}^{q_3}\left(\frac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}}\right)^{d\alpha_{3j}} + \exp\left\{-\sum\limits_{k=1}^{q_5}\alpha_{5k}\left(\frac{\bar{w}_{(2)k} - \bar{w}^*_{(1)k}}{\bar{w}_{(2)k} + \bar{w}^*_{(1)k}}\right)\right\}\right]$

(a) Special Case of Proposed Class using Qualitative Variables

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
1	1	0	1	1	0	1	1	0	1	Generalized class using qualitative variables	$t_{qual} = \left[\eta \bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \ \tau^*_{1\,i'} - \tau_{(2)i'} \right]\left[\prod_{j'=1}^{q_4}\left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}}\right)^{\alpha_{4j'}} + \exp\left\{ \sum_{k'=1}^{q_6} \alpha_{6k'}\left(\dfrac{\varepsilon_{(2)k'} - \varepsilon^*_{(1)k'}}{\varepsilon_{(2)k'} + \varepsilon^*_{(1)k'}}\right)\right\}\right]$
2	1	0	1	1	0	0	0	0	0	Regression Estimator	$t_{(reg)} = \left[\bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \ \tau^*_{1\,i'} - \tau_{(2)i'}\right]$
3	1	0	0	1	0	1	0	0	0	ratio estimator	$t_{(r)} = \bar{y}_2 \prod_{j'=1}^{q_4}\left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}}\right)^{\alpha_{4j'}}$
4	1	0	0	0	1	0	1	0	1	exponential estimator	$t_{(\exp)} = \exp\left\{ \sum_{k'=1}^{q_6} \alpha_{6k'}\left(\dfrac{\varepsilon_{(2)k'} - \varepsilon^*_{(1)k'}}{\varepsilon_{(2)k'} + \varepsilon^*_{(1)k'}}\right)\right\}$
5	1	0	1	1	0	1	0	0	0	Regression-cum ratio estimator	$t_{(rcr)} = \left[\bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \ \tau^*_{1\,i'} - \tau_{(2)i'}\right]\left[\prod_{j'=1}^{q_4}\left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}}\right)^{\alpha_{4j'}}\right]$
6	1	0	1	1	0	-1	0	0	0	Regression-cum product estimator	$t_{mix(rcp)} = \left[\bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \ \tau^*_{1\,i'} - \tau_{(2)i'}\right]\left[\prod_{j'=1}^{q_4}\left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}}\right)^{-\alpha_{4j}}\right]$
7	1	0	0	1	1	1	1	1	1	Ratio-cum expo Ratio estimator	$t_{(rcer)} = \left[\prod_{j'=1}^{q_4}\left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}}\right)^{\alpha_{4j'}} + \exp\left\{ \sum_{k'=1}^{q_6} \alpha_{6k'}\left(\dfrac{\varepsilon_{(2)k'} - \varepsilon^*_{(1)k'}}{\varepsilon_{(2)k'} + \varepsilon^*_{(1)k'}}\right)\right\}\right]$
8	1	0	0	1	1	1	1	-1	-1	Ratio-cum expo product estimator	$t_{mix(rcep)} = \left[\prod_{j'=1}^{q_4}\left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}}\right)^{\alpha_{4j'}} + \exp\left\{ -\sum_{k'=1}^{q_6} \alpha_{6k'}\left(\dfrac{\varepsilon_{(2)k'} - \varepsilon^*_{(1)k'}}{\varepsilon_{(2)k'} + \varepsilon^*_{(1)k'}}\right)\right\}\right]$

(b) Special Cases of the Generalized Class using Mixture of Auxiliary Variables:

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
1	1	1	1	1	0	0	0	0	0	Regression estimator for mixture	$t_{mix(reg)} = \left[\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_{1i}\left(\bar{x}_{(1)i}^* - \bar{x}_{(2)i}^* \right) + \sum_{i=1}^{q_2} \alpha_{2i}\left(\tau_{(1)i}^* - \tau_{(2)i}^* \right) \right]$
2	1	0	0	1	1	1	1	0	0	Ratio estimator for mixture	$t_{mix(r)} = \bar{y}_2^* \prod_{i=1}^{q_3}\left(\frac{\bar{z}_{(1)i}^*}{\bar{z}_{(2)i}^*} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4}\left(\frac{\omega_{(1)i}^*}{\omega_{(2)i}^*} \right)^{\alpha_{4i}}$
3	1	0	0	0	1	0	1	1	1	Exponential estimator for mixture	$t_{mix(\exp)} = \exp\left\{ \sum_{i=1}^{q_5} \alpha_{5i}\left(\frac{\bar{w}_{(2)i}^* - \bar{w}_{(1)i}^*}{\bar{w}_{(2)i}^* + \bar{w}_{(1)i}^*} \right) + \sum_{i=1}^{q_6} \alpha_{6i}\left(\frac{\varepsilon_{(2)i}^* - \varepsilon_{(1)i}^*}{\varepsilon_{(2)i}^* + \varepsilon_{(1)i}^*} \right) \right\}$
4	1	1	1	1	1	1	0	0	0	Regression-cum-ratio estimator for mixture	$t_{mix(rcr)} = \left[\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_{1i}\left(\bar{x}_{(1)i}^* - \bar{x}_{(2)i}^* \right) + \sum_{i=1}^{q_2} \alpha_{2i}\left(\tau_{(1)i}^* - \tau_{(2)i}^* \right) \right]\left[\prod_{i=1}^{q_3}\left(\frac{\bar{z}_{(1)i}^*}{\bar{z}_{(2)i}^*} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4}\left(\frac{\omega_{(1)i}^*}{\omega_{(2)i}^*} \right)^{\alpha_{4i}} \right]$
5	1	1	1	1	-1	-1	0	0	0	Regression-cum-product estimator for mixture	$t_{rcp} = \left[\bar{y}_2^* + \sum_{i=1}^{q_1} \alpha_{1i}\left(\bar{x}_{(1)i}^* - \bar{x}_{(2)i}^* \right) + \sum_{i=1}^{q_2} \alpha_{2i}\left(\tau_{(1)i}^* - \tau_{(2)i}^* \right) \right]\left[\prod_{i=1}^{q_3}\left(\frac{\bar{z}_{(2)i}^*}{\bar{z}_{(1)i}^*} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4}\left(\frac{\omega_{(2)i}^*}{\omega_{(1)i}^*} \right)^{\alpha_{4i}} \right]$
6	1	0	0	1	1	1	1	1	1	Ratio-cum-exponential ratio	$t_{rce} = \bar{y}_2^* \left[\prod_{i=1}^{q_3}\left(\frac{\bar{z}_{(1)i}^*}{\bar{z}_{(2)i}^*} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4}\left(\frac{\omega_{(1)i}^*}{\omega_{(2)i}^*} \right)^{\alpha_{4i}} + \exp\left\{ \sum_{i=1}^{q_5} \alpha_{5i}\left(\frac{\bar{w}_{(2)i}^* - \bar{w}_{(1)i}^*}{\bar{w}_{(2)i}^* + \bar{w}_{(1)i}^*} \right) + \sum_{i=1}^{q_6} \alpha_{6i}\left(\frac{\varepsilon_{(2)i}^* - \varepsilon_{(1)i}^*}{\varepsilon_{(2)i}^* + \varepsilon_{(1)i}^*} \right) \right\} \right]$
7	1	0	0	1	-1	-1	1	-1	-1	Ratio-cum-exponential product	$t_{rcep} = \bar{y}_2^* \left[\prod_{i=1}^{q_3}\left(\frac{\bar{z}_{(2)i}^*}{\bar{z}_{(1)i}^*} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4}\left(\frac{\omega_{(2)i}^*}{\omega_{(1)i}^*} \right)^{\alpha_{4i}} + \exp\left\{ \sum_{i=1}^{q_5} \alpha_{5i}\left(\frac{\bar{w}_{(1)i}^* - \bar{w}_{(2)i}^*}{\bar{w}_{(1)i}^* + \bar{w}_{(2)i}^*} \right) + \sum_{i=1}^{q_6} \alpha_{6i}\left(\frac{\varepsilon_{(1)i}^* - \varepsilon_{(2)i}^*}{\varepsilon_{(1)i}^* + \varepsilon_{(2)i}^*} \right) \right\} \right]$

A GENERAL CLASS OF MEAN ESTIMATORS USING MIXTURE OF AUXILIARY VARIABLES IN TWO-PHASE SAMPLING WHEN THE PRESENCE OF NON-RESPONSE ON SECOND PHASE

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ABSTRACT

In this paper we have suggested a general class of estimators in two-phase sampling to estimate the population mean of study variable in the case when non-response occur on second phase and on study and/or auxiliary variable(s). We are using several continuous and categorical auxiliary variables simultaneously while constructing the class. Also we are assuming that the information on all auxiliary variables is not available for population (no information case) as mostly the practical cases. The expressions of mean square error of suggested class have been derived. Furthermore several special cases of proposed class have been identified.

KEYWORDS

1. INTRODUCTION

In survey sampling, sometimes estimation of population mean of study variable on the basis of some auxiliary qualitative and quantitative variable(s) is required to enhance the efficiency of estimator when there is issue of non-response either on item or on unit.

From a number of issues related to sampling, it noted that sample size referred to the number of responses that need to obtain. However, no matter how well the sampling design planned, a poor response rate to a mail or telephone survey or to interviews can influence a study virtually useless.

Hansen and Hurwitz (1946) very first time developed an estimator to overcome the problem of non-response by sub-sampling the non-respondents. Under the sampling scheme by dividing the whole population into two units such as N into N_1 and N_2 for responding and non-responding units respectively, let n be the sample size of first phase drawn without replacement by simple random sampling (SRSWOR) from the N and n_1 shows the responding units while n_2 shows non-responding units. A sub sample is selected from the first phase by SRSWOR of size m and m_1 supply the information and m_2 that refuse to response. A sub-sample of r_2 units is randomly taken from the m_2 non-respondents this sub sample is specified by $r_2 = m_2 / k_2$, $k_2 > 1$.

Hansen and Hurwitz consider the problem of non-response and the suggested estimator is

$$\bar{y}_2^* = \frac{n_{21}\bar{y}_2 + n_{22}\bar{y}_{2r}}{n_2} \tag{1.1}$$

with

$$Var\ \bar{y}_2^* = \lambda_2 S_y^2 + \theta S_{y_2}^2\ ;$$

$$\lambda_1 = \frac{1}{n_1} - \frac{1}{N}\ ,\ \lambda_2 = \frac{1}{n_2} - \frac{1}{N}\ ,\ \lambda_3 = \frac{1}{n_2} - \frac{1}{n_1}\ \text{and}\ \theta = \frac{W_2\ \ k-1}{n_2}\ ;\ W_2 = \frac{N_2}{N}\ \text{and}\ S_y^2\ \text{and}$$

$S_{y_2}^2$ be the population variances based on N and N_2 units respectively.

The use of auxiliary information can increase the precision of an estimator when study variable y is highly correlated with the auxiliary variable x (Singh et al. (2010)). In two phase sampling when we take a large sample from the population and a sub sample from the large sample then there is an issue of less sample size and large non response, due to this the mean square error became larger. To compensate such an issue we use auxiliary variables that are highly correlated with study variable. Wu and Luan (2003) discussed that the major advantage of using two-phase sampling is the gain in high precision without substantial increase in cost.

The presence or absence of population auxiliary information plays important role in efficiency of estimators for two-phase sampling. Samiuddin and Hanif (2006) emphasize on possible three cases for the use of auxiliary information; no information case (when population information about none of auxiliary variables is available), partial information case (when population information of some auxiliary variables is available and on some is not) and full information case (when population information of all auxiliary variables is available). Ahmad and Hanif (2010) using different techniques such as regression and ratio compare the efficiency through mean square error considering above mentioned situation and proved in full information case we get efficient estimator as it is obvious.

In practical scenario, where the multi-auxiliary variables are highly correlated with study variable and that can be obtained by two phase sampling to increase the importance of an estimators. The above three situation also can be discussed in multi-auxiliary variables, recently under these three cases several classes of univariate and multivariate classes of ratio and regression estimators have been developed by Ahmad, et al. (2009a) Ahmad, et al. (2009b), Ahmad, et al. (2010) and Ahmad and Hanif (2010).

Naik and Gupta (1996), Jhajj et al. (2006), Shabbir and Gupta (2007) developed different estimators on the basis of one or more auxiliary attribute(s) to estimate population mean under two phase sampling. Shahbaz and Hanif (2009), developed shrinkage ratio and regression estimators to estimate population mean. Inam and Hanif (2009) proceeds the above mentioned researchers work in appropriate manner and developed some ratio and product as well as shrinkage estimator using multi-auxiliary attributes in two phase sampling.

Under the same sampling strategy of Hansen and Hurwitz, Singh and Kumar (2008) suggested the following estimators using two phase sampling and considering the non-response at second phase sample the ratio estimator is

$$t_d^r = \bar{y}_2^* \left(\frac{\bar{x}_1}{\bar{x}_2^*} \right) \left(\frac{\bar{x}_1}{\bar{x}_2} \right) \tag{1.2}$$

with

$$MSE(t_d^r) = \lambda_3 \ S_y^2 + 4RS_x^2(R-\beta) \ +\theta \ S_{y_2}^2 + RS_{x_2}^2(R-2\beta_{(2)}) \ +\lambda_1 S_y^2,$$

the product estimator is

$$t_d^p = \bar{y}_2^* \left(\frac{\bar{x}_2^*}{\bar{x}_1} \right) \left(\frac{\bar{x}_2}{\bar{x}_1} \right) \tag{1.3}$$

with

$$MSE(t_d^p) = \lambda_3 \ S_y^2 + 4RS_x^2(R+\beta) \ +\theta \ S_{y_2}^2 + RS_{x_2}^2(R+2\beta_{(2)}) \ +\lambda_1 S_y^2,$$

another product estimator is

$$t_{\alpha_1}^{\alpha_2} = \bar{y}_2^* \left(\frac{\bar{x}_2}{\bar{x}_2^*} \right)^{\alpha_1} \left(\frac{\bar{x}_1}{\bar{x}_2} \right)^{\alpha_2} \tag{1.4}$$

with

$$MSE(t_{\alpha_1}^{\alpha_2}) = \lambda_3 \ S_y^2 + R\alpha_2 S_x^2(R\alpha_2 - 2\beta) \ +\theta \ S_{y_2}^2 + R\alpha_1 S_{x_2}^2(R\alpha_1 - 2\beta_{(2)}) \ +\lambda_1 S_y^2$$

and the regression estimator is

$$t_d = \bar{y}_2^* + d_1 \ \bar{x}_2 - \bar{x}_2^* \ + d_2 \ \bar{x}_1 - \bar{x}_2 \tag{1.5}$$

with

$$MSE(t_d) = \lambda_3 \ S_y^2 + d_2 S_x^2(d_2 - 2\beta) \ +\theta \ S_{y_2}^2 + d_1 S_{x_2}^2(d_1 - 2\beta_{(2)}) \ +\lambda_1 S_y^2.$$

Singh and Kumar (2009) proposed the following general class of estimators with bias and MSE for population mean of the study variable in the presence of non response using auxiliary information under double sampling. The product estimator is

$$T_{DS} = \bar{y}_2^* \left(\frac{a\bar{x}_2^* + b}{a\bar{x}_1 + b} \right)^{\alpha} \left(\frac{a\bar{x}_2 + b}{a\bar{x}_1 + b} \right)^{\beta} \tag{1.6}$$

with

$$MSE \ T_{DS} \ = \bar{Y}^2 \left[\lambda_3 \ C_y^2 + \ \alpha+\beta \ \phi \ \ \alpha+\beta \ \phi+2K_{yx} \ C_x^2 \right.$$
$$\left. +\lambda_2^* \ C_{y \ 2}^2 \ + a\phi \ a\phi+2K_{yx \ 2} \ C_{x \ 2}^2 \ +\lambda_1 SC_y^2 \right].$$

Irsa (2011) formed a generalized class of estimator and estimate population mean when non –response is present at second phase and used multi-auxiliary quantitative variables and the generalized class is

$$t_1^m = \left(\overline{y}_2^* + a\sum_{i=1}^{p_1} \alpha_i \ \overline{x}_{1i} - \overline{x}_{2i}^* \right) \left[b \prod_{i=1}^{p_2} \left(\frac{\overline{x}_{1i}}{\overline{x}_{2i}^*} \right)^{c\beta_i} + d \exp \sum_{i=1}^{p_3} \gamma_i \left(\frac{e \ \overline{x}_{2i}^* - \overline{x}_{1i}}{\overline{x}_{2i}^* + \overline{x}_{1i}} \right) \right] \qquad (1.7)$$

with

$$MSE \ t_1^m = \lambda_2 S_y^2 + \theta S_{y_{(2)}}^2 - \phi'_{yx_{1\times3}} T_{x_{3\times3}}^{-1} \phi_{yx_{3\times1}}.$$

2. SUGGESTED CLASS OF ESTIMATORS

Consider the total population (denoted by U) of N units is divided into two sections: one is the section (denoted by U_1) of N_1 units, which would be available on the first attempt at the first stage and the other section (denoted by U_2) of N_2 units, which are not available on the first attempt at the first phase but will be available on the second attempt. From N units, a first phase sample (denoted by u_1) of n_1 units is drawn by simple random sampling without replacement (SRSWOR).A second phase sample (denoted by u_2) of n_2 units (i.e. $n_2 < n_1$) is drawn from n_1 by simple random sampling without replacement (SRSWOR) and the variable of interest y is measured at second phases. At second phase let m_1 units supply information which is denoted by v_1 and m_2 units refuse to response which is denoted by v_2, where $v_1 = u_2 \cap U_1$ and $v_2 = u_2 \cap U_2$. A sub-sample (denoted by v_{2m}) of r_2 units is randomly taken from the m_2 non-respondents by applying the strategy defined of Hansen and Hurwitz (1946) and this sub sample is specified by $r_2 = \dfrac{m_2}{k_2}$, k_2 >1. Assume that no non response is observed in this sub sample. Let x_i, z_j, w_k denotes the set of multi-auxiliary quantitative variables for $i = 1, 2, 3, ..., q_1$, $j = 1, 2, 3, ..., q_3$ and $k = 1, 2, 3, ..., q_5$ respectively and population mean $\overline{X}_i = N^{-1}\sum_{t=1}^{N} x_t$, $\overline{X}_{(1)i} = N_1^{-1}\sum_{t=1}^{N_1} x_t$,

$\overline{X}_{(2)i} = N_2^{-1}\sum_{t=1}^{N_2} x_t$, $\overline{Z}_j = N^{-1}\sum_{t=1}^{N} z_t$, $\overline{Z}_{(1)j} = N_1^{-1}\sum_{t=1}^{N_1} z_t$, $\overline{Z}_{(2)j} = N_2^{-1}\sum_{t=1}^{N_2} z_t$, $\overline{W}_k = N^{-1}\sum_{t=1}^{N} w_t$,

$\overline{W}_{(1)k} = N_1^{-1}\sum_{t=1}^{N_1} w_t$ and $\overline{W}_{(2)k} = N_2^{-1}\sum_{t=1}^{N_2} w_t$ denote the population means of the responding and non-responding units. Let $\overline{x}_{(1)i} = n_1^{-1}\sum_{t=1}^{n_1} x_t$, $\overline{z}_{(1)j} = n_1^{-1}\sum_{t=1}^{n_1} z_t$ and $\overline{w}_{(1)k} = n_1^{-1}\sum_{t=1}^{n_1} w_t$

denotes the sample mean of all n_1 units, and $\overline{x}'_{(1)i} = m_1^{-1}\sum_{t=1}^{m_1} x_t$, $\overline{x}'_{(2)i} = m_2^{-1}\sum_{t=1}^{m_2} x_t$,

$\overline{z}'_{(1)j} = m_1^{-1}\sum_{t=1}^{m_1} z_t$, $\overline{z}'_{(2)j} = m_2^{-1}\sum_{t=1}^{m_2} z_t$, $\overline{w}'_{(1)k} = m_1^{-1}\sum_{t=1}^{m_1} w_k$ and $\overline{w}'_{(2)k} = m_2^{-1}\sum_{t=1}^{m_2} w_t$ denote the

sample means of the m_1 responding and m_2 non-responding units. Further, let

$\overline{x}_{(r_2)i} = \dfrac{1}{r_2}\sum_{t=1}^{r_2} x_t$, $\overline{z}_{(r_2)j} = \dfrac{1}{r_2}\sum_{t=1}^{r_2} z_t$ and $\overline{w}_{(r_2)k} = \dfrac{1}{r_2}\sum_{t=1}^{r_2} w_t$ denotes the sample mean of the

$r_2 = \dfrac{m_2}{k_2}$, $k_2 > 1$ sub-sampled units. Let $t_3 = e\left\{1 + \dfrac{f}{2}\alpha_5^{t}{}_{1\times q_5}\, Z_{q_5\times q_5}\, d_{w_{q_5\times 1}} + \dfrac{l}{2}\alpha_6^{t}{}_{1\times q_6}\, E_{q_6\times q_6}\, d_{\varepsilon_{q_6\times 1}}\right\}$

denotes another set of multi-auxiliary qualitative variables for

$i' = 1,2,3,\ldots,q_2$, $j' = 1,2,3,\ldots,q_4$, $k' = 1,2,3,\ldots,q_6$ with population proportions $\Phi_{i'} = N^{-1}\sum\limits_{t=1}^{N}\tau_t$,

$\Phi_{(1)i'} = N_1^{-1}\sum\limits_{t=1}^{N_1}\tau_t$, $\Phi_{(2)i'} = N_2^{-1}\sum\limits_{t=1}^{N_2}\tau_t$, $\Psi_{j'} = N^{-1}\sum\limits_{t=1}^{N}\omega_t$, $\Psi_{(1)j'} = N_1^{-1}\sum\limits_{t=1}^{N_1}\omega_t$, $\Psi_{(2)j'} = N_2^{-1}\sum\limits_{t=1}^{N_2}\omega_t$,

$E_{k'} = N^{-1}\sum\limits_{t=1}^{N}\varepsilon_t$, $E_{(1)k'} = N_1^{-1}\sum\limits_{t=1}^{N_1}\varepsilon_t$ and $E_{(2)k'} = N_2^{-1}\sum\limits_{t=1}^{N_2}\varepsilon_t$ denote the population

proportions of the responding and non-responding groups. Let $\tau_{(1)i'} = n_1^{-1}\sum\limits_{t=1}^{n_1}\tau_t$,

$\omega_{(1)j'} = n_1^{-1}\sum\limits_{t=1}^{n_1}\omega_t$ and $\varepsilon_{(1)k'} = n_1^{-1}\sum\limits_{t=1}^{n_1}\varepsilon_t$ denotes the sample proportions of all n_1 units, and

$\tau'_{(1)i'} = m_1^{-1}\sum\limits_{t=1}^{m_1}\tau_t$, $\tau'_{(2)i} = m_2^{-1}\sum\limits_{t=1}^{m_2}\tau_t$, $\omega'_{(1)j'} = m_1^{-1}\sum\limits_{t=1}^{m_1}\omega_t$, $\omega'_{(2)j'} = m_2^{-1}\sum\limits_{t=1}^{m_2}\omega_t$, $\varepsilon'_{(1)k'} = m_1^{-1}\sum\limits_{t=1}^{m_1}\varepsilon_t$

and $\varepsilon'_{(2)k'} = m_2^{-1}\sum\limits_{t=1}^{m_2}\varepsilon_t$ denote the sample proportions of the m_1 responding and m_2 non-

responding units. Further, let $\tau_{(r_2)i'} = \dfrac{1}{r_2}\sum\limits_{t=1}^{r_2}\tau_t$, $\omega_{(r_2)j'} = \dfrac{1}{r_2}\sum\limits_{t=1}^{r_2}\omega_t$ and $\varepsilon_{(r_2)k'} = \dfrac{1}{r_2}\sum\limits_{t=1}^{r_2}\varepsilon_t$

denotes the sample proportions of the $r_2 = \dfrac{m_2}{k_2}$, $k_2 > 1$ sub-sampled units.

Let us define sampling errors for quantitative and qualitative auxiliary variables as

$$\bar{e}_{x\,1\,i} = \bar{x}_{1\,i} - \bar{X}_i,\quad \bar{e}^{*}_{x(2)i} = \bar{x}^{*}_{(2)i} - \bar{X}_i,\quad \bar{e}_{\tau(1)i'} = \tau_{1\,i'} - \Phi_{i'},\quad \bar{e}^{*}_{\tau(2)i'} = \tau^{*}_{(2)i'} - \Phi_{i'},$$

$$\bar{e}_{z(1)j} = \bar{z}_{(1)j} - \bar{Z}_j\ \bar{e}^{*}_{z(2)j} = \bar{z}^{*}_{(2)j} - \bar{Z}_j,\quad \bar{e}^{*}_{y(2)} = \bar{y}^{*}_{(2)} - \bar{Y},\quad \bar{e}_{\omega(1)j'} = \bar{\omega}_{1\,j'} - \Psi_{j'},$$

$$\bar{e}^{*}_{\omega(2)j'} = \bar{\omega}^{*}_{(2)j'} - \Psi_{j'},\quad \bar{e}_{w(1)k} = \bar{w}_{1\,k} - \bar{W}_k\ \bar{e}^{*}_{w(2)k} = \bar{w}^{*}_{(2)k} - \bar{W}_k,\quad \bar{e}_{\varepsilon(1)k'} = \bar{\varepsilon}_{1\,k'} - E_{k'}$$

$$,\ \bar{e}^{*}_{\varepsilon(2)k'} = \bar{\varepsilon}^{*}_{(2)k'} - E_{k'}.$$

Some useful expectations are

$$E(\bar{e}^{*}_{y\,2}) = E(\bar{e}^{*}_{x\,2\,i}) = E(\bar{e}_{x\,1\,i}) = E(\bar{e}^{*}_{\tau\,2\,i'}) = E(\bar{e}_{\tau\,1\,i'}) = E(\bar{e}^{*}_{z\,2\,j}) = E(\bar{e}_{z\,1\,j})$$

$$= E(\bar{e}^{*}_{\omega\,2\,j'}) = E(\bar{e}_{\omega\,1\,j'}) = E(\bar{e}^{*}_{w\,2\,k}) = E(\bar{e}_{w\,1\,k}) = E(\bar{e}^{*}_{\varepsilon\,2\,k'}) = E(\bar{e}_{\varepsilon\,1\,k'}) = 0$$

Let $d_{x_{q_1\times 1}} = \bar{e}^{*}_{x(2)i} - \bar{e}_{x(1)i}$, $d_{z_{q_3\times 1}} = \bar{e}^{*}_{z(2)j} - \bar{e}_{z(1)j}$ and $d_{w_{q_5\times 1}} = \bar{e}^{*}_{w(2)k} - \bar{e}_{w(1)k}$ are

differences of sampling errors for quantitative variables.

$$d_{\tau_{q_2 \times 1}} = \bar{e}^*_{\tau(2)i'} - \bar{e}_{\tau(1)i'}, \quad d_{\omega_{q_4 \times 1}} = \bar{e}^*_{\omega(2)j'} - \bar{e}_{\omega(1)j'} \text{ and } d_{\varepsilon_{q_6 \times 1}} = \bar{e}^*_{\varepsilon(2)k'} - \bar{e}_{\varepsilon(1)k'} \text{ are differences}$$

for qualitative variables. Further let $X_{q_1 \times q_1} = \left[\bar{X}_i^{-1} \right]_{q_1 \times q_1}$; $i = 1, 2, 3 \ldots q_1$,

$Z_{q_3 \times q_3} = \left[\bar{Z}_j^{-1} \right]_{q_3 \times q_3}$; $j = 1, 2, 3 \ldots q_3$ and $W_{q_5 \times q_5} = \left[\bar{W}_k^{-1} \right]_{q_5 \times q_5}$; $k = 1, 2, 3 \ldots q_5$ are diagonal

matrices for quantitative variables and $\Phi_{q_2 \times q_2} = \left[\Phi_{i'}^{-1} \right]_{q_2 \times q_2}$; $i' = 1, 2, 3 \ldots q_2$,

$\Psi_{q_4 \times q_4} = \left[\Psi_{j'}^{-1} \right]_{q_4 \times q_4}$; $j' = 1, 2, 3 \ldots q_4$ and $E_{q_6 \times q_6} = \left[E_{k'}^{-1} \right]_{q_6 \times q_6}$; $k' = 1, 2, 3 \ldots q_6$ are diagonal

matrices for qualitative variables.

The generalized class of regression-cum-ratio exponential estimator to estimate population mean in the presence of non-response when non-response occur on second phase is;

$$t_{mix} = t_1 \ t_2 + t_3 \tag{2.1}$$

where

$$t_1 = \left[\eta \bar{y}_2^* + a \sum_{i=1}^{q_1} \alpha_{1i} \ \bar{x}_{1\,i} - \bar{x}^*_{(2)i} + b \sum_{i'=1}^{q_2} \alpha_{2i'} \ \tau_{1\,i'} - \tau^*_{(2)i'} \right], \tag{2.2}$$

$$t_2 = c \prod_{j=1}^{q_3} \left(\frac{\bar{z}_{(1)j}}{\bar{z}^*_{(2)j}} \right)^{d\alpha_{3j}} \prod_{i=1}^{q_4} \left(\frac{\omega_{(1)j'}}{\omega^*_{(2)j'}} \right)^{h\alpha_{4j'}} \tag{2.3}$$

and

$$t_3 = e \exp \left[\sum_{k=1}^{q_5} f\alpha_{5k} \left(\frac{\bar{w}^*_{(2)k} - \bar{w}_{1\,k}}{\bar{w}^*_{(2)k} + \bar{w}_{(1)k}} \right) + \sum_{k'=1}^{q_6} l\alpha_{6k'} \left(\frac{\varepsilon^*_{(2)k'} - \varepsilon_{1\,k'}}{\varepsilon_{2\,k'} + \varepsilon_{(1)k'}} \right) \right]. \tag{2.4}$$

For deriving the expression of bias for estimators first of all we write (2.2), (2.3) and (2.4) in form of sampling errors as

$$t_1 = \left[\eta \ \bar{e}^*_{y(2)} + \bar{Y} - a \sum_{i=1}^{q_1} \alpha_{(1)i} \ \bar{e}^*_{x(2)i} - \bar{e}_{x(1)i} - b \sum_{i'=1}^{q_2} \alpha_{2\,i'} \ \bar{e}^*_{\tau(2)i'} - \bar{e}_{\tau(1)i'} \right], \tag{2.5}$$

$$t_2 = c \left[1 + d \sum_{j=1}^{q_3} \frac{\alpha_{(3)j}}{\bar{Z}_j} \ \bar{e}_{z(1)j} - \bar{e}^*_{z(2)j} + h \sum_{j'=1}^{q_4} \frac{\alpha_{(4)j'}}{\Psi_{j'}} \ \bar{e}_{\omega(1)j'} - \bar{e}^*_{\omega(2)j'} \right] \tag{2.6}$$

and

$$t_3 = e \exp \left[\sum_{k=1}^{q_5} \frac{f\alpha_{(5)k}}{2\bar{W}_k} \ \bar{e}^*_{w(2)k} - \bar{e}_{w(1)k} - \bar{e}^*_{w(2)k} - \bar{e}_{w(1)k} \left(\frac{\bar{e}^*_{w(2)k} + \bar{e}_{w(1)k}}{2\bar{W}_k} \right) \right.$$

$$\left. + \sum_{k'=1}^{q_6} \frac{l\alpha_{(6)k'}}{2E_{k'}} \ \bar{e}^*_{\varepsilon(2)k'} - \bar{e}_{\varepsilon(1)k'} - \bar{e}^*_{\varepsilon(2)k'} - \bar{e}_{\varepsilon(1)k'} \left(\frac{\bar{e}^*_{\varepsilon(2)k'} + \bar{e}_{\varepsilon(1)k'}}{2E_{k'}} \right) \right] \tag{2.7}$$

Simplifying and expressing t_1, t_2 and t_3 in matrix notation, we get

$$t_1 = \left[\eta\ \bar{e}^*_{y(2)} + \bar{Y}\ -a\alpha_1^t{}_{1\times q_1}\ d_{x_{q_1\times 1}} - b\alpha_2^t{}_{1\times q_2}\ d_{x_{q_2\times 1}} \right],\tag{2.8}$$

$$t_2 = c\left[1 - d\alpha_3^t{}_{1\times q_3}\ W_{q_3\times q_3} d_{z_{q_3\times 1}} - h\alpha_4^t{}_{1\times q_4}\ \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}} \right]\tag{2.9}$$

and

$$t_3 = e\left\{ 1 + \frac{f}{2}\alpha_5^t{}_{1\times q_5}\ Z_{q_5\times q_5} d_{w_{q_5\times 1}} + \frac{l}{2}\alpha_6^t{}_{1\times q_6}\ E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}} \right\}.\tag{2.10}$$

Simplifying and expressing t_1, t_2 and t_3 in matrix notation, we get

$$t_{mix} = \left[\eta\ \bar{e}_{y(2)} + \bar{Y}\ -a\alpha_1^t{}_{1\times q_1}\ d_{q_1\times 1} - b\alpha_2^t{}_{1\times q_2}\ d_{q_2\times 1} \right]$$

$$\left[1 - cd\alpha_3^t{}_{1\times q_3}\ Z_{q_3\times q_3} d_{1\times q_3} - ch\alpha_4^t{}_{1\times q_4}\ \Psi_{q_4\times q_4} d_{q_4\times 1} \right.$$

$$\left. + 2^{-1}ef\alpha_5^t{}_{1\times q_5}\ W_{q_5\times q_5} d_{q_5\times 1} + 2^{-1}el\alpha_6^t{}_{1\times q_6}\ E_{q_6\times q_6} d_{q_6\times 1} \right]$$

or

$$t_{mix} = \eta\bar{e}^*_{y(2)} - \eta\bar{e}^*_{y(2)} cd\alpha_3^t{}_{1\times q_3}\ Z_{q_3\times q_3} d_{z_{q_3\times 1}}$$

$$-\eta\bar{e}^*_{y(2)} ch\alpha_4^t{}_{1\times q_4}\ \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}} + \eta\bar{e}^*_{y(2)} 2^{-1}ef\alpha_5^t{}_{1\times q_5}\ W_{q_5\times q_5} d_{w_{q_5\times 1}}$$

$$+\ \eta\bar{e}^*_{y(2)} 2^{-1}el\alpha_6^t{}_{1\times q_6}\ E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}} + \eta\bar{Y} - \eta\bar{Y}cd\alpha_3^t{}_{1\times q_3}\ Z_{q_3\times q_3} d_{z_{q_3\times 1}}$$

$$-\eta\bar{Y}ch\alpha_4^t{}_{1\times q_4}\ \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}}$$

$$+\eta\bar{Y}2^{-1}ef\alpha_5^t{}_{1\times q_5}\ W_{q_5\times q_5} d_{w_{q_5\times 1}} + \eta\bar{Y}\ 2^{-1}el\alpha_6^t{}_{1\times q_6}\ E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}} - a\alpha_1^t{}_{1\times q_1}\ d_{x_{q_1\times 1}}$$

$$+acd\alpha_1^t{}_{1\times q_1}\ d_{x_{q_1\times 1}} \alpha_3^t{}_{1\times q_3}\ Z_{q_3\times q_3} d_{z_{q_3\times 1}} + ach\alpha_1^t{}_{1\times q_1}\ d_{x_{q_1\times 1}} \alpha_4^t{}_{1\times q_4}\ \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}}$$

$$-2^{-1}aef\alpha_1^t{}_{1\times q_1}\ d_{x_{q_1\times 1}} \alpha_5^t{}_{1\times q_5}\ W_{q_5\times q_5} d_{w_{q_5\times 1}} - 2^{-1}ael\alpha_1^t{}_{1\times q_1}\ d_{x_{q_1\times 1}} \alpha_6^t{}_{1\times q_6}\ E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}}$$

$$-b\alpha_2^t{}_{1\times q_2}\ d_{\tau_{q_2\times 1}} - bcd\alpha_2^t{}_{1\times q_2}\ d_{\tau_{q_2\times 1}} \alpha_3^t{}_{1\times q_3}\ Z_{q_3\times q_3} d_{z_{q_3\times 1}}$$

$$+bch\alpha_2^t{}_{1\times q_2}\ d_{\tau_{q_2\times 1}} \alpha_4^t{}_{1\times q_4}\ \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}}$$

$$-2^{-1}bef\alpha_2^t{}_{1\times q_2}\ d_{\tau_{q_2\times 1}} \alpha_5^t{}_{1\times q_5}\ W_{q_5\times q_5} d_{w_{q_5\times 1}} - 2^{-1}bel\alpha_2^t{}_{1\times q_2}\ d_{\tau_{q_2\times 1}} \alpha_6^t{}_{1\times q_6}\ E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}}$$

ignoring second and higher order terms, we have

$$t_{mix} = \eta\ \bar{e}^*_{y(2)} + \bar{Y}\ -a\alpha_1^t{}_{1\times q_1}\ d_{x_{q_1\times 1}} - b\alpha_2^t{}_{1\times q_2}\ d_{\tau_{q_2\times 1}}$$

$$-\eta\bar{Y}cd\alpha_3^t{}_{1\times q_3}\ Z_{q_3\times q_3} d_{z_{q_3\times 1}} - \eta\bar{Y}ch\alpha_4^t{}_{1\times q_4}\ \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}}\ \backslash$$

$$+2^{-1}\eta\bar{Y}ef\alpha_5^t{}_{1\times q_5}\ W_{q_5\times q_5} d_{w_{q_5\times 1}} + 2^{-1}m\bar{Y}el\alpha_6^t{}_{1\times q_6}\ E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}}$$

or

$$t_{mix} - \bar{Y} = \eta \bar{e}^{*}_{y(2)} + (\eta-1)\bar{Y} - a\alpha_1^{t}{}_{1\times q_1}\, d_{x_{q_1\times 1}}$$

$$-b\alpha_2^{t}{}_{1\times q_2}\, d_{\tau_{q_2\times 1}} - \eta\bar{Y}cd\alpha_3^{t}{}_{1\times q_3}\, Z_{q_3\times q_3}d_{z_{q_3\times 1}}$$

$$-\eta\bar{Y}ch\alpha_4^{t}{}_{1\times q_4}\, \Psi_{q_4\times q_4}d_{\omega_{q_4\times 1}} + 2^{-1}\eta\bar{Y}ef\,\alpha_5^{t}{}_{1\times q_5}\, W_{q_5\times q_5}d_{w_{q_5\times 1}}$$

$$+2^{-1}\eta\bar{Y}el\alpha_6^{t}{}_{1\times q_6}\, E_{q_6\times q_6}d_{\varepsilon_{q_6\times 1}}$$

or

$$t_{mix} - \bar{Y} = \eta\bar{e}^{*}_{y(2)} + (\eta-1)\bar{Y} - h^{t}{}_{1\times m}H_{m\times 1}, \tag{2.11}$$

where

$$h^{t} = \begin{bmatrix} \alpha_1^{t}{}_{1\times q_1} & \alpha_2^{t}{}_{1\times q_2} & \alpha_3^{t}{}_{1\times q_3} & \alpha_4^{t}{}_{1\times q_4} & \alpha_5^{t}{}_{1\times q_5} & \alpha_6^{t}{}_{1\times q_6} \end{bmatrix}_{1\times m}$$

and

$$H^{t} = \begin{bmatrix} ad_{x_{q_1\times 1}} & bd_{\tau_{q_2\times 1}} & \eta\bar{Y}cdZ_{q_3\times q_3}d_{z_{q_3\times 1}} & \eta\bar{Y}ch\Psi_{q_4\times q_4}d_{\omega_{q_4\times 1}} \\ & -2^{-1}\eta\bar{Y}efW_{q_5\times q_5}d_{w_{q_5\times 1}} & -2^{-1}\eta\bar{Y}elE_{q_6\times q_6}d_{\varepsilon_{q_6\times 1}} \end{bmatrix}_{1\times m}$$

$$;\sum_{i=1}^{6} q_i = m$$

from (2.11)

$$E(t_{mix} - \bar{Y})^2 = E(\eta\bar{e}^{*}_{y(2)} + (\eta-1)\bar{Y} - h^{t}{}_{1\times m}H_{m\times 1})^2 \tag{2.12}$$

Differentiating (2.11) w.r.t $h_{m\times 1}$ and equating it equal to zero.

$$\eta E(H_{m\times 1}\bar{e}^{*}_{y(2)}) + (\eta-1)\bar{Y}E(H_{m\times 1}) - h_{m\times 1}E(H_{m\times 1}H^{t}{}_{1\times m}) = 0 \tag{2.13}$$

or

$$\eta\Omega_{m\times 1} - h_{m\times 1}\Lambda_{m\times m} = 0$$

or

$$h_{m\times 1} = \eta\Lambda^{-1}{}_{m\times m}\Omega_{m\times 1},$$

where

$$\Omega_{m\times 1} = E(H_{m\times 1}\bar{e}_{y(2)}) \text{ and } \Lambda_{m\times m} = E(H_{m\times 1}H^{t}{}_{1\times m}) = \begin{bmatrix} \Delta_{ij} \end{bmatrix}_{m\times m}$$

with

$$\Delta_{11} = aE(d_{q_1\times 1}d^{t}_{1\times q_1})a^{t} = a^2\Delta_{q_1\times q_1} \quad,$$

$$\Delta_{12} = aE(d_{q_1\times 1}d^{t}_{1\times q_2})b = ab\Delta_{q_1\times q_2} \quad,$$

$$\Delta_{13} = acd\bar{Y}E(d_{q_1\times 1}d^{t}_{1\times q_3})Z_{q_3\times q_3} = acd\bar{Y}\Delta_{q_1\times q_3}Z_{q_3\times q_3},$$

$$\Delta_{14} = \bar{Y}achE(d_{q_1\times 1}d^{t}_{1\times q_4})\Psi_{q_4\times q_4} = \bar{Y}ach\Delta_{q_1\times q_4}\Psi_{q_4\times q_4},$$

$$\Delta_{15} = -aef\,2^{-1}\,\bar{Y}E(d_{q_1\times1}d^t_{1\times q_5})W_{q_5\times q_5} = -aef\,2^{-1}\bar{Y}\Delta_{q_1\times q_5}W_{q_5\times q_5},$$

$$\Delta_{16} = -ael\,2^{-1}\,\bar{Y}E(d_{q_1\times1}d^t_{1\times q_3})Z_{q_3\times q_3} = -ael\,2^{-1}\,\bar{Y}_{q_1\times q_3}Z_{q_3\times q_3},$$

$$\Delta_{22} = bE(d_{q_2\times1}d^t_{1\times q_2})b^t = b^2\Delta_{q_2\times q_2},$$

$$\Delta_{23} = bcd\bar{Y}E(d_{q_2\times1}d^t_{1\times q_3})Z_{q_3\times q_3}\,bcd\bar{Y}\Delta_{q_2\times q_3}Z_{q_3\times q_3},$$

$$\Delta_{24} = \bar{Y}bchE(d_{q_2\times1}d^t_{1\times q_4})\Psi_{q_4\times q_4} = \bar{Y}bch\Delta_{q_2\times q_4}\Psi_{q_4\times q_4},$$

$$\Delta_{25} = -bef\,2^{-1}\,\bar{Y}E(d_{q_2\times1}d^t_{1\times q_5})W_{q_5\times q_5} = -bef\,2^{-1}\,\bar{Y}\Delta_{q_2\times q_5}W_{q_5\times q_5},$$

$$\Delta_{26} = -bel\,2^{-1}\bar{Y}E(d_{q_2\times1}d^t_{1\times q_6})E_{q_6\times q_6} = -bel\,2^{-1}\,\bar{Y}\Delta_{q_2\times q_6}E_{q_6\times q_6},$$

$$\Delta_{33} = (cd\bar{Y})^2E(d_{q_3\times1}d^t_{1\times q_3})Z^2_{q_3\times q_3} = (cd\bar{Y})^2\Delta_{q_3\times q_3}Z^2_{q_3\times q_3},$$

$$\Delta_{34} = (\bar{Y}c)^2\,dhZ_{q_3\times q_3}E(d_{q_3\times1}d^t_{1\times q_4})\Psi_{q_4\times q_4} = (\bar{Y}c)^2\,dhZ_{q_3\times q_3}\Delta_{q_3\times q_4}\Psi_{q_4\times q_4},$$

$$\Delta_{35} = -cdef\,2^{-1}\,\bar{Y}^2Z_{q_3\times q_3}E(d_{q_3\times1}d^t_{1\times q_5})W_{q_5\times q_5} = -cdef\,2^{-1}\,\bar{Y}^2Z_{q_3\times q_3}\Delta_{q_3\times q_5}W_{q_5\times q_5},$$

$$\Delta_{36} = -cdel\,2^{-1}\,\bar{Y}^2Z_{q_3\times q_3}E(d_{q_3\times1}d^t_{1\times q_6})E_{q_6\times q_6} = -cdel\,2^{-1}\,\bar{Y}^2Z_{q_3\times q_3}\Delta_{q_3\times q_6}E_{q_6\times q_6},$$

$$\Delta_{44} = (ch\bar{Y})^2E(d_{q_4\times1}d^t_{1\times q_4})\Psi^2_{q_4\times q_4} = (ch\bar{Y})^2\Delta_{q_4\times q_4}\Psi^2_{q_4\times q_4},$$

$$\Delta_{45} = -chef\,2^{-1}\,\bar{Y}^2\,\Psi_{q_4\times q_4}E(d_{q_4\times1}d^t_{1\times q_5})W_{q_5\times q_5} = -chef\,2^{-1}\,\bar{Y}^2\Psi_{q_4\times q_4}\Delta_{q_4\times q_5}W_{q_5\times q_5},$$

$$\Delta_{46} = -ehcl\,2^{-1}\,\bar{Y}^2\Psi_{q_4\times q_4}E(d_{q_4\times1}d^t_{1\times q_6})E_{q_6\times q_6} = -ehcl\,2^{-1}\,\bar{Y}^2\Psi_{q_4\times q_4}\Delta_{q_4\times q_6}E_{q_6\times q_6},$$

$$\Delta_{55} = \ ef\,2^{-1}\ ^2\,W_{q_5\times q_5}\,\bar{Y}\ ^2\,E(d_{q_5\times1}d^t_{1\times q_5}) = \ ef\,2^{-1}\ ^2\,W_{q_5\times q_5}\,\bar{Y}\ ^2\,\Delta_{q_5\times q_5},$$

$$\Delta_{56} = e^2fl\,4^{-1}\,\bar{Y}^2W_{q_5\times q_5}E(d_{q_5\times1}d^t_{1\times q_6})E_{q_6\times q_6} = e^2fl\,4^{-1}\,\bar{Y}^2W_{q_5\times q_5}\Delta_{q_5\times q_6}E_{q_6\times q_6},$$

$$\Delta_{66} = \ efl\,2^{-1}\,\bar{Y}E_{q_6\times q_6}\ ^2\,E(d_{q_6\times1}d^t_{1\times q_6}) = \ efl\,2^{-1}\,\bar{Y}E_{q_6\times q_6}\ ^2\,\Delta_{q_6\times q_6},$$

and

$$MSE(t_{mix}) = E(\eta\bar{e}_{y(2)} + (\eta-1)\bar{Y} - h^t_{1\times m}H_{m\times1})(\eta\bar{e}_{y(2)} + (\eta-1)\bar{Y} - h^t_{1\times m}H_{m\times1}).$$

Using (2.13) the *MSE* is

$$MSE(t_{mix}) = E\ \eta\bar{e}_{y(2)} + (\eta-1)\bar{Y}\ \ \eta\bar{e}_{y(2)} + (\eta-1)\bar{Y} - h^t_{1\times m}H_{m\times1}\ \ ,$$

or

$$MSE(t_{mix}) = \eta^2E\ \bar{e}_{y(2)}\ ^2 + \ (\eta-1)\bar{Y}\ ^2 - \eta h^t_{1\times m}E\ H_{m\times1}\bar{e}_{y(2)}\ \ .$$

or

$$MSE(t_{mix}) = \eta^2\ \lambda_2S^2_y\ + \ (\eta-1)\bar{Y}\ ^2 - \eta h^t_{1\times m}\Omega_{m\times1},$$

Substituting $\ h_{m\times1} = \eta\Lambda^{-1}_{m\times m}\Omega_{m\times1}$

$$MSE(t_{mix}) = \left[\eta^2 - 2\eta + 1 \right] \overline{Y}^2 + \eta^2 \left[\lambda_2 S_y^2 - \Omega_{1\times m} \Lambda^{-1}_{m\times m} \Omega_{m\times 1} \right],$$

or

$$MSE(t_{mix}) = \left[\eta^2 - 2\eta + 1 \right] \overline{Y}^2 + \eta^2 \Gamma,$$

where

$$\Gamma = \left[\lambda_2 S_y^2 - \Omega_{1\times m} \Lambda^{-1}_{m\times m} \Omega_{m\times 1} \right],$$

Differentiating *MSE* with respect to η and equating to zero, we have

$$2\eta \overline{Y}^2 - 2\overline{Y}^2 + 2\eta \Gamma = 0$$

or

$$\eta = \frac{\overline{Y}^2}{\overline{Y}^2 + \Gamma} = \left[1 + \Gamma \overline{Y}^{-2} \right]^{-1}$$

After substituting the value of η the minimum *MSE* will be

$$MSE(t_{mix}) = \left[\left\{ \left[1 + \overline{Y}^{-2}\Gamma \right]^{-1} - 1 \right\} \overline{Y} \right] + \left[1 + \overline{Y}^{-2}\Gamma \right]^{-2} \Gamma \tag{2.14}$$

3. SPECIAL CASES

Table-1: For Mixture of Multi-Auxiliary Variables

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
1	η	a	b	c	d	h	e	f	l	Regression estimator for mixture	$t_{mix(reg)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i} \left(\bar{x}_{(1)i} - \bar{x}_{(2)i}^* \right) + \sum_{i=1}^{q_2} \alpha_{2i} \left(\tau_{(1)i} - \tau_{(2)i}^* \right) \right]$
2	1	0	0	1	1	1	1	0	0	Ratio estimator for mixture	$t_{mix(r)} = \bar{y}_2 \prod_{i=1}^{q_3} \left(\frac{\bar{z}_{(1)i}}{\bar{z}_{(2)i}^*} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega_{(1)i}}{\omega_{(2)i}^*} \right)^{\alpha_{4i}}$
3	1	0	0	0	1	0	1	1	1	Exponential estimator for mixture	$t_{mix(\exp)} = \exp\left\{ \sum_{i=1}^{q_5} \alpha_{5i} \left(\frac{\bar{w}_{(2)i}^* - \bar{w}_{(1)i}}{\bar{w}_{(2)i}^* + \bar{w}_{(1)i}} \right) - \sum_{i=1}^{q_6} \alpha_{6i} \left(\frac{\varepsilon_{(2)i}^* - \varepsilon_{(1)i}}{\varepsilon_{(2)i}^* + \varepsilon_{(1)i}} \right) \right\}$
4	1	1	1	1	1	1	0	0	0	Regression-cum ratio for mixture	$t_{mix(rcr)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i} \left(\bar{x}_{(1)i} - \bar{x}_{(2)i}^* \right) + \sum_{i=1}^{q_2} \alpha_{2i} \left(\tau_{(1)i} - \tau_{(2)i}^* \right) \right] \left[\prod_{i=1}^{q_3} \left(\frac{\bar{z}_{(1)i}}{\bar{z}_{(2)i}^*} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega_{(1)i}}{\omega_{(2)i}^*} \right)^{\alpha_{4i}} \right]$
5	1	1	1	1	-1	-1	0	0	0	Regression-cum product for mixture	$t_{mix(rcp)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i} \left(\bar{x}_{(1)i} - \bar{x}_{(2)i}^* \right) + \sum_{i=1}^{q_2} \alpha_{2i} \left(\tau_{(1)i} - \tau_{(2)i}^* \right) \right] \left[\prod_{i=1}^{q_3} \left(\frac{\bar{z}_{(1)i}}{\bar{z}_{(2)i}^*} \right)^{-\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega_{(1)i}}{\omega_{(2)i}^*} \right)^{-\alpha_{4i}} \right]$
6	1	0	0	1	1	1	1	1	1	Ratio-cum expo ratio for mixture	$t_{mix(rcer)} = \left[\prod_{i=1}^{q_3} \left(\frac{\bar{z}_{(1)i}}{\bar{z}_{(2)i}^*} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega_{(1)i}}{\omega_{(2)i}^*} \right)^{\alpha_{4i}} + \exp\left\{ \sum_{i=1}^{q_5} \alpha_{5i} \left(\frac{\bar{w}_{(2)i}^* - \bar{w}_{(1)i}}{\bar{w}_{(2)i}^* + \bar{w}_{(1)i}} \right) - \sum_{i=1}^{q_6} \alpha_{6i} \left(\frac{\varepsilon_{(2)i}^* - \varepsilon_{(1)i}}{\varepsilon_{(2)i}^* + \varepsilon_{(1)i}} \right) \right\} \right]$
7	1	0	0	0	1	1	1	-1	-1	Ratio-cum expo ratio for mixture	$t_{mix(rcep)} = \left[\prod_{i=1}^{q_3} \left(\frac{\bar{z}_{(1)i}}{\bar{z}_{(2)i}} \right)^{d\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega_{(1)i}}{\omega_{(2)i}^*} \right)^{h\alpha_{4i}} + \exp\left\{ -\sum_{i=1}^{q_5} \alpha_{5i} \left(\frac{\bar{w}_{(2)i}^* - \bar{w}_{(1)i}}{\bar{w}_{(2)i}^* + \bar{w}_{(1)i}} \right) - \sum_{i=1}^{q_6} \alpha_{6i} \left(\frac{\varepsilon_{(2)i}^* - \varepsilon_{(1)i}}{\varepsilon_{(2)i}^* + \varepsilon_{(1)i}} \right) \right\} \right]$

Table-2: For Quantitative Multi-Auxiliary Variables

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
1	1	1	0	1	0	0	0	0	0	Regression estimator	$t_{(reg)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i}\ \bar{x}_{(1)i} - \bar{x}^*_{(2)i} \right]$
2	1	0	0	1	1	0	0	0	0	Ratio estimator	$t_{(r)} = \bar{y}_2 \prod_{j=1}^{q_3} \left(\dfrac{\bar{z}_{(1)j}}{\bar{z}^*_{(2)j}} \right)^{\alpha_{3j}}$
3	1	0	0	0	1	0	1	1	0	Exponential estimator	$t_{(exp)} = \exp\left\{ \sum_{k=1}^{q_5} \alpha_{5k} \left(\dfrac{\bar{w}^*_{(2)k} - \bar{w}_{(1)k}}{\bar{w}^*_{(2)k} + \bar{w}_{(1)k}} \right) \right\}$
4	1	1	0	1	1	0	0	0	0	Regression-cum ratio estimator	$t_{(rcr)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i}\ \bar{x}_{(1)i} - \bar{x}^*_{(2)i} \right] \left[\prod_{j=1}^{q_3} \left(\dfrac{\bar{z}_{(1)j}}{\bar{z}^*_{(2)j}} \right)^{\alpha_{3j}} \right]$
5	1	1	0	1	-1	0	0	0	0	Regression-cum product estimator	$t_{(rcp)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i}\ \bar{x}_{(1)i} - \bar{x}^*_{(2)i} \right] \left[\prod_{j=1}^{q_3} \left(\dfrac{\bar{z}_{(1)j}}{\bar{z}^*_{(2)j}} \right)^{-d\alpha_{3j}} \right]$
6	1	0	0	1	1	1	1	1	1	Ratio-cum expo ratio estimator	$t_{(rcer)} = \left[\prod_{j=1}^{q_3} \left(\dfrac{\bar{z}_{(1)j}}{\bar{z}^*_{(2)j}} \right)^{d\alpha_{3j}} + \exp\left\{ \sum_{k=1}^{q_5} \alpha_{5k} \left(\dfrac{\bar{w}^*_{(2)k} - \bar{w}_{(1)k}}{\bar{w}^*_{(2)k} + \bar{w}_{(1)k}} \right) \right\} \right]$
7	1	0	0	1	1	1	1	-1	-1	Ratio-cum expo product estimator	$t_{(rcep)} = \left[\prod_{j=1}^{q_3} \left(\dfrac{\bar{z}_{(1)j}}{\bar{z}^*_{(2)j}} \right)^{d\alpha_{3j}} + \exp\left\{ -\sum_{k=1}^{q_5} \alpha_{5k} \left(\dfrac{\bar{w}^*_{(2)k} - \bar{w}_{(1)k}}{\bar{w}^*_{(2)k} + \bar{w}_{(1)k}} \right) \right\} \right]$

Table -3: For Qualitative Multi-Auxiliary Variables

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
1	1	0	1	1	0	0	0	0	0	Regression estimator	$t_{(reg)} = \left[\bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \left(\tau_{(1)i'} - \tau^*_{(2)i'} \right) \right]$
2	1	0	0	1	0	1	0	0	0	Ratio estimator	$t_{(r)} = \bar{y}_2 \prod_{j'=1}^{q_4} \left(\dfrac{\omega_{(1)j'}}{\omega^*_{(2)j'}} \right)^{\alpha_{4j'}}$
3	1	0	0	0	1	0	1	0	1	Exponential estimator	$t_{(\exp)} = \exp\left\{ \sum_{k'=1}^{q_6} \alpha_{6k'} \left(\dfrac{\varepsilon^*_{(2)k'} - \varepsilon_{(1)k'}}{\varepsilon^*_{(2)k'} + \varepsilon_{(1)k'}} \right) \right\}$
4	1	0	1	1	0	1	0	0	0	Regression-cum ratio estimator	$t_{(rcr)} = \left[\bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \left(\tau_{(1)i'} - \tau^*_{(2)i'} \right) \right] \left[\prod_{j'=1}^{q_4} \left(\dfrac{\omega_{(1)j'}}{\omega^*_{(2)j'}} \right)^{\alpha_{4j'}} \right]$
5	1	0	1	1	0	-1	0	0	0	Regression-cum product estimator	$t_{mix(rcp)} = \left[\bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \left(\tau_{(1)i'} - \tau^*_{(2)i'} \right) \right] \left[\prod_{j'=1}^{q_4} \left(\dfrac{\omega_{(1)j'}}{\omega^*_{(2)j'}} \right)^{-\alpha_{4j'}} \right]$
6	1	0	0	1	1	1	1	1	1	Ratio-cum expo ratio estimator	$t_{(rcer)} = \left[\prod_{j'=1}^{q_4} \left(\dfrac{\omega_{(1)j'}}{\omega^*_{(2)j'}} \right)^{\alpha_{4j'}} + \exp\left\{ \sum_{k'=1}^{q_6} \alpha_{6k'} \left(\dfrac{\varepsilon^*_{(2)k'} - \varepsilon_{(1)k'}}{\varepsilon^*_{(2)k'} + \varepsilon_{(1)k'}} \right) \right\} \right]$
7	1	0	0	1	1	1	1	-1	-1	Ratio-cum expo product estimator	$t_{mix(rcep)} = \left[\prod_{j'=1}^{q_4} \left(\dfrac{\omega_{(1)j}}{\omega^*_{(2)j}} \right)^{\alpha_{4j}} + \exp\left\{ -\sum_{k'=1}^{q_6} \alpha_{6k'} \left(\dfrac{\varepsilon^*_{(2)k'} - \varepsilon_{(1)k'}}{\varepsilon^*_{(2)k'} + \varepsilon_{(1)k'}} \right) \right\} \right]$

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A GENERAL CLASS OF MEAN ESTIMATORS USING MIXTURE OF AUXILIARY VARIABLES IN TWO-PHASE SAMPLING IN THE PRESENCE OF NON-RESPONSE ON FIRST PHASE

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ABSTRACT

In this paper we have suggested a general class of estimators in two-phase sampling to estimate the population mean of study variable in the case when non-response occur on first phase. We are using several continuous and categorical auxiliary variables simultaneously while constructing the class. Also we are assuming that the information on all auxiliary variables is not available for population (no information case) as mostly the practical cases. The expressions and mean square error of suggested class have been derived. Furthermore several special cases of proposed class have been identified.

KEYWORDS

Mixture of Multi-Auxiliary Variables; Non-response; Two Phase Sampling; Generalized Estimator; No Information Case.

1. INTRODUCTION

Precision of an estimator can be increased by using auxiliary information when observed study variable is highly correlated with the auxiliary variable x (Singh et al. (2010)). Two phase sampling scheme is used when such information is not available in real life situation. In different situations auxiliary information is required in quantitative as well as qualitative form to increase the precision of an estimator there is vital use of auxiliary information in two phase sampling. Presence and absence of auxiliary information plays very important role on the efficiency of estimators in two phase sampling.

Samiuddin and Hanif (2006) have discussed first time three possibilities of availability of population information of auxiliary variable(s).These possibilities are ; no information case (NIC) when population information on all auxiliary variables is not available, partial information case (PIC) when population information on few auxiliary variables are available but not available on few other variables, full information case (FIC) when population information on all auxiliary variables is available. Ahmad and Hanif(2010) developed estimators using different techniques like ratio and regression considered these three cases and compare the efficiency of estimators on the basis of mean square error.

In practical situation where data on multiple auxiliary variables which is related to observed study variable y and that can be obtained by using two phase sampling to increase the importance of estimators. Ahmad, et al. (2009), Ahmad, et al. (2010) and Ahmad and Hanif (2010) developed different estimators of two phase sampling using different techniques of estimators for example ratio and regression considering three possibilities of auxiliary information using multi-auxiliary variables when auxiliary information is quantitative in nature. As mentioned above the auxiliary information is also available in qualitative nature. Recently Inam and Hanif (2009) proposed different estimators for three possibilities availability of auxiliary information using multi-auxiliary qualitative variables.

If the information is not supplied by variable of interest from all the selected sampling units then problem of non-response occur that effect the precision of estimators basically there are two types of non-response first is unit non-response and second is item non-response. During the discussion of sampling, it was noted that sample size referred to the number of responses that need to be obtained. But no matter how well the sampling design is planned, a poor response rate to a mail or telephone survey or to interviews can render a study virtually useless.

To address the problem of non-response Hansen and Hurwitz used the following sampling scheme. The total population of N units is divided into two units: N_1 units are the responding units and N_2 are the non responding units. From N units, a first phase sample of n_1 units is drawn by simple random sampling without replacement (SRSWOR). A second phase sample of n_2 units (i.e. $n_2 < n_1$) is drawn n_1 by simple random sampling without simple random sampling and the variable of interest y is measured on it. At second phase let v_1 units supply information and v_2 units refuse to response. A sub-sample of r_2 units is randomly taken from the m_2 non-respondents by and this sub sample is specified by $r_2 = m_2 / k_2$, $k_2 > 1$. Under this sampling scheme Hansen and Hurwitz (1946) suggested the estimator with *MSE* when non-response at second phase.

$$t_{1(2)} = w_1 \bar{y}_{1\,2} + w_2 \bar{y}_{2r\,2} \tag{1.1}$$

with

$$V(t_{1(2)}) = \left(\frac{1}{n_2} - \frac{1}{N} \right) S_y^2 + \frac{W_2(k_2 - 1)}{n_2} S_{y_2}^2 \,.$$

Following the same strategy of Hansen and Hurwitz (1946), Singh and Kumar (2008) suggested following estimator with bias and *MSE* when non-response at second phase. The ratio estimator is

$$t_{2(2)} = \bar{y}_2^* \left(\frac{\bar{x}_1}{\bar{x}_2^*} \right) \left(\frac{\bar{x}_1}{\bar{x}} \right) \tag{1.2}$$

with

$$MSE(t_{2(2)}) = \left(\frac{1}{n_2} - \frac{1}{n_1} \right) S_y^2 + 4RS_x^2 (R - \beta)$$

$$+\frac{W_2(k_2-1)}{n_2}\ S_{y_2}^2+RS_{x_2}^2(R-2\beta_{(2)})\ +\left(\frac{1}{n_1}-\frac{1}{N}\right)S_y^2,$$

the product estimator with *MSE* is

$$t_{3(2)}=\bar{y}_2^*\left(\frac{\bar{x}_2^*}{\bar{x}_1}\right)\left(\frac{\bar{x}}{\bar{x}_1}\right) \tag{1.3}$$

with

$$MSE(t_{3(2)})=\left(\frac{1}{n_2}-\frac{1}{n_1}\right)\ S_y^2+4RS_x^2(R+\beta)$$

$$+\frac{W_2(k_2-1)}{n_2}\ S_{y_2}^2+RS_{x_2}^2(R+2\beta_{(2)})\ +\left(\frac{1}{n_1}-\frac{1}{N}\right)S_y^2$$

and the regression estimator

$$t_{4(2)}=\bar{y}_2^*+d_1\ \bar{x}-\bar{x}_2^*\ +d_2\ \bar{x}_1-\bar{x} \tag{1.4}$$

with

$$MSE(t_{4(2)})=\left(\frac{1}{n_2}-\frac{1}{n_1}\right)\ S_y^2+d_2S_x^2(d_2-2\beta)$$

$$+\frac{W_2(k_2-1)}{n_2}\ S_{y_2}^2+d_1S_{x_2}^2(d_1-2\beta_{(2)})\ +\left(\frac{1}{n_1}-\frac{1}{N}\right)S_y^2$$

Khare and Srivastava (1993, 1995) proposed following estimators in the presence of non-response at second phase. The product estimator is

$$t_{5(2)}=\frac{\bar{y}_2^*}{\bar{x}_1}\ \bar{x}_2^* \tag{1.5}$$

with

$$MSE(t_{5(2)})=\left(\frac{1}{n_1}-\frac{1}{N}\right)S_y^2+\left(\frac{1}{n_2}-\frac{1}{n_1}\right)S_p^2+\frac{W_2(k_2-1)}{n_2}S_{p_2}^2,$$

and the regression estimator

$$t_{6(2)}=\bar{y}_2^*+b^*(\bar{x}_1-\bar{x}_2^*) \tag{1.6}$$

with

$$MSE(t_{6(2)})=\left(\frac{1}{n_1}-\frac{1}{N}\right)S_y^2+\left(\frac{1}{n_2}-\frac{1}{n_1}\right)S_y^2\ 1-\rho^2\ +\frac{W_2(k_2-1)}{n_2}S_{\beta_2}^2.$$

Tabasum and Khan (2004) revisited the ratio estimator t_1 by Khare and Srivastava (1993) and found that the cost of this estimator is less than the cost gained by Hansen-Hurwitz estimator under some conditions.

Iqra (2011) suggested class of generalized estimator for estimating the population mean using multi- auxiliary quantitative variables when non-response at first phase. The generalized class is

$$t_m = \left[\bar{y}_2 + a \sum_{i=1}^{q_1} \alpha_i \; \bar{x}_{(1)i}^* - \bar{x}_{(2)i} \right] \left[b \prod_{i=1}^{q_2} \left(\frac{\bar{x}_{(1)i}^*}{\bar{x}_{(2)i}} \right)^{c\beta_i} + d \exp \left\{ \sum_{i=1}^{q_3} e\gamma_i \left(\frac{\bar{x}_{(1)i}^*}{\bar{x}_{(2)i}} \right) \right\} \right] \quad (1.7)$$

with

$$MSE(t_m) = \lambda_2 S_y^2 - Q_{1\times p}^t T_{p\times p}^{-1} Q_{p\times 1}.$$

2. SUGGESTED CLASS OF ESTIMATORS

Consider the total population (denoted by U) of N units is divided into two sections: one is the section (denoted by U_1) of N_1 units, which would be available on the first attempt at the first stage and the other section (denoted by U_2) of N_2 units, which are not available on the first attempt at the first phase but will be available on the second attempt. From N units, a first phase sample (denoted by u_1) of n_1 units is drawn by simple random sampling without replacement (SRSWOR).A second phase sample (denoted by u_2) of n_2 units (i.e. $n_2 < n_1$) is drawn from n_1 by simple random sampling without replacement (SRSWOR) and the variable of interest y is measured at second phases. At second phase let m_1' units supply information which is denoted by v_1' and m_2' units refuse to response which is denoted by v_2', where $v_1' = u_2 \cap U_1$ and $v_2' = u_2 \cap U_2$. A sub-sample (denoted by v_{2m}') of r_1 units is randomly taken from the m_2' non-respondents by applying the strategy defined of Hansen and Hurwitz (1946) and this sub sample is specified by $r_1 = \dfrac{m_2'}{k_1}$, $k_1 > 1$. Assume that no non response is observed in this sub sample.

Let x_i, z_j, w_k denotes the set of multi-auxiliary quantitative variables for $i = 1, 2, 3, ..., q_1$, $j = 1, 2, 3, ..., q_3$, $k = 1, 2, 3, ..., q_5$ respectively with population mean $\bar{X}_i = N^{-1} \sum_{t=1}^{N} x_t$, $\bar{X}_{(1)i} = N_1^{-1} \sum_{t=1}^{N_1} x_t$, $\bar{X}_{(2)i} = N_2^{-1} \sum_{t=1}^{N_2} x_t$, $\bar{Z}_j = N^{-1} \sum_{t=1}^{N} z_t$, $\bar{Z}_{(1)j} = N_1^{-1} \sum_{t=1}^{N_1} z_t$, $\bar{Z}_{(2)j} = N_2^{-1} \sum_{t=1}^{N_2} z_t$, $\bar{W}_k = N^{-1} \sum_{t=1}^{N} w_t$, $\bar{W}_{(1)k} = N_1^{-1} \sum_{t=1}^{N_1} w_t$ and $\bar{W}_{(2)k} = N_2^{-1} \sum_{t=1}^{N_2} w_t$ denote the population means of the response and non-response groups. Let $\bar{x}_{(1)i} = n_1^{-1} \sum_{t=1}^{n_1} x_t$, $\bar{z}_{(1)j} = n_1^{-1} \sum_{t=1}^{n_1} z_t$ and $\bar{w}_{(1)k} = n_1^{-1} \sum_{t=1}^{n_1} w_t$ denotes the sample mean of all n_1 units, and $\bar{x}_{(1)i}' = m_1'^{-1} \sum_{t=1}^{m_1'} x_t$, $\bar{x}_{(2)i}' = m_2'^{-1} \sum_{t=1}^{m_2'} x_t$, $\bar{z}_{(1)j}' = m_1'^{-1} \sum_{t=1}^{m_1'} z_t$, $\bar{z}_{(2)j}' = m_2'^{-1} \sum_{t=1}^{m_2'} z_t$, $\bar{w}_{(1)k}' = m_1'^{-1} \sum_{t=1}^{m_1'} w_k$

and $\bar{w}'_{(2)k} = m_2'^{-1} \sum_{t=1}^{m_2'} w_t$ denote the sample means of the m_1' responding and m_2' non-

responding units. Further, let $\bar{x}_{(r_1)i} = \frac{1}{r_1} \sum_{t=1}^{r_1} x_t$, $\bar{z}_{(r_1)j} = \frac{1}{r_1} \sum_{t=1}^{r_1} z_t$ and $\bar{w}_{(r_1)k} = \frac{1}{r_1} \sum_{t=1}^{r_1} w_t$

denotes the sample mean of the $r_1 = m_2' / k_1$, $k_1 > 1$ sub-sampled units. Let $\tau_{i'}, \omega_{j'}, \varepsilon_{k'}$ denotes another set of multi-auxiliary qualitative variables for $i' = 1, 2, 3, ..., q_2$, $j' = 1, 2, 3, ..., q_4$, $k' = 1, 2, 3, ..., q_6$ respectively with population

proportions $\Phi_{i'} = N^{-1} \sum_{t=1}^{N} \tau_t$, $\Phi_{(1)i'} = N_1^{-1} \sum_{t=1}^{N_1} \tau_t$, $\Phi_{(2)i'} = N_2^{-1} \sum_{t=1}^{N_2} \tau_t$, $\Psi_{j'} = N^{-1} \sum_{t=1}^{N} \omega_t$,

$\Psi_{(1)j'} = N_1^{-1} \sum_{t=1}^{N_1} \omega_t$, $\Psi_{(2)j'} = N_2^{-1} \sum_{t=1}^{N_2} \omega_t$, $E_{k'} = N^{-1} \sum_{t=1}^{N} \varepsilon_t$, $E_{(1)k'} = N_1^{-1} \sum_{t=1}^{N_1} \varepsilon_t$ and

$E_{(2)k'} = N_2^{-1} \sum_{t=1}^{N_2} \varepsilon_t$ denote the population proportions of the response and non-response

groups. Let $\tau_{(1)i'} = n_1^{-1} \sum_{t=1}^{n_1} \tau_t$, $\omega_{(1)j'} = n_1^{-1} \sum_{t=1}^{n_1} \omega_t$ and $\varepsilon_{(1)k'} = n_1^{-1} \sum_{t=1}^{n_1} \varepsilon_t$ denotes the sample

proportions of all n_1 units, and $\tau'_{(1)i'} = m_1'^{-1} \sum_{t=1}^{m_1'} \tau_t$, $\tau'_{(2)i} = m_2'^{-1} \sum_{t=1}^{m_2'} \tau_t$, $\omega'_{(1)j'} = m_1'^{-1} \sum_{t=1}^{m_1'} \omega_t$,

$\omega'_{(2)j'} = m_2'^{-1} \sum_{t=1}^{m_2'} \omega_t$, $\varepsilon'_{(1)k'} = m_1'^{-1} \sum_{t=1}^{m_1'} \varepsilon_k$ and $\varepsilon'_{(2)k'} = m_2'^{-1} \sum_{t=1}^{m_2'} \varepsilon_t$ denote the sample

proportions of the m_1' responding and m_2' non-responding units. Further, let

$\tau_{(r_1)i'} = \frac{1}{r_1} \sum_{t=1}^{r_1} \tau_t$, $\omega_{(r_1)j'} = \frac{1}{r_1} \sum_{t=1}^{r_1} \omega_t$ and $\varepsilon_{(r_1)k'} = \frac{1}{r_1} \sum_{t=1}^{r_1} \varepsilon_t$ denotes the sample proportions

of the $r_1 = m_2' / k_1$, $k_1 > 1$ sub-sampled units.

Let us define sampling errors for quantitative auxiliary variables and qualitative auxiliary variables as

$$\bar{e}_{y(2)} = \bar{y}_{(2)} - \bar{Y}, \; \bar{e}^*_{x(1)i} = \bar{x}^*_{(1)i} - \bar{X}_i, \; \bar{e}_{x(2)i} = \bar{x}_{(2)i} - \bar{X}, \; \bar{e}^*_{z(1)j} = \bar{z}^*_{(1)j} - \bar{Z}_j,$$

$$\bar{e}_{z(2)j} = \bar{z}_{(2)j} - \bar{Z}_j, \; \bar{e}^*_{w(1)k} = \bar{w}^*_{(1)j} - \bar{W}, \; \bar{e}_{w(2)j} = w_j - \bar{W}, \; \bar{e}^*_{\tau(1)i'} = \tau^*_{(1)i'} - \Phi_{i'},$$

$$\bar{e}_{\tau(2)i'} = \tau_{(2)i'} - \Phi_{i'}, \; \bar{e}^*_{\omega(1)j'} = \bar{\omega}^*_{(1)j'} - \Psi_{j'}, \; \bar{e}_{\omega(2)j'} = \omega_{(2)j'} - \Psi_{j'},$$

$$\bar{e}^*_{\varepsilon(1)k'} = \bar{\varepsilon}^*_{(1)k'} - E_{k'} \text{ and } \bar{e}_{\varepsilon(2)k'} = \bar{\varepsilon}_{(2)k'} - E_{k'}.$$

The expectations of sampling errors and their functions are

$$E(\bar{e}_{y(2)}) = E(\bar{e}_{x(2)i}) = E(\bar{e}^*_{x\,1\,i}) = E(\bar{e}_{\tau(2)i'}) = E(\bar{e}^*_{\tau\,1\,i'}) = E(\bar{e}_{z(2)j}) = E(\bar{e}^*_{z\,1\,j})$$

$$= E(\bar{e}_{\omega(2)j'}) = E(\bar{e}^*_{\omega\,1\,j'}) = E(\bar{e}_{w(2)k}) = E(\bar{e}^*_{w\,1\,k}) = E(\bar{e}_{\varepsilon(2)k}) = E(\bar{e}^*_{\varepsilon\,1\,k'}) = 0$$

Let $d_{x_{q_1 \times 1}} = \overline{e}_{x(2)i} - \overline{e}^*_{x(1)i}$, $d_{z_{q_3 \times 1}} = \overline{e}_{z(2)j} - \overline{e}^*_{z(1)j}$ and $d_{w_{q_5 \times 1}} = \overline{e}_{w(2)k} - \overline{e}^*_{w(1)k}$ are differences of sampling errors for quantitative variables.

Let $d_{\tau_{q_2 \times 1}} = \overline{e}_{\tau(2)i'} - \overline{e}^*_{\tau(1)i'}$, $d_{\omega_{q_4 \times 1}} = \overline{e}_{\omega(2)j'} - \overline{e}^*_{\omega(1)j'}$ and $d_{\varepsilon_{q_6 \times 1}} = \overline{e}_{\varepsilon(2)k'} - \overline{e}^*_{\varepsilon(1)k'}$ are differences of sampling errors for qualitative variables. Further let $X_{q_1 \times q_1} = \left[\overline{X}_i^{-1}\right]_{q_1 \times q_1}$;
$i = 1, 2, 3...q_1$, $Z_{q_3 \times q_3} = \left[\overline{Z}_j^{-1}\right]_{q_3 \times q_3}$; $j = 1, 2, 3...q_3$ and $W_{q_5 \times q_5} = \left[\overline{W}_k^{-1}\right]_{q_5 \times q_5}$; $k = 1, 2, 3...q_5$
are diagonal matrices for quantitative variables and $\Phi_{q_2 \times q_2} = \left[\Phi_{i'}^{-1}\right]_{q_2 \times q_2}$; $i' = 1, 2, 3...q_2$,
$\Psi_{q_4 \times q_4} = \left[\Psi_{j'}^{-1}\right]_{q_4 \times q_4}$; $j' = 1, 2, 3...q_2$ and $E_{q_6 \times q_6} = \left[E_{k'}^{-1}\right]_{q_6 \times q_6}$; $k' = 1, 2, 3...q_6$ are
diagonal matrices for qualitative variables.

The generalized class of regression-cum-ratio-exponential estimators for estimating population mean in the presence of non-response at first phase using mixture of mlti-auxiliary variables can be suggested as

$$t_{mix} = t_1 \ t_2 + t_3 \tag{2.1}$$

where

$$t_1 = \left[\eta \overline{y}_2 + a \sum_{i=1}^{q_1} \alpha_{1i} \left(\overline{x}^*_{1\,i} - \overline{x}_{(2)i}\right) + b \sum_{i'=1}^{q_2} \alpha_{2i'} \left(\tau^*_{1\,i'} - \tau_{(2)i'}\right)\right], \tag{2.2}$$

$$t_2 = c \left[\prod_{j=1}^{q_3} \left(\frac{\overline{z}^*_{(1)j}}{\overline{z}_{(2)j}}\right)^{d\alpha_{3j}} \prod_{j'=1}^{q_4} \left(\frac{\omega^*_{1\,j'}}{\omega_{(2)j'}}\right)^{h\alpha_{4j'}}\right] \tag{2.3}$$

and

$$t_3 = e \exp\left[\sum_{k=1}^{q_5} f\alpha_{5k} \left(\frac{\overline{w}_{(2)k} - \overline{w}^*_{(1)k}}{\overline{w}_{(2)k} + \overline{w}^*_{(1)k}}\right) + \sum_{k'=1}^{q_6} l\alpha_{6k'} \left(\frac{\varepsilon_{(2)k'} - \varepsilon^*_{(1)k'}}{\varepsilon_{(2)k'} + \varepsilon^*_{(1)k'}}\right)\right]. \tag{2.4}$$

For deriving the expression of bias for estimators first of all we write (2.2), (2.3) and (2.4) in form of sampling errors as

$$t_1 = \left[\eta \ \overline{e}_{y(2)} + \overline{Y} - a \sum_{i=1}^{q_1} \alpha_{(1)i} \left(\overline{e}_{x(2)i} - \overline{e}^*_{x(1)i}\right) - b \sum_{i'=1}^{q_2} \alpha_{2\,i'} \left(\overline{e}_{\tau(2)i'} - \overline{e}^*_{\tau(1)i'}\right)\right], \tag{2.5}$$

$$t_2 = c \left[1 + d \sum_{j=1}^{q_3} \frac{\alpha_{(3)j}}{Z_j} \left(\overline{e}^*_{z(1)j} - \overline{e}_{z(2)j}\right) + h \sum_{j'=1}^{q_4} \frac{\alpha_{(4)j'}}{\Psi_{j'}} \left(\overline{e}^*_{\omega(1)j'} - \overline{e}_{\omega(2)j'}\right)\right] \tag{2.6}$$

and

$$t_3 = e \exp\left[\sum_{k=1}^{q_5} \frac{f\alpha_{(5)k}}{2\overline{W}_k} \left(\overline{e}_{w(2)k} - \overline{e}^*_{w(1)k}\right) - \left(\overline{e}_{w(2)k} - \overline{e}^*_{w(1)k}\right) \left(\frac{\overline{e}_{w(2)k} + \overline{e}^*_{w(1)k}}{2\overline{W}_k}\right)\right]$$

$$+\sum_{k'=1}^{q_6} \frac{l\alpha_{(6)k'}}{2E_{k'}} \left[\bar{e}_{\varepsilon(2)k'} - \bar{e}^*_{\varepsilon(1)k'} \; - \; \overline{e_{\varepsilon(2)k'} - \bar{e}^*_{\varepsilon(1)k'}} \left(\frac{\bar{e}_{\varepsilon(2)k'} + \bar{e}^*_{\varepsilon(1)k'}}{2E_{k'}} \right) \right] \quad (2.7)$$

In matrics form the eq (2.5),(2.6) and (2.7) can be written as

$$t_1 = \left[\eta\, \bar{e}_{y(2)} + \overline{Y} \; -a\alpha_1^t{}_{1\times q_1}\, d_{x_{q_1\times 1}} -b\alpha_2^t{}_{1\times q_2}\, d_{\tau_{q_2\times 1}} \right] \quad (2.8)$$

$$t_2 = c\left[1 - d\alpha_3^t{}_{1\times q_3}\, Z_{q_3\times q_3} d_{z_{q_3\times 1}} -h\alpha_4^t{}_{1\times q_4}\, \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}} \right], \quad (2.9)$$

and

$$t_3 = e\left[1 + \frac{f}{2}\alpha_5^t{}_{1\times q_5}\, W_{q_5\times q_5} d_{w_{q_5\times 1}} +\frac{l}{2}\alpha_6^t{}_{1\times q_6}\, E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}} \right]. \quad (2.10)$$

Substituting (2.8), (2.9) and (2.10) in (2.1), we get

$$t_{mix} = \left[\eta\, \bar{e}_{y_{(2)}} +\overline{Y} \; -a\alpha_1^t{}_{1\times q_1}\, d_{x_{q_1\times 1}} -b\alpha_2^t{}_{1\times q_2}\, d_{\tau_{q_2\times 1}} \right]$$

$$\left[1 -cd\alpha_3^t{}_{1\times q_3}\, Z_{q_3\times q_3} d_{z_{q_3\times 1}} -ch\alpha_4^t{}_{1\times q_4}\, \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}} \right.$$

$$\left. +2^{-1}ef\,\alpha_5^t{}_{1\times q_5}\, W_{q_5\times q_5} d_{w_{q_5\times 1}} +2^{-1}el\alpha_6^t{}_{1\times q_6}\, E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}} \right]$$

or

$$t_{mix} = \eta\bar{e}_{y_{(2)}} -\eta\bar{e}_{y_{(2)}} cd\alpha_3^t{}_{1\times q_3}\, Z_{q_3\times q_3} d_{z_{q_3\times 1}}$$

$$-\eta\bar{e}_{y_{(2)}} ch\alpha_4^t{}_{1\times q_4}\, \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}} +\eta\bar{e}_{y_{(2)}} 2^{-1}ef\,\alpha_5^t{}_{1\times q_5}\, W_{q_5\times q_5} d_{w_{q_5\times 1}}$$

$$+ \eta\bar{e}_{y_{(2)}} 2^{-1}el\alpha_6^t{}_{1\times q_6}\, E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}} +\eta\overline{Y}$$

$$-\eta\overline{Y}cd\alpha_3^t{}_{1\times q_3}\, Z_{q_3\times q_3} d_{z_{q_3\times 1}} -\eta\overline{Y}ch\alpha_4^t{}_{1\times q_4}\, \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}}$$

$$+\eta\overline{Y}2^{-1}ef\,\alpha_5^t{}_{1\times q_5}\, W_{q_5\times q_5} d_{w_{q_5\times 1}}$$

$$+\eta\overline{Y}\, 2^{-1}el\alpha_6^t{}_{1\times q_6}\, E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}} -a\alpha_1^t{}_{1\times q_1}\, d_{x_{q_1\times 1}}$$

$$+acd\alpha_1^t{}_{1\times q_1}\, d_{x_{q_1\times 1}} \alpha_3^t{}_{1\times q_3}\, Z_{q_3\times q_3} d_{z_{q_3\times 1}}$$

$$+ach\alpha_1^t{}_{1\times q_1}\, d_{x_{q_1\times 1}} \alpha_4^t{}_{1\times q_4}\, \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}}$$

$$-2^{-1}aef\,\alpha_1^t{}_{1\times q_1}\, d_{x_{q_1\times 1}} \alpha_5^t{}_{1\times q_5}\, W_{q_5\times q_5} d_{w_{q_5\times 1}}$$

$$-2^{-1}ael\alpha_1^t{}_{1\times q_1}\, d_{x_{q_1\times 1}} \alpha_6^t{}_{1\times q_6}\, E_{q_6\times q_6} d_{\varepsilon_{q_6\times 1}}$$

$$-b\alpha_2^t{}_{1\times q_2}\, d_{\tau_{q_2\times 1}} -bcd\alpha_2^t{}_{1\times q_2}\, d_{\tau_{q_2\times 1}} \alpha_3^t{}_{1\times q_3}\, Z_{q_3\times q_3} d_{z_{q_3\times 1}}$$

$$+bch\alpha_2^t{}_{1\times q_2}\, d_{\tau_{q_2\times 1}} \alpha_4^t{}_{1\times q_4}\, \Psi_{q_4\times q_4} d_{\omega_{q_4\times 1}}$$

$$-2^{-1}bef\,\alpha_2^{t}{}_{1\times q_2}\;d_{\tau_{q2\times1}}\;\alpha_5^{t}{}_{1\times q_5}\;W_{q_5\times q_5}\,d_{w_{q5\times1}}$$

$$-2^{-1}bel\,\alpha_2^{t}{}_{1\times q_2}\;d_{\tau_{q2\times1}}\;\alpha_6^{t}{}_{1\times q_6}\;E_{q_6\times q_6}\,d_{\varepsilon_{q6\times1}}$$

ignoring second and higher order terms , we have

$$t_{mix}=\eta\;\bar{e}_{y_{(2)}}+\bar{Y}\;-a\alpha_1^{t}{}_{1\times q_1}\;d_{x_{q1\times1}}-b\alpha_2^{t}{}_{1\times q_2}\;d_{\tau_{q2\times1}}$$

$$-\eta\bar{Y}cd\,\alpha_3^{t}{}_{1\times q_3}\;Z_{q_3\times q_3}\,d_{z_{q3\times1}}-\eta\bar{Y}ch\alpha_4^{t}{}_{1\times q_4}\;\Psi_{q_4\times q_4}\,d_{\omega_{q4\times1}}$$

$$+2^{-1}\eta\bar{Y}ef\,\alpha_5^{t}{}_{1\times q_5}\;W_{q_5\times q_5}\,d_{w_{q5\times1}}+2^{-1}m\bar{Y}el\alpha_6^{t}{}_{1\times q_6}\;E_{q_6\times q_6}\,d_{\varepsilon_{q6\times1}}$$

or

$$t_{mix}-\bar{Y}=\eta\bar{e}_{y_{(2)}}+(\eta-1)\bar{Y}-a\alpha_1^{t}{}_{1\times q_1}\;d_{x_{q1\times1}}$$

$$-b\alpha_2^{t}{}_{1\times q_2}\;d_{\tau_{q2\times1}}-\eta\bar{Y}cd\,\alpha_3^{t}{}_{1\times q_3}\;Z_{q_3\times q_3}\,d_{z_{q3\times1}}$$

$$-\eta\bar{Y}ch\alpha_4^{t}{}_{1\times q_4}\;\Psi_{q_4\times q_4}\,d_{\omega_{q4\times1}}+2^{-1}\eta\bar{Y}ef\,\alpha_5^{t}{}_{1\times q_5}\;W_{q_5\times q_5}\,d_{w_{q5\times1}}$$

$$+2^{-1}\eta\bar{Y}el\alpha_6^{t}{}_{1\times q_6}\;E_{q_6\times q_6}\,d_{\varepsilon_{q6\times1}}$$

or

$$t_{mix}-\bar{Y}=\eta\bar{e}_{y_{(2)}}+(\eta-1)\bar{Y}-h^{t}{}_{1\times m}H_{m\times1},\tag{2.11}$$

where

$$h^{t}=\begin{bmatrix}\alpha_1^{t}{}_{1\times q_1}&\alpha_2^{t}{}_{1\times q_2}&\alpha_3^{t}{}_{1\times q_3}&\alpha_4^{t}{}_{1\times q_4}&\alpha_5^{t}{}_{1\times q_5}&\alpha_6^{t}{}_{1\times q_6}\end{bmatrix}_{1\times m}$$

and

$$H^{t}=\begin{bmatrix}ad_{x_{q1\times1}}&bd_{\tau_{q2\times1}}&\eta\bar{Y}cdZ_{q_3\times q_3}\,d_{z_{q3\times1}}&\eta\bar{Y}ch\Psi_{q_4\times q_4}\,d_{\omega_{q4\times1}}\end{bmatrix}$$

$$\begin{bmatrix}-2^{-1}\eta\bar{Y}efW_{q_5\times q_5}\,d_{w_{q5\times1}}&-2^{-1}\eta\bar{Y}elE_{q_6\times q_6}\,d_{\varepsilon_{q6\times1}}\end{bmatrix}_{1\times m}$$

with

$$\sum_{i=1}^{6}q_i=m\,.$$

From (2.11)

$$E(t_{mix}-\bar{Y})^2=E(\eta\bar{e}_{y_{(2)}}+(\eta-1)\bar{Y}-h^{t}{}_{1\times m}H_{m\times1})^2\tag{2.12}$$

Differentiating (2.11) w.r.t $h_{m\times1}$ and equating it equal to zero, we get

$$\eta E(H_{m\times1}\bar{e}_{y_{(2)}})+(\eta-1)\bar{Y}E(H_{m\times1})-h_{m\times1}E(H_{m\times1}H^{t}{}_{1\times m})=0\tag{2.13}$$

or

$$\eta\Omega_{m\times1}-h_{m\times1}\Lambda_{m\times m}=0$$

or

$$h_{m\times1}=\eta\Lambda^{-1}{}_{m\times m}\Omega_{m\times1},$$

where $\Omega_{m\times 1} = E(H_{m\times 1}\bar{e}_{y_{(2)}})$ and $\Lambda_{m\times m} = E(H_{m\times 1}H^t_{1\times m}) = \left[\Delta_{ij}\right]_{m\times m}$ with

$$\Delta_{11} = aE(d_{q_1\times 1}d^t_{1\times q_1})a^t = a^2\Delta_{q_1\times q_1},$$

$$\Delta_{12} = aE(d_{q_1\times 1}d^t_{1\times q_2})b = ab\Delta_{q_1\times q_2},$$

$$\Delta_{13} = acd\bar{Y}E(d_{q_1\times 1}d^t_{1\times q_3})Z_{q_3\times q_3} = acd\bar{Y}\Delta_{q_1\times q_3}Z_{q_3\times q_3},$$

$$\Delta_{14} = \bar{Y}achE(d_{q_1\times 1}d^t_{1\times q_4})\Psi_{q_4\times q_4} = \bar{Y}ach\Delta_{q_1\times q_4}\Psi_{q_4\times q_4},$$

$$\Delta_{15} = -aef2^{-1}\bar{Y}E(d_{q_1\times 1}d^t_{1\times q_5})W_{q_5\times q_5} = -aef2^{-1}\bar{Y}\Delta_{q_1\times q_5}W_{q_5\times q_5},$$

$$\Delta_{16} = -ael2^{-1}\bar{Y}E(d_{q_1\times 1}d^t_{1\times q_3})Z_{q_3\times q_3} = -ael2^{-1}\bar{Y}_{q_1\times q_3}Z_{q_3\times q_3},$$

$$\Delta_{22} = bE(d_{q_2\times 1}d^t_{1\times q_2})b^t = b^2\Delta_{q_2\times q_2},$$

$$\Delta_{23} = bcd\bar{Y}E(d_{q_2\times 1}d^t_{1\times q_3})Z_{q_3\times q_3} bcd\bar{Y}\Delta_{q_2\times q_3}Z_{q_3\times q_3},$$

$$\Delta_{24} = \bar{Y}bchE(d_{q_2\times 1}d^t_{1\times q_4})\Psi_{q_4\times q_4} = \bar{Y}bch\Delta_{q_2\times q_4}\Psi_{q_4\times q_4},$$

$$\Delta_{25} = -bef2^{-1}\bar{Y}E(d_{q_2\times 1}d^t_{1\times q_5})W_{q_5\times q_5} = -bef2^{-1}\bar{Y}\Delta_{q_2\times q_5}W_{q_5\times q_5},$$

$$\Delta_{26} = -bel2^{-1}\bar{Y}E(d_{q_2\times 1}d^t_{1\times q_6})E_{q_6\times q_6} = -bel2^{-1}\bar{Y}\Delta_{q_2\times q_6}E_{q_6\times q_6},$$

$$\Delta_{33} = (cd\bar{Y})^2 E(d_{q_3\times 1}d^t_{1\times q_3})Z^2_{q_3\times q_3} = (cd\bar{Y})^2\Delta_{q_3\times q_3}Z^2_{q_3\times q_3},$$

$$\Delta_{34} = (\bar{Y}c)^2 dhZ_{q_3\times q_3}E(d_{q_3\times 1}d^t_{1\times q_4})\Psi_{q_4\times q_4} = (\bar{Y}c)^2 dhZ_{q_3\times q_3}\Delta_{q_3\times q_4}\Psi_{q_4\times q_4},$$

$$\Delta_{35} = -cdef2^{-1}\bar{Y}^2 Z_{q_3\times q_3}E(d_{q_3\times 1}d^t_{1\times q_5})W_{q_5\times q_5} = -cdef2^{-1}\bar{Y}^2 Z_{q_3\times q_3}\Delta_{q_3\times q_5}W_{q_5\times q_5},$$

$$\Delta_{36} = -cdel2^{-1}\bar{Y}^2 Z_{q_3\times q_3}E(d_{q_3\times 1}d^t_{1\times q_6})E_{q_6\times q_6} = -cdel2^{-1}\bar{Y}^2 Z_{q_3\times q_3}\Delta_{q_3\times q_6}E_{q_6\times q_6},$$

$$\Delta_{44} = (ch\bar{Y})^2 E(d_{q_4\times 1}d^t_{1\times q_4})\Psi^2_{q_4\times q_4} = (ch\bar{Y})^2\Delta_{q_4\times q_4}\Psi^2_{q_4\times q_4},$$

$$\Delta_{45} = -chef2^{-1}\bar{Y}^2\Psi_{q_4\times q_4}E(d_{q_4\times 1}d^t_{1\times q_5})W_{q_5\times q_5} = -chef2^{-1}\bar{Y}^2\Psi_{q_4\times q_4}\Delta_{q_4\times q_5}W_{q_5\times q_5},$$

$$\Delta_{46} = -ehcl2^{-1}\bar{Y}^2\Psi_{q_4\times q_4}E(d_{q_4\times 1}d^t_{1\times q_6})E_{q_6\times q_6} = -ehcl2^{-1}\bar{Y}^2\Psi_{q_4\times q_4}\Delta_{q_4\times q_6}E_{q_6\times q_6},$$

$$\Delta_{55} = ef2^{-1}{}^2 W_{q_5\times q_5}\bar{Y}^2 E(d_{q_5\times 1}d^t_{1\times q_5}) = ef2^{-1}{}^2 W_{q_5\times q_5}\bar{Y}^2\Delta_{q_5\times q_5},$$

$$\Delta_{56} = e^2 fl4^{-1}\bar{Y}^2 W_{q_5\times q_5}E(d_{q_5\times 1}d^t_{1\times q_6})E_{q_6\times q_6} = e^2 fl4^{-1}\bar{Y}^2 W_{q_5\times q_5}\Delta_{q_5\times q_6}E_{q_6\times q_6},$$

$$\Delta_{66} = efl2^{-1}\bar{Y}E_{q_6\times q_6}{}^2 E(d_{q_6\times 1}d^t_{1\times q_6}) = efl2^{-1}\bar{Y}E_{q_6\times q_6}{}^2\Delta_{q_6\times q_6}$$

and

$$MSE(t_{mix}) = E(\eta\bar{e}_{y_{(2)}} + (\eta-1)\bar{Y} - h^t_{1\times m}H_{m\times 1})(\eta\bar{e}_{y_{(2)}} + (\eta-1)\bar{Y} - h^t_{1\times m}H_{m\times 1}).$$

Using (2.13) the *MSE* is

$$MSE(t_{mix}) = E\ \eta\bar{e}_{y(2)} + (\eta-1)\bar{Y}\ \ \eta\bar{e}_{y(2)} + (\eta-1)\bar{Y} - h^t_{1\times m}H_{m\times 1}\ ,$$

or

$$MSE(t_{mix}) = \eta^2 E\ \bar{e}_{y(2)}{}^2 + (\eta-1)\bar{Y}{}^2 - \eta h^t_{1\times m}E\ H_{m\times 1}\bar{e}_{y(2)}\ .$$

or

$$MSE(t_{mix}) = \eta^2 \ \lambda_2 S_y^2 \ + \ (\eta-1)\overline{Y}^{\,2} - \eta h^t_{1\times m} \Omega_{m\times 1},$$

Substituting $\ h_{m\times 1} = \eta \Lambda^{-1}_{m\times m} \Omega_{m\times 1}$

$$MSE(t_{mix}) = \ \eta^2 - 2\eta + 1 \ \overline{Y}^2 + \eta^2 \ \lambda_2 S_y^2 \ - \Omega_{1\times m} \Lambda^{-1}_{m\times m} \Omega_{m\times 1} \ ,$$

or

$$MSE(t_{mix}) = \ \eta^2 - 2\eta + 1 \ \overline{Y}^2 + \eta^2 \Gamma, \text{where} \quad \Gamma = \ \lambda_2 S_y^2 \ - \Omega_{1\times m} \Lambda^{-1}_{m\times m} \Omega_{m\times 1} \ ,$$

Differentiating *MSE* with respect to *η* and equating to zero, we have

$$2\eta\overline{Y}^{\,2} - 2\overline{Y}^{\,2} + 2\eta\Gamma = 0$$

or

$$\eta = \frac{\overline{Y}^{\,2}}{\overline{Y}^{\,2} + \Gamma} = \ 1 + \Gamma\overline{Y}^{\,-2} \ ^{-1}$$

After substituting the value of *η* the minimum *MSE* will be

$$MSE(t_{mix}) = \left[\left\{ \ 1 + \overline{Y}^{-2}\Gamma \ ^{-1} -1 \right\} \overline{Y} \right] + \ 1 + \overline{Y}^{-2}\Gamma \ ^{-2} \Gamma \qquad (2.14)$$

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3. SOME SPECIAL CASES

Table-1: For Quantitative Auxiliary Variables

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
1	1	1	0	1	0	0	0	0	0	Regression estimator	$t_{(reg)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i} \left(\bar{x}^*_{(1)i} - \bar{x}_{(2)i} \right) \right]$
2	1	0	0	1	1	0	0	0	0	Ratio estimator	$t_{(r)} = \bar{y}_2 \prod_{j=1}^{q_3} \left(\dfrac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}} \right)^{\alpha_{3j}}$
3	1	0	0	0	1	0	1	1	0	Exponential estimator	$t_{(\exp)} = \exp \left\{ \sum_{k=1}^{q_5} \alpha_{5k} \left(\dfrac{\bar{w}_{(2)k} - \bar{w}^*_{(1)k}}{\bar{w}_{(2)k} + \bar{w}^*_{(1)k}} \right) \right\}$
4	1	1	0	1	1	0	0	0	0	Regression-cum ratio estimator	$t_{(rcr)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i} \left(\bar{x}^*_{(1)i} - \bar{x}_{(2)i} \right) \right] \left[\prod_{j=1}^{q_3} \left(\dfrac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}} \right)^{\alpha_{3j}} \right]$
5	1	1	0	1	-1	0	0	0	0	Regression-cum product estimator	$t_{(rcp)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i} \left(\bar{x}^*_{(1)i} - \bar{x}_{(2)i} \right) \right] \left[\prod_{j=1}^{q_3} \left(\dfrac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}} \right)^{-\alpha_{3j}} \right]$
6	1	0	0	1	1	1	1	1	1	Ratio-cum expo ratio estimator	$t_{(rcer)} = \left[\prod_{j=1}^{q_3} \left(\dfrac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}} \right)^{d\alpha_{3j}} + \exp \left\{ \sum_{k=1}^{q_5} \alpha_{5k} \left(\dfrac{\bar{w}_{(2)k} - \bar{w}^*_{(1)k}}{\bar{w}_{(2)k} + \bar{w}^*_{(1)k}} \right) \right\} \right]$
7	1	0	0	1	1	1	1	-1	-1	Ratio-cum expo product estimator	$t_{(rcep)} = \left[\prod_{j=1}^{q_3} \left(\dfrac{\bar{z}^*_{(1)j}}{\bar{z}_{(2)j}} \right)^{d\alpha_{3j}} + \exp \left\{ -\sum_{k=1}^{q_5} \alpha_{5k} \left(\dfrac{\bar{w}_{(2)k} - \bar{w}^*_{(1)k}}{\bar{w}_{(2)k} + \bar{w}^*_{(1)k}} \right) \right\} \right]$

Table-2: For Qualitative Auxiliary Variables

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
1	1	0	1	1	0	0	0	0	0	Regression estimator	$t_{(reg)} = \left[\bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \; \tau^*_{1\,i'} - \tau_{(2)i'} \right]$
2	1	0	0	1	0	1	0	0	0	Ratio estimator	$t_{(r)} = \bar{y}_2 \prod_{j'=1}^{q_4} \left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}} \right)^{\alpha_{4j'}}$
3	1	0	0	0	1	0	1	0	1	Exponential estimator	$t_{(exp)} = \exp\left\{ \sum_{k'=1}^{q_6} \alpha_{6k'} \left(\dfrac{\varepsilon_{(2)k'} - \varepsilon^*_{(1)k'}}{\varepsilon_{(2)k'} + \varepsilon^*_{(1)k'}} \right) \right\}$
4	1	0	1	1	0	1	0	0	0	Regression-cum ratio estimator	$t_{(rcr)} = \left[\bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \; \tau^*_{1\,i'} - \tau_{(2)i'} \right] \left[\prod_{j'=1}^{q_4} \left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}} \right)^{\alpha_{4j'}} \right]$
5	1	0	1	1	0	-1	0	0	0	Regression-cum product estimator	$t_{mix(rcp)} = \left[\bar{y}_2 + \sum_{i'=1}^{q_2} \alpha_{2i'} \; \tau^*_{1\,i'} - \tau_{(2)i'} \right] \left[\prod_{j'=1}^{q_4} \left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}} \right)^{-\alpha_{4j'}} \right]$
6	1	0	0	1	1	1	1	1	1	Ratio-cum expo ratio estimator	$t_{(rcer)} = \left[\prod_{j'=1}^{q_4} \left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}} \right)^{\alpha_{4j'}} + \exp\left\{ \sum_{k'=1}^{q_6} \alpha_{6k'} \left(\dfrac{\varepsilon_{(2)k'} - \varepsilon^*_{(1)k'}}{\varepsilon_{(2)k'} + \varepsilon^*_{(1)k'}} \right) \right\} \right]$
7	1	0	0	1	1	1	1	-1	-1	Ratio-cum expo product estimator	$t_{mix(rcep)} = \left[\prod_{j'=1}^{q_4} \left(\dfrac{\omega^*_{1\,j'}}{\omega_{(2)j'}} \right)^{\alpha_{4j'}} + \exp\left\{ -\sum_{k'=1}^{q_6} \alpha_{6k'} \left(\dfrac{\varepsilon_{(2)k'} - \varepsilon^*_{(1)k'}}{\varepsilon_{(2)k'} + \varepsilon^*_{(1)k'}} \right) \right\} \right]$

Table-3: For Mixture of Auxiliary Variables

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
1	1	1	0	1	1	0	1	1	0	Generalized class using quantitative variables	$t_{quan} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i}\ \bar{x}^*_{(1)i} - \bar{x}_{(2)i} \right] \left[\prod_{i=1}^{q_3} \left(\frac{\bar{z}^*_{(1)i}}{\bar{z}_{(2)i}} \right)^{\alpha_{3i}} + \exp\left\{ \sum_{i=1}^{q_5} \alpha_{5i} \left(\frac{\bar{w}_{(2)i} - \bar{w}^*_{(1)i}}{\bar{w}_{(2)i} + \bar{w}^*_{(1)i}} \right) \right\} \right]$
2	1	0	1	1	0	1	1	0	1	Generalized class using qualitative variables	$t_{qual} = \left[\bar{y}_2 + \sum_{i=1}^{q_2} \alpha_{2i}\ \tau^*_{(1)i} - \tau_{(2)i} \right] \left[\prod_{i=1}^{q_4} \left(\frac{\omega^*_{(1)i}}{\omega_{(2)i}} \right)^{\alpha_{4i}} + \exp\left\{ \sum_{i=1}^{q_6} \alpha_{6i} \left(\frac{\varepsilon_{(2)i} - \varepsilon^*_{(1)i}}{\varepsilon_{(2)i} + \varepsilon^*_{(1)i}} \right) \right\} \right]$
3	1	1	1	1	0	0	0	0	0	Regression estimator for mixture	$t_{mix(reg)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i}\ \bar{x}^*_{(1)i} - \bar{x}_{(2)i} + \sum_{i=1}^{q_2} \alpha_{2i}\ \tau^*_{(1)i} - \tau_{(2)i} \right]$
4	1	0	0	1	1	1	1	0	0	Ratio estimator for mixture	$t_{mix(r)} = \bar{y}_2 \prod_{i=1}^{q_3} \left(\frac{\bar{z}^*_{(1)i}}{\bar{z}_{(2)i}} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega^*_{(1)i}}{\omega_{(2)i}} \right)^{\alpha_{4i}}$
5	1	0	0	0	1	0	1	1	1	Exponential estimator for mixture	$t_{mix(exp)} = \exp\left\{ \sum_{i=1}^{q_5} \alpha_{5i} \left(\frac{\bar{w}_{(2)i} - \bar{w}^*_{(1)i}}{\bar{w}_{(2)i} + \bar{w}^*_{(1)i}} \right) + \sum_{i=1}^{q_6} \alpha_{6i} \left(\frac{\varepsilon_{(2)i} - \varepsilon^*_{(1)i}}{\varepsilon_{(2)i} + \varepsilon^*_{(1)i}} \right) \right\}$
6	1	1	1	1	1	1	0	0	0	Regression-cum ratio for mixture	$t_{mix(rcr)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i}\ \bar{x}^*_{(1)i} - \bar{x}_{(2)i} + \sum_{i=1}^{q_2} \alpha_{2i}\ \tau^*_{(1)i} - \tau_{(2)i} \right] \left[\prod_{i=1}^{q_3} \left(\frac{\bar{z}^*_{(1)i}}{\bar{z}_{(2)i}} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega^*_{(1)i}}{\omega_{(2)i}} \right)^{\alpha_{4i}} \right]$
7	1	1	1	1	-1	-1	0	0	0	Regression-cum product for mixture	$t_{mix(rcp)} = \left[\bar{y}_2 + \sum_{i=1}^{q_1} \alpha_{1i}\ \bar{x}^*_{(1)i} - \bar{x}_{(2)i} + \sum_{i=1}^{q_2} \alpha_{2i}\ \tau^*_{(1)i} - \tau_{(2)i} \right] \left[\prod_{i=1}^{q_3} \left(\frac{\bar{z}^*_{(1)i}}{\bar{z}_{(2)i}} \right)^{-\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega^*_{(1)i}}{\omega_{(2)i}} \right)^{-\alpha_{4i}} \right]$

S#	η	a	b	c	d	h	e	f	l	Estimator type	Estimator name
8	1	0	0	1	1	1	1	1	1	Ratio-cum expo ratio for mixture	$$t_{mix(rcer)} = \left[\prod_{i=1}^{q_3} \left(\frac{\bar{z}_{(1)i}^{*}}{\bar{z}_{(2)i}} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega_{(1)i}^{*}}{\omega_{(2)i}} \right)^{\alpha_{4i}} + \exp\left\{ \sum_{i=1}^{q_5} \alpha_{5i} \left(\frac{\bar{w}_{(2)i} - \bar{w}_{(1)i}^{*}}{\bar{w}_{(2)i} + \bar{w}_{(1)i}^{*}} \right) - \sum_{i=1}^{q_6} \alpha_{6i} \left(\frac{\varepsilon_{(2)i} - \varepsilon_{(1)i}^{*}}{\varepsilon_{(2)i} + \varepsilon_{(1)i}^{*}} \right) \right\} \right]$$
9	1	0	0	0	1	1	1	-1	-1	Ratio-cum expo ratio for mixture	$$t_{mix(rcep)} = \left[\prod_{i=1}^{q_3} \left(\frac{\bar{z}_{(1)i}^{*}}{\bar{z}_{(2)i}} \right)^{\alpha_{3i}} \prod_{i=1}^{q_4} \left(\frac{\omega_{(1)i}^{*}}{\omega_{(2)i}} \right)^{\alpha_{4i}} + \exp\left\{ -\sum_{i=1}^{q_5} \alpha_{5i} \left(\frac{\bar{w}_{(2)i} - \bar{w}_{(1)i}^{*}}{\bar{w}_{(2)i} + \bar{w}_{(1)i}^{*}} \right) - \sum_{i=1}^{q_6} \alpha_{6i} \left(\frac{\varepsilon_{(2)i} - \varepsilon_{(1)i}^{*}}{\varepsilon_{(2)i} + \varepsilon_{(1)i}^{*}} \right) \right\} \right]$$

BAYESIAN ANALYSIS OF GUMBEL TYPE II DISTRIBUTION UNDER NON-INFORMATIVE PRIORS USING ASYMMETRIC LOSS FUNCTIONS

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ABSTRACT

In the recent study, the Bayesian estimation of the single parameter of Gumbel type II distribution has been discussed. Uniform and Jeffreys priors have been assumed under different asymmetric loss functions for estimation. In order to compare the performance of the estimators, the risk associated with each estimate has been calculated. Bayesian credible intervals and Highest Posterior Density (HPD) intervals have been derived by using the posterior distribution under each prior. The Bayesian hypotheses testing have been done using corresponding Bayes factor for testing the hypotheses concerning different values of the parameter. The posterior predictive distribution has been derived to predict the future values of the variable. All the results have been obtained under complete and right censored samples. The inverse transformation technique of simulation along with a real life data has been used to illustrate the numerical aspects of the study.

KEY WORDS

Bayes estimators, posterior predictive intervals (P.P.I), credible intervals (C.I), highest posterior density (HPD) intervals, right censoring, entropy loss function (ELF), LINEX loss function (LLF), squared logarithmic loss function (SLLF) and precautionary loss function (PLF).

1. INTRODUCTION

The Gumbel distribution produced by Emil Julius Gumbel (1891-1966) is used to model the distribution of the maximum (or the minimum) of a number of samples of various distributions. It is useful in predicting the chance that an extreme earthquake, flood or other natural disaster will occur. In hydrology, the Gumbel distribution is used to analyze such variables as monthly and annual maximum values of daily rainfall and river discharge volumes. Kotz and Nadarajah (2000) have given some applications of this distribution.

Chechile (2001) obtained the posterior distribution assuming that the random sample is taken from the Gumbel distribution using the conjugate prior. Wu and Lin (2001) derived an exact confidence interval for the shape parameter and an exact joint confidence region for the shape and scale parameters of the Weibull and Gumbel distributions under censored samples. Corsini et al. (2002) discussed the maximum likelihood (ML) algorithms and Cramer-Rao (CR) bounds for the location and scale

parameters of the Gumbel distribution. Mousa (2002) obtained the Bayesian estimation for the two parameters of the Gumbel distribution based on record values. Miladinovic (2008) investigated kernel density estimation (KDE) and its application to the Gumbel probability distribution. Saleem and Aslam (2009) obtained the maximum likelihood and Bayes estimators of the parameter of Rayleigh distribution under random censored data. Ali et al. (2010) estimated the Laplace model using uniform and Jeffreys prior under different loss functions using complete and censored data. Saleem and Raza (2011) derived the classical and Bayes estimator for the exponential distribution under random censored data.

2. POSTERIOR DISTRIBUTION UNDER COMPLETE SAMPLES

The pdf of the gumbel type II distribution for a random variable X is:

$$f\left(x;\alpha,\beta\right)=\alpha\beta x^{-\left(\alpha+1\right)}e^{-\beta x^{-\alpha}}, \ x>0, \ \alpha>0, \ \beta>0 \tag{1}$$

The likelihood function for a random sample of n observations is:

$$L(\underline{x};\alpha,\beta)=\alpha^{n}\beta^{n}\left(\prod_{i=1}^{n}x_{i}\right)^{-(\alpha+1)}e^{-\beta\sum_{i=1}^{n}x_{i}^{-\alpha}}, \ \text{where} \ \underline{x}=x_{1},x_{2},...,x_{n} \tag{2}$$

The non-informative uniform prior is assumed to be:

$$p\left(\beta\right)\propto 1 \tag{3}$$

The posterior distribution of β given data under uniform prior is:

$$p(\beta|\underline{x})=\frac{\left(\sum_{i=1}^{n}x_{i}^{-\alpha}\right)^{n+1}\beta^{n}e^{-\beta\sum_{i=1}^{n}x_{i}^{-\alpha}}}{\Gamma(n+1)} ; \ \beta>0 \tag{4}$$

Jeffreys Prior for shape parameter of Gumbel type II distribution can be obtained as:

Jeffreys prior is defined as: $p_{j}\propto\sqrt{\left|I\left(\beta\right)\right|}$; here; $p_{j}\propto\sqrt{\left|\beta\right|}=\dfrac{1}{\beta}$ \hfill (5)

The posterior distribution of β given data using Jefferys prior is obtained as:

$$p(\beta|\underline{x})\propto\beta^{n-1}e^{-\beta\sum_{i=1}^{n}x_{i}^{-\alpha}}, \ \beta>0 \tag{6}$$

3. THE BAYES ESTIMATOR AND RISK UNDER COMPLETE SAMPLES

The Bayes estimator and risks under the assumption of uniform and Jeffreys priors using different asymmetric loss functions for complete samples are presented in the followings. Where $\gamma=0.57721$ is an Euler constant.

Loss Function		Uniform Prior	Jeffreys Prior
ELF	Estimator	$\dfrac{n}{\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)}$	$\dfrac{n-1}{\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)}$
	Risk	$\displaystyle\sum_{k=1}^{n}\frac{1}{k}-\gamma-\ln n$	$\displaystyle\sum_{k=1}^{n-1}\frac{1}{k}-\gamma-\ln n-1$
SLLF	Estimator	$\exp E \log\beta$	$\exp E \log\beta$
	Risk	$E \log\beta^2 - E \log\beta^2$	$E \log\beta^2 - E \log\beta^2$
LLF	Estimator	$-(n+1)\ln\left[\dfrac{\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}}{\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}+1}\right]$	$-n\ln\left[\dfrac{\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}}{\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}+1}\right]$
	Risk	$E \beta - \beta_{LLF}$	$E \beta - \beta_{LLF}$
PLF	Estimator	$\left[\dfrac{(n+1)(n+2)}{\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)^2}\right]^{\frac{1}{2}}$	$\left[\dfrac{n(n+1)}{\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)^2}\right]^{\frac{1}{2}}$
	Risk	$\dfrac{2\left[(n+1)(n+2)^{\frac{1}{2}}-(n+1)\right]}{\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)}$	$\dfrac{2\left[n(n+1)^{\frac{1}{2}}-n\right]}{\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)}$

4. BAYESIAN CREDIBLE AND HIGHEST POSTERIOR DENSITY INTERVALS FOR COMPLETE SAMPLES

The Bayesian credible intervals under uniform and Jeffreys priors are:

$$\frac{\chi^2_{1-k/2,\ 2n+1}}{2\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)} < \beta < \frac{\chi^2_{k/2,\ 2n+1}}{2\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)} \quad \text{and} \quad \frac{\chi^2_{1-k/2,\ 2n}}{2\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)} < \beta < \frac{\chi^2_{k/2,\ 2n}}{2\left(\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}\right)}$$

respectively.

where k is level of significance

The HPD interval for β under uniform prior can be obtained by solving the following two equations simultaneously. $\displaystyle\int_{\beta_1}^{\beta_2} p \beta|\underline{x} \, d\beta = 1-k$ and $p \beta_1|\underline{x} = p \beta_2|\underline{x}$ which give:

$$n \ln\left(\frac{\beta_1}{\beta_2}\right) - \sum_{i=1}^{n} x_i^{-\alpha} \ \beta_1 - \beta_2 \ = 0 \tag{7}$$

$$\Gamma\left(n+1, \beta_2 \sum_{i=1}^{n} x_i^{-\alpha}\right) - \Gamma\left(n+1, \beta_1 \sum_{i=1}^{n} x_i^{-\alpha}\right) - k - 1 \ \Gamma(n+1) = 0 \tag{8}$$

The HPD interval under U.P can be obtained by solving (7) and (8) simultaneously.

The HPD interval for β under Jeffreys prior can be obtained by solving the following two equations simultaneously:

$$(n-1) \ln\left(\frac{\beta_1}{\beta_2}\right) - \sum_{i=1}^{n} x_i^{-\alpha} \ \beta_1 - \beta_2 \ = 0 \tag{9}$$

$$\Gamma\left(n, \beta_2 \sum_{i=1}^{n} x_i^{-\alpha}\right) - \Gamma\left(n, \beta_1 \sum_{i=1}^{n} x_i^{-\alpha}\right) - k - 1 \ \Gamma(n) = 0 \tag{10}$$

5. POSTERIOR PREDICTIVE DISTRIBUTION AND PREDICTIVE INTERVALS FOR COMPLETE SAMPLES

The posterior predictive distribution under the assumption of uniform prior can be obtained as:

$$p \ y|x \ = \int_0^\infty p(\beta|\underline{x}) f \ y; \beta \ d\beta \ => \ p \ y|x \ = \frac{\alpha \ n+1 \ \left(\sum_{i=1}^{n} x_i^{-\alpha}\right)^{n+1} y^{-(\alpha+1)}}{\left(\sum_{i=1}^{n} x_i^{-\alpha} + y^{-\alpha}\right)^{n+2}} \tag{11}$$

The posterior predictive interval using U.P can be obtained by solving the following equations:

$$\left(\frac{\sum_{i=1}^{n} x_i^{-\alpha}}{\sum_{i=1}^{n} x_i^{-\alpha} + L^{-\alpha}}\right)^{n+1} = 1 - \frac{k}{2} \ \text{ and } \ \left(\frac{\sum_{i=1}^{n} x_i^{-\alpha}}{\sum_{i=1}^{n} x_i^{-\alpha} + U^{-\alpha}}\right)^{n+1} = \frac{k}{2}$$

where k is level of significance

The posterior predictive distribution under the assumption of Jeffreys prior can be obtained as:

$$p \ y|x \ = \int_0^\infty p(\beta|\underline{x}) f \ y; \beta \ d\beta \ => \ p \ y|x \ = \frac{n\alpha \left(\sum_{i=1}^{n} x_i^{-\alpha}\right)^{n} y^{-(\alpha+1)}}{\left(\sum_{i=1}^{n} x_i^{-\alpha} + y^{-\alpha}\right)^{n+1}} \tag{12}$$

The predictive interval using J.P can be obtained by soling the following two equations:

$$\left(\frac{\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}}{\displaystyle\sum_{i=1}^{n} x_i^{-\alpha} + L^{-\alpha}}\right)^n = 1 - \frac{k}{2} \quad \text{and} \quad \left(\frac{\displaystyle\sum_{i=1}^{n} x_i^{-\alpha}}{\displaystyle\sum_{i=1}^{n} x_i^{-\alpha} + U^{-\alpha}}\right)^n = \frac{k}{2}$$

6. LIKELIHOOD FUNCTION UNDER RIGHT CENSORED SAMPLES

Let $f_T\ t;\beta$ and $R_T\ t;\beta$ be pdf and survivor function of random variable T, respectively, where β is an unknown parameter. Assuming T to follow a Gumbel type II distribution, so that

$$f_T\ t;\beta\ = \alpha\beta t^{-(\alpha+1)} e^{-\beta t^{-\alpha}}; \text{ where } t > 0 \text{ and } \beta > 0 \tag{13}$$

Assuming that the censoring variable X is observed, let Y represent an observation (whether it is failure or a censored observation). Then for a random sample of n–r failures y_1, y_2, …, y_{n-r} and the right censored observations y_{n-r+1}, y_{n-r+2}, …, y_n the likelihood function, as discussed by Phillips (2004) is:

$$L\ \underline{y}|\beta,\lambda\ = \left[\alpha^n \lambda^r \beta^{n-r}\left\{\prod_{i=1}^{n} y_i^{-(\alpha+1)}\right\} e^{-(\beta+\lambda)\sum_{i=1}^{n} y_i^{-\alpha}}\right] \tag{14}$$

7. POSTERIOR DISTRIBUTION UNDER CENSORED DATA

Under the assumption of uniform prior the posterior distribution for β,λ is:

$$p\ \beta,\lambda|\underline{y}\ = \frac{\left(\displaystyle\sum_{i=1}^{n} y_i^{-\alpha}\right)^{n-r+1}\left(\displaystyle\sum_{i=1}^{n} y_i^{-\alpha}\right)^{r+1}}{\Gamma(n-r+1)\Gamma(r+1)}\lambda^r \beta^{n-r} e^{-(\beta+\lambda)\sum_{i=1}^{n} y_i^{-\alpha}} \tag{15}$$

The marginal distributions of λ and β are:

$$p\ \lambda|\underline{y}\ = \frac{\left(\displaystyle\sum_{i=1}^{n} y_i^{-\alpha}\right)^{r+1}}{\Gamma(r+1)}\lambda^r e^{-\lambda\sum_{i=1}^{n} y_i^{-\alpha}} \tag{16}$$

$$p\ \beta|\underline{y}\ = \frac{\left(\displaystyle\sum_{i=1}^{n} y_i^{-\alpha}\right)^{n-r+1}}{\Gamma(n-r+1)}\beta^{n-r} e^{-\beta\sum_{i=1}^{n} y_i^{-\alpha}} \tag{17}$$

The posterior distribution under Jeffreys prior (assuming independence) is:

$$p\ \beta,\lambda|\underline{y}\ = \frac{\left(\displaystyle\sum_{i=1}^{n} y_i^{-\alpha}\right)^{n-r}\left(\displaystyle\sum_{i=1}^{n} y_i^{-\alpha}\right)^{r}}{\Gamma\ n-r\ \Gamma\ r}\lambda^{r-1}\beta^{n-r-1} e^{-(\beta+\lambda)\sum_{i=1}^{n} y_i^{-\alpha}} \tag{18}$$

The marginal distribution of β and λ are:

$$p\left(\beta\middle|\underline{y}\right) = \frac{\left(\sum_{i=1}^{n} y_i^{-\alpha}\right)^{n-r}}{\Gamma(n-r)}\beta^{n-r-1}e^{-\beta\sum_{i=1}^{n} y_i^{-\alpha}} \tag{19}$$

$$p\left(\lambda\middle|\underline{y}\right) = \frac{\left(\sum_{i=1}^{n} y_i^{-\alpha}\right)^{r}}{\Gamma(r)}\lambda^{r-1}e^{-\lambda\sum_{i=1}^{n} y_i^{-\alpha}} \tag{20}$$

8. THE BAYES ESTIMATOR AND RISK UNDER UNIFORM PRIOR USING CENSORED DATA

The Bayes estimator and risks under uniform prior using different loss functions for censored data are:

Loss Function		Parameter β	Parameter λ
ELF	Estimator	$\dfrac{n-r}{\sum_{i=1}^{n} y_i^{-\alpha}}$	$\dfrac{r}{\sum_{i=1}^{n} y_i^{-\alpha}}$
	Risk	$\sum_{k=1}^{n-r}\dfrac{1}{k}-\gamma-\ln(n-r)$	$\sum_{k=1}^{r}\dfrac{1}{k}-\gamma-\ln(r)$
LLF	Estimator	$-(n-r+1)\ln\left(\dfrac{\sum_{i=1}^{n} y_i^{-\alpha}}{\sum_{i=1}^{n} y_i^{-\alpha}+1}\right)$	$-(r+1)\ln\left(\dfrac{\sum_{i=1}^{n} y_i^{-\alpha}}{\sum_{i=1}^{n} y_i^{-\alpha}+1}\right)$
	Risk	$E(\beta)-\beta_{LLF}$	$E(\lambda)-\lambda_{LLF}$
PLF	Estimator	$\left[\dfrac{(n-r+1)(n-r+2)}{\left(\sum_{i=1}^{n} y_i^{-\alpha}\right)^2}\right]^{\frac{1}{2}}$	$\left[\dfrac{(r+1)(r+2)}{\left(\sum_{i=1}^{n} y_i^{-\alpha}\right)^2}\right]^{\frac{1}{2}}$
	Risk	$2\left(\beta_{PLF}-E(\beta)\right)$	$2\left(\lambda_{PLF}-E(\lambda)\right)$

9. THE BAYES ESTIMATOR AND RISK UNDER JEFFREYS PRIOR USING CENSORED DATA

The Bayes estimator and risks under Jeffreys prior using different loss functions for censored data are:

Loss Function	Parameter β	Parameter λ

ELF	Estimator	$\dfrac{n-r-1}{\sum\limits_{i=1}^{n} y_i^{-\alpha}}$	$\dfrac{r-2}{\sum\limits_{i=1}^{n} y_i^{-\alpha}}$
	Risk	$\sum\limits_{k=1}^{n-r-1}\dfrac{1}{k}-\gamma-\ln\ n-r-1$	$\sum\limits_{k=1}^{r-2}\dfrac{1}{k}-\gamma-\ln\ r-2$
LLF	Estimator	$-\ n-r\ \ln\left(\dfrac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha}+1}\right)$	$-\ r\ \ln\left(\dfrac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha}+1}\right)$
	Risk	$E\ \beta\ -\beta_{LLF}$	$E\ \lambda\ -\lambda_{LLF}$
PLF	Estimator	$\left[\dfrac{(n-r+1)(n-r)}{\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)^2}\right]^{\frac{1}{2}}$	$\left[\dfrac{r(r+1)}{\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)^2}\right]^{\frac{1}{2}}$
	Risk	$2\ \beta_{PLF}-E\ \beta$	$2\ \lambda_{PLF}-E\ \lambda$

10. BAYESIAN CREDIBLE AND HIGHEST POSTERIOR DENSITY INTERVALS FOR CENSORED DATA

The Bayesian credible intervals for λ and β under uniform prior are:

$$\frac{\chi^2_{1-k/2\ \ 2\ n-r+1}}{2\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)} < \beta < \frac{\chi^2_{k/2\ \ 2\ n-r+1}}{2\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)}\ \ \text{and}\ \ \frac{\chi^2_{1-k/2\ \ 2\ r+1}}{2\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)} < \lambda < \frac{\chi^2_{k/2\ \ 2\ r+1}}{2\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)}$$

The Bayesian HPD interval for λ can be obtained by solving the following two equations simultaneously

$$r\ln\left(\frac{\lambda_1}{\lambda_2}\right)-(\lambda_1-\lambda_2)\sum\limits_{i=1}^{n} y_i^{-\alpha}=0 \tag{21}$$

$$\Gamma\left(r+1,\lambda_1\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)-\Gamma\left(r+1,\lambda_2\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)-(k-1)\Gamma(r+1)=0 \tag{22}$$

The Bayesian HPD interval for β can be obtained by solving the following two equations simultaneously

$$n-r\ \ln\left(\frac{\beta_1}{\beta_2}\right)-\sum\limits_{i=1}^{n} y_i^{-\alpha}\ \beta_1-\beta_2\ =0 \tag{23}$$

$$\Gamma\left\{n-r+1,\beta_1\sum\limits_{i=1}^{n} y_i^{-\alpha}\right\}-\Gamma\left\{n-r+1,\beta_2\sum\limits_{i=1}^{n} y_i^{-\alpha}\right\}-(1-k)\Gamma(n-r+1)=0 \tag{24}$$

The credible intervals for β and λ under Jeffreys prior are:

$$\frac{\chi^2_{1-k/2, \; 2n-r}}{2\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)} < \beta < \frac{\chi^2_{k/2, \; 2n-r}}{2\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)} \quad \text{and} \quad \frac{\chi^2_{1-k/2, \; 2r}}{2\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)} < \lambda < \frac{\chi^2_{k/2, \; 2r}}{2\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)}$$

The HPD interval for β under Jeffreys prior can be obtained by solving the following two equations simultaneously.

$$n - r - 1 \; \ln\left(\frac{\beta_1}{\beta_2}\right) - \sum\limits_{i=1}^{n} y_i^{-\alpha} \; \beta_1 - \beta_2 \; = 0 \tag{25}$$

$$\Gamma\left\{n - r, \beta_1 \sum\limits_{i=1}^{n} y_i^{-\alpha}\right\} - \Gamma\left\{n - r, \beta_2 \sum\limits_{i=1}^{n} y_i^{-\alpha}\right\} - (1 - k)\Gamma(n - r) = 0 \tag{26}$$

The HPD interval for λ under Jeffreys can be obtained by solving the following two equations simultaneously:

$$r - 1 \; \ln\left(\frac{\lambda_1}{\lambda_2}\right) - (\lambda_1 - \lambda_2)\sum\limits_{i=1}^{n} y_i^{-\alpha} = 0 \tag{27}$$

$$\Gamma\left(r, \lambda_1 \sum\limits_{i=1}^{n} y_i^{-\alpha}\right) - \Gamma\left(r, \lambda_2 \sum\limits_{i=1}^{n} y_i^{-\alpha}\right) - (1 - k)\Gamma(r + 1) = 0 \tag{28}$$

11. POSTERIOR PREDICTIVE DISTRIBUTION AND PREDICTIVE INTERVALS FOR CENSORED DATA

The posterior predictive distribution using posterior distribution of β under the assumption of uniform prior is:

$$p(x \mid y) = \int\limits_{0}^{\infty} p\; \beta \mid \underline{y} \; p(y; \beta)d\beta \; => p(x \mid y) = \frac{\alpha(n - r + 1)\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)^{n-r+1} x^{-(\alpha+1)}}{\left(\sum\limits_{i=1}^{n} y_i^{-\alpha} + x^{-\alpha}\right)^{n-r+2}} \tag{29}$$

The posterior predictive interval using posterior distribution of β under uniform prior can be obtained as:

$$\left(\frac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha} + L^{-\alpha}}\right)^{n-r+1} = 1 - \frac{k}{2} \quad \text{and} \quad \left(\frac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha} + U^{-\alpha}}\right)^{n-r+1} = \frac{k}{2}$$

where k is level of significance

The posterior predictive distribution using posterior distribution of λ under the assumption of uniform prior is:

$$p(x|y) = \int_0^\infty p\ \lambda|y\ f\ x;\lambda\ d\lambda \Rightarrow p\ y|x\ = \frac{\alpha(r+1)\left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)^{r+1}}{x^{(\alpha+1)}\left(\sum\limits_{i=1}^{n} y_i^{-\alpha} + x^{-\alpha}\right)^{r+2}} \quad (30)$$

The predictive interval using posterior distribution of λ under uniform prior can be obtained by solving the following two equations:

$$\left(\frac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha} + L^{-\alpha}}\right)^{r+1} = 1 - \frac{k}{2} \quad \text{and} \quad \left(\frac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha} + U^{-\alpha}}\right)^{r+1} = \frac{k}{2}$$

The posterior predictive distribution using posterior distribution of β under Jeffreys prior is:

$$p\ x|y = \int_0^\infty p\ \beta|y\ f\ x;\beta\ d\beta \Rightarrow p\ x|y\ = \frac{\alpha\ n-r\ \left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)^{n-r} x^{-(\alpha+1)}}{\left(\sum\limits_{i=1}^{n} y_i^{-\alpha} + x^{-\alpha}\right)^{n-r+1}} \quad (31)$$

The posterior predictive distribution using posterior distribution of λ under Jeffreys prior is:

$$p\ x|y = \int_0^\infty p\ \lambda|y\ f\ x;\lambda\ d\lambda \Rightarrow p\ x|y\ = \frac{\alpha\ r\ \left(\sum\limits_{i=1}^{n} y_i^{-\alpha}\right)^{r} x^{-(\alpha+1)}}{\left(\sum\limits_{i=1}^{n} y_i^{-\alpha} + x^{-\alpha}\right)^{r+1}} \quad (32)$$

The posterior predictive interval using posterior distribution of β under Jeffreys prior can be obtained by solving the following two equations:

$$\left(\frac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha} + L^{-\alpha}}\right)^{n-r} = 1 - \frac{k}{2} \quad \text{and} \quad \left(\frac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha} + U^{-\alpha}}\right)^{n-r} = \frac{k}{2}$$

The posterior predictive interval using posterior distribution of λ under Jeffreys prior can be obtained by solving the following two equations:

$$\left(\frac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha} + L^{-\alpha}}\right)^{r} = 1 - \frac{k}{2} \quad \text{and} \quad \left(\frac{\sum\limits_{i=1}^{n} y_i^{-\alpha}}{\sum\limits_{i=1}^{n} y_i^{-\alpha} + U^{-\alpha}}\right)^{r} = \frac{k}{2}$$

12. SIMULATION STUDY

In this section, the results of simulation for the Bayes estimates and corresponding risks for different loss functions using complete and censored data have been presented. The simulation has been carried out for $\beta = 5$ and for sample sizes 50, 150, 300 and 500 under 10,000 replications. The simulation has also been done for $\beta = 10$, 15 and 20 and for sample sizes 100, 200 and 400 but the results have not been presented; however, the results follow the similar patterns. The risks associated with each estimate have been presented in the brackets. The Bayesian credible intervals and posterior predictive intervals have also been calculated.

Table 1:
Simulation using B. E and Risks under U. P and J.P for Complete Data (β= 5)

Sample size	Uniform Prior				Jeffreys Prior			
	ELF	SLLF	LLF	PLF	ELF	SLLF	LLF	PLF
50	5.8884	5.9474	5.6780	6.0647	5.7706	5.8290	5.5667	5.9470
	(0.0100)	(0.0198)	(0.3281)	(0.0586)	(0.0102)	(0.0205)	(0.3217)	(0.0586)
150	4.6666	4.6822	4.6262	4.7133	4.6355	4.6511	4.5955	4.6822
	(0.0033)	(0.0066)	(0.0716)	(0.0155)	(0.0034)	(0.0067)	(0.0711)	(0.0155)
300	5.2734	5.2822	5.2451	5.2998	5.2559	5.2646	5.2276	5.2822
	(0.0017)	(0.0033)	(0.0460)	(0.0088)	(0.0017)	(0.0034)	(0.0458)	(0.0088)
500	5.0325	5.0375	5.0173	5.0476	5.0224	5.0264	5.0073	5.0375
	(0.0010)	(0.0020)	(0.0252)	(0.0050)	(0.0010)	(0.0027)	(0.0252)	(0.0050)

Table 2:
Simulation using B. E and Risks under U. P and J.P for 10% Censored (β= 5)

Sample size	Uniform Prior				Jeffreys Prior			
	ELF	SLLF	LLF	PLF	ELF	SLLF	LLF	PLF
50	6.3484	6.4134	6.0332	6.5262	6.4895	6.5659	6.1931	6.7129
	(0.0111)	(0.0220)	(0.4563)	(0.0734)	(0.0108)	(0.0225)	(0.4464)	(0.0734)
150	4.6351	4.7452	4.5880	4.6783	4.5688	4.6610	4.5219	4.6116
	(0.0037)	(0.0074)	(0.0815)	(0.0175)	(0.0037)	(0.0074)	(0.0809)	(0.0175)
300	5.3069	5.3356	5.2934	5.3315	5.3265	5.3159	5.2739	5.3512
	(0.0019)	(0.0037)	(0.0520)	(0.0099)	(0.0018)	(0.0037)	(0.0518)	(0.0099)
500	5.0858	5.1180	5.0948	5.0100	5.0971	5.1067	5.0835	5.1112
	(0.0011)	(0.0022)	(0.0289)	(0.0057)	(0.0011)	(0.0022)	(0.0288)	(0.0057)

Table 3:
Simulation using B. E and Risks under U. P and J.P for 20% Censored (β = 5)

Sample size	Uniform Prior				Jeffreys Prior			
	ELF	SLLF	LLF	PLF	ELF	SLLF	LLF	PLF
50	6.4275	6.3327	5.9876	6.6331	6.5881	6.6517	6.1629	6.7938
	(0.0125)	(0.0247)	(0.6005)	(0.0900)	(0.0122)	(0.0253)	(0.5859)	(0.0900)
150	4.7303	4.7565	4.6844	4.7960	4.3104	4.3171	4.2457	4.3593
	(0.0042)	(0.0083)	(0.0919)	(0.0197)	(0.0041)	(0.0084)	(0.0911)	(0.0197)
300	5.2868	5.3808	5.2495	5.3145	5.3089	5.3584	5.2717	5.3365
	(0.0021)	(0.0042)	(0.0594)	(0.0112)	(0.0021)	(0.0042)	(0.0592)	(0.0112)
500	5.0073	5.1169	4.9873	5.0023	5.1698	5.2541	5.1582	5.1856
	(0.0013)	(0.0025)	(0.0325)	(0.0064)	(0.0013)	(0.0025)	(0.0324)	(0.0064)

The simulation study has been conducted for two priors and under four loss functions. From the study it can be assessed that Bayes estimate of the parameter β converges to the true value of β by increasing the sample size. Under both priors the convergence of the estimates towards the true value of the parameter is faster in case of LLF. The rate of convergence is random under remaining loss functions. Using LLF the estimated value of the parameter is always less than the estimates obtained under other loss functions. The risk associated with estimates under ELF is the minimum for each estimate. The patterns of risks are similar almost for each prior and under every loss function. It can also be observed that the risk associated with SLLF estimate is approximately double than that of ELF estimate for most of the cases. It is interesting to note that the magnitude of risks under ELF and SLLF are not very much affected by the choice of different priors, censoring rate and magnitude of true parameter value. However, the performance of Bayes estimator under ELF is the best, for each prior, as the risk associated with the estimate is the minimum among other estimates. However, the size of over or under estimation is directly proportional to the censoring rate and magnitude of true parameter value while, it is inversely proportional to the sample size. Finally, it can be concluded that the ELF may be preferred for obtaining the Bayes estimates of parameter (β) of Gumbel Type II distribution, for each prior, as the risks associated with these estimates are the minimum. However, the convergence of the estimate towards the true value of the parameter under LLF estimators is comparatively faster for non-informative priors.

Table 4:
95% C.I under Different Priors for Complete and Censored Data (β = 5)

S.S	Prior	Complete data			10% censoring			20% censoring		
		LL	UL	UL–LL	LL	UL	UL–LL	LL	UL	UL–LL
50	U.P	4.4716	7.7632	3.2916	4.9479	9.0534	4.1054	5.4804	10.0701	4.5896
	J.P	4.3704	7.6290	3.2586	4.9693	8.8844	3.9151	5.3274	9.8619	4.5344
150	U.P	3.9978	5.4740	1.4762	4.0273	5.6102	1.5829	3.9992	5.7070	1.7078
	J.P	3.9495	5.4424	1.4929	3.9940	5.5681	1.5741	3.9633	5.6640	1.7007
300	U.P	4.7101	5.9036	1.1936	4.7479	6.0250	1.2771	4.7991	6.2288	1.4297
	J.P	4.6935	5.8869	1.1934	4.7331	5.9952	1.2621	4.7420	6.2075	1.4655
500	U.P	4.6108	5.4924	0.8817	4.6710	5.6196	0.9486	4.6455	5.6516	1.0061
	J.P	4.6012	5.4829	0.8817	4.6607	5.6071	0.9464	4.6340	5.6350	1.0010

The width of credible intervals is directly proportional to the censoring rate and true value of the parameter while, it is inversely proportional to the sample size for each prior. It is interesting to note that each credible interval contains the true and estimated (under each loss function) value of the parameter. The credible intervals are generally skewed to the left side of the true value of the parameter under each prior. However, the skewness is lesser in case of uniform prior. While, in case of Jeffreys prior the intervals are always narrower.

Table 5:
95% P.P.I under Different Priors for Complete and Censored Data (β = 5)

S.S	Prior	Complete data			10% censoring			20% censoring		
		LL	UL	UL–LL	LL	UL	UL–LL	LL	UL	UL–LL
50	U.P	2.1623	6.1900	4.0277	2.2529	6.6730	4.4201	2.3414	7.1441	4.8027
	J.P	2.1543	6.1492	3.9949	2.2444	6.6291	4.3847	2.3323	7.0971	4.7648
150	U.P	2.0795	5.7035	3.6240	2.1229	5.9330	3.8101	2.1687	6.1746	4.0059
	J.P	2.0771	5.6909	3.6138	2.1204	5.9199	3.7995	2.1660	6.1609	3.9949
300	U.P	2.4255	5.9342	3.5087	2.4695	6.1666	3.6971	2.4197	6.4111	3.9914
	J.P	2.4242	5.9277	3.5035	2.4682	6.1598	3.6916	2.4183	6.4040	3.9857
500	U.P	2.5085	5.8399	3.3314	2.5541	6.0805	3.5264	2.6002	6.3232	3.7230
	J.P	2.5077	5.8360	3.3283	2.5533	6.0764	3.5231	2.6994	6.3190	3.6196

The posterior predictive intervals have been calculated under different priors. The width of these intervals decreases with increasing the sample size while, it increases with increase in true parameter value. It can also be observed that the width of posterior predictive intervals is more than that of corresponding credible intervals which is in accordance with the theory. It is interesting to note that both posterior predictive intervals and credible intervals contain the Bayes estimated value of the parameter β under each prior.

Table 6:
Posterior Probabilities and Bayes Factor for Different Hypotheses
under Uniform and Jeffreys priors

Null Hypothesis	Alternative Hypothesis	Uniform Prior			Jeffreys Prior		
		Posterior Probabilities		Bayes Factor	Posterior Probabilities		Bayes Factor
H0	H1	P(H0)	P(H1)	B	P(H0)	P(H1)	B
β ≤ 4.0	β > 4.0	0.0037	0.9963	0.0037	0.0057	0.9943	0.0057
β ≤ 5.0	β > 5.0	0.1106	0.8894	0.1244	0.1405	0.8595	0.1635
β ≤ 6.0	β > 6.0	0.5157	0.4843	1.0648	0.5715	0.4285	1.3337
β ≤ 7.0	β > 7.0	0.8786	0.1214	7.2372	0.9042	0.0958	9.4384
β ≤ 8.0	β > 8.0	0.9859	0.0141	69.9220	0.9931	0.0069	143.9275
β ≤ 9.0	β > 9.0	0.9992	0.0008	1249.0000	0.9997	0.0003	3332.3333

The above analysis indicates that the trend of information to support the null hypothesis starts (at least) from H_0: β ≤ 6.0 and H_1: β > 6.0 under each prior. The evidence in favor of H_0 increases with increase in the value of the parameter and vice versa. Also the Bayes factor is greater in case of Jefferys prior for each hypothesis.

13. REAL LIFE ANALYSIS

Following data regarding monthly wind speed in Cameron Highland from year 2004-2006 has been used to illustrate the applicability of the results obtained in previous sections. The analysis has been carried out under complete and censored observations.

Table 7:
Monthly Wind Speed in Cameron Highland, 2004-2006

Year/Month	1	2	3	4	5	6	7	8	9	10	11	12
2004	2.76	1.56	2.00	1.38	0.91	2.02	1.12	1.34	1.27	1.95	1.63	2.61
2005	3.00	1.88	2.25	1.55	1.66	1.66	1.67	1.66	1.83	1.69	1.79	2.69
2006	2.31	3.11	1.92	1.73	1.61	2.01	1.61	1.98	1.62	1.59	2.27	2.77

In the following tables, the Bayes estimates and the corresponding risks along with 95% credible intervals and posterior predictive intervals obtained for complete and censored (10% censored and 20% censored) data have been presented. The risks associated with each estimate have been presented in the brackets.

Table 8:
Bayes Estimates and Risks under Complete and Censored Data

Prior	Complete Samples				20% Censored Data			
	ELF	SLLF	LLF	PLF	ELF	SLLF	LLF	PLF
Uniform	3.9702	4.0255	3.8707	4.1352	3.7967	3.8647	3.6876	3.9995
	(0.0138)	(0.0278)	(0.2097)	(0.0548)	(0.0178)	(0.0357)	(0.2447)	(0.0672)
Jeffreys	3.8599	3.9152	3.7661	4.0249	3.6611	3.7292	3.5604	3.8639
	(0.0142)	(0.0286)	(0.2040)	(0.0548)	(0.0184)	(0.0370)	(0.2363)	(0.0672)

Table 9:
95% C.I under Different Priors Using Complete and Censored Data

S.S	Data Type								
	Complete data			10% Censoring			20% Censoring		
	LL	UL	UL–LL	LL	UL	UL–LL	LL	UL	UL–LL
Uniform	2.8730	5.4964	2.6233	2.8074	5.5831	2.7757	2.6336	5.4874	2.8538
Jeffreys	2.7807	5.3682	2.5875	2.7051	5.4382	2.7331	2.5229	5.3267	2.8038

Table 10:
95% P.P.I under Different Priors Using Complete and Censored Data

Prior	Data Type								
	Complete data			10% censoring			20% censoring		
	LL	UL	UL–LL	LL	UL	UL–LL	LL	UL	UL–LL
Uniform	1.0519	7.6358	6.5839	1.0450	7.6342	6.5892	0.9996	7.5234	6.5238
Jeffreys	1.0221	7.5525	6.5305	1.0115	7.5407	6.5292	0.9629	7.4185	6.4556

In order to assess the applicability of the results under real life data the estimates using ELF can again be preferred as the concerned magnitude of risk is smaller. The 95% credible intervals and posterior predictive intervals are shorter under the assumption of Jeffreys prior.

14. CONCLUSIONS AND RECOMMENDATIONS

The study has been conducted to assess the patterns and performance of the Bayes estimates of the shape parameter of Gumbel type II distribution under different loss functions using non-informative priors. From the simulation study it can be concluded that

the performance of estimates under ELF is better for each prior, as the risk associated with these estimates is the minimum. However, the convergence of the estimate towards the true value of the parameter under LLF estimators is comparatively faster. The size of over or under estimation is directly proportional to the censoring rate and magnitude of true parameter value, while it is inversely proportional to the sample size. The width of credible intervals and posterior predictive intervals decreases with increase in the sample size. The credible intervals are generally left aligned under each prior; however, the skewness is lesser in case of uniform prior. The credible intervals and posterior predictive intervals are shorter under Jeffreys prior. Similarly, in hypothesis testing scheme the Bayes factor is always greater under the assumption of Jeffreys prior. For real life data, the ELF can again be preferred as the corresponding risk is the minimum. The width of credible intervals and posterior predictive intervals are again lesser for Jeffreys prior. Therefore in order to estimate the said parameter of the Gumbel type II distribution, the use of ELF under the assumption of Jeffreys prior can be preferred. The study can further be extended for informative priors and considering both scale and shape parameters. Some other loss functions and censoring techniques can also be included in the further study.

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ANALYSIS OF STATISTICAL LITERACY IN PAKISTANI STUDENTS

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ABSTRACT

This paper reports on an investigation of 9601 university applicants' conceptual understanding of basic statistics concepts learnt at secondary school level tested in a university admission test. The hypothesis is confirmed that there is a significant difference in the conceptual understanding of statistics between applicants from the Pakistani examination system and those who appeared from the British examination system in favour of the latter.

KEY WORDS

Differential item functioning; readiness for learning; university selection; basic statistical knowledge

BACKGROUND

Aga Khan University Examination Board is a Federal Board of Intermediate and Secondary Education that offers qualifications in both English and Urdu. AKU-EB's primary purpose is to improve the quality of education. It seeks to do so by making examinations of reputable standard more accessible to Pakistani students.

Since 2009, the Examination Board has constructed the University Admission Test for all entities of the university which is widely spread from undergraduate to post graduate programmes in medical education (MBBS, MHPM, M.Sc.Epi.Bio-Statistics) through nursing (B.ScN, Post RN B.ScN, M.ScN) to education (M.Ed. & M.A). The admission test is intended to select a better mix of appropriate students for each programme. In 2010, EB extended development of the entrance test to an Institute of Business Administration in rural Sindh offering MBA and BBA programmes.

1. INTRODUCTION

This paper reports on an investigation of students' conceptual understanding of basic statistics concepts learnt at secondary school level tested in the admission test. The hypothesis is that there is a significant difference in the conceptual understanding of statistics between applicants from the Pakistani examination system as compared to those applicants who are appeared from British examination system.

The study was carried out on 9601 applicants for undergraduate, graduate and post graduate programmes. The two educational groups are self selected and in the absence of firm data on their respective selection ratios no sound conclusion can be drawn about the relative effectiveness of the two systems of education. However, the DIF Statistic

(Kubaik, 1990) discounts overall difference in establishing departures from the norm. Differences in the relative facility of different types of question are expected to reveal instructional quality.

2. RATIONALE

Despite widespread acceptance of the notion that improving student performance may have a high economic and social payoff, policy analysts in all countries have surprisingly little empirical data on which to base educational strategies for raising achievement (Sorto, 2010). In Pakistan as well this question is all the more pressing. Applicants appeared from Pakistani examination system applied for the University Admission Test 2010-2012 scored very low in the Mathematics component of the test when compared with the applicants from the British system of examination.

Further, the AKU-UAT's own validity and reliability study on last three years of applicant performance in different programmes shows little improvement in the ratio of successful applicants from Pakistani educational background to O-Level applicants. While some reasons for this poor performance may be evident, and there is widespread agreement that the main challenge in Pakistan is the quality of education, there is little empirical analysis that helps policy makers understand the reasons for the low level of student performance in Pakistani schools or how to improve it (Carnoy et al., 2008). This study will compare the syllabus concepts based upon the knowledge of Statistics at SSC level with the O-Level syllabus and applicants' responses on the items used to determine the understanding of the concepts learnt.

3. LITERATURE REVIEW

In the last two decades several research papers were written internationally to drawn attention to the importance of the basic statistics concepts taught at secondary school level or O-Level in Mathematics.

Starkings (1997) unfolds the realities of the data analysis presented in the National Curriculum of Europe, America (North and South) and third world countries as 'Within the developed countries there appears to be a coming together of secondary school data analysis techniques.' Many areas such as stem and leaf or box plots are often ignored, even after several experts in the area have demonstrated, at various conferences, the benefit of including these topics. Starkings goes on further to state; 'The Education Reform Act in 1988 become law and for the first time England and Wales had a statutory National Curriculum which represented a turning point in the history of education for these countries. The statutory curriculum for mathematics contained an attainment target called Data Handling'. Attainment target 5- Handling Data: Pupils should collect process and interpret data and should understand, estimate and use probabilities.' (Sweetman, 1991). In third world countries, like Pakistan, the move towards the teaching and learning of data analysis techniques is prevalent (Starkings, 1997).

Developing countries such as Pakistan rely '… heavily on text book material and still promote rote learning of definitions and formulas rather that real understanding of statistical concepts. In Pakistan a series of Statistics Teachers' Education Program (STEP) provides a forum for the enhancement of teachers' knowledge through

participation in sessions containing (a) lectures by expert statisticians/ professors, (b) open discussion and (c) group work regarding both course content and teaching methodology (Habibullah, 1995).

On the basis of these researches in 2000 Curriculum Wing, Ministry of Education – Pakistan published a revised National Curriculum-Mathematics for Grade I-XII included Information Handling from Grade III onwards as a standard followed by an updated version with students' learning outcomes in 2006.

On the other hand, Rahbar and Vellani (2001) reported on a cohort of 374 medical students who were admitted during 1989 -1994 to the medical university under consideration when approximately 50% were from the Pakistani examination system, 23% from the British system, 21% from a mix of the two systems (OHS). It is noteworthy that the selection ratio for applicants from the Pakistani examination system was 0.02, for the British system was 0.18, and for the OHS group, 0.10.

4. METHODOLOGY

In this study different approaches were used to validate the hypothesis that in admission test applicants from O-A level examination system have better understanding of the basic statistics concepts as compared to applicants from SSC-HSSC examination system. The discussion was based upon the

- *item analysis* of the items used to tests basic statistics concepts of applicants from different system of education for undergraduate and graduate admission test
- *comparison of the scores* of applicants from O-A level Vs SSC-HSSC examination system by using box plots
- *ANOVA- comparison of means* for the set of items defined for each group.
- *DIF- statistics* to identify the item bias between groups.

For the preliminary analysis of the items, for each programme separate item analyses of classical test theory was used with the help of CONQUEST. The results of applicants' performance in these test items were then used to determine the difficulty index and discrimination index of each item in the test (Mitra N K &Nagaraja H, 2009).In this study, the item difficulty index (P) refers to the percentage of the total number of correct responses to the test item. It is calculated by the formula $P = R/T$, where, R is the number of total correct responses and T is the total number of responses (correct + incorrect + blank responses). For the calculation of discrimination index our study used the method adopted by Kelley (1939) where, the percentage in the upper group (P_U) and lower group (P_L), 27% performers were selected, thus D= P_U - P_L (Mitra N K &Nagaraja H, 2009).

A graphical representation of applicants' performances is given in the box plots (Figure 2) while ANOVA was used to test for significant differences among the groups.

5. DISCUSSION AND CONCLUSION

In 2010, 11 & 12 Admission test, total 17 multiple choice questions with four options test items on statistics concepts were used in the Mathematics component of the tests to select candidates for medical education in the two undergraduate and four graduate programmes and in one graduate and one post graduate business studies programme.

Candidate sample

Table 1 shows the candidates count in each programme according to the system of education. In this study applicants were categorized in two groups of 11969 medical education institution applicants which include 3448 applicants of MBBS & B.Sc. N programme 2012, whose profile for system of education is unavailable and 1081 business administration institution applicants.

Table 1			
Programme	**Education - SSC**	**Education - HSSC**	**# of applicants**
Undergraduate	O-Level	A-Level	1756
		HSSC	437
	SSC	HSSC	6457
	American System		58
	OTH		95
Graduate	SSC	HSSC	788
	OTH		10
Grand Total			**9601**

Table 1 also shows that in Pakistan for the admission in professional institution students preferred to stay with HSSC examination offered by local boards as OHS group. This group expands system of education to another group in which after O-Level students preferred to take the advantage of HSSC examination for better prospects at the time of entrance at university education. However, data reveals that Cambridge University examination system for Advanced Level is still in demand in best educational institutions in Pakistan.

Learning outcomes item sample

An inquest approach was used to investigate, what exactly is provided to the learners to build the concepts – that is what curriculum is set for learners? Table 2 describes the learning outcomes of the National Curriculum of Pakistan (NC) Mathematics (2000, 2006) for Grade IX-X in comparison with Cambridge O-Level Mathematics Syllabus 2010. In the NC sub-subject areas are categorized as standards. In Pakistani mathematics curriculum the standard "*Information Handling*" refers to learning outcomes based upon the basic concepts of Statistics where as in Cambridge O-Level syllabus the concepts of Statistics and Probability are highly structured. The Cambridge University O-Level Mathematics syllabus covers not only basic concepts but simultaneously provides opportunity to students to study the concepts for in-depth understanding .e.g. National Curriculum- Mathematics 2006, Pakistan has given the weightage of 5% for Information Handling in Grade VI-VII curriculum and 10% for Basic Statistics in Grade IX-X. (NC-2006, pg. 140-142). There is no opportunity for SSC students to study the concepts of probability as none of the learning outcomes of NC-2006 included the concepts in Grade VI-X curriculum. However, NC-2006 added learning out comes for graphical estimation of median, quartiles and mode.

Table 2 shows the similarities and differences in the desired learning outcomes of both the syllabus.

Syllabus	NC –Pakistan Learning standard (2006)	Cambridge O-Level Mathematics Syllabus (2010)
Sub-content area	Information Handling	Statistics and Probability
Learning outcomes	• Construct grouped frequency table. • Construct histograms with equal and unequal class intervals. • Construct a frequency polygon. • Construct a cumulative frequency table. • Draw a cumulative frequency polygon.	• collect, classify and tabulate statistical data; read, interpret and • draw simple inferences from tables and statistical diagrams; • construct and use bar charts, pie charts, pictograms, simple frequency distributions and frequency polygons; • use frequency density to construct and read histograms with equal and unequal intervals;
	• Find measures of central tendency and dispersion to draw conclusions. • Calculate (for ungrouped and grouped data): • arithmetic mean by definition and using deviations from assumed mean, • median, mode, geometric mean, harmonic mean. • recognize properties of arithmetic mean. • calculate weighted mean and moving averages. • estimate median, quartiles and mode graphically. • measure range, variance and standard deviation.	• calculate the mean, median and mode for individual data and • distinguish between the purposes for which they are used; • construct and use cumulative frequency diagrams; estimate the median, percentiles, quartiles and inter-quartile range; • calculate the mean for grouped data; identify the modal class from a grouped frequency distribution.
		• Probability: calculate the probability of a single event as either a fraction or a decimal (not a ratio); • Calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate. (In possibility diagrams outcomes will be represented by points on a grid and in tree diagrams outcomes will be written at the end of branches and probabilities by the side of the branches.)

These analyses are an indication of the basic statistics concepts taught at SSC level in Pakistan. Text books based upon NC- 2006 have been published by Punjab Text Book Board alone but with limited exploration and practice exercises of the concepts. Pakistani students are facing real challenges in this regard.

Comparison of the performance of the applicants from different system of education was possible based on three sets of items. Set I (item # 1, 2, 3, 9, 10 &11) covered concepts drawn from both systems

- *read, interpret and draw simple inferences from tables and statistical diagrams.*
- *estimate the median, quartiles and mode graphically;*
- *recognize properties of arithmetic mean;*
- *calculate the probability of a single event as either a fraction or a decimal (not a ratio);*
- *calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate.*

Item Set II (item # 4, 5, 6) was attempted by undergraduate and graduate applicants from SSC-HSSC background only. These items were on the concepts of

- *calculate the mean, median and mode for individual data , for grouped data and distinguish between the purposes for which they are used;*
- *read, interpret bar charts, pie charts, pictograms, simple frequency distributions and frequency polygons;*
- *estimate the median, percentiles, quartiles and inter-quartile range;*

In addition to the above, Item Set III (item #, 13, 14 and 15) were attempted by undergraduate and graduate applicants from SSC-HSSC background, Item Set IV (item # 16 & 17) were attempted by undergraduate MBBS programme applicants only from O–A Level, OHS and SSC-HSSC background. These items were on the concepts of

For Item Set III (item #, 13, 14 and 15)
- *draw simple inferences from tables and statistical diagrams;*
- *read, interpret and draw simple inferences from cumulative frequency diagrams.*

For Item Set IV (item # 16 & 17)
- *read, interpret a frequency distribution and*
- *find the range, variance and standard deviation of the given data;*

and item # 7, 8 & 12 separately were attempted by B.Sc.N, MBA & BBA programmes from SSC-HSSC background only. These items were on the concepts of

- *estimate the median, percentiles, quartiles and inter-quartile range;*
- *draw simple inferences from tables and statistical diagrams;*
- *read, interpret and draw simple inferences from cumulative frequency diagrams.*

Relationship between difficulty index and examination group

The Figure 1 Scatter plot shows that the common items have proved far too difficult for the average candidate. 29.4% of the test items with discrimination index> 0.28 had the difficulty index above 0.35. 7 out of 17 test items showed difficulty index ranging in between 0.20 to 0.30 with poor and negative discrimination index. 30 % of the test items had the difficulty index below 0.20 with discrimination index below 0.19.
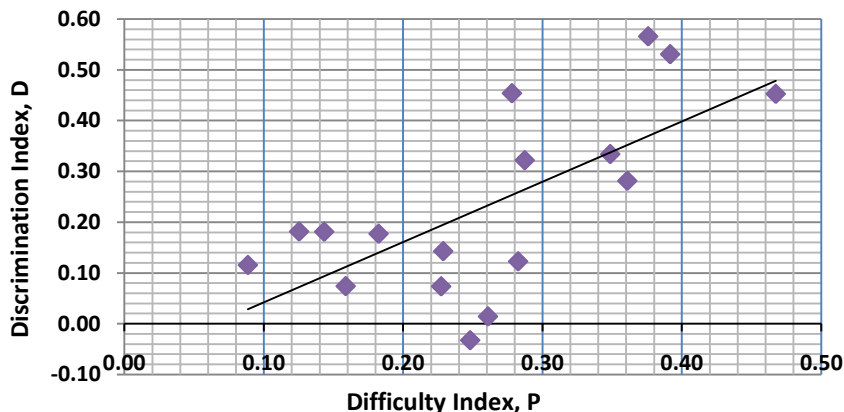
Figure 1: Difficulty index and discrimination index of Set I

Further analysis of the data indicated in Table 3 that there was a wide spectrum of level of difficulty among the test items on the concepts of basic statistics. Item Set I which was attempted by all groups measured skills of reading and interpreting of statistical table and diagrams such as *estimation of mean, median and mode graphically.* Applicants from O-A level background attempted the Item Set I (items # 1, 2 & 3) with item facility ranging in between 0.43 to 0.67 with discrimination index > 0.28, in OHS the same items were remained moderately difficult items with item facility index in between 0.35 to 0.44. However, the same set of items attempted by applicants from SSC-HSSC background were proven to be very difficult items with item facility ranging from 0.16 to 0.25 with discrimination index < 0.26. Item Set I (item # 9, 10 &11), which calls to tests the concepts of *measure of central tendency and probability* was problematic for all candidates. O-A level applicants scored best, with item facility ranging between 0.23 to 0.29 and OHS with 0.14 to 0.23 but SSC-HSSC with 0.006 to 0.16. The Item Set II (Item 4, 5 & 6) measured the concept of *recognize properties of arithmetic mean and estimate median, quartiles and 50th percentile* still proven to be complicated for some reasons which reflects the classroom teaching method with emphasis on rote learning rather than understanding in the Pakistani schools. In addition to the previous example Item # 6 calls for the estimation of 50th percentile. Here applicants were unable to link the concept of median, quartiles and 50th percentile.

In this comparison, it is concluded that O-A level applicants proficiency on the concepts of basic statistics is at acceptable level, the applicants from OHS group also attempted the MCQs with understanding but the hypothesis for the SSC-HSSC group was getting accepted as the performance of the applicants was very low on these items. Item # 2 as shown in the Table 6(See appendix) calls for the understanding of the concept of the "*measure of the central tendency*" where SSC-HSSC group unable to grasp of the concept accurately. Here, the benefit of the exposure to the type of the test item went to O level group as BISE test papers did not consider the cognitive level of *higher order thinking* for test item designing for SSC–HSSC examinations.

Table 3

System of Education	Item #	# of applicants	Item Facility	Item discrimination
O – A Level	1	819	0.43	0.28
	2	819	0.55	0.4
	3	819	0.65	0.54
	9	819	0.25	0.21
	10	819	0.23	0.26
	11	819	0.29	0.46
OHS	1	206	0.39	0.44
	2	206	0.35	0.17
	3	206	0.44	0.53
	9	206	0.21	0.26
	10	206	0.23	0.2
	11	206	0.14	0.07
SSC - HSSC	1	2332	0.25	0.26
	2	2332	0.31	0.19
	3	2332	0.16	0.18
	4	822	0.24	**-0.04**
	5	822	0.28	0.12
	6	822	0.26	0.01
	7	519	0.09	0.12
	8	188	0.23	0.14
	9	2332	0.1	0.11
	10	2332	0.16	0.17
	11	2332	0.06	0.03
	12	286	0.35	0.33
AMR	1	23	0.34	**-0.08**
	2	23	0.37	0.25
	3	23	0.54	0.67
	9	23	0.43	0.38
	10	23	0.35	0.13
	11	23	0.3	0.63
OTH	1	39	0.27	0.18
	2	39	0.51	0.62
	3	39	0.38	0.54
	9	39	0.23	0.21
	10	39	0.29	**-0.05**
	11	39	0.2	0.32
O – A Level, OHS, SSC - HSSC, AMR & OTH	13	3467	0.46	0.57
	14	3467	0.35	0.33
	15	3467	0.47	0.45
	16	3162	0.39	0.53
	17	3162	0.38	0.57

Comparison of Scores by box plots

Figure 2 shows Box plots, used to strength the argument presented above to compare the scores of each group on Item Set I and evidence is there that applicants from O-A level and AMR system (GRE and SAT) background 50 % of the applicants scored in the range of 1 to 4 marks out of total of 6 marks with outliers in O-A level group at the both ends and in AMR system on the upper end. Whereas, OHS and OTH system applicants were performed on the items in the same way i.e. 50% of the

Figure 2
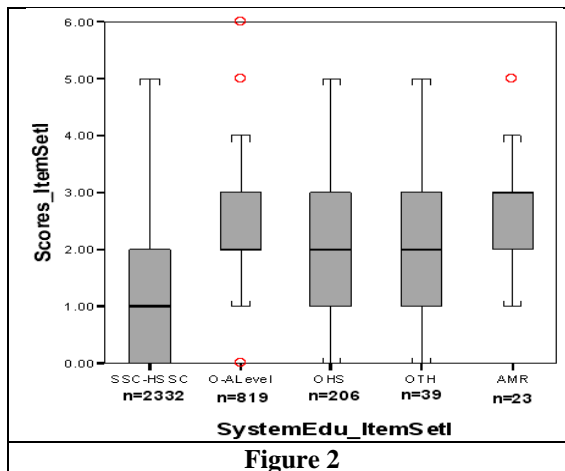
applicants scores between 1 to 3 marks out of 6 marks and upper one quarter scored between 3 to 5 marks and lower quarter scored 0 or 1 marks. In SSC-HSSC group, 50% of the applicants scored 0 or 1 mark and upper quarter ranged the score of 5 marks out of 6 marks with the median score at 1 mark, saved the group for the range of the score.

ANOVA – analysis of variance approach

These differences are confirmed in a one way ANOVA in which items are treated as random and merged with the item X group interaction.

Table 4

ANOVA: Single Factor				
SUMMARY (Item SET I)				
Groups	*Count*	*Sum*	*Average*	*Variance*
O-A Level	819	2075	2.53	1.79
OHS	206	380	1.84	1.68
SSC-HSSC	2332	2781	1.19	0.98
AMR	23	61	2.65	1.23
OTH	39	82	2.1	1.56
ANOVA (Item SET I)				
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*
Between Groups	1146.184	4	286.54	233.35	<0.001
Within Groups	4192.213	3414	1.22		
Total	5338.397	3418			

The items were the focus of a DIF analysis of the item bias between the groups specifically O-A level and SSC-HSSC level for item Set I by using DIFAS software. In the DIFAS analysis data, O-A level were marked as reference group (1) and SSC-HSSC group as focal group(2). The results are shown below in Table 5.

Table 5

DIF analysis: Nonparametric tests for dichotomous items
Stratifying variable: Sum of item responses
Stratum size: 1
Number of strata: 7
Number of reference group members: 819
Number of focal group members: 819
Grouping variable: Var7
Reference Value = 1, Focal Value = 2

STRATUM-LEVEL INFORMATION

Stratum	Reference Frequency	Focal Frequency
0	49	95
1	155	276
2	209	280
3	195	129
4	146	36
5	53	3
6	12	0

DIF STATISTICS: DICHOTOMOUS ITEMS

Name	MH CHI	MH LOR	LOR SE	LOR Z	BD	CDR	ETS
Var 1	40.9313	-0.7989	0.1251	-6.3861	2.17	Flag	C
Var 2	11.5495	-0.4253	0.1235	-3.4437	1.178	Flag	A
Var 3	172.3384	1.6544	0.1313	12.6002	0.052	Flag	C
Var 4	0.1227	-0.0628	0.1478	-0.4249	0	OK	A
Var 5	45.8657	-0.9962	0.149	-6.6859	0.073	Flag	C
Var 6	24.2297	0.8527	0.1719	4.9604	5.688	Flag	C

In the Table -5 Mantel-Haenszel chi-square statistic (Holland & Thayer, 1988; Mantel & Haenszel, 1959) is distributed as chi-square with one degree of freedom. Critical values of this statistic are 3.84 for a Type I error rate of 0.05 and 6.63 for a Type I error rate of 0.01. The Mantel-Haenszel common log-odds ratio (Camilli & Shepard, 1994; Mantel & Haenszel, 1959) is asymptotically normally distributed. Positive values indicate DIF in favor of the reference group, and negative values indicate DIF in favor of the focal groups. The standard error is the non-symmetric estimator presented by Robins, Breslow and Greenland (1986) and LOR Z is the Mantel-Haenszellog-odds ratio divided by the estimated standard error. A value greater than 2.0 or less than –2.0 may be considered evidence of the presence of DIF.

The Breslow-Day chi-square test of trend in odds ratio heterogeneity (Breslow& Day, 1980; Penfield, 2003) is distributed as chi-square with one degree of freedom. The combined decision rule (CDR) flags any item for which either the Mantel-Haenszel chi-square or the Breslow-Day chi-square statistic is significant at a Type I error rate of 0.025 (Penfield, 2003). The message OK is printed if neither statistic is significant, and the message FLAG is printed if either statistic is significant.

(DIFAS 4.0, User Manual © 2007)

The above table and description shows that the item # 3 and 6 call to tests the concept of *probability* favors the O-A level applicants and item (1, 2, &5) measured the concept of *measure of central tendency* favors SSC-HSSC group.

Suggestions for bridging the gaps:

The finding of this study leads to some suggestion for bridging the gaps of concepts of basic statistics identified in different groups of applicants from the Pakistani system of examination and the British system of examination in university admission test.

- Need to update the NC- 2006 Mathematics i.e. weightage given for the *Information Handling* from 10% to 20% at SSC level.
- In SSC examination the mathematics MCQ items should be written on the higher order thinking skills.
- To promote active learning during the classroom teaching, mathematics text books should be written with exploration task for the given learning outcomes.
- Adapt the standard system of education and provide opportunities to premedical students for studying Statistics as additional subject at HSSC level to remain in touch with basic statistics concepts.

APPENDIX

Table 6

Item # 2

In a class of girls and boys, the average (arithmetic mean) height of the girls is 48.3 inches, what is the average height of all of the students in the class?

Which two of the following statements together provide sufficient additional information to answer the question?

i)   The number of girls in the class is 18.
ii) The sum of the heights of all of the students in the class is 1,379.4 inches.
iii) The average height of the boys in the class is 48.5 inches.
iv) The ratio of the number of girls to the number of boys in the class is 3 to 2.
v)   The difference between the number of girls and the number of boys in the class is 6.
   A)   (i) and (ii)
   B)   (ii) and (ii)
   C)   (iii) and (iv)
   D)   (iv) and (v)

Item #10

A list consists of five different numbers in increasing order.
   (I)   The average of the five numbers on the list is greater than 14.
   (II) The second number on the list is greater than 14.
   (III) The median of the five numbers on the list is greater than 14.
Which of the following statements is true?
   A.   If (I) then (II)
   B.   If (I) then (III)
   C.   If (II) then (I)
   D.   If (III) then (I)

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IMPACT OF TERRORISM ON THE PSYCHOLOGY OF WORKING WOMEN IN PAKISTAN: A CASE STUDY OF SINDH

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ABSTRACT

The impact of terrorism is dangerous and creates a great number of problems for working women and demolishes possessions, autonomy and brings monetary problems and destroys human psychology. Working women in Pakistan are very much afraid and disturb because of sudden suicide bomb attacks, assignation, and planned armed robberies. Now-a-days it is very difficult for working women to move from home to workplace because of uncertain security situations. They feel insecure and their motivational working capabilities are weakening day by day as they suffer psychologically, socially, economically, ethically and religiously. Primary and secondary research methodology is used and sample is Karachi is selected in Sindh region in Pakistan. Terrorist activities sabotage the working environment in this city. The effect of terrorism dangerously damages her personality plus weakens her abilities. A schedule as open ended questionnaire of 10 questions were asked by 100 working women from different sectors like education, health, multinational organizations, Ngo's daily wage earners, working women in beauty parlors, tuition centers etc. This study is based on qualitative and quantative research and results are tabulated and analyzed with the help of statistics.

KEY WORDS

1. INTRODUCTİON

The meaning of terrorism is bomb blast, suicide terror, firing at innocent. These are the fierce acts and harm working women activities and create a sense of uncertainty. This state demoralizes and de-motivates the working women in working environment. Working women sense fear of being killed at any time and this caused steady mess that results in the form of blood pressure, psyche problems, mental disorder and heart diseases. Currently Pakistan is facing biased terrorist's attacks. Pakistan is handicapped because of distressed financial aid state of affairs. The economic growth of the nation depends on monetary aid time-honored from worldwide fiscal organizations. Pakistan is facing discriminating sticky situation because of mounting financial stress on family head especially worsen due to corruption, inflation, and power shortage. Because of terrorist activities foreign investors are reluctant to invest in Pakistan, domestic harmony, and stability is threatened, suicide bomb attacks create insecurity among people that deteriorate their confidence in the government. At present Pakistan is facing the problem of domestic safety and threats [Pakistan Press Review, 2010, p.21]. The terrorist attacks in Karachi are giving intimidation as inhabitants that terrorist set-up is becoming stronger in the cities [Pakistan Press Review, 2010, p.65].

2. LITERATURE REVIEW

Terror is the aim of terrorism [Sandman & Lanard, 2003]. Work tension has pessimistic impact on the psychological and corporeal fitness of the personnel [Cooper & Marshall, 1976]. The feeling of nervousness and restlessness at job can be in the shape of dysphoria [Olff, Sijbrandij, Opmeer, Carlier & Gersons, 2009]. The name fear described as sadistic world-shattering events [Kurtz, 1987]. Freedman [1983] said that brutal society need biased profits via terrorism at the same time as Long in 1990 said that features terrorism exist in this ear. Benjamin [1996] admitted that Pakistan and Afghanistan are not in a situation to support any terrorist activity. Terrorism is a form of aggressive rebelliousness [Sondhi, 2000]. Terrorism is the fact of this epoch. This expression is derived from Latin word Terror which means horror [Mishra, 2004]. According to Mishra, current terrorism originated from the French revolution. This term was considered as aggressive activities made by labor organizations in 1800s and early 1900s [Gutteridge, 1987]. After World War II, the term was used for prejudiced groups [Combs, 1987]. According to Eilliot [1978] the expression was used by unkind left wing group. In 1970s there was a staged increase in terrorism all over the world [Kurtz, 1987]. Terrorist groups at present apply computer know-how to ease actions [Combs, 1987]. It is not only leads serious psychological health problems but also capability to destroying manners. It builds up such a mind-set in character that holds back capacity to function efficiently. The most common state of affairs are depression, anxiety, psychometric evils as insomnia, back or stomach aches [world health organizations, 2001]. Terrorism planned and designed actions that are used to attain politically enforced targets [Ruby, 2002]. global use of violent behavior in real or just warning ensuing unsympathetic health effects ranging from loss of well being or security to injury, illness or health [Arnold, Ortenwell, Binbaum, Sunda and Anantharaman, 2003]. It effects and creates depressive psychological disorder, nervousness, heart ache reactions [Bleich, Gelkopf and Solomon, 2003]. Terror creates tension, posttraumatic stress disorder, anxiety, depression, regressive behavior, separation problems, difficulties in sleep [Wanda, 2004]. It badly affects girls in the shape of depression than boys [DiMaggios and Galea, 2008]. It also creates psychological poor health [Steel, Silovo, Phan and Bauman, 2009].

3. PROBLEM STATEMENT

Working women in Pakistan are very much frightened and upset because of unexpected suicide bomb attacks, assignation, and intended armed robberies. Now- a-days it is very hard for working women to travel from home to workplace because of uncertain security situations. They feel insecure and their motivational working capabilities are weakening day by day as they suffer psychologically, socially, economically, ethically and religiously. Terrorist activities sabotage the working environment in this city. The ultimate effect of terrorism on the psychology of working women and damage her personality plus weaken her abilities.

4. PURPOSE OF RESEARCH

The main aim of this research is to find out the problems faced by working women who are working in different capacities at different working places but facing the same fears and threats in today's insecure world. No matter weather they are executives in

reputable organizations or worker at beauty parlor, maid at home, or street cleaner, factory worker or sales girls. Their contribution to develop the country is not negligible.

5. OBJECTIVES OF THE STUDY

The main objective of this research is to the make the working environment protected and safe and sound for every working woman no matter where they are working. The additional step must be taken by the organizations as to provide secure transportation facility. After that there is a need to fit hidden electronic cameras from street to organization not only at main gate or entrance but also inside the organizations. The next one is to appoint private security guards who must have commando training. The other one is to provide safety training to every working woman where they can learn how to face uncertain situation but also can hit the one who can be dangerous or can catch the person who is suspicious.

6. LIMITATIONS OF STUDY

Few limitations must be considered when involves in this research study and that is only focused on working women whose age start from 20 to end at 40. Karachi is the selected area for study in which, Clifton, Gulshan-e- Iqbal, North Nazimabad, Malir, Suhrab Goth, are selected.

7. RESEARCH METHODOLOGY

Primary and secondary research methodology is used and sample is Karachi is selected in Sindh region in Pakistan. A schedule as open ended questionnaire of 10 questions were asked by 100 working women from different sectors like education, health, multinational organizations, Ngo's, daily wage earners, working women in beauty parlors, tuition centers etc. this study is based on qualitative and quantative research and results are tabulated and analyzed with the help of statistics.

8. FINDINGS

In this research study ten open ended questions were asked as a schedule. The respondents who are in 100 in numbers were interviewed face to face from working women in different sectors. As teachers from academic institutions especially school and colleges, doctors and nurses from health sector executives and administrators from public and private organizations/Ngo's/daily wage earners/beauty parlors /tuitions centers. The focused research areas are Clifton, Gulshan-i-Iqbal, North Nazimabad, Malir and Suhrab Gouth. The focused age group is between 20-40.

Working women plays very important role in the economic development of the country. Working women are major supporters of their families as economic conditions becoming worst day by day. As terrorist activities hit Pakistan, it is very difficult for working women to contribute their potential for the economic development.

In Sindh, Karachi is the business hub where terrorist activities sabotage the working environment. Because of terrorism working women feel insecure that results many psychological problems in female such as blood pressure, depression, anxiety, insomnia, back ache, stomach ache, nervousness, heart ache, aggressive attitude, tension, post

traumatic stress, mental disorder, regressive behavior, damage of organs, separation problems, allergy, sleeping disorder, feeling of fright, upset attitude, disorganization, insecurity and also deterioration in social relations. It is hard point that most of the working women are unable to get complete treatment to be healthy because of financial problems as doctor's fees high and medicines are expensive while in government sector hospitals medicines and doctors are not available and if doctors are available then the patients are not properly treated.

The respondents inform that they are facing terrorism in the for of sudden suicide bomb attacks, sudden firing, robberies, snatching of cars, mobiles, money etc. these terrorism activities effects badly on the performance of the working women that is declining at work results decrease in the productivity of the organization that ultimately effects the economic conditions of the country. It causes economic turn down as well as investment and GDP of the county decline. Less investment means fewer jobs available in the market and that ultimately create stress. It also causes stress at work place that disturbs the overall working environment.

Following table shows the Impact of Terrorism on the Psychology of Working Women in Sindh [Effects on Psychology that results ill health]

Table 1:
Impact of Terrorism on the Psychology of Working Women in Sindh
[Effects on Psychology that results ill health]

S.No	Effects of Terrorism on Mental Health results ill health
1	Nervousness
2	Tension
3	Mental Disorder
4	Fright
5	Insecurity
6	Blood Pressure
7	Depression
8	Back Ache
9	Stomach Ache
10	Insomnia
11	Heart Ache
12	Aggressive Attitude
13	Posttraumatic Stress
14	Demage of Tissues
15	Regressive Behavior
16	Demage of Organ
17	Seperation Problems
18	Allergy
19	Sleep Problems
20	Breathing Problems
21	Disorganization
22	Anxiety

Survey Note: The above table shows that because of terrorism activities working women are suffering different psyche and physical health problems that results negatively on the performance of the organizations and also GDP goes down and economic development also declines.

Following table-2 shows the data collected from respondents in the form of percentage.

Table 2:
Impact of Terrorism on the Psychology of Working Women in Sindh
[Effects on Psychology that results ill health in Percentages]

S.No.	Effects on Mental Health results ill health	Percentage
1	Nervousness	2%
2	Tension	5%
3	Mental disorder	2%
4	Fright	10%
5	Insecurity	1%
6	Blood Pressure	5%
7	Depression	3%
8	Back Ache	10%
9	Stomach Ache	8%
10	Insomnia	5%
11	Heart Ache	1%
12	Aggressive Attitude	8%
13	Posttraumatic Stress	1%
14	Demage of Tissues	1%
15	Regressive Behavior	8%
16	Demage of Organ	3%
17	Seperation Problems	5%
18	Allergy	5%
19	Sleep Problems	7%
20	Breathing Problems	5%
21	Disorganization	3%
22	Anxiety	2%

Survey Note: The above table shows the different diseases and its percentages of the suffering working women who mental health as deteriorate that effects their physical health badly. Working women now feel less motivated at work place that is the loss of not only economy but they are also facing financial problems as they are the major financial supporters of their families.

9. CONCLUSION

Terrorism hit Pakistan destructively caused miserable death in different incidents. Terrorist strike Pakistan as a state and people as nation Terrorism severely effected

Pakistani people psychologically and destruct economy, policies, social life, religiously, Pakistan, now stand alone without friends at international level. New electronic weapons cause failure of organ, or even death or destroy senses of breathing problems, damage of tissues or organ or allergy.

10. RECOMMENDATIONS

7   It is recommended that there is a need to make the working environment protected and safe and sound for every working woman no matter where they are working. The additional step must be taken by the organizations as to provide secure transportation facility. After that there is a need to fit hidden electronic cameras from street to organization not only at main gate or entrance but also inside the organizations. The next one is to appoint private security guards who must have commando training. The other one is to provide safety training to every working woman where they can learn how to face uncertain situation but also can hit the one who can be dangerous or can catch the person who is suspicious. Table talk would be fruitful to rectify grievances and negotiations must be fruitful. Effective dialogue is a need of time with political / religious parties and with superpowers and neighboring countries also. Try to bring investment in the country and start up new projects that generate employment opportunities in the country in different areas that can reduce disparity, hopelessness that leads reduction in terrorism activities.

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DISTRIBUTIONAL PROPERTIES OF GENERALIZED ORDER STATISTICS FOR EXTENDED EXPONENTIAL DISTRIBUTION

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ABSTRACT

In this paper we have derived the distributional properties of generalized order statistics of extended exponential distribution. We have derived the p.d.f.s of generalized order statistics for extended exponential distribution, sth positive and negative moments, expressions for means and variances and finally the joint p.d.f.s of rth and sth generalized order statistics for the cases m=-1 and m$\neq$ -1 has been derived.

KEYWORDS

Generalized Order Statistics; Extended Exponential Distribution.

1. INTRODUCTION

In 2010 Nadarajah and Haghighi introduced a new distribution as an extension to the exponential distribution which serves as an alternative to the Gamma, Weibull, and Exponentiated Exponential (EE) distributions. This new distribution can be used for modeling the data in situations where monotone hazard rate occur in the life time data analysis problems, like Gamma, Weibull and EE distributions which are also used for such situations. For $\alpha > 0$, $\lambda > 0$ and $t > 0$. The corresponding distribution function and probability density function of extended exponential distribution are:

$$F\ t\ = 1 - \exp\left[1 - \ 1 + \lambda t\ ^{\alpha}\right] \tag{1.1}$$

$$f\ t\ = \alpha\lambda\ 1 + \lambda t\ ^{\alpha-1}\exp\left[1 - \ 1 + \lambda t\ ^{\alpha}\right] \tag{1.2}$$

(Nadarajah and Haghighi; 2010)

Nadarajah and Haghighi mentioned that the new extended exponential distribution can also be viewed as truncated Weibull distribution truncated at 0. For the shape parameter $\alpha = 1$ the new extended distribution reduces to exponential distribution.

Kamps in 1995 introduced the concept of generalized order statistic (GOS), starting with the joint density function of n uniform generalized order statistics on a cone of n-dimensional euclidean space, given as:

$$f^{U(1,n,\tilde{m},k),\dots,U(n,n,\tilde{m},k)}(u_1,\dots,u_n) = k\left(\prod_{j=1}^{n-1}\gamma_j\right)\left(\prod_{i=1}^{n-1}\ 1 - u_i\ ^{m_i}\right)\ 1 - u_n\ ^{k-1} \tag{1.3}$$

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where $0 \leq u_1 \leq ... \leq u_n \leq 1$ of $\mathbb{R}^n$, n $\in N$, $k \geq 1$ and $M_i = \sum_{j=1}^{n-1} m_j \geq 1, 1 \leq i \leq n-1$ and all

$m_j \in \mathbb{R}$ be the parameters such that $\gamma_j = k+n-i+M_i \geq 1, \forall i \in 1,...,n$ and suppose

$\tilde{m} = m_1,...,m_{n-1}$, if $n \geq 2$ ($\tilde{m} \in \mathbb{R}$ arbitrary, if n=1) (Kamps; 1995).

Generalized order statistics was presented as unification to the several models of ordered random variables. Giving different values to the parameters we can have these models. As Kamps (1995) showed that in the model of generalized order statistics if we have $\gamma_r = n-r+1$ we get the model of ordinary order statistics, for $\gamma_r = k$ the resulting model will be for kth record values, with $\gamma_r = n-r+1$ α_r we have the model of sequential order statistics. When $\gamma_r = \beta_r$ the resulting model will represent for Pfeifer's record values. Many other models like k_n-records from non-identical distributions, progressive type-II censoring and others can be obtained. However it is noticed that upper records from discrete distributions do not serve as a sub-model of generalized order statistics. One important feature is that many results obtained for generalized order statistics can be generalized to its sub-models.

Generalized order statistics provides a much flexible approach in reliability theory, statistical modeling, inference and in life testing phenomena.

Over the years a lot of work has been done on generalized order statistics since their introduction by Kamps in 1995. Kamps and Gather (1997) used the distributional properties of generalized order statistics to give the characterization of exponential distributions. Burkschat et al. (2003) introduced the concept of Dual Generalized Order Statistics as a unification of the models of descendingly ordered random variables like reversed order statistics, lower k-records and lower Pfeifer records. Aleem and Pasha (2006) derived the distribution of the ratio of two generalized order statistics from Pareto distribution using Mellin transformation. Ahmad (2008) obtained the explicit expressions of single and product moments of generalized order statistics from linear exponential distribution. Beg and Ahsanullah (2004) derived the distribution of the concomitants of generalized order statistics for the Farlie-Gumbel-Morgenstern (FGM) family of bivariate distributions. Faizan and Athar (2008) derived the explicit expressions for single and product moments for generalized order statistics of a family of distributions expressed as $\bar{F}(x) = [ax^p + b]^c$, $p > 0, \eta < x < \omega$ where a, b and c are so chosen that F(x) is a *df* over (η, ω). From Kumaraswamy distribution Garg (2009) derived the joint distribution of two generalized order statistics and the distributions of product and ratio of two generalized order statistics using Mellin transformation and its inverse. For a doubly Type II censored informative sample following a general class of continuous distributions, using one sample scheme, Ahmad (2010) obtained general Bayesian prediction intervals for future generalized order statistics when the future sample was also assumed to follow the general class of continuous distributions.

2. PROBABILITY DENSITY FUNCTION OF GENERALIZED ORDER STATISTICS:

If $X(1,n,m,p),....,X(n.n.m.p)$ are n generalized order statistics where ($p \geq 1$, m is a real number) then the rth generalized order statistics has the p.d.f as:

$$f_{r,n,m,p}(x) = \begin{cases} \dfrac{c_{r-1}}{r-1!}[1-F(x)]^{\gamma_r-1} f(x) g_m^{r-1}[F(x)] & F^{-1}(0) <x< F^{-1}(1) \\ 0 & \textit{Otherwise} \end{cases} \tag{2.1}$$

where

$$\gamma_j = p + (n-j)(m+1)$$

$$c_{r-1} = \prod_{j=1}^{r} \gamma_j$$

$$g_m(x) = -\ln(1-x) \qquad\qquad \text{for } m = -1$$

$$g_m(x) = \frac{1}{m+1}\left[1-(1-x)^{m+1}\right] \qquad \text{for } m \neq -1$$

(Ahsanullah and Nevzorov; 2001)

2.1 Probability density function of generalized order statistics for extended exponential distribution when m=-1:

For m=-1

$$g_m[F(x)] = (1+\lambda x)^{\alpha} -1 \tag{2.2}$$

$$c_{r-1} = \prod_{j=1}^{r} p = p^r \tag{2.3}$$

$$\gamma_r = p \tag{2.4}$$

Using (1.1), (1.2), (2.2), (2.3) and (2.4) in (2.1) we get:

$$f_{r,n,m,p}(x) = \frac{p^r}{r-1!}\left[1-\left[1-\exp\left[1-(1+\lambda x)^{\alpha}\right]\right]\right]^{p-1}$$

$$\alpha\lambda(1+\lambda x)^{\alpha-1}\left[\exp\left[1-(1+\lambda x)^{\alpha}\right]\right]\left[(1+\lambda x)^{\alpha}-1\right]^{r-1}$$

Simplifying this expression we get

$$f_{r,n,m,p}(x) = \begin{cases} \dfrac{\alpha\lambda p^r}{r-1!}(1+\lambda x)^{\alpha-1}\left[(1+\lambda x)^{\alpha}-1\right]^{r-1}\left[\exp\left[1-(1+\lambda x)^{\alpha}\right]\right]^{p} & x>0 \\ 0 & \text{Otherwise} \end{cases} \tag{2.5}$$

Using

$$a+b^{\;n} = \sum_{i=0}^{n}\binom{n}{x} a^i b^{n-i} \tag{2.6}$$

We get:

$$f_{r,n,m,p}\;x\; = \frac{\alpha\lambda p^r}{r-1\;!}\exp\left[p\left[1-\;1+\lambda x^{\;\alpha}\right]\right]\sum_{i=0}^{r-1}\binom{r-1}{i}\;1+\lambda x^{\;r\alpha-\alpha i-1}\;-1^{\;i}\quad x>0 \tag{2.7}$$

2.2 Probability Density Function of generalized order statistics for extended exponential distribution when m ≠ -1:

Now for m ≠ -1

$$g_m[F(x)] = \frac{1}{(m+1)}\left[1-\left[\exp[1-(1+\lambda x)^{\alpha}]\right]^{m+1}\right] \tag{2.8}$$

As

$$c_{r-1} = \prod_{j=1}^{r}\gamma_j \tag{2.9}$$

and

$$\gamma_j = p+\;n-j\quad m+1 \tag{2.10}$$

Now using (1.1), (1.2), (2.8), (2.9) and (2.10) in (2.1) we get:

$$f_{r,n,m,p}\;x\; = \prod_{j=1}^{r}\frac{\gamma_j}{r-1\;!}\left[1-\left[1-\exp\left[1-\;1+\lambda x^{\;\alpha}\right]\right]\right]^{\gamma_r-1}\alpha\lambda\;1+\lambda x^{\;\alpha-1}\exp\left[1-\;1+\lambda x^{\;\alpha}\right]$$

$$\frac{1}{m+1^{\;r-1}}\left[1-\exp\;m+1\left[1-\;1+\lambda x^{\;\alpha}\right]\right]^{r-1}$$

Hence the p.d.f of generalized order statistics for Extended Exponential distribution when m ≠ -1 is given as:

$$f_{r,n,m,p}\;x\; = \begin{cases}\dfrac{\alpha\lambda\;1+\lambda x^{\;\alpha-1}}{m+1^{\;r-1}}\prod_{j=1}^{r}\dfrac{\gamma_j}{r-1\;!}\left[\exp\left[1-\;1+\lambda x^{\;\alpha}\right]\right]^{p+\;n-r\;\;m+1}\left[1-\exp\;m+1\left[1-\;1+\lambda x^{\;\alpha}\right]\right]^{r-1} & x>0 \\ 0 & \text{Otherwise}\end{cases}$$

$$\tag{2.11}$$

Using the binomial expansion (2.6), we can write (2.11) as:

$$f_{r,n,m,p}\left(x\right)=\begin{cases}\dfrac{\alpha\lambda\left(1+\lambda x\right)^{\alpha-1}}{\left(m+1\right)^{r-1}}\prod_{j=1}^{r}\gamma_{j}\sum_{i=0}^{r-1}\dfrac{\left(-1\right)^{i}}{\left(r-i-1\right)!i!}\left[\exp\left[1-\left(1+\lambda x\right)^{\alpha}\right]\right]^{p+n+i-r\left(m+1\right)} & x>0\\0 & \text{Otherwise}\end{cases}$$

$$(2.12)$$

3. MOMENTS OF GENERALIZED ORDER STATISTICS FOR EXTENDED EXPONENTIAL DISTRIBUTION

The sth moment can be derived as:

$$\mu_{r,n,m,p}^{s}=E\left(X_{r,n,m,p}^{s}\right)=\int_{-\infty}^{\infty}x^{s}f_{r,n,m,p}\left(x\right)dx$$

$$(3.1)$$

3.1 Moments for the case m=-1:

Using (2.5) in (3.1) we get:

$$\mu_{r,n,m,p}^{s}=\int_{0}^{\infty}\dfrac{\alpha\lambda p^{r}}{\left(r-1\right)!}x^{s}\left(1+\lambda x\right)^{\alpha-1}\left[\left(1+\lambda x\right)^{\alpha}-1\right]^{r-1}\left[\exp\left[1-\left(1+\lambda x\right)^{\alpha}\right]\right]^{p}dx$$

$$(3.2)$$

Let

$$\left.\begin{aligned}u&=\left(1+\lambda x\right)^{\alpha}-1\\x&=\dfrac{\left(u+1\right)^{\frac{1}{\alpha}}-1}{\lambda}\\du&=\alpha\lambda\left(1+\lambda x\right)^{\alpha-1}dx\end{aligned}\right\}$$

$$(3.3)$$

Substituting in (3.2) we get:

$$\mu_{r,n,m,p}^{s}=\dfrac{p^{r}}{\left(r-1\right)!\lambda^{s}}\int_{0}^{\infty}\left[\left(u+1\right)^{\frac{1}{\alpha}}-1\right]^{s}u^{r-1}\exp\left(-pu\right)^{s}du$$

Using binomial expansion (2.6) and simplifying we get:

$$\mu_{r,n,m,p}^{s}=\dfrac{p^{r}}{\left(r-1\right)!\lambda^{s}}\sum_{i=0}^{s}\binom{s}{i}\left(-1\right)^{i}\int_{0}^{\infty}u^{r-1}\left(u+1\right)^{\frac{s-i}{\alpha}}\exp\left(-pu\right)du$$

$$(3.4)$$

Since

$$\left(1+z\right)^{\alpha}=\sum_{j=0}^{\alpha}\dfrac{\overline{\left|\alpha+1\right.}}{\overline{\left|\alpha-j+1\right.}}\dfrac{z^{j}}{j!}\quad\text{where }\alpha\epsilon\mathcal{R}$$

$$(3.5)$$

Using in (3.4):

$$\mu_{r,n,m,p}^{s} = \frac{1}{r-1\ !\lambda^{s}} \sum_{i=0}^{s} \sum_{j=0}^{s-i} \frac{\binom{s}{i}\left(-1\right)^{i}\overline{\frac{s-i}{\alpha}+1}\ \overline{r+j}}{p^{j}\,j!\ \overline{\frac{s-i}{\alpha}-j+1}} \tag{3.6}$$

Hence (3.6) provides the sth moment of Generalized Order Statistics for Extended Exponential distribution for m=-1.

Putting s=1 in (3.6) we get:

$$\mu_{r,n,m,p} = \frac{1}{r-1\ !\lambda}\left[\overline{\frac{1}{\alpha}+1}\ \sum_{j=0}^{\frac{1}{\alpha}} \frac{\overline{r+j}}{p^{j}\,j!\ \overline{\frac{1}{\alpha}-j+1}} - \overline{r}\right] \tag{3.7}$$

which is the mean of the distribution of GOS for Extended Exponential distribution for m=-1.

Now putting s=2 in (3.6) we get:

$$\mu_{r,n,m,p}^{2} = \frac{1}{r-1\ !\lambda^{2}}\left[\overline{\frac{2}{\alpha}+1}\ \sum_{j=0}^{\frac{2}{\alpha}} \frac{\overline{r+j}}{p^{j}\,j!\ \overline{\frac{2}{\alpha}-j+1}} - 2\,\overline{\frac{1}{\alpha}+1}\ \sum_{j=0}^{\frac{1}{\alpha}} \frac{\overline{r+j}}{p^{j}\,j!\ \overline{\frac{1}{\alpha}-j+1}} + \overline{r}\right] \tag{3.8}$$

Hence variance can be obtained by using (3.7) and (3.8).

3.2 Moments for case m ≠ -1:

Using (2.11) in (3.1) we get:

$$\mu_{r,n,m,p}^{s} = \int_{0}^{\infty} x^{s}\,\frac{\alpha\lambda\ \overline{1+\lambda x}^{\,\alpha-1}}{m+1^{\,r-1}} \prod_{j=1}^{r} \frac{\gamma_{j}}{r-1\ !}\left[\exp\left[1-\ \overline{1+\lambda x}^{\,\alpha}\right]\right]^{p+\ n-r\ m+1}$$

$$\left[1-\exp\ m+1\left[1-\ \overline{1+\lambda x}^{\,\alpha}\right]\right]^{r-1} dx \tag{3.9}$$

Using substitution (3.3) and binomial expansion (2.6) we get:

$$= \prod_{j=1}^{r} \sum_{i=0}^{r-1} \sum_{k=0}^{s} \frac{\binom{s}{k}\gamma_{j}\ \overline{-1}^{\,i+k}}{m+1^{\,r-1}\ r-i-1\ !i!\lambda^{s}} \int_{0}^{\infty} \overline{u+1}^{\,\frac{s-k}{\alpha}} \exp\left[-u\left[p+\ n-r+i\ m+1\right]\right] du \tag{3.10}$$

Since

$$\int_0^\infty \left(x+\beta\right)^{\nu} e^{-\mu x} dx = \mu^{-\nu-1} e^{\beta\mu} \overline{)\left(v+1,\beta\mu\right)} \qquad \left[\left|\arg\beta\right|<\pi, \operatorname{Re}\ \mu>0\right] \qquad (3.11)$$

<div align="right">(I.S. Gradshteyn, I.M. Ryzhik; 2007)</div>

(3.10) becomes:

$$\mu_{r,n,m,p}^s = \prod_{j=1}^{r}\sum_{i=0}^{r-1}\sum_{k=0}^{s} \frac{\binom{s}{k}\gamma_j\left(-1\right)^{i+k}\exp\left[p+\left(n-r+i\right)\left(m+1\right)\right]}{\lambda^s\left(m+1\right)^{r-1}\left(r-i-1\right)!i!\left[p+\left(n-r+i\right)\left(m+1\right)\right]^{\frac{s-k}{\alpha}+1}\overline{)\left(\frac{s-k}{\alpha}+1,\left[p+\left(n-r+i\right)\left(m+1\right)\right]\right)}}$$

<div align="right">(3.12)</div>

Hence (3.12) provides the sth moment of the distribution of Generalized Order Statistics for Extended Exponential distribution for $m \neq -1$.

Putting s=1 in (3.12) we get:

$$\mu_{r,n,m,p} = \prod_{j=1}^{r}\sum_{i=0}^{r-1}\frac{\gamma_j\left(-1\right)^{i}\exp\left[p+\left(n-r+i\right)\left(m+1\right)\right]}{\lambda\left(m+1\right)^{r-1}\left(r-i-1\right)!i!\left[p+\left(n-r+i\right)\left(m+1\right)\right]}\left[\frac{\overline{)\left(\frac{1}{\alpha}+1,\left[p+\left(n-r+i\right)\left(m+1\right)\right]\right)}}{\left[p+\left(n-r+i\right)\left(m+1\right)\right]^{\frac{1}{\alpha}}}-\overline{)\left(1,\left[p+\left(n-r+i\right)\left(m+1\right)\right]\right)}\right]$$

<div align="right">(3.13)</div>

Putting s=2 we get:

$$\mu_{r,n,m,p}^2 = \prod_{j=1}^{r}\sum_{i=0}^{r-1}\left[\frac{\gamma_j\left(-1\right)^{i}\exp\left[p+\left(n-r+i\right)\left(m+1\right)\right]}{\lambda^2\left(m+1\right)^{r-1}\left(r-i-1\right)!i!\left[p+\left(n-r+i\right)\left(m+1\right)\right]}\left[\frac{\overline{)\left(\frac{2}{\alpha}+1,\left[p+\left(n-r+i\right)\left(m+1\right)\right]\right)}}{\left[p+\left(n-r+i\right)\left(m+1\right)\right]^{\frac{2}{\alpha}}}-2\frac{\overline{)\left(\frac{1}{\alpha}+1,\left[p+\left(n-r+i\right)\left(m+1\right)\right]\right)}}{\left[p+\left(n-r+i\right)\left(m+1\right)\right]^{\frac{1}{\alpha}}}+\overline{)\left(1,\left[p+\left(n-r+i\right)\left(m+1\right)\right]\right)}\right]\right]$$

<div align="right">(3.14)</div>

Hence using (3.13) and (3.14) variance can be obtained.

4. NEGATIVE MOMENTS OF GENERALIZED ORDER STATISTICS FOR EXTENDED EXPONENTIAL DISTRIBUTION

The sth negative moment can be obtained using:

$$\mu_{r,n,m,p}^{-s} = E\left(X_{r,n,m,p}^{-s}\right) = \int_{-\infty}^{\infty} x^{-s} f_{r,n,m,p}(x)\, dx \tag{4.1}$$

4.1 Negative Moments for the case m=-1:

Using (2.5) in (4.1) we get:

$$\mu_{r,n,m,p}^{-s} = \int_{0}^{\infty} x^{-s} \frac{\alpha\lambda p^{r}}{(r-1)!}(1+\lambda x)^{\alpha-1}\left[(1+\lambda x)^{\alpha}-1\right]^{r-1}\left[\exp\left[1-(1+\lambda x)^{\alpha}\right]\right]^{p} dx \tag{4.2}$$

Making use of substitution (3.3) in (4.2) we obtain:

$$= \frac{p^{r}}{(r-1)!}\int_{0}^{\infty} \frac{(-1)^{-s}\left[1-(u+1)^{\frac{1}{\alpha}}\right]^{-s}}{\lambda^{-s}} u^{r-1} \exp[-pu]\,du$$

Since

$$(1-x)^{-n} = \sum_{i=0}^{\infty}\binom{-n}{i}(-x)^{i} = \sum_{i=0}^{\infty}\binom{n+i-1}{i}x^{i} \tag{4.3}$$

So

$$\mu_{r,n,m,p}^{-s} = \frac{p^{r}\lambda^{s}(-1)^{-s}}{(r-1)!}\int_{0}^{\infty}\sum_{i=0}^{\infty}\binom{s+i-1}{i}(u+1)^{\frac{i}{\alpha}}u^{r-1}\exp[-pu]\,du$$

Using expansion (3.5) we get:

$$\mu_{r,n,m,p}^{-s} = \frac{p^{r}\lambda^{s}(-1)^{-s}}{(r-1)!}\sum_{i=0}^{\infty}\binom{s+i-1}{i}\int_{0}^{\infty}\sum_{j=0}^{\infty}\frac{\left|\frac{i}{\alpha}\right|\frac{i}{\alpha}+1}{\left|\frac{i}{\alpha}-j+1\right.}\frac{u^{j}}{j!}u^{r-1}\exp[-pu]\,du$$

$$\mu_{r,n,m,p}^{-s} = \frac{\lambda^{s}(-1)^{-s}}{(r-1)!}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\frac{\left.\frac{i}{\alpha}\right|\binom{s+i-1}{i}\left|\frac{i}{\alpha}+1\right.\left|\overline{r+j}\right.}{p^{j}j!\left|\frac{i}{\alpha}-j+1\right.} \tag{4.4}$$

which is the sth negative moment for GOS of extended exponential distribution when m=-1.

4.2 Negative Moments for the case $m \neq -1$:

The sth negative moment can be derived using (2.11) in (4.1).

$$\mu_{r,n,m,p}^{-s} = \int_0^{\infty} x^{-s} \frac{\alpha\lambda \left(1+\lambda x\right)^{\alpha-1}}{\left(m+1\right)^{r-1}} \prod_{j=1}^{r} \frac{\gamma_j}{r-1\,!} \left[\exp\left[1-\left(1+\lambda x\right)^{\alpha}\right]\right]^{p+\,n-r\,\,m+1}$$

$$\left[1-\exp\left(m+1\right)\left[1-\left(1+\lambda x\right)^{\alpha}\right]\right]^{r-1} dx \quad (4.5)$$

Using substitution (3.3) and binomial expansion (2.6) we have:

$$= \prod_{j=1}^{r} \frac{\gamma_j \, \lambda^s}{\left(m+1\right)^{r-1}} \sum_{i=0}^{r-1} \frac{\left(-1\right)^{i-s}}{i!\,\,\text{r-i-1}\,!} \int_0^{\infty} \left[1-\left(u+1\right)^{\frac{1}{\alpha}}\right]^{-s} \exp\left[-u\left[\left(p+\,n-r+i\right)\,m+1\right]\right] du$$

Using expansion (4.3) and result (3.13) we get:

$$\mu_{r,n,m,p}^{-s} = \prod_{j=1}^{r} \frac{\gamma_j \, \lambda^s}{\left(m+1\right)^{r-1}} \sum_{i=0}^{r-1} \sum_{k=0}^{\infty} \frac{\left(-1\right)^{i-s} \binom{s+k-1}{k} \exp\left[\left(p+\,n-r+i\right)\,m+1\right]}{i!\,\,\text{r-i-1}\,!\,\left[\left(p+\,n-r+i\right)\,m+1\right]^{\frac{k}{\alpha}+1}} \left(\frac{k}{\alpha}+1,\left[\left(p+\,n-r+i\right)\,m+1\right]\right)$$

$$(4.6)$$

The expression (4.6) provides the sth negative moment for GOS of Extended Exponential distribution when $m \neq -1$.

5. JOINT DISTRIBUTION OF GENERALIZED ORDER STATISTICS FOR EXTENDED EXPONENTIAL DISTRIBUTION

If $X\left(r,n,m,p\right)$ and $Y\left(r,n,m,p\right)$ are rth and sth generalized order statistics respectively then their joint density function is:

$$f_{r,s,n,m,p}\left(x,y\right) = \begin{cases} \dfrac{c_{s-1}}{r-1\,!\,s-r-1\,!} \left[\bar{F}\left(x\right)\right]^{m} \left[g_m\left(F\left(x\right)\right)\right]^{r-1} \left[g_m\left(F\left(y\right)\right)-g_m\left(F\left(x\right)\right)\right]^{s-r-1} \left[\bar{F}\left(y\right)\right]^{\gamma_s-1} f\left(x\right) f\left(y\right) \\ \qquad\qquad F^{-1}\left(0\right) < x < y < F^{-1}\left(1\right) \\ 0 \qquad\qquad\qquad\qquad \text{Otherwise} \end{cases}$$

$$(5.1)$$

(Ahsanullah and Nevzorov; 2001)

5.1 Joint Probability Density Function for m=-1:

We have

$$\bar{F}\left(x\right) = 1-F\left(x\right) \qquad\qquad (5.2)$$

So for m=-1

$$\left[\overline{F}\ x\ \right]^{\ m} = \exp\left[\ 1+\lambda x\ ^{\alpha}-1\right] \tag{5.3}$$

$$\left[\overline{F}\ y\ \right]^{\ m} = \exp\left[\ 1+\lambda y\ ^{\alpha}-1\right] \tag{5.3 continued}$$

$$c_{s-1} = p^{s}\ \text{and}\ \gamma_{s} = p$$

$$\left[\overline{F}\ y\ \right]^{\gamma_{s}-1} = \left[\ \exp\left[1-\ 1+\lambda y\ ^{\alpha}\ \right]\ \right]^{p-1} \tag{5.4}$$

Now using (1.1), (1.2), (2.2), (2.3), (2.4), (5.3) and (5.4) in (5.1) we get:

$$f_{r,s,n,m,p}\ x,y\ = \frac{p^{s}}{r-1\ !\ s-r-1\ !}\exp\left[\ 1+\lambda x\ ^{\alpha}-1\right]\left[\ 1+\lambda x\ ^{\alpha}-1\right]^{r-1}\left[\ 1+\lambda y\ ^{\alpha}-1-\ 1+\lambda x\ ^{\alpha}+1\right]^{s-r-1}$$

$$\left[\ \exp\left[1-\ 1+\lambda y\ ^{\alpha}\ \right]\ \right]^{p-1}\alpha\lambda\ 1+\lambda x\ ^{\alpha-1}\exp\left[1-\ 1+\lambda x\ ^{\alpha}\right]\alpha\lambda\ 1+\lambda y\ ^{\alpha-1}\exp\left[1-\ 1+\lambda y\ ^{\alpha}\right]$$

Thus

$$f_{r,s,n,m,p}\ x,y\ = \begin{cases} \dfrac{\alpha^{2}\lambda^{2}\,p^{s}}{r-1\ !\ s-r-1\ !}\ 1+\lambda x\ ^{\alpha-1}\ 1+\lambda y\ ^{\alpha-1}\left[\ 1+\lambda x\ ^{\alpha}-1\right]^{r-1}\left[\ 1+\lambda y\ ^{\alpha}-\ 1+\lambda x\ ^{\alpha}\right]^{s-r-1} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\left[\ \exp\left[1-\ 1+\lambda y\ ^{\alpha}\ \right]\ \right]^{p} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0 < x < y < \infty \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{Otherwise} \end{cases} \tag{5.5}$$

which is the joint density of rth and sth generalized order statistics for extended exponential distribution when m=-1.

5.2 Joint Probability Density Function for $m \neq -1$:

We have

$$\left[\overline{F}\ x\ \right]^{\ m} = \left[\ \exp\left[1-\ 1+\lambda x\ ^{\alpha}\ \right]\ \right]^{m} \tag{5.6}$$

and

$$\left[\overline{F}\ y\ \right]^{\gamma_{s}-1} = \left[\ \exp\left[1-\ 1+\lambda y\ ^{\alpha}\ \right]\ \right]^{p+\ n-s\ m+1\ -1} \tag{5.7}$$

Using (2.8) we get:

$$g_{m}\left[F\ y\ \right] - g_{m}\left[F\ x\ \right] = \frac{1}{m+1}\left[\ \exp\ m+1\ \left[1-\ 1+\lambda x\ ^{\alpha}\right]-\exp\ m+1\ \left[1-\ 1+\lambda y\ ^{\alpha}\right]\right] \tag{5.8}$$

Using (1.1), (1.2), (2.8), (2.9), (2.10), (5.6), (5.7) and (5.8) in (5.1) we get:

$$f_{r,s,n,m,p}(x,y) = \prod_{j-1}^{s} \frac{\gamma_j}{(r-1)!(s-r-1)!} \left[\exp\left[1-(1+\lambda x)^{\alpha}\right]\right]^m \frac{1}{(m+1)^{r-1}} \left[1-\exp(m+1)\left[1-(1+\lambda x)^{\alpha}\right]\right]^{r-1}$$

$$\frac{1}{(m+1)^{s-r-1}} \left[\exp(m+1)\left[1-(1+\lambda x)^{\alpha}\right]-\exp(m+1)\left[1-(1+\lambda y)^{\alpha}\right]\right]^{s-r-1}$$

$$\left[\exp\left[1-(1+\lambda y)^{\alpha}\right]\right]^{p+(n-s)(m+1)-1} \alpha\lambda(1+\lambda x)^{\alpha-1}\exp\left[1-(1+\lambda x)^{\alpha}\right]$$

$$\alpha\lambda(1+\lambda y)^{\alpha-1}\exp\left[1-(1+\lambda y)^{\alpha}\right]$$

Using binomial expansion (2.6) we get:

$$= \prod_{j=1}^{s} \frac{\gamma_j \alpha^2 \lambda^2 (1+\lambda x)^{\alpha-1}(1+\lambda y)^{\alpha-1}}{(r-1)!(s-r-1)!(m+1)^{s-2}} \sum_{i=0}^{r-1}\binom{r-1}{i}(-1)^i \left[\exp(m+1)\left[1-(1+\lambda x)^{\alpha}\right]\right]^i$$

$$\sum_{k=0}^{s-r-1}\binom{s-r-1}{k}\left[\exp(m+1)\left[1-(1+\lambda x)^{\alpha}\right]\right]^{s-r-1-k}\left[\exp(m+1)\left[1-(1+\lambda y)^{\alpha}\right]\right]^k (-1)^k$$

$$\exp(m+1)\left[1-(1+\lambda x)^{\alpha}\right]\left[\exp\left[1-(1+\lambda y)^{\alpha}\right]\right]^{p+(n-s)(m+1)}$$

Hence the joint density is

$$f_{r,s,n,m,p}(x,y) = \begin{cases} \prod_{j=1}^{s}\sum_{i=0}^{r-1}\sum_{k=0}^{s-r-1}\frac{\gamma_j \alpha^2 \lambda^2 (1+\lambda x)^{\alpha-1}(1+\lambda y)^{\alpha-1}(-1)^{i+k}}{i!k!(m+1)^{s-2}(r-i-1)!(s-r-k-1)!}\left[\exp\left[1-(1+\lambda x)^{\alpha}\right]\right]^{(m+1)(i+s-k-r)} \\ \left[\exp\left[1-(1+\lambda x)^{\alpha}\right]\right]^{p+(n-s+k)(m+1)} \qquad\qquad 0<x<y<1 \\ \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{Otherwise} \end{cases}$$

$$(5.9)$$

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FUTURE OF MOBILE APPLICATIONS FOR EDUCATION IN PAKISTAN

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ABSTRACT

In the era of mobile revolution everywhere in schools and colleges students are using mobile phones. The mobile application development is taking place which will give new look and shape to education system in the world and as well as in Pakistan. Many new mobile applications are developed for the simple hand-held devices and these are adding new dimensions to the education system. With development of many mobile applications for education it is now possible to make the schools completely book free, thus textbooks will be replaced slowly by eBooks in the educational institutions.

Mobile learning involves using mobile devices as learning tools and accessing learning resources outside an academic institutions premise to allow the learning process to be more dynamic, flexible and collaborative. The objective of this study is to study the possibility of mobile education system in Pakistan. The study is based on secondary data collected from Pakistan Telecommunication Authority, Islamabad and World Wireless Quick Facts. After analysis of data, it is concluded that Pakistan is eighth mobile user in the world so there is bright future of mobile applications for education in Pakistan.

KEY WORDS

Mobile revolution, Mobile applications, eBooks.

INTRODUCTION

The term mobile learning is defined as any sort of learning that happens when the learner is not at a fixed, predetermined location, or learning that happens when the learner takes advantage of the learning opportunities offered by mobile technologies. The term covers learning with portable technologies including but not limited to handheld computers, MP3 players, notebooks and mobile phones.

M-learning focuses on the mobility of the learner, interacting with portable technologies, and learning that reflects a focus on how society and its institutions can accommodate and support an increasingly mobile population. M-Learning also brings strong portability by replacing books and notes with small RAMs, filled with tailored learning contents.

OBJECTIVES

The objective of this paper is to evaluate the possibility of m-learning in Pakistan and to suggest measures and means for its implementation in Pakistan.

BACKGROUND OF STUDY

Pakistan has progressed rapidly in the field of information technology since 2000 when for the first time; Information Technology Policy was officially announced by the Government of Pakistan. Now, over 2100 dialing stations are in place nation-wide. The backbone has recently been upgraded to Dense Wavelength Division Multiplexing (DWDM) with the capacity of 10 gigabytes per second. Bandwidth has been increased from 215.2 Mbps to 610 Mbps and the number of Internet Service providers (ISPs) is 127. Pakistan owns a communication satellite now in orbit, and has extensive international connectivity. About 600 cities have been connected through optical fiber, and 1707 cities have been linked to the Internet. However, one of the main challenges is that the physical and human infrastructure necessary for the implementation of various e-learning and M- learning programs are not uniformly distributed throughout the country due to rural and urban segmentation of the country.

It is estimated that 1.5 billion mobile phones are in operation in the world today (Prensky, 2004). This is more than three times the number of personal computers (PCs), and today's most sophisticated phones have the processing power of a mid-1990s PC. These facts, and the range of computer-like functionality offered by top-of-the-range devices, are leading some observers to speculate that many people in the future not so distant future will start to see the mobile phone as an alternative to a PC. For example Jeff Hawkins, inventor of the Palm Pilot, was recently quoted (Stone 2004) as saying, 'One day, 2 or 3 billion people will have cell phones, and they are not all going to have PCs .The mobile phone will become their digital life'.

The fact that the use of smart phones and tablet PCs in a professional and private context is becoming increasingly common allows learning processes to be transferred to these devices. With an internet connection it is possible to learn anywhere at any time and to call up and internalize contents in the context of concrete questions. Keeping in view the increasing growth and utilization of mobile equipments, we have selected this topic for consideration in Pakistan.

HISTORY OF MOBILE DEVICES

Modern mobile devices began with the Apple Newton in 1993, followed by the Palm Pilot in 1996. Five years later the Pocket PC and the introduction of flash player were the next significant introduction, and have since been used for educational purposes. The next major development occurred when cell phones gained the capabilities of personal digital assistants (PDAs) and merged connectivity. The different types of connectivity available through mobile devices are: wide area network (WAN), local area network (LAN), and personal area network (PAN). Within the field of education it was originally envisioned that handheld devices could serve as computer replacements in which full courses could be delivered. However, to date only individual applications and teacher training has been successful, as well as data collection, mainly in the scientific and medical fields. Currently, there are a multitude of devices available for mobile learning, ranging from PDAs to video players to cell phones. Add-ons to mobile devices such as cameras, barcode readers, and Global Positioning Systems (GPS) are also popular.

APPLICATION OF MOBILE DEVICES

Students are mostly keen about new mobile devices when they are introduced in the classroom. There is a wide range of applications available, several of which are suitable for the classroom. The University of North Carolina has developed a mobile device classroom response system in which all students can answer the teacher's questions, thus enabling a teacher to monitor the level of each student's understanding.

Mobile devices are also being used to improve communication and efficiency at the University of California, San Diego where location-based information is available on handheld devices, enabling staff and students on campus to locate each other immediately. At some universities, mobile devices are actively encouraged. Medical schools are especially active is utilizing handheld devices. At the University of South Dakota, medical school students were given handhelds and the use of these devices has been successful in several subjects. Similarly, at Duke University all freshmen were given Apple iPods, which were used to store course content, music appreciation, poetry, and readings. Other possibilities for the use of mobile devices in education are e-books, GPS and audio devices.

CONTRIBUTION OF MOBILE DEVICES IN EDUCATION

M-learning enables the extension of learning such that it weaves itself into a person's work, when and where they need it. People with time constraints need small portable learning objects that suit low bandwidth conditions, and these devices must suit a new generation of learners with different expectations. Mobile devices open the possibility for new types of learning activities and can help person for doing quick lessons in free time while traveling. Thus, mobile devices provide functionalities in three key areas which are given below.

1. Notification System: the first area is the notification system, a function that instantly sends an SMS or e-mail. This system is a means of informing and reminding the learner, and communicates information about the next learning activity. This function was used in a project in South Africa. In this project mobile devices were used to complement postal-based distance learning. An SMS was the only means of informing an individual living in a remote area that the course materials were ready to be picked up at the post office. Thus, this simple notification system enabled individuals to keep up with distance-learning activities, proving to be useful and relevant for people living in remote rural areas.
2. Learning Management System: second, mobile capabilities can provide access to a learning management system. For example, some mobile phone devices can support a text-based portal through which a student can log-inland complete enrolment for a class or browse a course catalogue.
3. Interactive Applications: third, mobile devices are increasingly providing interactive applications. For example, these devices can facilitate interactive lectures, application-simulations and online discussion boards.

MOBILE-LEARNING TOOLS

This paper provides an overview of the tools needed to create, offer, and access mobile learning. Mobile learning tools are the result of two converging technologies: computers and mobile phones. However, tools are emerging that are specifically designed for mobile learning; for instance, providing authoring capability for audio learning content (e.g., spoken word, podcasts) along with associated interactive assessments and surveys. Other tools are optimized to provide e-learning content through the phone's web browsing capability.

There are several e-learning content authoring tools on the market that offer a mobile-friendly output version of your content. However, some of them are designed to run within their own platform and stand-alone portability isn't always possible. Some of the tools that we've seen only target one screen size. This is not a definitive list of authoring tools. Numerous platforms are available, each with its own advantages, technical specifications, and cost are given in following Table-1.

Table-1:
Typical Technical Specifications

Characters	Laptop	Tablets	PDA	Smart phone	Mobile phone
Weight	2 kg	1.8 kg	0.2 kg	0.1 kg	0.1 kg
Memory	2 GB	1GB	192 MB	64 MB	4 MB
Storage	89 GB	60 GB	2 GB	*	*
Display	1400x1280	1024x768	640x480	220x176	200x150
Battery	3-5 hr	3.5 hr	6-8 hr	8-12 hr	4-10 hr
Cost USD	2000	1500	800	400	200

- Wireless Laptop Computer: A wireless laptop computer offers the greatest capabilities, including maximum storage and a standard PC platform that enables conventional e-learning and web content. However, a wireless laptop is often not suitable for use by small children, and does not permit e-learning while moving.
- Tablet Computer: A tablet computer has full computer capabilities without the keyboard and has been especially successful for teaching and learning of visual subjects. Its major drawback is that the screen scratches too easily.
- Personal Digital Assistant: A personal digital assistant (PDA) and Pocket PC are portable and can have many add-ons, but might not be compatible and incur high costs. The smart phone is a PDA with some mobile phone features or a mobile phone with PDA features. It is advantageous because it is a small device to carry, but the display size is limited. The mobile phone is the least expensive alternative, adequate for exchange of simple messages.
- Wireless Networking: Wireless networking is a feature that can be added or built into the system. Bluetooth wireless is a wireless connection to a local device. It exchanges and synchronizes data so can be utilized for interactions between students and teachers, and is useful for connecting to peripheral devices.
- A Global Positioning System (GPS) is a hardware device that guides learners to locations and objects, records data, and teaches navigation skills. Another

hardware device is a data probe, which hooks onto the device and is used for real-time data collection with sensors for temperature, air and light.

- Radio Frequency Identification (RFID), another hardware device that can be used for m-learning, is a reader on the mobile device that will detect and retrieve data such as lesson plans from a designated object. Still and video cameras can also be used in m-learning as they offer multiple functions to capture data.
- Web browsers and Macromedia flash are the most common type of software used in mobile devices. Additional programs to consider are Acrobat PDF readers for MS office, Power Point converters and media players.

CHALLENGES TO MOBILE LEARNING

There are so many areas in Pakistan which do not have access to electricity and telecommunication links. There is non-availability of software in local and regional languages; lack of e-learning content for schools; lack of know-how in developing digital content for children. In addition there are cost implications involved in implementing the Mobile learning policy in Pakistan.

MOBILE DEVICE INDUSTRY TRENDS

It is visualized that within 15 to 20 years there will be one global mobile campus. Mobile devices are rapidly evolving, size is decreasing, capabilities are increasing, and cost is decreasing. However, battery life time is stills an issue for it needs to be extended. The imminent arrival of the Windows Mobile 5 will include hard drive support and a 30 per cent increase in battery life which will alleviate the problem of loss of information in Pocket PCs.

ADVANTAGES OF M. LEARNING

It will bring new technology into the classroom, there will be more light weight devices as compare to books, PCs, mobile learning supports the learning process rather than being integral to it, mobile learning will be used according to the groups of students involved, mobile learning can be a useful add-on tool for students with special needs.

GROWTH OF MOBILE TECHNOLOGY

The Mobile technology can be used successfully in the following area such as: testing, surveys, job aids and just-in-time (J.I.T.) learning. Location-based and contextual learning, social-networked mobile learning, m mobile educational gaming, deliver M-Learning to cellular phones using two way SMS messaging and voice-based Cell Casting.

Thus, cell or smart phones, multi-game devices, personal media players (PMPs), personal digital assistants (PDAs), or wireless single-purpose devices can help deliver coaching and mentoring, conduct assessments and evaluations, provide on-the-job support and access to information, education and references, and deliver podcasts, update alerts, forms and checklists. In these ways, mobile learning can enhance and support more traditional learning modes, making it more portable and accessible. Mobile devices can also serve as powerful data collection tools and facilitate the capture of user created content.

Mobile devices that support mobile learning include: E-book, OutStart, Handheld audio and multimedia guides, in museums and galleries, Handheld game console, modern gaming consoles such as Sony PSP or Nintendo DS, Personal audio player, e.g. for listening to audio recordings of lectures (podcasting), Personal Digital Assistant, in the classroom and outdoors, Tablet computer, UMPC, mobile phone, camera phone and Smart Phone.

DATA ANALYSIS

Country wise data regarding, numbers mobile phones, population of the country was collected from various official resources and compiled, percentage were calculated. Summary of data are presented in the following Table .2.

Table 2:
List of Countries by Number of Mobile phones in Use

Rank	Country	M. phones	Population	% of pop.	Last updated
	World	Over 5.6 bil	7,012,000,000	79.86	2011
1	China	1,010,000,000	1,341,000,000	75.32	Mar. 2012
2	India	919,170,000	1,210,193,422	76.00	Mar. 2012
3	USA	327,577,529	310,866,000	103.9	June 2011
4	Brazil	250,800,000	192,379,287	130.36	April 2012
5	Indonesia	250,100,000	237,556,363	105.28	May 2009
6	Russia	224,260,000	142,905,200	154.5	July 2011
7	Japan	121,246,700	127,628,095	95.1	June 2011
8	Pakistan	114,610,000	178,854,781	66.5	Jan 2012
9	Germany	107,000,000	81,882,342	130.1	2009
10	Nigeria	90,583,306	140,000,000	64.7	Feb. 2011
11	Mexico	88,797,186	112,322,757	79.8	Sep. 2010
12	Italy	88,580,000	60,090,400	147.4	Dec. 2008
13	Bangladesh	86,550,000	148,090,000	58.5	Apr. 2012
14	Philippines	86,000,000	94,013,200	91.5	Oct. 2011
15	U.K	75,750,000	61,612,300	122.9	Dec. 2008

Source: World Population Clocks — POP Clocks. census.gov.

This list ranks the countries of the world by the number of mobile phones in use. We have given only 15 countries of the world, but there are 60 countries of world whose statistics are given in Wikipedia. It is observed that Pakistan is the 8th user of mobile phones in the world. Number one user of mobile phone is China followed by India, It is seen that 67% people in Pakistan are using mobile phones.
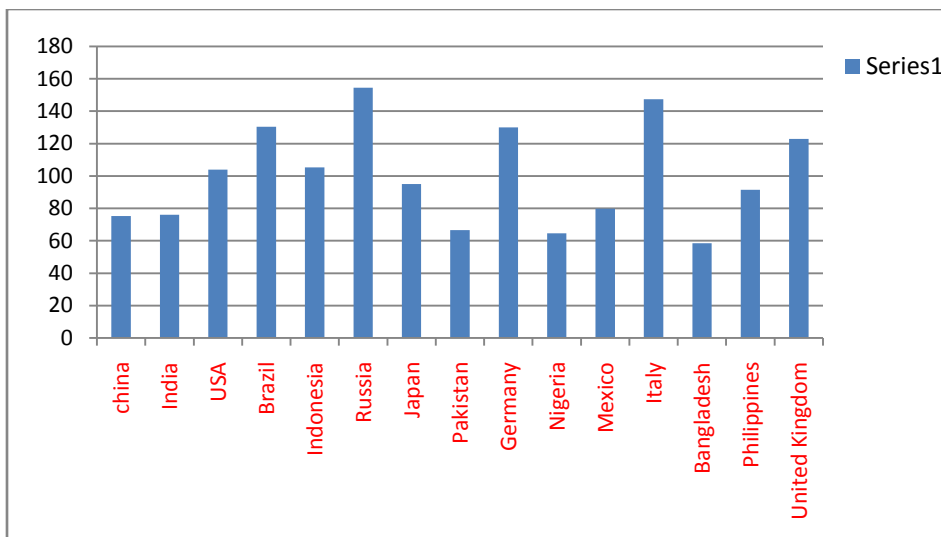
Fig. 1: Bar chart showing country wise percentage of mobile utilization

It is observed from the Bar chart that, Russia is the highest user of mobile phone, 155%, followed by Italy 147%, Brazil 130%, Germany 130%, UK 123%, Indonesia, 105%, USA 104%, respectively. The lowest user in this series is Bangladesh, 59% peoples are using mobile phones. About 67% peoples of Pakistan are using mobile phones, within few years; there will be possibility that more people will use the mobile phones. This trend shows that there is possibility of mobile phones application for education sector in Pakistan.

CONCLUSION

It is observed that through Mobile Learning, students will be able to experience learning through listening, reading, predicting, reflecting and responding. This entire experience can take place via a smart phone or other mobile technology media. Thus, learning can take place anywhere at any time, but most importantly, all the time.

M-learning has the potential to improve efficiency in the education sector and expand educational opportunities to underserved communities in remote areas. However, there will be challenge to be faced by the management in implementing m-learning. It is widely believed that mobile learning could be a huge factor in getting disaffected young adults to engage in learning, where more traditional methods have failed. As mobile phones combine PDA functions with cameras, video and MP3 players, and as tablets combine the portability of PDAs with the functionality of desktops, the world of learning becomes more mobile, more flexible and more exciting. It is concluded that there is future of mobile phone, technology applications in education sector in Pakistan.

RECOMMENDATIONS

Before m-learning programs can be implemented in Pakistan, infrastructure must be established, ICT services be expanded, innovative policies should be administered, curriculum and content should be developed, and teacher training may be arranged.

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WHY FORGIVENESS WORKS AS AN ORGANIZATIONAL CONFLICT-RESOLUTION STRATEGY: ROLE OF EMOTIONAL EMPATHY AND GENDER

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ABSTRACT

It has been observed that women leaders tend to use forgiveness as a problem-solving strategy to solve organizational conflict because they are more interested in maintaining relationships than men (Grace-Odeleye, 2007). Contrary to this gender-specific model, the present study was planned to extend the existing model by (a) investigating the contribution of gender and emotional empathy in order to understand the role of forgiveness in conflict resolution and (b) proposing that the suggested model equally applies to men and women leaders as it has been found that both genders become more forgiving when they have innate ability to empathically understand others (Exline, Baumeister, Zell, Kraft, & Witvliet 2008). The sample will comprise of 45 men and 45 women administrative-level individuals belonging to higher education sector. The inclusion criteria for the participants were at least 5 years of: a) administrative experience, b) teaching experience, and c) supervisory experience. Heartland Forgiveness Scale (HFS; Thompson, Snyder, Hoffman, et al, 2005), Emotional Empathy Scale (Caruso, & Salovey, 1998) and Conflict Resolution Questionnaire (Henning, 2003) were used to measure the study variables. The results of the study showed no gender differences on forgiveness and emotional empathy. However, working women ($M = 149$) were found to be significantly different from men ($M = 133$) on total conflict resolution strategies ($t = -4.84$, $p < .000$). Hierarchical Multiple Regression analyses verified the proposals that forgiveness and gender interact ($R^2 = .32$; F (8, 81) = 5.10, $p < .000$) and forgiveness and emotional empathy interact ($R^2 = .23$; F (8, 81) = 3.41, $p < .01$) to predict conflict resolution at workplace. The results implied that forgiveness and emotional empathy are important predictors of organizational conflict management that have been scantly addressed in Pakistan and that gender interplays with such variables to impact conflict-related behaviors at workplace.

1. INTRODUCTION

It is only in the last two decades that forgiveness has been conceptualized as a construct with profound implications for physiological and psychosocial well-being. For instance, evidence implies that forgiveness is strongly related with mental and emotional health (Anderson, 2007; Bono & McCullough, 2006; Legaree, Turner, & Lollis, 2007), physiological and psychological well-being (Lamb, 2005; Harvard Women's Health Watch Newsletter, 2005), reduced anxiety and depression (Zechmeister, Garcia, Romero, & Vas, 2004), and enhanced personal happiness and interpersonal relationships (Harvard Women's Health Watch Newsletter, 2005).

Enright and Coyle (1998) stated "in genuine forgiveness, one who has suffered an unjust injury chooses to abandon his or her right to resentment and retaliation, and instead offers mercy to the offender" (p. 140). Similarly, McCullough, Fincham, and Tsang (2003) maintain that forgiveness involves a prosocial change regarding a transgressor on the part of the transgression recipient.

Forgiveness and Organizational Conflict Resolution

It has been reiterated time and again that organizational conflict, though disliked, is a pervasive part of the work life in organizations (Henning, 2003). Evidence suggests that conflict affects the organization adversely in terms of poor performance, lack of cooperation, wasting of resources and productivity (see, for instance, Hotepo, Asokere, & Ajemunigbohun, 2010), delays of work, disinterest and lack of action and in extreme cases complete breakdown of the group (Parker, 1974). Madsen, Gygi, Plowman, and Hammond (2009) noted that 'Conflicts in the workplace may include situations such as coworkers having minor disagreements, departments at war with each other, hurtful rumors being spread, accurate or inaccurate performance appraisals, ethical and legal issues, employment decisions (e.g., hiring, firing, promotions), lack of support on initiatives or decisions, and more.'

It follows, then, if organizational conflict is an interpersonal problem, it can be managed either focusing on interpersonal relationships or through structural changes (Hotepo, Asokere, & Ajemunigbohun, 2010). As highlighted by significant number of researchers, 'forgiveness' is one dynamic that has not been much explored in organizational setting (Madsen et al, 2009), yet is an important construct that is important to address in the workplace environment (Cameron & Caza, 2002).

Recently, Madsen, Gygi, Plowman and Hammond (2009) presented a thorough review on the potential benefits of intra- and interpersonal forgiveness, emphasizing on the role of forgiveness in promoting the performance and productivity of employees and organizations. In fact, they have proposed a theoretical framework, in which forgiveness is offered as a workplace intervention strategy, "that allows for effective learning, development, and contribution of all employees." Previously, Aquino, Grover, Goldman, and Folger (2003) had also pointed out the significance of forgiveness in organizational settings. Through scientific inquiry they concluded that forgiveness is a successful technique for repairing damaged relationships, which in turn, influences the overall functioning of the organization.

Similarly, Stone (2002) observed that, 'An organizational culture that does not promote forgiveness will be engaged in negative and destructive politics… which will eventually decrease an organization's effectiveness." In other words, Stone accentuated that a forgiving culture reduces job turnover and aggressive and passive-aggressive behavior on the part of the individual. Following these theoretical arguments, research has provided substantial impact of forgiveness in organizational settings. For instance, a research conducted by Bradfield, Aquino, and Stanwyck (1997) yielded positive association between forgiveness and organizational conflict management and restoration

of relationships. In a qualitative analysis, Grace-Odeleye and Osula (2007) also identified interpersonal forgiveness as a successful means of managing conflicts at workplace.

Why Forgiveness Works in Resolving Conflict at Workplace?

Several arguments have been presented explaining why forgiveness works in organizations. Scobie and Scobie (1998) stated, "forgiveness is reported to be a treatment which offers a means to overcome anger, resentment, the 'debilitating repletion of negative action' and to alleviate a persistent negative state" (p. 374). While, Stone (2002) explained that when employees and managers have an in-depth understanding of the value of forgiveness, it actually provides opportunities to "use mistakes, failures, flaws and breakdowns of life as opportunities to awaken greater wisdom, compassion and capability in our co- workers and ourselves" (p. 279). On the other hand, McCullough and Witvliet (2000) maintain that the capacity to forgive is every bit intrinsic to human nature.

According to Aquino et al (2003), forgiveness motivates employees to "extend acts of conciliation and goodwill toward the offender and to overcome social estrangement" (p. 213), which makes the working relationship between individuals more effective and productive. They further pointed out that forgiveness is actually a type of "problem-solving coping strategy in that it reconciles conflicting parties and salvages the social relationship for future interactions" (p. 213).

Forgiveness and Gender Differences

It is a general contention that women are more forgiving than men (Miller, Worthington, Jr., & McDaniel, 2008). Some studies have provided support to this assertion. For instance, a meta-analytic review by Miller et al (2008) of 70 studies reported an effect size of .28, supporting the stereotype that women are more forgiving than men. The same study also explored various reasons for gender differences in forgiveness. Firstly, the analyses identified that gender differences on forgiveness may be moderated by the type of measure used. That is, it was found that when forgiveness was measured in terms of vengeance, gender differences were observed. Stuckless and Goranson (1992) proposed that men tend to score high on vengeance as they are socialized to be more aggressive in solving conflicts, whereas women are socialized to maintain interpersonal relationships in conflict management and are therefore, less vengeful. Secondly, dispositional moderators such as agreeableness and empathy were also recognized as potential dispositional moderators. In addition, based on Kohlberg (1984) and Gilligan's (1994) theorizing, the study proposed that women may be more forgiving than men as women are more motivated to maintain relationships and are responsive to others' needs whereas men are driven to maintain social order through either revenge or other social mechanisms, which in turn, may incline women more towards forgiving behavior (2008). Similarly, religion's contribution to forgiveness among women was also assessed. It was proposed that since women are more religious (Freese, 2004) and religion considers forgiveness as a value (Rye, 2005), it can be argued then these women would be more forgiving. The authors of the study have also presented biological evidence to support gender differences on forgiveness. They proposed that gender differences on forgiveness might arise because of differences in perception of transgressions. Support for this argument was taken from a study conducted by Sani et al

(2007), in which fMRI technology was used to study whether there were differences in brain activity of men and women engaging in forgiving and non-forgiving acts. The results of the study were used to conclude that 'males and females process and react to emotionally hurtful events in functionally different ways' (Miller et al, 2008).

Grace-Odeleye and Osula (2007) studied the role of gender and forgiveness in conflict resolution at workplace. Her research showed that 'women leaders were found to utilize forgiveness more often than men leaders.' This study, as the above-cited study, explained that the gender differences on forgiveness are observed because women are more interested in maintaining relationships than men. The same research also showed that women leaders tend to utilize forgiveness more as a problem-solving strategy to solve organizational conflict as compared to men. Other researches also support association between forgiveness and women managers preference to use it in conflict resolution, compared to men. For instance, Belenky, Clinchy, Goldberger, and Tarule, (1986) and Gilligan (1982) found that compared to men, women managers tend to focus on care and concern for others, building relationships, and communicating and resolving conflicts.

However, contradictory evidence also exists documenting no significant differences between men and women on forgiveness (see, for example, Berry, Worthington, Parrott, O'Connor, & Wade, 2001; Kalbfleisch, 1997). Similarly, Toussaint and Webb (2005) conducted a cross-sectional research to investigate gender differences on forgiveness in general population. Their study also yielded no gender difference on forgiveness. Toussaint and Webb (2005) suggested that 'there appears to be no straightforward gender differences in levels of forgiveness.'

Forgiveness and Emotional Empathy

Emotional empathy allows one to develop vicarious connection with another person so that one can feel the way the other is feeling. According to Ashraf and Tariq (2007), emotional empathy is a 'unique capacity of human beings to recognize and understand emotions in others.' Moore (1990) believes that empathy is "an organizer and regulator of a variety of behaviors" (p. 75). Research has found that emotional empathy plays a role in the survival of a group and in social bonding (Hoffman, 1981), inhibits aggressive behavior (Eisenberg, Zhou, & Koller, 2001), enhances emotional intelligence (Salovey & Mayer, 1997), and affiliative tendency (Ashraf & Tariq, 2007), and is an important component of conflict resolution (Greenspan, 1997). For Hoffman (1990), emotional empathy is vital for moral development and justice, thus serving as a means for societal harmony and unity as positive interpersonal relationships are dependent upon one's ability to take another's perspective (Johnson, Cheek, & Smither, 1983).

It follows, then, if emotional empathy is crucial for developing and maintaining interpersonal relationships, it must be tied with forgiveness also. As explained by Toussaint and Webb (2005), 'it is likely that an ability to understand others, to relate to others, and to treat others as one would like to be treated would enable a person to forgive others. The empathic person tends to focus on others' experiences in a fairly objective or unselfish manner rather than focusing on one's own experiences in a selfish manner.' Substantial evidence supports this argument. For example, Enright and Coyle (1998) and McCullough

(2000) and Worthington (1998) have found positive association between forgiveness and emotional empathy. Other studies have also come up with similar links between forgiveness and emotional empathy (see, for example, Konstam, Chernoff, & Deveney, 2001; Macaskill, Maltby, & Day, 2002; Zechmeister & Romero, 2002).

There has been a steady trend in understanding and applying the concept of emotional empathy as well as emotional intelligence in enhancing interpersonal relationships at workplace. As pointed out by Kurzynski (1998), 'good working relationships are essential to organizational effectiveness.' Evidence suggests that emotional empathy and emotional intelligence plays constructive role in smoothing out interpersonal conflicts in organizations. For instance, a study conducted to determine the moderating role of emotional intelligence between conflict resolution strategies and organizational citizenship behavior (OCB) found that emotional intelligence allows one to take the perspective of another person and use withdrawing strategies to manage conflict (Salami, 2010).

In organizational settings, the interactive role of emotional empathy and forgiveness has also been proposed to resolve conflicts. It has been noted that socially problematic situations usually emerge because of self-control failures (Baumeister & Exline, 1999). However, Baumeister and Exline (1999) asserted that a trait of emotional empathy would allow successful leaders to supersede self-interest and become cognizant of others' needs. In an interpersonal conflict situation, emotional empathy may lend the offended party a cognitive framework to understand their own propensity for wrongdoing and past need for forgiveness, thereby making way for forgiveness and reconciliation in resolving social problems.

The Present Study

All the studies mentioned afore, have been conducted in western culture. The present study is unique in the sense that it aimed to establish whether gender differences in forgiveness also exist in Pakistani culture and to understand why forgiveness works in resolution of organizational conflicts.

Thus, in the light of the above discussion, the present study was designed to address following questions: (1) Are working women in Pakistan also more forgiving than men? (2) Does forgiveness and gender interact to impact conflict resolution at workplace? (3) Does forgiveness and emotional empathy interact to predict conflict resolution at workplace?

2. METHOD

Participants

The sample for the present study comprised of 45 men and 45 women, administrative-level individuals belonging to higher education sector. The inclusion criteria for the participants were at least 5 years of: a) administrative experience, b) teaching experience, and c) supervisory experience. Their ages ranged from 30 to 45. 60% of the participants belonged to social sciences departments and 40% to natural sciences departments.

Instruments

Details of the instruments used to measure the study variables are presented below.

(a) Emotional Empathy Scale (Caruso, & Salovey, 1998)

Emotional Empathy Scale (Mayer et al, 1999) consists of 30 items. Each item is anchored on a five-point Likert type rating scale, where a score of '1' is assigned to '*strongly disagree*,' '2' to '*disagree*,' '3' to '*uncertain*,' '4' to '*agree*,' and '5' to '*strongly agree*.' Six items are negatively worded. An Alpha Coefficient of .88 has been reported for the scale. It is recommended for use with adult as well as adolescent population (Mayer et al, 1999).

(b) Heartland Forgiveness Scale (HFS; Thompson, Snyder, Hoffman, et al, 2005)

The Heartland Forgiveness Scale (HFS) measures a person's dispositional forgiveness of self, others, and situations beyond anyone's control (e.g., a natural disaster or illness). It is a self-report questionnaire and can be completed with paper and pencil or on the computer. HFS consists of 18 items, with half items negatively worded. All items are placed on 7-point rating scale, ranging from '*almost always false of me*' to '*almost always true of me*.' It has demonstrated desirable psychometric properties, including Convergent validity, satisfactory internal consistency reliability and strong test-retest reliability (Thomson et al, 2005).

(c) Conflict Resolution Questionnaire (Henning, 2003)

The Conflict Resolution Questionnaire (Henning, 2003) was developed to measure a person's ability to create mutually beneficial resolutions to conflict for all participants. CRQ is based on the theories of Weeks (1994) and Fisher and Ury (1991). It consists of 40 items, out of which 11 items are negatively worded. The items are rated on a five-point Likert-type scale, ranging from 'almost never' with a vlaue of '1' to 'almost always' with a value of '5'. The scale satisfies all the required psychometric requirements including high internal consistency and validity protocols. In the present study, four sub-factors of the Conflict Resolution Questionnaire as presented by Marcus Henning (2003) were utilized. These sub-factors are described as following: (1) factor one, 'power', it relates to aspects of control and influence; (2) factor two, 'consideration', it is connected with the notion of giving careful reflection to conflict issues; (3) factor three, 'doubt', the items within this factor describe features of unsureness; and (4) factor four, 'atmosphere', the items in this factor allude to qualities of environmental significance and its impact on conflict resolution.

(d) Demographic Information Proforma

Information on background variables was acquired through a demographic information proforma. Follwoing information was obtained: age, gender, department, teaching, administrative, and supervisory experience.

Procedure

The particpants were approached individually and after ensuring complete confidentiality and taking their informed consent, the questionnaires were handed over.

Specific instructions were given separately for completing the forms. Only those individuals were included who had an experience of at least five years in teaching, administartive, and supervisory roles. The scales were applied on 100 working men and women. However, upon scrutiny it was found that 10 forms were not completely filled. These forms were discarded for final analyses. Later, the data was entered into SPSS 17 for statistical verification.

3. RESULTS

In order to answer the research question related to gender differences in forgiveness, *t*-test was applied. In addition, *t*-value was also calculated to investigate gender differences for other study variables. In order to determine whether gender interacts with forgiveness to predict conflict resolution, Hierarchical Multiple Regression was utilized. In the first step, all the background variables were entered, followed by gender and forgiveness to see their contribution as main effects in the second step. Lastly, gender and forgiveness were jointly entered to yield their interactive impact. Interaction of forgiveness and emotional empathy was also determined by the same procedure and steps using Hierarchical Multiple Regression.

Study Variables and Gender Differences

The results yielded a non-significant difference between men and women on forgiveness as well as emotional empathy. However, significant differences were observed on organizational conflict resolution between men and women, including differences on sub-dimensions of conflict resolution namely, consideration and atmosphere.

Table 1
Mean, Standard Deviations, and t-values for Gender Differences on Study Variables (N=90)

	Gender		*t*	*p*
	Men	**Women**		
Forgiveness	83	84	-.48	.63
Emotional Empathy	105	103	.68	.50
Power	30	30	.78	.90
Consideration	30	36	-5.21	.000
Doubt	17	16	1.48	.07
Atmosphere	16	18	- 2.90	.01
Total Conflict	133	149	- 4.84	.000

Results shown in Table 1 indicate that men and women are equally forgiving and emotionally empathic. While, mean values obtained on conflict resolution of men and women signify that women are more interested in resolving conflicts than men. Similarly, results also suggested that women are more considerate of others and careful of the conducive workplace environment in managing conflicts. Non-significant gender differences on 'power' and 'doubt' specified that both men and women equally use power and are sometimes are doubtful in dealing with conflicts at workplace.

Forgiveness, Gender Differences, and Organizational Conflict Resolution

Table 2 exhibits β values for all the predictors. For the background predictors, only supervision was found to correlate significantly to conflict resolution. After controlling for these factors, gender differences were found on 'consideration' and 'atmosphere,' and total conflict resolution. While forgiveness was significantly found to impact 'power' and 'doubt.' Significant interaction was observed for all the dimensions of conflict resolution including total, except, doubt. The results confirmed the hypothesis that gender interacts with forgiveness to predict conflict management [F (8, 81) = 5.10, $p<.000$].

Figure 1 shows that men who have greater tendency for forgiveness are better at conflict resolution. Interestingly opposite was observed for women. It was found that working women did not rely on 'forgiveness' as a strategy for conflict resolution.

Table 2:
Summary of Hierarchical Regression Analyses Predicting Organizational Conflict Resolution from Background Variables, Gender, Forgiveness (Standardized Beta Coefficients) (N=89)

	Power	Consideration	Doubt	Atmosphere	Total Conflict
Step 1					
Age	.18	-.06	-.06	.08	-.03
Department	.02	-.22	-.14	-.16	-.19
Administration	-.27	-.03	.00	-.05	-.12
Teaching	-.01	-.06	.01	.12	.13
Supervision	-.06	.30**	.14	-.19	.33***
R^2	.09	.14**	.05	.08	.20**
Step 2					
Gender (male)	.06	-.48***	-.18	-.33**	-.38**
Forgiveness	-.28**	-.22*	-.05	.24**	-.05
R^2	.17	.33***	.07	.20*	.29***
Step 3					
Gender (male) X Forgiveness	.78**	.41***	-1.5	-2.62**	-1.8**
R^2	.18**	.34***	.09	.31*	.32***

Note: Power: $F(8, 81) = 2.15$, $p<.000$; Consideration: F (8, 81) = 5.1, $p<.000$;
Doubt: F (8, 81) = .96, $p<.67$; Atmosphere: F (8, 81) = 4.60, $p<.000$;
Total Conflict: F (8, 81) = 5.10, $p<.000$

$^{***}p<.000$; $^{**}p<.01$; $^{*}p<.05$
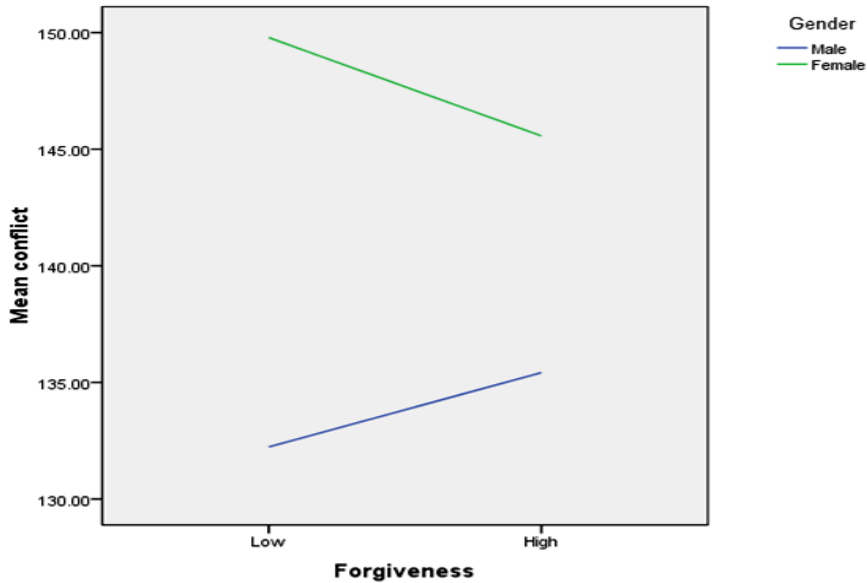
Fig.1: Interaction between Forgiveness and Gender to Predict Conflict Resolution

Forgiveness, Emotional Empathy and Conflict Resolution

Table 3 displays β values for all the predictors.   As for the first model, only supervision was found to correlate significantly to conflict resolution. Controlling for these factors, it was found that emotional empathy does not contribute to conflict resolution independently, while forgiveness was found to be significantly related with 'power' (negative correlation) and 'atmosphere.' However, the interactive effect of forgiveness and emotional empathy was found to significantly predict 'power,' 'consideration,' and total conflict management. The results confirmed that forgiveness and emotional empathy interact to predict conflict management [$F (8, 81) = 3.41$, $p<.01$].

The results can be taken to mean that though emotional empathy is not related with conflict resolution directly, when combined with forgiveness it supports conflict resolution tendency (see Graph 2). That is, individuals who are emotionally empathic as well as forgiving are inclined towards solving workplace problems.

Table 3:
Summary of Hierarchical Regression Analyses Predicting Organizational
Conflict Resolution from Background Variables, Emotional Empathy, Forgiveness
(Standardized Beta Coefficients) (N=89)

|	Power	Consideration	Doubt	Atmosphere	Total Conflict
Step 1					
Age	.18	-.06	-.07	.08	-.03
Department	.02	-.22	-.14	-.16	-.19
Administration	-.27	-.03	.00	-.05	-.12
Teaching	-.01	-.06	.01	.12	.13
Supervision	-.06	.30**	.14	-.19	.33***
R^2	.09	.14**	.05	.08	.20**
Step 2					
Forgiveness	-.23*	-.18	-.01	.25**	.00
Emotional Empathy	.21	.16	.03	.01	.18
R^2	.20*	.21**	.05	.14	.20**
Step 3					
Emotional Empathy X Forgiveness	2.24	4.17***	4.8***	2.24	-1.8**
R^2	.22**	.28***	.15**	.14	.23**

Note: Power: $F(8, 81) = 2.61$, $p<.01$; Consideration: $F(8, 81) = 3.98$, $p<.000$;
Doubt: $F(8, 81) = 1.72$, $p<.12$; Atmosphere: $F(8, 81) = 1.69$, $p<.14$;
Total Conflict: $F(8, 81) = 3.41$, $p<.01$

$^{***}p<.000$; $^{**}p<.01$; $^{*}p<.05$
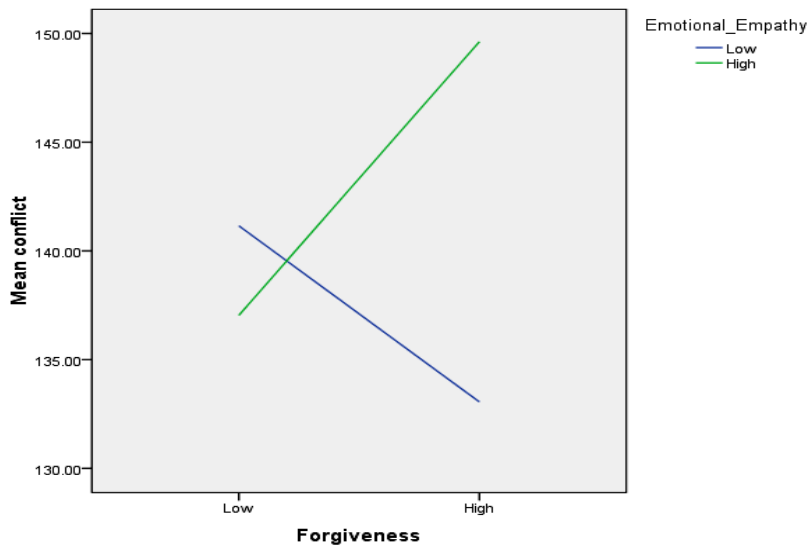
Fig. 2: Interaction between Forgiveness and Emotional Empathy to
Predict Conflict Resolution

4. DISCUSSION

Forgiveness is a variable, which has positive interpersonal and psychological implications. Though all religions and collectivistic as well as individualistic cultures applaud forgiving behavior (Grace-Odeleye & Osula, 2007), it is a construct on which scant research literature exists. The present study, at least to the knowledge of authors, is the first attempt to study forgiveness in Pakistan. The results of our study have established the significance of forgiveness in working environment.

One of the objectives of the present study was to investigate gender differences in forgiveness in Pakistan. As discussed above no conclusive data exists in this regard in other countries. A substantial number of researches have shown that women are more forgiving than men (see, for instance, Miller et al, 2008) while some studies also suggests no gender differences on this variable. (see, for instance, Toussaint & Webb, 2005). The results of the present study also found that men and women do not differ in forgiveness. One explanation may be related with the type of the sample used. Our participants consisted of working men and women with atleast five years of experience in teaching, administrative, and supervisory role at university level, with 30-45 age range. İt remains to be seen whether similar results would be obtained in general population and in different age groups in Pakistan.

Similarly, non-significant results were observed for emotional empathy and gender. This was contrary to expectations. For instance, a study conducted by Ashraf and Tariq (2007) on University students revealed that women are emotionally more empathic than men. With regards, total conflict resolution, significant gender differences were observed. İt was found that working women were more inclined to resolve workplace conflict than working men. İn addition, the analyses also revealed that women were more considerate and desired a conflict-free working environment than men. These results support earlier speculations. As pointed out by Grace-Odeleye and Osula (2007), 'some researchers assert that women leaders bring greater benefits to organizations than men in the leadership practices exhibited (see, Bales, 2000; Gatteau, 2000).' According to Billing and Alvesson (1993), "Women look at problems in a different way. They can point to different models for solving problems. They also have intuition" (p. 130). Women also tend to focus on care and concern for others, building relationships, and communicating and resolving conflicts (Belenky, et al., 1986; Gilligan, 1982).

The prime purpose of the present study was to understand the moderators, which link forgiveness with conflict resolution at workplace. We proposed two models. The first model tested the interactive contribution of forgiveness and gender in organizational conflict management. The results verified the model and showed that forgiveness and gender combined together explained 32% variance in conflict resolution at workplace. Analysis of the graphic presentation of the data exhibited some interesting insights in the issue. Contrary to expectations, men were found to be more forgiving in conflict resolution than women. The present study proposes a couple of explanations for this discrepancy. First, it may be that women leaders and managers are afraid of being labeled as 'soft' in dealing with problems and challenges at workplace so perhaps they tend to

avoid using this strategy. Secondly, it is also obvious from the results that working women tend to be more considerate and conscious of keeping the working environment conducive for everyone, it may that they use other strategies to manage conflicts than forgiveness.

The second model proposed that individuals, who are forgiving and have the ability to take the perspective of other people, are more capable of solving workplace conflicts. This model was also proved by the results of the study. It was found that forgiveness and emotional empathy interacted to account for 23% of the variance in conflict resolution. This agrees with previous speculations. It has been argued that being able to understand other's problems and mistakes as one's own allows one to forgive the other person (Toussaint & Webb, 2008). This was also supported by graphic presentation (Figure 2).

Generally, the results showed that forgiveness contributes significantly to total conflict resolution and more so than gender and emotional empathy. In both models, it was observed that 'power' was negatively associated with forgiveness, while 'consideration' and 'atmosphere' were found to be positively related with forgiveness. This provided support to general models of leadership theory (see, for review, Grace-Odeleye & Osula, 2007). Huggins (2000) speculating on relationship between leadership and forgiveness pointed out 'forgiveness simply offers us all a way to learn from and move on" (p. 9). Similarly, emphasizing upon various positive outcomes associated with forgiveness in leadership practices, Nelson (2001) said " When, as a leader, you forgive, you implicitly establish a climate of compassion, a place where everyone feels free to try new things, make mistakes - and relax. If others know that you utilize forgiveness in your leadership practices this reassures people that they can come to you as the leader with problems, challenges and mistakes (p. 8).

5. COMMENTS AND CONCLUSION

The present study points toward the complex nature of factors involved in organizational conflict resolution. As asserted by Madsen et al (2008), "improving individuals, team, and organizational performance is a primary focus of management and human resource development (HRD)." They further stated, "HRD researchers and practitioners should consider the influence of individual and workplace forgiveness on the performance and productivity of employees and organization." In addition, they proposed workplace forgiveness as an intervention strategy for overcoming interpersonal hurts and disputes. In support of the previous research, the results of present study offered some profound implications for workplace environment. The study showed that forgiveness and emotional empathy may be considered as important response to conflict solution, i.e., jointly may be considered another way to enhance performance of organizations and improve the workplace culture (Bradfield, Aquino, & Stanwyck, 1997). The results of the present study also shed some light on the role of gender in conflict management at workplace. It appears that though working women are more interested in solving organizational conflicts and disputes, men tend to use forgiveness as a strategy to resolve workplace problems. It may be concluded that men and women both bring unique benefits to organizational efficiency and development.

As noted before, this study was perhaps the first one of its sort in Pakistan. More researches on similar patterns need to be undertaken to fully comprehend the principles of forgiveness, gender, and emotional empathy in relation to conflict management at workplace. Future researches must elaborate on the models proposed by the present study to completely understand the dynamics involved in forgiveness and strategies that can be developed to promote forgiveness, and emotional empathy to restore relationships in workplace. Research using different samples and age groups in workplace may also add to the existing data. Moreover, cross-cultural studies on forgiveness and gender can also provide insight to the issue at hand. Direct relationship between forgiveness and productivity of an organization also needs to be addressed. As it is obvious from the above discussion, if forgiveness brings such positive benefits to organizations and interpersonal relationships, attempts should also be made to identify the determinants of forgiveness. This will help in formulating modules on learning how to be forgiving in workplace environment. Future research work should also address the implications of using forgiveness as a value and strategy in daily life.

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DETERMINANTS OF FERTILITY
(A CASE STUDY OF DISTRICT MARDAN KHYBER PAKHTUNKHWA)

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ABSTRACT

There is abundance of literature on the factors determining the fertility level. This also includes macro or aggregate level studies to probe the phenomenon. This paper is a micro level attempt to find the factors determining fertility level. The finding reveals that literacy of the head of household (LHH), use of contraceptives (UC) and female literacy rate (FLR) are negatively related with fertility rate (FR). While female labor force participation rate (FLFP) significantly affects fertility rate (FR) with positive coefficient as revealed in other previous studies. Similarly per capita income (PCI) also has positive relation with fertility. The findings would help the policy makers regarding any policy to population control and effective use of various techniques used for decline in fertility.

1. INTRODUCTION

Population is the main focus of economics since Malthusian time. One way to study the ratio of population growth is by understanding the elements of fertility rates. The understanding of these elements could be taken as guide line for making socio- economic policies useful for poverty alleviation.

During the second half of the twentieth century, medical and health sectors advanced a lot, which reduced the mortality rate and lead to faster increase in world population. World population more than doubled growing from 2.51 billion to 6.07 billion in the era of 1950 to 2000, respectively (Hakim, 2000). This is mainly because of above average growth rate observed in less developed countries. Approximately, population growth in less developed countries has been recorded at about three times since 1950. However, efforts have been made over last few decades to control the population growth in developing and least developing countries (LDC). Developing countries like South Korea (0.9%), Indonesia (1.3%), India (1.7%), Iran (1.2%) and Bangladesh (1.7%) have somewhat controlled the population growth in last thirty years (Population Reference Bureau, 2001). This fertility decline has been made possible by progammes like Family Planning Programs sponsored either by government or by non-governmental organizations, and anti-natalist policies (UNO, 2000).

Leading Female Indicators in South Aisa

Country	Contraceptive Prevalence Rate	Female Labor Force Participation	Female Literacy Rate (+15)
India	41	38	47
Pakistan	24	24	32
Bangladesh	49	26	66
Sri Lanka	66	47	81

UN Report (2002)

Education brings socio economic changes in different societies which lead to development. Increase in population has been controlled in underdeveloped countries by adopting certain methods through education and awareness. Pakistan is facing problems in reducing population growth rate because of less female education as compared to India, Sri Lanka and Bangladesh. The role of women in economic development in Pakistan is 24% which is less than India, Bangladesh and Sri Lanka that is 38%, 26% and 47%, respectively. The high fertility rate in Pakistan is the product of low literacy rate and ineffective contraceptive prevalence methods. Similarly, the ratio of contraceptive prevalence methods in Pakistan is as less as 24% compared to 41% in India, 49% in Bangladesh and 66% in Sri Lanka.

There are many reasons by which Pakistan is behind in such developments. For instance, Pakistani society is not only strictly religious but also traditional hindrances are present. Female participation is hardly encouraged in socio economic activities and they are preferred and forced to stay within their houses. Lack of education and awareness has caused high fertility rate especially in rural areas.

Another reason of high population growth rate is the large family size. Families with large number of members are considered powerful in rural areas. Sons are given preference against daughters which sometimes increases the fertility as well as the family size.

While studying population growth of Pakistan, it is observed that Pakistan has consistently maintained high fertility and population growth in last several decades.

This study is a micro level learning being focused on a small tehsil Takht-Bhai of district Mardan in province of Khyber Pakhtoon Khwa. The selection of the tehsil is based on its highest fertility rate and population amongst other districts of province. The tehsil Takht-Bhai is located at 34°12'0N 72°1'60E with an altitude of 283 meters (931 feet) lying the south-west of the district. Its population according to 1998 census growth rate is estimated at 1.96 million in 2009. The literacy rate of the tehsil was recorded at 48.27% in 2009.

Research Objectives

- To find out the socio and economic determinants of fertility; To find out the impact of each variable on fertility in the study area; To suggest policies for reduction in fertility in the study area.

Research Hypothesis

H_0: Social and economic factors do not cause fertility
H_1: Socio and economic factors cause fertility

2. REVIEW OF LITERATURE

There is rich literate about fertility rate & their impact on socio-economic factor of developing countries. But keeping in view the objectives, the relevant literature has been reviewed & mentioned here in this paper:

Akman (2002), observed the relationship between education and fertility rate in Bangladesh. The result shows that education is an important determinant of fertility in Bangladesh. The study explains that the education in general and secondary education in particular put positive impact on reducing fertility rate and also give equal & good status to women in every walk of life. The inverse relationship between fertility & education states that if the level of education is high then the fertility rate will be low & vice versa. He suggests that education should be provided by the government to every citizen irrespective of their gender.

Fauzia et al. (1999), has conducted a survey to compare the fertility rate in urban areas and rural areas. She finds that the women in the urban areas had lower fertility than the women in the rural areas. The face behind this difference is due to the prevalence of family planning programs in urban areas. Secondly, the awareness of uses of contraceptive methods is more in urban areas as these are more accessible through government hospitals & clinics. Despite such facilities also available in rural areas, there are still some obstacle in the wide spread of family planning programs. She suggested that awareness about family planning programmes in rural areas would allow them to use different methods for reducing fertility rate. And the government should provide facilities to the rural areas & also encourage the services of private institutions which can play an important role in this respect.

M M Masih (1998) in his study finds the determinants of high fertility. He used time series data & cointegration techniques for long run & vector correction model for short run dynamics. The results suggest that female education, family planning negatively affect fertility rate in the underdevelopment economy like India.

Mohsin et al. (2003), in their of female education, marriages at lower age, ignorance of the contraceptive methods & other religious & traditional values. They use econometric techniques multiple regression model & co-integration. They suggest in their report that the government of Pakistan should focus on female education, family planning programs, encourage & create awareness regarding contraceptive methods to overcome the problem of rapidly rising population.

Shaheen et al. (2007), used cross section data to find out relationship between education & fertility & also the impact of contraceptive use of the fertility. They observed that higher will be female education the lower will be the fertility rate. The use of contraceptive is also negatively related with fertility.

Soomro G Y et al. (2000), conduct the impact of the female education & fertility implication for family planning programme in Pakistan. The data which they collected from their survey shows that the educated women have lesser children than illiterate women. They were of the view that the educated women can control their fertility even in the absence of family planning progarmme. The result suggests that the government of Pakistan should make investment in female education.

Toor IA (2007), observed that education at any level can impact the fertility rate. He uses the cross section data for the study area & finds the impact of different variables on fertility rate like school; life expectancy; education of married women; index of development; health status; female labor force participation; infant mortality rate & some religious & traditional values. He observed that fertility may be positive because of weak relationship between health facilities & infant motility rates. He also suggested that only higher education can influence the fertility rate.

3. METHODOLOGY AND DEFINITION OF VARIABLES

This part deals with the research methodology of the study "Impact of socio economic factors on fertility rate in Tehsil Takht-i-Bhai district Mardan".

3.1 Fertility Model and Definition of Variables.

Economic theories of fertility assume that parents have the number of children they do because they desire approximately that number, given the costs of birth control. This demand for more children is affected by many socio-economic factors such as the level of human capital of family members, family income and the experience of child mortality.

The main objective of the current study is to find out the significances of explanatory variables, viz. (LHH, LR….) with the dependent variable FR. For this, we have estimated the linear econometric model as

$$FR = b_0 + b_1 LHH + b_2 FLR + b_3 UC + b_4 PCI + b_5 FLFP + ui$$

where

FR = FERTILITY RATE
LHH = LITERACY OF THE HOUSEHOLD HEAD
FLR = FEMALE LITERACY RATE
PCI = PER CAPITA INCOME
UC = USE OF CONTRACEPTIVES
FLFP=FEMALE LABOR FORCE PARTICIPATION
and
$b_{i,s}$ *are parameters to be estimated.*

3.1.1 Education of the Head of Household and Fertility

A literate household head also having inverse impact on fertility. While being literate, he is relatively well aware of the adverse impact of the large family size.

3.1.2 Education and Fertility

Education not only important for individual but also to the society as such because it enable individual adaptable to changing conditions. Women with education favor low fertility due to several reasons. The educated mother prefers child schooling rather than

involvement in domestic or agriculture activities. Educated women also take care of her health as well as of their child. As a result of these the infant mortality rate also becomes low. Thus we can say that there is inverse relationship between fertility and female literacy rate.

3.1.3 Economic Conditions and Fertility

The relationship between economic conditions and fertility is not clear. There are two different views about income−fertility relationship. One view is that there is positive relationship between income and fertility while second view is that there is inverse relationship between income and fertility. The rationale behind this positive income-fertility relationship is that, holding everything else constant, higher income implies greater resources available to support a large family. In contrast the supporters of negative income−fertility relationship argued that most of the people with higher incomes stress upon quality of Child not on quantity and thus rise in income might lead to a reduction in fertility. Furthermore, a majority of the later studies tend to support this negative association.

3.1.4 Female Labour Force and Fertility

Female labour force participation portrays a negative relationship with the fertility rate. The literature shows that self-employment can allow women to generate income while taking care of their children and other household responsibilities. Public sector employment often involves shorter hours and the presence of child care facilities that makes it more compatible with child rearing. Most studies suggest that in underdeveloped countries the women with some job wish to have more children. However, a−priori we can't say anything about relationship between Female labor force participation and fertility rate.

3.1.5 Contraceptives and Fertility

The use of contraceptives has inverse relationship with fertility as when use of contraceptives rises, fertility will decline.

Following table explain the expected signs of different variables used in this study in relation to fertility rate in selected study area:

Variables	Expected Signs
LHH	—
FLR	—
PCI	+ / —
UC	—
FLFP	+ / —

3.2 Universe of the Study

According to objectives of the research study, a comprehensive interview schedule comprising 16 questions was developed. Considering the sensitivity of the issue in the study area, some irrelevant questions were dropped from the interview schedule and only necessary questions were asked for obtaining meaningful responses.

3.3 Sampling and Sample Size

The sample is selected from Tehsil Takht-i-Bhai district Mardan and 100 respondents were selected by simple random sampling.

3.4 Data Analysis

After data collection, tally sheets were prepared and data were further analyzed and interpreted using appropriate software (Eviews-5) widely used for statistical analysis.

4. RESULT AND DISCUSSION

The parameter estimates of the preferred econometric model are presented in Table below along with their significances as shown by the t-ratios and the Adj. R^2 of the regression. All the parameters with the exception of the intercept term which is usually added to the econometric model for computational convenience have economic meanings. The coefficients of explanatory variables are statistically significant at 5 percent level.

DEPENDENT VARIABLE: FR			
Variable	**Coefficient**	**t-Statistic**	**Prob.**
LHH	-0.174194	-3.173047	0.0038
FLR	-0.934956	-5.055045	0.0000
PCI	0.162257	2.017674	0.0068
UC	-0.713774	-2.025760	0.0074
FLFP	1.306341	2.104599	0.0380
C	4.657941	8.750563	0.0000
R-squared 0.714181			
Adjusted R-squared 0.569935			
Durbin-Watson stat 1.932871			

The coefficient of LHH, FLR and UC are negatively related with FR. It implies that a one percent change in these variables will affect the explanatory variable FR by 17%, 93% and 71% inversely and separately. In the case of PCI and FLFP, positive relation with FR is observed means a one per cent change in these variables will increase the explanatory variable FR by 16% and 130%, respectively and separately.

The Adj. R^2 provides evidence in favor of all regressors. This is a joint test of significance and is useful to know how close the estimated observations to the scattered observations are. The value of adj. R^2 is 0.71. It implies that overall 71 percent of the sample variations in the FR are explained by the selected regressors and the rest variation appears to be due to change in the factor captured by the disturbance term (e_i).

The data also do not have problem of autocorrelation as clear from the value of Durban Watson statistic above. Similarly, there is no problem of multico-linearity in the data as indicated by the value of R^2 and significant t-values.

5. CONCLUSIONS AND POLICY RECOMMENDATIONS

The study analyses the determinants of fertility at micro level. It binds the relationship between fertility and socio-economic determinants. The finding appears to be consistent with macro level findings which states that education is negatively related to fertility. Educated women have fewer children than uneducated i.e. when the level of education is high the fertility rate will be low and vice versa. Apart from female's education, the education of the head of the family also significantly affects the fertility rate. Other than these, the use of contraceptive prevailing measures negatively relate to fertility. On the other hand, per capita income and female labor force participation contributing significantly and positively to fertility rate in the study area.

These findings would definitely help the policy makers regarding policy to control population growth. This study being at micro level is considered to be important as micro level studies always found effective planning. While population is growing Pakistan at an alarming rate, the population planning authority in Pakistan is in process of adopting measures which could really help in controlling the issue. This study reveals that fertility decline not only requires structural change but also requires considerable awareness program in addition to other family planning programs.

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MEASURING QUALITY OF AGE REPORTING DATA IN PAKISTAN

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ABSTRACT

The objective of the study was to determine the age heaping phenomena in two census data (1981, 1998) by using modified Whipple's index method. The results of this method was compared with other summary measures of age reporting quality ; Whipple's index and Myer's blended index. The quality of age reporting for 1998 census was improved if compared to the 1981 census data. Age heaping occurred most often in the female population compared to male and in rural areas compared to urban areas. The results showed that total modified Whipple's index account well for the quality of age reporting and it takes account of preference and avoidance of all ten digits. So it is a fair and reliable measure and produces the same results as Myer's blended index. The main advantage of its use lies in the simplicity of its calculation.

INTRODUCTION

A population census is a complex, large scale operation usually undertaken only once in every decade. Although appropriate scientific methods are adopted for data collection, however, the responses may not be accurate. The census data serve many purposes especially in national policy making and demographic research studies. Consequently, the scope of research in these areas and validity of their findings depends upon the accuracy of census data. Among the large volume of data collected in census, Age-sex structure is a fundamental component and plays an important role in demographic analysis (1). Good age reporting is a crucial prerequisite for accurate estimates of basic component of population change (i.e. fertility, mortality and migration measures). So misreporting of age seriously affects the quality of data (2). That's why, age reporting error has been a perennial cause of concern for many social scientist, demographers, sociologist and economists in developing countries (3). It is therefore important to examine where in the age distribution and in which population subgroups the misreporting occurs, and with what magnitude.

Data on age in developing countries like Pakistan are subject to errors. A common error in age reporting is the tendency of rounding the ages to the nearest figure ending in '0' or '5' or to a lesser extent, in even number.

Several techniques have been developed (mostly by demographers and epidemiologists) to measure the magnitude of errors emanating from digit preference as well as techniques of correcting or adjusting the information. Some of the techniques are robust while others are not. Whipple index is a classic measurement for evaluating the

age heaping. This index has been widely used to assess the quality of age reporting in census and survey data. This index measures only the attraction for age ending in 0 and 5. On the basis of modifications to the original Whipple's index, Spoorenberg (4) proposed a new synthetic index which accounts the attraction/repulsion of all ten digits. This is known as the total modified Whipple's index and is based on the same assumption of the original Whipple's index.

The objective of this study is to investigate the pattern of digit preference by applying the total modified Whipple's index on the data reported in two census of Pakistan (1981, 1998). To evaluate this advance method we will compare it with the other commonly used summary measure of age reporting.

MATERIALS AND METHODS

We studied the single year age data from two census reported by Pakistan Census Organization. These censuses were carried out in 1981 and 1998 and the population composition was described according to age, sex and residence status (urban and rural). Three standard indices are used to detect the digit preference, namely, the Whipple's index (WI), total modified Whipple's index (MWI), and Myer's blended index.

Whipple's index is simple to calculate and most widely applied (5). It detect age heaping on terminal digit '0' and '5' in the range from 23 to 62 both years inclusive. The Whipple's index is calculated as

$$WI = \frac{\left(\sum_{j=0}^{7} \Upsilon_{25+j\times 5} \right) \times 100 \times 5}{\sum_{i=23}^{62} \Upsilon_i} \tag{1}$$

where Υ_x is the population of age x in completed year.

The Whipple's index varies from 100 to 500. Where there is avoidance for '0' and '5' then WI=100. On the other hand, WI=500 indicating that only ages ending in '0' and '5' were reported (6).

Roger (7) proposed first change in original formulation of WI and calculate the two measures of index as

$$W_0 = \frac{\left(\sum_{j=0}^{3} \Upsilon_{30+j\times 10} \right) \times 10}{\sum_{i=23}^{62} \Upsilon_i} \tag{2}$$

and

$$W_5 = \frac{\left(\sum_{j=0}^{3} \Upsilon_{25+j\times10}\right) \times 10}{\sum_{i=23}^{62} \Upsilon_i} \tag{3}$$

By taking average of (2) and (3) we return to the original Whipple's index (WI).

Later Noumbissi (8) has advocated the following modification in the calculation of the two indices.

$$W_0 = \frac{5 \, \Upsilon_{30} + \Upsilon_{40} + \Upsilon_{50} + \Upsilon_{60}}{(_5\Upsilon_{28} + _5\Upsilon_{38} + _5\Upsilon_{48} + _5\Upsilon_{58})} \tag{4}$$

and

$$W_5 = \frac{5 \, \Upsilon_{25} + \Upsilon_{35} + \Upsilon_{45} + \Upsilon_{55}}{(_5\Upsilon_{23} + _5\Upsilon_{33} + _5\Upsilon_{43} + _5\Upsilon_{53})} \tag{5}$$

The above modifications are based on the same assumptions as of the original Whipple's index (linearity and rectangularity) and allow to measure age heaping for all terminal digits. For each digit, the degree of preference or avoidance can be determined as follows:

$$W_1 = 5 \, \Upsilon_{31} + \Upsilon_{41} + \Upsilon_{51} + \Upsilon_{61} \Big/ (_5\Upsilon_{29} + _5\Upsilon_{39} + _5\Upsilon_{49} + _5\Upsilon_{59}) \tag{6}$$

$$W_2 = 5 \, \Upsilon_{32} + \Upsilon_{42} + \Upsilon_{52} + \Upsilon_{62} \Big/ (_5\Upsilon_{30} + _5\Upsilon_{40} + _5\Upsilon_{50} + _5\Upsilon_{60}) \tag{7}$$

$$W_3 = 5 \, \Upsilon_{23} + \Upsilon_{33} + \Upsilon_{43} + \Upsilon_{53} \Big/ (_5\Upsilon_{21} + _5\Upsilon_{31} + _5\Upsilon_{41} + _5\Upsilon_{51}) \tag{8}$$

$$W_4 = 5 \, \Upsilon_{24} + \Upsilon_{34} + \Upsilon_{44} + \Upsilon_{54} \Big/ (_5\Upsilon_{22} + _5\Upsilon_{32} + _5\Upsilon_{42} + _5\Upsilon_{52}) \tag{9}$$

$$W_6 = 5 \, \Upsilon_{26} + \Upsilon_{36} + \Upsilon_{46} + \Upsilon_{56} \Big/ (_5\Upsilon_{24} + _5\Upsilon_{34} + _5\Upsilon_{44} + _5\Upsilon_{54}) \tag{10}$$

$$W_7 = 5 \, \Upsilon_{27} + \Upsilon_{37} + \Upsilon_{47} + \Upsilon_{57} \Big/ (_5\Upsilon_{25} + _5\Upsilon_{35} + _5\Upsilon_{45} + _5\Upsilon_{55}) \tag{11}$$

$$W_8 = 5 \, \Upsilon_{28} + \Upsilon_{38} + \Upsilon_{48} + \Upsilon_{58} \Big/ (_5\Upsilon_{26} + _5\Upsilon_{36} + _5\Upsilon_{46} + _5\Upsilon_{56}) \tag{12}$$

$$W_9 = 5 \, \Upsilon_{29} + \Upsilon_{39} + \Upsilon_{49} + \Upsilon_{59} \Big/ (_5\Upsilon_{27} + _5\Upsilon_{37} + _5\Upsilon_{47} + _5\Upsilon_{57}) \tag{13}$$

where Υ_x is the population of completed age x and $_5\Upsilon_x$ the population of the age range (x, x+4).

The problems with the extension proposed by Noumbissi is that it is not convenient to compare changes through time and across countries. There is still need of a summary index which computed the variability of age reporting. To overcome this Spoorenberg proposed a new synthetic index called the Modified total whipple's index (W_{tot}). It is

computed as the sum of the absolute difference between the digit specific modified whipple's index and 1 and summarizes all age preference and avoidance effects. The total modified whipple's index (W_{tot}) is written as below:

$$W_{tot} = \sum_{i=0}^{9} \left| W_i - 1 \right| \tag{14}$$

If no preference is observed, then, $W_{tot} = 0$. The W_{tot} reaches the maximum value of 16. This index can thus be used as a general measure of the quality of age reporting and gives a more accurate and sensitive measure of overall age reporting quality. Moreover, W_{tot} is also based on the same assumptions as original Whipple's index.

The other standard index used in this study is Myers blended index (9). Myer's index measures preference for all terminal digits 0 to 9. For the calculation of Myers index, select the age range for which the digital preference has to be measured. Normally it is based on single year age data from 10 to 89 year. Using this range, take the sum of number of people whose age ends with a particular digit for the population aged 10 and over, and then for the population aged 20 and over. Apply weights to each series and the results are added to obtain a blended population. A summary index is obtained by summing the absolute deviations between the aggregate and theoretical distribution (10%). Theoretical range of Myer's blended index is from 0 to 90. An index of 0 represents no heaping and an index of 90 represents a heaping of all reported ages at a single digit (10).

RESULTS AND DISCUSSION

The degree of digit preference for total, urban and rural population of two censuses separated for males and females are presented in table 1. From the examination of table 1, it is noted that two census have quite different pattern in age reporting. Based on United Nations Standard (2), age reporting in 1981 is very poor and the strong preference of age ending in 0 and 5 is reflected. The situation in 1998 is less extreme. This indicates that digit preference declined over time and general quality of age reporting has improved from one census to the next which indicates that poor reporting of age in 1981 census was notices by the technical people of the census organization and they may have trained their enumerators to obtain better information while collecting data on age.

Table 1:
Comparison of the original Whipple's index, total modified Whipple's index
and Myers' blended index for 1981 and 1998 Census of Pakistan

		Original Whipple's index		Total Modified Whipple's index		Myer's blended Index	
		Males	**Female**	**Males**	**Female**	**Males**	**Female**
1981	Total	3.32	3.27	9.35	9.16	73.54	74.74
	Urban	3.40	3.27	9.65	9.13	77.15	76.48
	Rural	3.14	3.29	8.67	9.25	65.66	70.43
1998	Total	1.72	2.01	3.09	4.34	29.82	38.86
	Urban	1.51	1.76	2.28	3.32	22.22	29.30
	Rural	1.85	2.14	3.59	4.86	34.14	43.77

The total digit specific modified Whipple's index for each sex is reported in 5th and 6th columns of table 1. The findings are similar with the above findings based on original Whipple's index. But the results reported by the original Whipple's index is partial because only digit preference for age ending in 0 and 5 is taken into account.

To evaluate the performance of total modified Whipple's index, we use it for comparisons between two census. The results indicate that both male and females tend to misreport their age. In 1981, males have a higher tendency of age heaping than females in urban areas. Whilst the reverse was observed in rural areas. The results of 1998 census indicate that overall females have a higher tendency of age misreporting. However, the change in W_{tot} values overtime showed that general quality of age reporting has improved both gender and residence wise.

Last, Myers blended index was calculated over the same range as the total modified Whipple's index. The results vary in an identical manner as total modified Whipple's index which is confirming the pertinence and validity of the W_{tot} index. The differences simply reflect the methods and assumptions upon which the two indices are based. The calculation method for the total modified Whipple's index is simpler than that used for Myer's blended index.

In short, while the original Whipple's index only measures preference for ages ending in digits 0 and 5, the modified total Whipple's index (W_{tot}) takes account of preference and avoidance of all ten digits using all the information obtained via the specific W_i indices. Moreover, it produces practically the same results as Myer's index. Its main advantages are the simplicity of its calculation methods and its comparability with the original Whipple's index. Hence by taking account of the effects of all ten digits, the W_{tot} index provides an essential complements to the specific W_i indices and a more accurate measure of overall age reporting quality.

Figure 1 below gives the preference for each digit as calculated by the digit-specific modified Whipple's index (W_i) for both census. Based on the results presented in figure 1, one can clearly identify that attraction for ages ending by 0 and 5 are the main causes to low quality of age reporting in 1981. At the same time, because the importance of the attraction on 0 and 5 age digit reduces in 1998, the age reported on the other digit gains significance, explaining the better quality of age reporting.
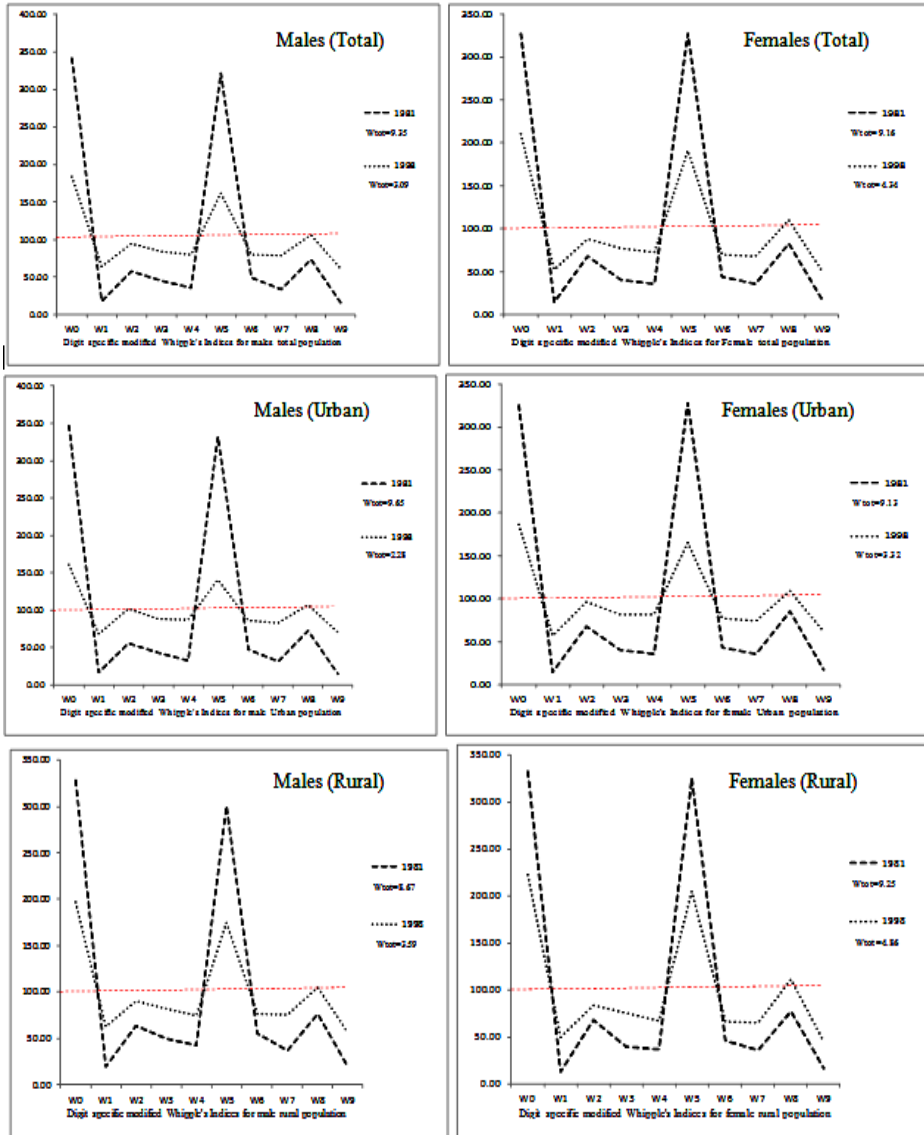
Figure 1: Quality of age reporting by sex: digit-specific modified indices and total modified Whipple's Index in 1981 and 1998 Census in Pakistan.

CONCLUSIONS

On the basis of modifications to the original Whipple's index, this paper proposes a general measure of age reporting quality- the total modified Whipple's index- in complement to the developments proposed by Noumbissi. To test its pertinence and validity, the new index is applied to sex-specified reported age data in two census data collected at 1981 and 1998. It can be deduced from the analysis that quality of age reporting in Pakistan was poor. The quality of age reporting in 1998 was better than 1981 census data. It is suggested that an expert enumerator is appointed for interview of female's population. Further whenever, any data gathering regarding age information takes place, it is recommended to refer to an ID and B-form in preference of the person's self report.

The results obtained are then compared with the myer's blended indices and the original Whipple's index obtained with the same data. This comparison shows that because the total modified Whipple's index is more sensitive than the original Whipple's index, it provides a more accurate measure of age reporting quality and produces results identical to those obtained with myers's blended index. So if one wants to assess the quality of age reporting with more precision and its changes through time, the total modified Whipple's index offers a simple alternative which fully accounts for the changes in the attraction/repulsion of all age digits. The limitation of this study is that we did not have access to the original data base from other census (conducted in 1951, 1961 and 1972) in order to calculate the indexes in detail and develop a full comparison among all censuses.

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CONSTRUCTION OF SEVERAL DIAMOND MODELS

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ABSTRACT

In this study we made classification of diamond stones towards cost and developed different prediction models for predicting diamond cost on the basis of stones characters by using different variable selection procedures such as Prediction sum of square (PRESS), Akaik information criterion (AIC) and Schwartz bayesian criterion (SBC). In classification of stones, findings demonstrate that all the stones such as: weight (X_1), clarity s12 (X_5), clarity ws1 (X_6), weight cut premium (X_1X_7) have positive effect towards diamond cost where as remaining stones characters contributes negatively. It is also observed that diamond stone having maximum weight (X_1) and maximum clarity ws1 (X_6) are more costly.

1. INTRODUCTION

Diamond stones are consumed in many jewelry forms like rings, bracelets, and nucleus. When people shop for diamonds, they learn that stones vary in cut, clarity, color, and weight. Production of any metal can be increased by improving color, clarity, or by the use of efficient cut, or by the both. In this study we evaluate several results on diamond model, which is a rich source of quality stones. Chu (1996) argued that the price of diamond jewelry depends on the four C's: caratage, cut, colour and clarity of the diamond stone. A good cut gives a diamond more sparkle. Colourless diamonds are the most prized. A flawless diamond has maximum clarity because the passage of light is unimpeded through the stone.

Suich and Derringer (1980) purposed the estimator of the unknown parameters in the regression model and proved that this estimator was uniformly inferior to the James-Stein estimator. They elucidated that if the usual regression F-ratio was significant and lack of fit was not, the regression equation was often judge to be an adequate predictive model, and then the factors such as examination of residuals and size of the mean square error also enter into this judgment. Their decision rule outlined was not an estimation technique and they concluded, it was another aid which the experimenter may find helpful in evaluating the adequacy of regression equation.

Freedman (1983) provided useful information about the regression equation. The objective of his study was to quantify the significance levels of conventional statistical tests both through simulation and asymptotic calculation. He developed a regression model in content where substantive theory was weak. He focused on extreme cases by assuming that dependent variables and the explanatory variables had no relation. His results showed that if explanatory variables with small t-statistics are dropped and the equation refitted then R^2 were high and the overall F were also highly significant.

Snee (1997) studied comparison of model prediction and coefficients with theory to determine the validity of regression models. He also used the new data to check model predictions in this respect, he used portions of the data to estimate the model coefficients and used the remaining data for measuring the accuracy of the model. His results showed that the data splitting was an effective method of model validation when it was not practically too possible to collect new data to test the model. He also used the appropriate examples to illustrate the various methods of model validation. He also presented some new methodology for model validation and concludes data splitting or cross-validations an effective method of evaluating a regression model.

Chu (2001) argued that many statistical problems were handled in the linear regression framework. He developed the pricing model for diamond stones by using the multiple linear regression and his results indicated that the log transformation was appropriate for the model.

Yasunori and Fujikoshi (2002) studied the different problems for variable selection in multivariate analysis. For this instance they expressed that a set of variable is sufficient and the remaining set of variable was redundant. They also used some model selection criterion like AIC, Cp in their study for selecting an appropriate model. They also discussed about recent development and outlook on the selection of variable approach. Their selection criterion was the expected mean squared error of prediction that was an unbiased estimator of its risk function. Actually, their apprehension was to discover modified criteria that reduced the bias, focused on a general setting between the true model and candidate model, distributional assumption and framework of asymptotic approximations.

2. METHODOLOGICAL CONSIDERATION

2.1 *Description about Diamond Stones Data*

In this study we used the Diamond data. The reason of taking this diamond data was to provide the key statistics to the diamond shopper which can helpful to decide the diamond is bargain or it is overpriced with different fixed diamond stones. Diamond data obtained from www.1diamondsource.com. This data set consists of 133 observations; four different characters related to diamond stones are taken and detailed in table 1.

Table 1: Diamond Stones	
Serial number	**Diamond Stones**
1	Weight
2	Cut
3	Color
4	Clarity
Source: www.1diamondsource.com	

In the following table 2, the variables under study are described.

Table 2: Diamond Stones	
Serial number	**Diamond Stones**
1	Weight
2	Color e
3	Color f
4	Color g
5	Clarity s12
6	Clarity ws1
7	Cut premium
8	Cost

(Here we are taken color d, clarity IF(internally flawless), and cut good as base for the color, clarity and cut respectively.)

2.2 Model Selection Procedure

In this section we have discussed the model selection procedures for our diamond Stones data set, such as: PRESS selection procedure, Akaik Information Criterion (AICp) and Sewart Bayesian Criterion (SBC), R^2_p and Adjusted R^2 Criterion.

According to a rough rule of thumb if we have $(p - 1)$ set of predictors then 2^{p-1} models can be constructed, Kutner et.al. (2004) *model selection procedures, also known as subset selection or variable selection procedures to identify a small group of regression models that are good according to a specified criterion.*

$PRESS_p$ Statistic

The PRESS selection procedure was proposed by Allen (1974) in the "Prediction Sum of Squares" as a Criterion for selecting predictor variables.

The $PRESS_p$ (prediction sum of squares) is a criterion to measure of how well the use of the fitted values for a subset model can predict the observed response Yi.

Montgomery et al. (2004) argued that frequently regression equations were used for prediction of future observations or estimation of the mean response. In general; we were select the regressors such that the mean square error of prediction is minimized. One could use the $PRESS_p$ statistic.

The PRESS is obtained by deleting the ith case from the data set, the regression function for the subset model from the regression function for the subset model from the remaining $(n : 1)$ cases is estimated and then by using the fitted regression function obtained the predicted values for i^{th} case, so it differs from SSE. If prediction error is defined as:

$$Y_i : \hat{Y}_{(i)}$$

Then the $PRESS_p$ statistic is defined as:

$$PRESS_p = \sum [Y_i - \hat{Y}_{(i)}]^2$$

$$PRESS_p = \sum [e_i / (1 : h_{ii})]^2 \tag{1}$$

The models which have the small $PRESS_p$ values are considered good candidate models. So, one can use the $PRESS_p$ values for model validation, and it is potentially useful for discriminating between alternative models.

AIC and SBC Criterion

Akaik Information Criterion (AIC_p) and Sewart Bayesian Criterion (SBC_p) are also providing penalties for adding predictors. In these criterions we search for models that have small values of AIC_p or SBC_p. These criterions may define as:

$$AIC_p = n \ln SSE_p : n \ln n + 2p \tag{2}$$

and

$$SBC_p = n \ln SSE_p : n \ln n + p(\ln n) \tag{3}$$

Here we observe that as p increases the first term (n ln SSE_p) (which is same in both models) decreases, the second term n ln n is fixed for a given sample size and the third term increases as the p (number of parameters) increases in the model. It is clear that the models with small SSE_p will do well, as long as the penalties 2p for AIC_p and p (ln n) for SBC_p are not too large.

R^2_p Criterion and Adjusted R^2_{adj} or Criterion

Examination of the coefficient of multiple determination is infect R^2_p criteria and defined as:

$$R^2_p = SSR_p / SST_o$$

or

$$R^2_p = 1 : SSE_p / SST_o \qquad (4)$$

where SSR_p is the regression sum of squares when p regressors are included,

SST_o represents the total sum of squares when all regressors are included and SSE_p explains the residual sum of squares when p regressors are included.

An alternative criterion is R^2_{adj} is used in contrast of R^2_p criterion because R^2_p does not take account of the number of parameters in the regression model and since can never decrease, as additional regressors are included into the model R^2_{adj} is defined as:

$$R^2_{adj} = 1 : \{(n : 1) / (n : p)\} \, SSEp / SST_o$$

$$R^2_{adj} = 1 : \{(n : 1) / (n : p)\} \, (1 : R^2_p)$$

$$R^2_{adj} = 1 : (n : 1) \, MSE / SST_o \qquad (5)$$

hence R^2_{adj} provide equivalent information. Note the largest R^2_{adj} for the parameters, as p increases max R^2_{adj} can indeed decrease.

3. DATA ANALYSIS

3.1 Verification of Regression Model's Assumptions

In this section we discuss some simple graphic and statistical methods for studying the appropriateness of the model for given data before drawing further conclusion based on the model.

In diamond stones data, cost (Y) as response variable and remaining variables $X_1,...,X_7$ (define in table 2) used as predictor variables.

The result of regression equation is:

$$Y = 426.530719 + 6608.720261X_1 - 455.9038321X_2 - 462.1398975X_3$$
$$- 799.6692359X_4 - 3222.056879X_5 - 1152.614798X_6 + 118.1495192X_7$$

In this model none of the assumption of regression is satisfied so we adopt the appropriate transformation for the given data set which is logarithm. After taking the log of Y we have the following regression model:

$$Y* = 3.356461708 + 0.4714298874X_1 - 0.02333538668X_2 - 0.0240435949X_3$$
$$- 0.05823937426X_4 - 0.273182729X_5 - 0.06910597165X_6 + 0.01333823078X_7$$

$U_i \sim NID (0,\sigma^2 I)$

Normality of Residuals

Regression model of diamond data is fitted by assuming that the residual follows normal distribution. We use the Jarque-Bera test for testing the normality.

The test statistic has the value:

J.B=25.18495

The following table 3 reports the result of Jarque-Bera (by using the Statistical package Eviews)

Table 3: Jarque-Bera Test of Diamond Stones

Mean	Median	Skewness	Kurtosis	Jarque-Bera	probability
1.96E-15	-0.007428	0.790296	4.430518	25.18495	0.000003

The tabulated value of χ^2 is 5.991 which is less than calculated J.B value, indicates the lack of normality.

Linearity of the Regression Model

The plot of the studentized residuals against the corresponding fitted values $\hat{y}$ provided useful information about the appropriateness of the model. This plot shows the random scatter of points along the horizontal line so it indicates that we can use the linear regression models.

Homoscedasticity or Equal Variance of Error

A plot of the residuals against fitted values is helpful to examine the constancy of error variance. This indicates that there is hetroscedasticity in the data. We also use the White hetroscedastic test for testing this assumption. The null hypothesis of White test is stated as:

H_0: There is no hetroscedasticity.

With test statistic result are as:

$nR^2 = 35.35642$

(We use the statistical package Eviews-3 for finding the results.)

The white hetrosceadasticity test (with cross terms) results

F-statistic	1.810484	Probability	0.023945
Obs*R-square	35.35642	Probability	0.035547

Here tabulated value of χ^2 is 14.067, which is less than computed value. We set $\alpha = 0.05$. As $P<\alpha$ and $nR^2 > \chi^2$ we reject H_0 at the 5% level of significance and conclude that the data is hetroscedastic.

Autocorrelation

As autocorrelation means correlation between successive values of residuals. This problem is more meaningful in time series data rather than cross sectional data. In this study we test the autocorrelation with the help of Durbin Watson test. Minitab provides Durbin Watson statistic that is 1.575758 in this study. As the tabulated value of Durbin Watson statistic is $d_l = 1.5412$, $d_u = 1.832$, so we conclude that there is no autocorrelation problem in the regression model.

Multicollinarity

We use the variance inflation factor as a device of detection of multicollinarity among the fixed wheat traits. It may define as:

$$\textbf{VIF}_i = 1 / (1-R_j^2)$$

According to the rule of thumb no traits have greater than 5 or 10 VIF (variance inflation factor), which means that there is no multicollinarity problem in the wheat data. (These VIF are obtained by statistical package Minitab-11 and reports in Appendix table.)

3.2 Prediction Model Selection

Different criterions are used for model selection. In this section we use the $PRESS_p$ statistic, AIC_p and SBC_p criterions (basic criterion of model selection) for diamond stones data set.

For this study we choose the following four models which satisfied the basic assumptions of regression:

Model 1

$$Y^* = 3.594386392 - 2.380791014X_7 + 2.371676358X_1X_7$$

$$U_i \sim NID\ (0,\sigma^2I)$$

The white hetrosceadasticity Test (with cross terms) Results

F-statistic	0.743758	Probability	0.527876
Obs*R-square	2.261347	Probability	0.519966

Here tabulated value of χ^2 is 5.991, which is greater than computed value. We set $\alpha = 0.05$. As $P>\alpha$ and $nR^2 < \chi^2$ we do not reject H_0 at the 5% level of significance and conclude that the data is homoscedastic.

Jarque-Bera Test of Diamond Stones

Mean	Median	Skewness	Kurtosis	Jarque-Bera	probability
1.38E-14	-0.004839	0.550190	3.907696	11.27590	0.003560

The tabulated value of χ^2 is 5.991 which is less than calculated J.B value, indicates the lack of normality.

Model 2

$$Y^{**} = - 2.931509865 + 6.609731125X_1 + 7.184953172X_5 + 7.323083547X_6$$
$$- 1.796679719X_7 - 7.321444324X_1X_5 - 7.272319277X_1X_6$$
$$+ 1.782170043X_1X_7$$

The white hetrosceadasticity test (with cross terms) results

F-statistic	1.453214	Probability	0.139845
Obs*R-square	19.55896	Probability	0.144671

Here tabulated value of χ^2 is 19.675, which is greater than computed value. We set $\alpha = 0.05$. As P>α and $nR^2 < \chi^2$ we do not reject H_0 at the 5% level of significance and conclude that the data is homoscedastic.

The serial correlation LM test results

F-statistic	3.787846	Probability	0.053925
Obs*R-square	4.005025	Probability	0.045365

As P>α we do not reject H_0 at the 5% level of significance and conclude that there is no serial correlation in the data set.

Model 3

$$Y^{***} = -1.918469756 + 5.637447532X_1 + 6.168514863X_5 + 6.224851367X_6$$
$$- 1.932755437X_7 - 0.03571651474X_1X_4 - 6.334606541X_1X_5$$
$$- 6.192836796X_1X_6 + 1.912818307X_1X_7$$

The white hetrosceadasticity test (with cross terms) results

F-statistic	1.200497	Probability	0.267387
Obs*R-square	23.47859	Probability	0.265910

We set $\alpha = 0.05$. As P>α we do not reject H_0 at the 5% level of significance and conclude that the data is homoscedastic.

Jarque-Bera Test of Diamond Stones

Mean	Median	Skewness	Kurtosis	Jarque-Bera	probability
1.49E-14	-0.002848	0.518830	4.052595	13.70682	0.001056

The tabulated value of χ^2 is 5.991 which is less than calculated J.B value, indicates the lack of normality.

The serial correlation LM test results

F-statistic	2.519967	Probability	0.114980
Obs*R-square	2.670138	Probability	0.102247

As P>α we do not reject H_0 at the 5% level of significance and conclude that there is no serial correlation in the data set.

Model 4

$$Y^{****} = -2.617555066 + 6.29733829X_1 + 6.947295459X_5 + 6.502681099X_6$$
$$- 1.933185834X_7 - 7.087889547X_1X_5 - 6.451504352X_1X_6$$
$$+ 1.925315542X_1X_7 - 0.04551660682X_1X_4X_6X_7$$

The white hetrosceadasticity test (with cross terms) results

F-statistic	1.318197	Probability	0.197819
Obs*R-square	20.46174	Probability	0.200140

We set $\alpha = 0.05$. As P>α we do not reject H_0 at the 5% level of significance and conclude that the data is homoscedastic.

The serial correlation LM test results

F-statistic	1.979837	Probability	0.161929
Obs*R-square	2.106886	Probability	0.146637

As $P > \alpha$ we do not reject H_0 at the 5% level of significance and conclude that there is no serial correlation in the data set.

Now by adopting the following criterions we choose the best model.

PRESS Statistic

Montgomery et al. (2004) idea is that *PRESS statistic can also use to evaluate candidate equations produced by a subset generation procedure.*

Here we use the set of equations that satisfied the basic assumptions and according to equation (1): $PRESS_p = 0.2281$

When predictors are as weight(X1), clarity s12 (X5), clarity ws1 (X6), cut premium (X7), weight color g (X1X4), weight clarity s12 (X1X5), weight clarity ws1 (X1X6), weight cut premium (X1X7).

The results of PRESS statistic are reported in following table as we know that PRESS statistic selects the subset regression model having the smallest PRESS statistic.

PRESSp statistic

Model	PRESSp
1	1.4454
2	0.2645
3	0.2281
4	0.2425

Here the smallest $PRESS_p = $ with model-3, so we select the following model:

$$Y^* = -1.918469756 + 5.637447532 X_1 + 6.168514863 X_5 + 6.224851367 X_6$$
$$- 1.932755437 X_7 - 0.03571651474 X_1 X_4 - 6.334606541 X_1 X_5$$
$$- 6.192836796 X_1 X_6 + 1.912818307 X_1 X_7$$

Predictors are as weight(X1), clarity s12 (X5), clarity ws1 (X6), cut premium (X7), weight color g (X1X4), weight clarity s12 (X1X5), weight clarity ws1 (X1X6), weight cut premium (X1X7), as best prediction model for diamond data set.

AIC_p and SBC_p Criterions

Two popular criterions that also provide penalties for adding predictors are Akaike's information criterion (AIC_p) and Schwarz' Bayesian criterion (SBC_p). As we have discussed in chapter 3 that the models with small values of AIC_p or SBC_p are considerd best model.

According to equation (2)

        $AIC_p = -3.395146$

and according to equation (3)

        $SBC_p = -3.199558$

Following table reports these values of AIC_p and SBC_p.

AIC_p and SBC_p

Model	AIC_p	SBC_p
1	-1.638957	-1.573761
2	-3.262178	-3.088322
3	-3.395146	-3.199558
4	-3.310188	-3.114600

As we know that the model having small values of AIC_p or SBC_p is considerd best model, so the model-3 having minimum value of AIC (-3.395146) is:

$$Y^* = -1.918469756 + 5.637447532X_1 + 6.168514863X_5 + 6.224851367X_6$$
$$- 1.932755437X_7 - 0.03571651474X_1X_4 - 6.334606541X_1X_5$$
$$- 6.192836796X_1X_6 + 1.912818307X_1X_7$$

Predictors are as weight(X_1), clarity s12 (X_5), clarity ws1 (X_6), cut premium (X_7), weight color g (X_1X_4), weight clarity s12 (X_1X_5), weight clarity ws1 (X_1X_6), weight cut premium (X_1X_7), as best prediction model for cost of diamond stones.

Here note that the SBC have minimum value (-3.199558). Hence one can choose the same model for prediction of cost of diamond stones.

R^2 and Adjusted R^2 Criterions

The results of R^2 and Adjusted R^2 are reported in following table as we know that R^2 and Adjusted R^2 select the model having the largest R^2 and Adjusted R^2.

R^2 and Adjusted R^2

Model	R^2	Adjusted R^2
1	0.066232	0.051866
2	0.829144	0.819576
3	0.852650	0.843143
4	0.843327	0.831863

Here the model-3 having largest value of R^2 and Adjusted R^2 (0.852650 and 0.843143) is:

$$Y = -1.918469756 + 5.637447532X_1 + 6.168514863X_5 + 6.224851367X_6$$
$$- 1.932755437X_7 - 0.03571651474X_1X_4 - 6.334606541X_1X_5$$
$$- 6.192836796X_1X_6 + 1.912818307X_1X_7$$

3.4 Comparison

All the criterions discussed above select the same prediction model of cost of diamond stones with predictors as weight(X_1), clarity s12 (X_5), clarity ws1 (X_6), cut premium (X_7), weight color g (X_1X_4), weight clarity s12 (X_1X_5), weight clarity ws1 (X_1X_6), weight cut premium (X_1X_7).

From the above prediction models, it is clear that cost of diamond stones with cut premium (X_7), weight color g (X_1X_4), weight clarity s12 (X_1X_5), weight clarity ws1 (X_1X_6), on the average decrease. Hence we can conclude the stones characters like cut premium (X_7), weight color g (X_1X_4), weight clarity s12 (X_1X_5), weight clarity ws1 (X_1X_6), contributes negatively to cost of diamond stones. On the other hand, cost

increases as the stones characters like weight (X1), clarity s12 (X5), clarity ws1 (X6), weight cut premium (X1X7) increases. The effect of clarity ws1 (X6) is positive and maximum i.e. 6.224851367 on the cost of diamond stones. So our study recommends that any diamond stone having maximum clarity ws1 (X6) can be more costly.

3.5 Summary

The basic regression assumptions on diamond stones data was verified by considering cost as response variable and other diamond stones as independent variables and results indicate linear regression model is appropriate for diamond data.

In the selection of best prediction model for predicting diamond cost, we developed different models. On the basis of different criterions such as R^2, R^2_{adj}, $PRESS_p$ Statistic, AIC and SBC we recommend that prediction model is the best one because it satisfies all the above criterions.

The basic assumptions of regression on diamond stones was verified by taking the log transformation of diamond cost as response variable and other diamond stones as independent variables. Results show the log transformation is appropriate for fitting the linear regression model.

4. CONCLUSIONS

The recommendation made in this study will helpful for diamond shopper for the selection of diamond stones that is bargain and not overpriced This research work will also facilitate the diamond stones seller to evaluate the customer adoptability for diamond stones.

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IMPACT OF FIRM PERFORMANCE ON CORPORATE GOVERNANCE

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ABSTRACT

Purpose – The purpose of this paper is to investigate the effects of prior firm performance on corporate governance practices

Design/methodology/approach – A total of 40 companies listed on Karachi Stock Exchange were used for this study. Hypothesis was tested using SEM by using PLS Graph Software.

Findings – The results show that prior change in firm performance was significantly related to the improvement in corporate governance practices.

Research limitations/implications – The sample size used in this study was relatively small and the focus was on six sectors, so the findings here may not apply to all sectors other than those used in our sample.

Originality/value – This study is first one to examine the effects of trends in prior firm performance on corporate governance practices in Pakistan

Keywords – Governance, Performance,

Paper type– Research paper

1. INTRODUCTION

In the beginning of this millennium many companies like Enron, worldwide were thought to be collapsed due to absence of good corporate governance. Traditionally, corporate governance focuses on the problems of separation of ownership and control, now it is perceived that firms should consider the all the stakeholders. To create a culture of consciousness, transparency and openness good corporate governance is needed by corporations. Corporate governance results in high customer satisfaction and maximizes the shareholders' wealth. Corporate governance assures that management is acting in the best interest of stakeholders.

Corporate governance is explained by four basic theories. First, *Agency Theory* explains the relationship between the shareholders (principals) and the management (Agents). It is considered that management would carry out their fiduciary duty but sometimes management deviate from their duty. These agency problems can be mitigated through better governance by fair disclosure of financial data and independent board of directors. Second, *Stewardship Theory* describe that manager are trustworthy and they value their personal reputations. Third, *Stakeholder Theory* emphasizes on the ethics, fiduciary relationship, social contract, property rights, stakeholders as investors, communication

ethics etc. Last, *Sociological Theory* focuses on board composition and distribution of wealth to realize the socio-economic objective of companies.

Corporate governance advocates claim that good governance practices are essential for high performance. Researchers and practitioners argue that if a firm is considering protecting the interests of its shareholders, the assets of the firm will be employed in a way to minimize misuse and maximize profit, resulting in awesome retunes to shareholders. Core et al. (1999) found that firms with weak governance structure have high agency problems, CEO's at these firms which have more agency problems obtain high rewards. They also find that firms with greater agency problems are unable to outperform. Numerous researches using an overall score of governance have found an association between governance and shareholder gain (Gompers et al., 2003) Companies with good governance are rewarded by better stock performance (Bradley, 2004). According to resource dependence view, directors can mitigate environmental uncertainty due to their relations with stakeholders (Pfeffer and Salancik, 1978).

This particular study is going to add in literature in two ways. First we examine the relationship between prior firm performance and good corporate governance. As it was examined earlier by Baysinger and Butler (1985), they investigated the relationship between prior performance and the board of directors. But they found no relationship between prior year performance and corporate governance. In their study only single characteristic of board, the independence of board of directors from the management of the companies, was considered. This study examines a variety of governance and performance variables to check the impact of prior firm performance on good corporate governance. Secondly, this study is a pioneering attempt focusing on manufacturing firms listed on Karachi Stock Exchange (KSE).

We proceed as follows: second section examines the comprehensive literature review related to the impact of firm performance on corporate governance. Third section includes methodology, hypotheses that are developed to test the relationship between prior year firm performance and corporate governance, third section also includes model that shows the proposed relationship between variables. Data analysis and results are discussed in section four. Last section contains discussion on results and suggestions for further research on this relationship.

2. LITERATURE REVIEW

According to agency view, outside directors can monitor management in a better way because they are independent from company's management (Fama and Jensen, 1983). Outsiders are in more preferable on insiders as insider-dominated board's results weak accountability of CEO as CEO has a power to influence the career of insider directors (Zajac and Westphal, 1994). Rechner et al., (1993) find that in contrast to the executive directors, there is little chance that careers of outsiders would be affected by the results of their decisions so they can make more accurate solution. Outside directors can add to company's performance due to their expertise from their prior experience (Mace, 1986). Outside directors also bring impartiality in evaluation of decision made by management (Baysinger and Hoskisson, 1990). Pfeffer (1973) find that Changes in environment directly affect the composition of board of director. Board includes the independent directors who bring managerial wisdom and external connections that help firm to

outperform (Baysinger and Butler, 1985). Hillman et al. (2000) investigated that utility companies made changes in board of directors to make board more responsive to aggressive conditions during industry deregulations. Baysinger and Butler (1985) find a connection between the nature of directors and performance of the firm taking outsider directors into considerations.

CEO duality is a very important issue in corporate governance. Agency theorists advise that that firm should avoid CEO duality in order to avoid managerial entrenchment and limit the CEO's power (Mallete and Fowler, 1992). Goyal and Park (2002) find that it becomes very hard to change CEO for unsatisfactory financial performance if CEO is also the chairman of board of directors. Fama and Jensen (1983) argue that duality means that there is no separation between decision management and decision control. Absence of duality helps in objective evaluation organizational and management performance (Weidenbaum, 1986). Pi and Timme (1993) conducted a research in banking industry and find that cost efficiency and return on assets (ROA) were lower for banks with duality and results were higher for banks that have different CEO and Chairman. Boyd (1995) concludes that there is a weak inverse relationship between duality and firm performance. Worrell et al. (1997) find that in case of CEO duality firm's stock market performance was unfavorable. Finkelstein and D'aveni (1994) named CEO duality as "double edged sword". They also argue that agency issues relate to CEO duality can be minimized by resource dependence advantages attached with CEO duality.

Mak and Kusnadi (2005) found that there is a negative relation between board size and firm value in Singapore and Malaysia. Yermack (1996) find consistent results with theories that small boards of directors are extra efficient. This study finds an inverse relationship between board size and firm value. Eisenberga et al.(1998) find a considerable negative correlation between board size and profitability in a sample of small and midsize firms. Cheng (2008) provide evidence that firms with large board have less variability of corporate performance. Conyon et. al. (1998) concludes negative effect of board size on performance. Belkhir (2009) finds a positive association between board size and performance, as measured by Tobins'q and the ROA. The author also concludes that change in board size is not affected by prior performance.

Vafeas (1999) Found that board meeting frequency is associated to corporate governance and ownership in a way that is reliable with agency theory. The yearly figure of board meetings is negatively related to firm value. These results suggested board meeting frequency, is a significant measurement of board operations. The association between ownership structure and company performance has been the focus of intense research in recent years. Claessens and Djankov(1999) conclude that firms with more concentrated ownership have high profitability and labor efficiency. Kuznetsov and Muravyev (2001) find that ownership concentration positively affects labor productivity, but has a negative impact on Tobins'q.

Institutional investors consider governance as important as performance while taking investment decision. Duggala and Millar (1999) find out a positive relation between the institutional ownership and corporate performance. Cornetta et al. (2007) suggest that institutional investors with potential business relationships can be considered as monitors of the firm. That's why they can be taken as an element of corporate governance. Brunello (2001) finds evidence that an increase in net profits of firm by 1 billion lire increases the

compensation of upper and middle managers by only 31 thousand. Brick (2006) finds that excess compensation of director and CEO are related to firm underperformance.

3. HYPOTHESIS AND METHODOLOGY

3.1. Hypotheses Development

Most of the researchers give emphasis on logical way of developing and testing the hypothesis. Following their view, this study aims to develop a hypotheses which deals with the relationship of financial performance and corporate governance variables

H_1:   All else being equal, companies with higher performance tend to have better corporate governance

3.2. Structural Model

3.2.1. Variables

Latent Exogenous Variables: Corporate Governance is dependent variable for this study. To measure good corporate governance, Percentage of outside directors on board, board size, CEO duality, frequency of board meetings, ownership concentration, Institutional ownership and management compensation are used. Ownership concentration is measured by Herfindahl-Hirschman Index (HHI).

Latent Endogenous Variables: Financial Performance is used as independent variable in this study. While selecting performance variables Literature shows that a number of different measurements of the financial performance are used. Literature uses return on assets (ROA), return on equity (ROE), earning per share (EPS), price to earnings (P/E) ratio, net profit after tax (NPAT) and Tobins'q as measure of financial performance. In this study all these variables are used to measure financial performance

Using PLS based SEM, the following Figure shows structural connection among the variables in this study
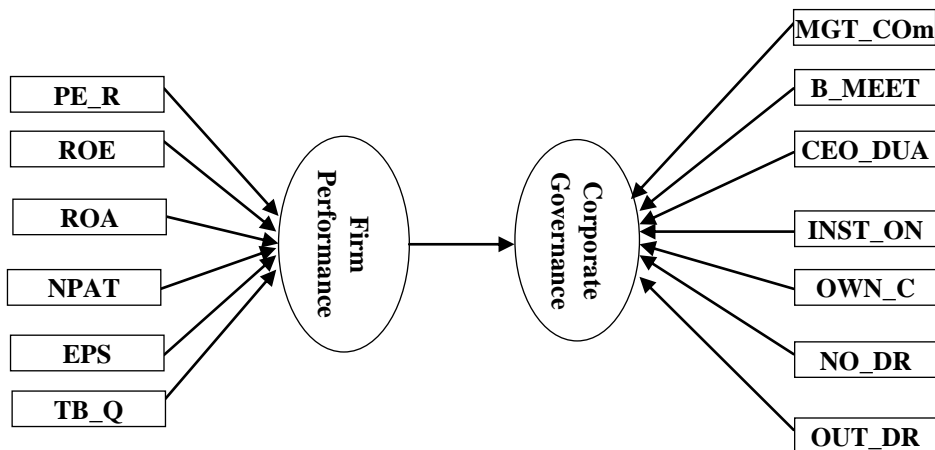
Figure-1

3.3. Measurement Model

All indicators (shown in squares) build and influence their respective latent constructs (shown in circles). The latent constructs can be measured in mathematical terms as:

$$\xi = \gamma_{x1}X1 + \gamma_{x2}X2 + \gamma_{x3}X3 + \gamma_{x4}X4 + \gamma_{x5}X5 + \gamma_{x6}X6 + \zeta$$
$$\eta = \gamma_{y1}Y1 + \gamma_{y2}Y2 + \gamma_{y3}Y3 + + \gamma_{x4}Y4 + \gamma_{x5}Y5 + \gamma_{x6}Y6 + \gamma_{x7}Y7 + \zeta$$

The hypothesis H_1 impact of latent exogenous variables, Firm Performance (ξ) on latent endogenous variables, Corporate Governance (η) would be measured through:

$$\eta = \beta\xi + \zeta$$

Table 1
Description of Exogenous and Endogenous Variables and Symbols

No.	Symbol	Abbreviation	Description
1	ξ	FIN_PERF	Latent Exogenous Variable, Financial Performance
3	η	COPR_GOV	Latent Endogenous Variable, Corporate Governance Measures
4	ζ		Random Disturbance Term
5	γ_{x1}	PE_R	Path Coefficient of X_1, Price to earnings Ratio
6	γ_{x2}	ROE	Path Coefficient of X_2, Return on Equity
7	γ_{x3}	ROA	Path Coefficient of X_3, Return on Assets
8	γ_{x4}	NPAT	Path Coefficient of X_4, Net Profit after Tax
9	γ_{x5}	EPS	Path Coefficient of X_5, Earning per Share
10	γ_{x6}	TB_Q	Path Coefficient of X_6, Tobins'q
11	γ_{y1}	MGT_COM	Path Coefficient of Y_1, Management Compensation
12	γ_{y2}	B_MEET	Path Coefficient of Y_2, Number of Board Meetings
13	γ_{y3}	CEO_DUA	Path Coefficient of Y_3, CEO Role Duality
14	γ_{y4}	INST_ON	Path Coefficient of Y_4, Institutional Ownership
15	γ_{y5}	OWN_C	Path Coefficient of Y_5, Ownership Concentration
16	γ_{y6}	NO_DR	Path Coefficient of Y_6, Number of Directors on Board
17	γ_{y7}	OUT_DR	Path Coefficient of Y_7, Number of outside Directors

4. DATA ANALYSIS AND RESULTS

4.1. Sample and Data

This study attempts to draw link between prior year firm financial performance and corporate governance based on random sample from manufacturing firms listed at Karachi Stock Exchange (KSE). It was not possible to collect the data for whole population, so it was decided to select a random sample. The is a quantitative study based on 6 years data gathered from audited annual reports of the companies covering the period 2005-10. Random sample of KSE listed companies cover five manufacturing sectors. The final representative sample from the target population includes sixteen textile companies, twelve companies from food producers, seven cement enterprises, two industrial engineering enterprises, three household enterprises and reaming are from automobile and parts sector.

Table 2
KSE-Sample Sector Wise

No.	Sector	Firms
1	Personal Goods	13
2	Food Producers	12
3	Constructions and metals	7
4	Industrial Engineering	2
5	Automobile and Parts	3
6	Household goods	3
	Total	40

4.2. Descriptive Statistics

To better understand the data, basic descriptive statistics applied, eeven if descriptive do not tell the whole story, but describe minimum, maximum, means and standard deviation of all variables. Table 3 shows descriptive of firm performance for the year 2009 and corporate governance for the year 2010.

Descriptive statistics in 2009 disclose that the average ROA is 6.4610 which indicate that KSE listed firms have normal return on assets. However standard deviation of 18.94721 suggests that there is considerable variation in return. In 2009 53by Dewan Auto Engineering shows minimum ROA of -42.53 and Pangrio Sugar Mills have maximum ROA of 78.37. The average return on assets during the period 2005-09 remains almost consistent (4.7458, 5.7583, 4.1230, 6.4610, 6.4610).

In 2009 Average ROE is 9.6152 which shows that KSE listed firm are performing well and earning a handsome return on equity investment however standard deviation of 22.523 indicate a notable variation in the ratio. For the year minimum ROE is -45.67 and maximum is 67.88 by Nestle Pakistan. The average ROE shows an improvement which indicate that firms are improving performance over the period (4.0258, 20.4697, 54.8480, 42.2058, 42.20).Average EPS is 11.5265 in the year 2009 which indicate the good performance of the KSE listed firms. Standard deviation in EPS is 25.32275 which show that there is a considerable variation in data. In 2009 Loss per share is -23.70 maximum EPS is 140.43 by Rafhan Maize Product. Five year average figures of EPS shows a normal variation (7.2370, 10.3565, 9.1375, 11.5265, 11.5265).

Average Tobins'q ratio in 2009 is 0.7978, standard deviation is 1.62971. This ratio shows consistency for the period except 2007 which might be due to the boom in stock exchange (0.8418, 0.9213, 5.1913, 0.7978, 0.7978). In 2009 Average price to earnings ratio is 4.6418 and standard deviation 28.64330. This ratio shows ups and downs in the data (8.3727, 16.1152, 12.8985, 4.6418, 4.6418). Average NPAT in 2009 is PKR 301410000 which shows that the KSE listed firms are making reasonable profit however standard deviation of PKR 1006960000 indicates that there is a high variation in the return from firm to firm. The Maximum Net loss is PKR 1960000000 by Shakarganj Mills and maximum NPAT is earned by Lucky Cement Ltd 4600000000 for the year 2009. The NPAT shows a consistency over the period under consideration (315650000, 334110000, 379840000, 234000000, 301410000).

In 2010 average Number of Directors on board is 7.7250 with a standard deviation of 90547. Minimum number of directors on the board is 7 and maximum 10.These descriptive shows that firms have minimum 7 directors one the board. Average number of directors on borad remains consistent over the period (7.7000, 7.7250, 7.7500, 7.7500, 7.7250). Average Number of outside Directors on board is 4.5250 with a standard deviation of 1.88091. Minimum number of outside director on board is 0 and maximum 8. Descriptive indicate that average number of outside director on board remain more than 4 over the period under consideration (4.5250, 4.4500, 4.4500, 4.5500, 4.4750).Average management compensation is PKR 8576045 with a standard deviation of PKR 150178000.The average compensation shows an increase over the period (49263225, 46109175, 57360275, 73363150, 85760450). This increase is due to increase in number of executives.

Average number of board meetings during the year 2010 is 5.1000 with a standard deviation of 1.90546. Minimum number of meeting during 2010 is 4 and maximum 14 meeting were held by Quetta Textile Mills Ltd. Descriptive shows that average number meeting during the period (5.1000, 4.8000, 4.8000, 4.9750, 4.9250). Average for CEO role duality is 0.3250 with a standard deviation of 47434. Descriptive result shows that average remains near o which indicates that there is low rule duality in KSE listed firms (0.3250, 0.3750, 0.3750, 0.3750, 0.3750). Average institutional ownership is 48.8557 percent with a standard deviation of 33.33702 percent. Average over the period shows that institutional ownership remains near about 50 percent (48.8557, 51.8933, 51.8933, 45.2115, 44.3950).

Table 3
Descriptive Statistics

		N	Minimum	Maximum	Mean	S.D
Firm Performance	ROA	40	-42.53	78.37	6.4610	18.94721
	ROE	40	-45.67	67.88	9.6152	22.523
	EPS	40	-23.70	140.43	11.5265	25.32275
	TOBINS_Q	40	.00	8.21	.7978	1.62971
	PE_RATIO	40	-136.25	66.67	4.6418	28.64330
	NPAT	40	-1960000000	4600000000	301410000	1006960000
Corporate Governance	B_SIZE	40	7.00	10.00	7.7250	.90547
	OUTSIDE_DIR	40	.00	8.00	4.5250	1.88091
	OWN_CONCEN	40	155.00	7073.00	1844.7500	1893.38761
	MGT_COMP	40	696000	920000000	85760450	150178000
	FREQ_BM	40	4.00	14.00	5.1000	1.90546
	CEO_DUAL	40	.00	1.00	.3250	.47434
	INST_OWNER	40	.00	98.45	48.8557	33.33702

The quality of the model is tested on the basis of significance of relationship among latent constructs and goodness of fit (R^2). To find the impact of prior year firm performance on corporate governance it is suitable to use structural equation modeling based on partial least square to measure the affect of independent variables on dependent variables.

4.3. Validity and Reliability test

Bootstrapping technique through 100 resamples with replacement was applied to check the individual indicator's significance to prove validity in measurement model. Validity test proves that EPS indicator from construct firm performance, two indicators Board Size and Ownership concentration from construct corporate governance remain significant in all years 2006-10. While, the others indicators were found significant in some years inconsistently.

Andreev et al. (2009) suggest that construct reliability of formative indicators should be performed through multicollinearity test. For this purpose, collinearity test was run through using SPSS to calculate VIF after taking each indicator of corporate governance construct one by one. Results show that VIF remains below 5 in all five years 2006-2010 which indicate that none of the independent indicators is substantially explained by other independent indicator

4.4. Analysis of Measurement Model

The SEM based on PLS gives an estimation of the impact of firm performance on corporate governance.

$$\eta = \beta\xi + \zeta$$

Left side of equation specifies the outcome variable corporate governance (η) while the right side (β) specifies the coefficient of latent endogenous variable financial performance. Figure 1 to 5 shows results including weights, loading, path coefficient and coefficient of determination after applying PLS based SEM using PLS graph 3.0 software.

In order to have the statistical significance of path coefficients (β) a bootstrapping technique through 100 resamples with replacement was applied

Table

Year	Beta Coefficient	t-value	Significance
2009-10	0.575***	6.0408	p<0.01
2008-09	0.575***	3.8908	p<0.01
2007-08	0.615***	3.9159	p<0.01
2006-07	0.651***	12.170	p<0.01
2005-06	0.621***	3.9530	p<0.01

* Significance at 10% (1.645)
** Significance at 5% (1.96)
*** Significance at 1% (2.576)

Tables list the beta coefficients and t values including their level of significance from the year 2005 to 2010. In all the years, we find strongly significant path coefficients between firm performance and corporate governance.
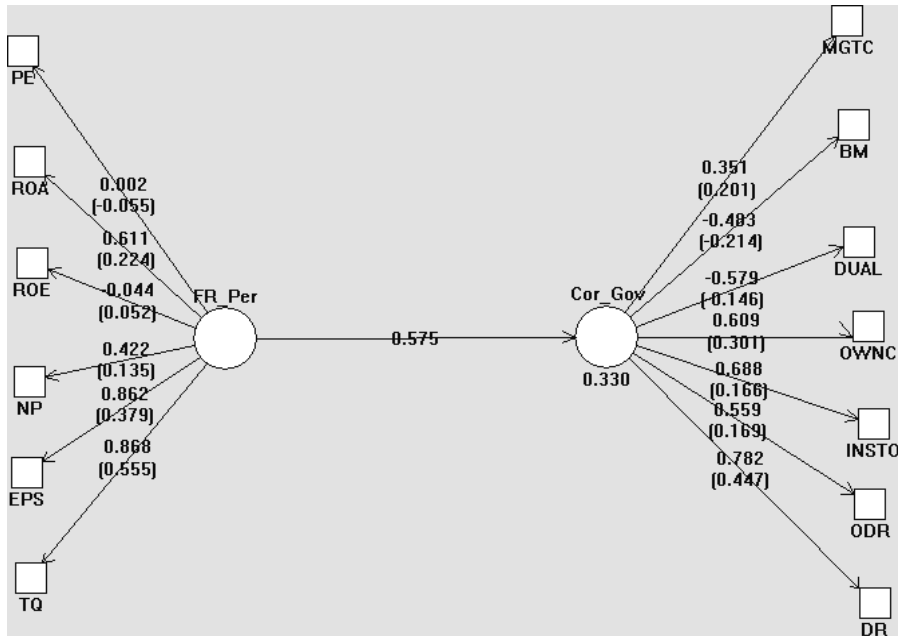
Figure 2: (PLS Results 2010)

Results in the above tables show that the firm performance has significant impact on corporate governance ($\beta = 0.575$, t-value = 6.0408, $p < 0.01$; $\beta = 0.575$, t-value = 3.8908, $p < 0.01$; $\beta = 0.615$, t-value = 3.9159, $p < 0.01$; $\beta = 0.651$, t-value = 12.170, $p < 0.01$; $\beta = 0.621$, t-value = 3.9530, $p < 0.01$) in all years 2005-10. In this way, it can be concluded that prior year financial performance exerts its positive influence on corporate governance.

R^2 measures the amount of the variance in dependent latent construct explained by independent latent constructs. R^2 values of the corporate governance construct explained by change in firm performance remain satisfactory (33%, 33.1%, 42.4%, 37.8%, 38.5%) over the five year period (table).

The above data, analyses and results support the model proposed in the study that financial performance affects the corporate governance practices. Overall findings suggest that corporate governance practices can be predicted successfully through firm performance up to 42%.

4.5. Hypothesis Testing

The study tests the hypothesis formulated in the beginning on the bases of strength of path coefficients by calculating multiple path values through PLS Graph Version 3.0. The standardized path coefficient (β) shows the significance of relations between latent constructs and permit the fulfillment of the proposed hypotheses to be analyzed. Chin (1998) suggests value of path coefficient should be at least 0.20 and may ideally exceed 0.30 to analyze the hypothesis meaningfully.

Table

Year	Hypothesis	Suggested effect	R-Square	Path coefficient	Significance	Confirmed
2009-10	H_1	+	33%	0.575	p<0.01	Yes
2008-09	H_1	+	33.1%,	0.575	p<0.01	Yes
2007-08	H_1	+	42.4%,	0.615	p<0.01	Yes
2006-07	H_1	+	37.8%,	0.651	p<0.01	Yes
2005-06	H_1	+	38.5%	0.621	p<0.01	Yes

* Significance at 10% (1.645)
** Significance at 5% (1.96)
*** Significance at 1% (2.576)

All (β) values of the impact of firm performance on corporate governance practices (H_1) have been significant and fall between 0.575 and 0.651, that is much higher than 0.30 over the five year period. Forthcoming table shows hypotheses testing for the period 2005-10.

The hypothesis H_1 suggests that prior year firm performance has positive impact on corporate governance practices. Above tables demonstrates significant values for the parameter of this relationship (0.575, $p < 0.01$; 0.575, $p < 0.01$; 0.615, $p < 0.01$; 0.651, $p < 0.01$; 0.621, $p < 0.01$) over the period 2005-10. This indicates good support for H_1 relating to the impact of prior year firm performance on corporate governance practices. It proves that firm performance construct is relevant antecedent and exerts significant impact in improving the corporate governance practices of business enterprises, thus H_1 is supported.

Given the above analyses and results, the study supports r acceptance of H_1. It further supports the overall model proposed in the study that the financial performance positively and significantly impact corporate governance practices.

5. CONCLUSION

The main purpose of this study was to find out the structural link and its impact of firm's financial performance on corporate governance practices. The study was positioned to conclude Financial Performance-corporate governance relationship through empirical research. The path coefficient values (β) confirm the premise that there is a significant positive correlation between financial performance and CG measures. The hypothesis has also been assessed through coefficient of determination (R^2). Enough empirical support has been provided to accept the proposed hypothesis H_1.

After considering the reliability and validity of measurement model and path coefficients, coefficient of determination, it can be concluded that through our model up to 44.4% variance in corporate governance practices can be explained by financial performance successfully.

Due The relatively small size of the sample and only few sectors this study may not be generalizable at large. Future studies are needed expanding both the scope of the research as well as tailoring both the performance and governance variables.

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DESIGNING SOFTWARE MAINTENANCE SERVICE LEVEL AGREEMENT IN OUTSOURCING CONTRACT

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ABSTRACT

Service Level Agreement is legal contract between services supplier and customer which exactly specifying terms and conditions and all standards of services. Although agreement defined set of terms and conditions, scope of services, Service level monitoring, Complaint mechanism, support activities, tool and technologies Service level plan, violation and penalties and support services etc, but could not manage and gain values.

The purpose of this paper to address designing and management issues pertaining to Software Maintenance Service Level Agreements (SLAs) and help organizations to gain maximum befits with less management efforts. Moreover the paper proposed new techniques to increase the performance of the Service Level Agreement.

1. INTRODUCTION

In advance and distributed IT world, Service Level Agreement (SLA) is becoming more complicated tool to define, measure and monitor the services that our organizations expected from venders. Although several efforts has already been made in make it valuable to obtain desired services but still organizations failed to get targeted values.

Many organizations hired the services of outsource software houses for their software development and often hired the same development house services for its maintenance but in some cases, hired third party services for their software's maintenance. This outsourcing always offer great potential, like access to particular expertise, cost minimization, and improved overall business processes [8].

At present, many organizations intend to outsource services to improve management performance and cost saving. Although, several challenges and no particular rules exist in outsource management. Among all these challenges, drafting and designing Service Level Agreement is core challenge [1].

Software Maintenance Services Level outsourcing always produced good results as outsourcing venders can execute these functions in good manners and efficiently in relation to time and cost [2]. The main aspect of outsourcing agreement is always based on terms and conditions in which both parties agreed upon. Mostly customers invite service level agreement from outsource venders with products and negotiate on it. In this critical phase, outsourcing risks, better performance, consultation with relative lawyers and discussion with suitable forums always leads to a good SLA. The clear Service Level Agreements in Software maintenance outsourcing agreement is vital component of

outsourcing success. Software maintenance outsourcing requires maintenance and might be up-gradation of existing custom-developed software or packages i.e. regulatory changes, up-gradation, new release installations, and etc [2].

Although, several methods / techniques have already been introduced to formulate effective service level agreement for outsource contract but this paper introduced new management techniques to increase SLAs performance in best interest of customer and outsource vender.

2. SERVICE LEVEL AGREEMENT

A Service Level Agreement is mutual contract between customers and outsources venders in which the vender is bound to provide all requisite quality based services mentioned in contract document and both parties are bound to obey all terms and conditions [3]. In case of either side violation, penalty will be imposed accordingly and both parties have also reserved the rights to file civil suit against each others in relevant legal forum [3].

3. SPECIFICATION OF SERVICE LEVEL AGREEMENT

A well verse Service Level Agreement (SLA), provides detailed specification of each services, Service Level Monitoring, Support activities, Complaints mechanism, Response time, Tools and Technologies, Service level targets, Service level plan, Service level objects, Violations and penalties, Methods to increase scope of services, roles and Responsibilities, Training services and etc [1].

The core objectives of detailed specifications is to achieve high performance, minimize risk, increase throughput, focus on quality base services required by the customer, pricing policies, penalties & rewards and authorities. A good Service Level Agreement should visibly define both parties' responsibilities with relevant actions [4].

4. STRUCTURE OF SERVICE LEVEL AGREEMENT

Structuring of Service Level Agreement (SLA) in outsourcing contract is very important, multi-level and multi-step process by involving both the customer and outsource vender [14]. The main focus is on business objectives, both parties (client and vender) group effort to estimation client's existing application software suit, new software functions, all internal and external processes and currently delivered baseline service levels [5]. This thing will provide crystal clear vision for software to effectively handle, organize, develop, management and to enhance quality of services by utilizing given resources. It is necessary to identify software services during analysis and designing of system or at least before integration of system. The detailed assessment of the software always provides costs and efforts relates with maintaining applications [14]. By applying this approach, proposal can be made that leads to feasibility roadmap for the software maintenance entire outsourcing plan.

During above mentioned assessment in phased approach, the relevant staff should:
- Collect and analyze information from different applications.
- Highlight all potential issues.

- Discuss same issues with relevant forums / companies to clarify issues to gain better understanding regarding applications.
- Identify software applications which are ready and easily outsourced.
- Maintain baseline report on findings.
- Forward strong recommendations with supporting documents to relevant management for approval.

Now, after the identification of business objectives, the customer and outsource vender should collect baseline performance metrics for applications that will be outsourced [14].

5. KEY AREAS OF SERVICE LEVEL AGREEMENT

Though two SLAs are exactly same because every organizations services, priorities and policies are vary time to time, to make an effective Software maintenance Service Level Agreement (SLA) for outsource contract, the following Key points should be considered while designing Service Level Agreement [6].

5.1 Description of Service

This very important section should list down all services you can provide or required by the customer with detailed description of each service. The customers have to scrutinize their actual present and future business needs [12].

5.2 Secure Important Stuff

In outsourcing contract, while analysis, it is necessary for organization to hire the services of IT specialists to check the possibility of misleading terms which may affect overall performance of business.

5.3 Performance Measurement

Another very important section about SLA is its performance measurement. There are various methods which are already available to quantify and report service levels, some organizations also hired third party services for the said purpose [11].

5.4 Identify Parties to the Agreement

This section should identify all individuals and groups by name or title of job with the physical location. All individuals involved should sign the contract document to fix responsibility [12].

5.5 Customer and Outsource Vender Duties

Definition of customer and outsource vender duties is mandatory in SLA, This section will fix client duties under the terms of contract for better cooperation because without client coordination and cooperation, it is impossible to achieve quality of services.

5.6 Include Best and Worst Condition

During Monitoring of Services performance, some time worst situation may arise, for example, Server computer become out of order during peak time. Such type of examples needs careful analysis for severe cases [11].

5.7 Problem Management Strategy

This section will describe methods to report problems and quick response time against each problem. In software maintenance outsourcing, it is necessary to define different

severity levels against each problem and also assign response time against each problem. This is very sensitive section of SLAs because usually performance is based on rapid response against problem. Therefore, always focus on response time [11].

5.8 Demand of Service Over Time

At the time of specifying service demand, the volume of demand and time distribution should also be cleared in SLA. In addition to this, the customer also should to define the expected volume of services to be required in future. This thing will be very beneficial for both parties to estimate growth of workload [12].

5.9 Warranties and Remedies

Software maintenance outsource Service Level Agreement should describe all about legal remedies like compensation for damages, exceptions and third-party claims. If an organization does not contain this portion, then other party can take its advantage and you may not be happy by their decision [7].

5.10 Violations and Penalty

The Service Level Agreement should describe all terms and conditions and accordingly their respective penalties. There are no hard and fast software maintenance service violation and penalty rules but when both parties agreed then they have to obey all terms and conditions mentioned in contract otherwise penalty will be imposed on the basis on violations. While deciding terms and conditions, it is necessary to include legal, ethical, justified able penalty [7].

5.11 Termination of Contract

In this section, your Service Level Agreement (SLA) must contain circumstances under which you or your customer can terminate contract. These circumstances could be passage of annual renewal date or failure of either side duties mentioned in Service Level Agreement (SLA). In case of termination of contract, the term either side one or two months notifications also must be include for proper termination of contract.

5.12 Renegotiation of Contract

Under normal circumstance, the contract must be renegotiated at least once a year due to budget cycle. In special circumstances like (price hiking, modification in services, change in physical location, change in industry standards, imposition of new taxes, fluctuation in foreign currency and etc) Service Level Agreement (SLA) also should provide renegotiation flexibility.

6. STEPS FOR DESIGNING SERVICE LEVEL AGREEMENT

Service Level Agreement designing is critical process and in order to create effective Service Level Agreement, the following given below major points should be under consideration.

6.1 Talk to Peers

The first step in designing is to arrange discussion to firms who already did Service Level Agreement for their software maintenance. This thing will give you an initial idea and in case you have no relationships then join IT seller peer groups [9], [14].

6.2 Hire A Lawyer

For legal matter like services violation and penalty, the service of lawyer is essential for customer if he does not have sufficient knowledge of legal matters. It is worth mentioning here that the lawyer should have sufficient IT knowledge. It is not matter wherever lawyer has been living [14].

6.3 Organization Self Assessment

It is necessary for customer to do detail assessment of his company's capabilities and mark every strong and weak point. Examine operational procedure of each service you offer and try to explore company history about these services. This information will help you to determine what you need in Service Level Agreement and what you don't need [14].

6.4 Prepare A Draft

This is not as a whole process. In this, both parties frame a structure of the Service Level Agreement on the basis of their requirements and then made discussion, debate, negotiation and finally prepare contents of the agreement. The duration of this process may vary depends upon the number of services and both parties experience with SLA [9].

6.5 Generate Buy-In

After preparation of draft, both parties have opportunity to review the draft, raise questions, check agreement clauses and make discussion with Technical teams before its final implementation. After this step, SLA developers can further negotiate it to make it quality of final document [1].

6.6 Pre-Implementation Tasks

Before implementation of Service Level Agreement, both parties have to establish tracking mechanism, reporting procedure, communication expectations and training to all the concerned staff members [1], [9].

6.7 Implement and Manage Agreement

The Service Level Agreement (SLA) implementation and management requires both parties' responsibilities. Management responsibilities includes highlight problem areas, performance evaluation, facilitate other party, coordination with each others to improve performance, maintaining ongoing contract, conduct service reviews and modify SLA for improvement [1].

7. COMMON PITFALLS AND ITS AVOIDANCE

Service Level Agreement for outsourcing is very critical success factor. The IT management of both parties (customer, vender) should to make criteria to measure performance of services. There are different types of pitfalls which can be expected in such services. We show you how to avoid such pitfalls.

7.1 Incomplete SLA

It is most common mistake while making SLA for outsource contract. Always provide detail description of quality and quantity of each service. Most of the SLAs are failed due to incomplete detailed description of each outsource service and it's happened due to non-technical / expert IT staff [10].

7.2 Incorrect Business Goals

The Very important thing in Service Level Agreement is to define Business Goals, most of the organizations could not define exact goals of the business and accordingly their Service Level Agreement also could not well define service level attributes. Therefore, it is mandatory for customer to define objectives which leads to service level attributes to exactly evaluate whether service commitments have been met [10].

7.3 Unclear and Incomplete Service Specification

All the services mentioned is Service Level Agreement should be in clear and complete format. Always not try to use ambiguous and confusing words. The unclear service specifications can create ambiguity and confusion for the outsource vender and leads to ineffective SLA [11].

7.4 Wrong Cost Measurement

The core problem in SLA is to measure wrong cost of each service. Normally, prices of services are fixed at the time of signing SLA for a particular year but some services prices vary over the time. Therefore, accurate cost estimation should be made at the designing of SLA [11].

8. COMPONENTS OF SERVICE LEVEL AGREEMENT

This template serves as guide for designing of software maintenance of service level agreement in outsource contract between customers and outsource vender for single service. For more than one services, copy of same template may be used.

The SLA should not to be in lengthy written format; it should be concise with salient details. For one Service base SLA, the following points should be under consideration:

S#	Area to address	Description
1	**Overview**	This section is simply executive summary of the SLA and key components of this document.
2	**Contract Date**	Mention starting and ending dates of agreement.
3	**Contract Number**	This is unique and mandatory in SLA. It should be change after renegotiation. They simply identify specific contracts.
4	**Goals and Objectives**	The detailed goals and objectives of the Service Level Agreement.
5	**Owner**	Detailed description about owner of the SLA and group of users / representatives involved in the scope of SLA. Avoid using the names of representatives.
6	**Project Description**	Brief Description about the project to be served. It may also include purpose, goals and functions of the Project.
7	**Service Name**	Detailed description and primary functions of the service to which SLA is created.

S#	Area to address	Description
8	**Service Expectation Level**	The user expectations about the service quality. Some time, the reader could not understand what the writer wants to say due to wrong interpretation. Use this section to set the expectations of the readers.
9	**Service Security Considerations**	Enlist all service security considerations like security policies and procedures.
10	**Proactive Maintenance**	This section describe about the responsibilities of both parties during SLA execution. Proactive maintenance is of paramount importance to both customer and vender. It will save time and money for both if problems are identified in advance and appropriate actions are taken before there is a major issue that results in downtime.
11	**Support Activities**	Detailed explanation of Support activities for software maintenance like development and updates of new patches, backup and restoration of database, diagnosing hardware problems, resolving environment configuration issues, installation of windows and etc.
12	**Support Mechanism**	Support mechanism includes all the methodologies used to support service
12.1	**Problem Reporting**	This section briefly narrate all ways to report a problem to client or vender like help desk, line of action after receiving problem, Problem assessment and updating of problem status.
12.2	**Support Coverage Time**	Required Support Timing of the both parties, ways to provide service beyond office timing and in gazette holidays
12.3	**Communication and Reporting Mechanism**	Technologies and techniques are used for communication like email, fax, mobile and etc. It also describe about monitoring and tracking of problem reports.
13	**Tools and Technologies**	Tools and Technologies used for the maintenance of software like OS Platform, DB Server, Project Management tools, visual modeler and reporting tools.
14	**Service Charging Policy**	This section include service charging policies like How present service will be charged, revise cost of service if modify, revised service charges if addition of service.
15	**Time Line and Payment Schedule**	Split the Time line and payment schedule in lieu of Service.
16	**Service Level Plan**	The Service Level Plan describes all terms and conditions and narrates how SLA will execute.
16.1	**The Solution**	It describe about already deployed software at customer side.

S#	Area to address	Description
16.2	**The Site**	The sites of the customers to be covered for software maintenance.
16.3	**The Equipment**	This section enlists all types of equipments used at customer sites like hardware, networks, internet and third party softwares to support customer deployed softwares.
16.4	**The Documentation**	Means the operating manuals, user instructions, technical literature and all other related materials in printed form for aiding the use and application of the Licensed Programs.
16.5	**Maintenance Services and charges**	Description of each maintenance and charges of each maintenance service.
16.6	**Date of Commencement**	The date on which this Agreement shall become effective.
16.7	**Pulsating Resources**	The customer's employees concerning to the performance of notified solution.
17	**Outsource Vender Obligations**	Enlist all liabilities of outsource venders like its Hardware, Network, Internet, Debagging and troubleshooting, online support, help desk and backup & recovery support.
18	**Debugging and Troubleshooting**	How to deal with problem by debugging and troubleshooting it. Define Severity Level against each problem and accordingly line of action, time frame to resolve it and priority level.
19	**Service Metrics for Performance**	There are number of Service Level Agreement performance Metrics to measure the performance of the party that other party expected performance be higher than in the SLA. Some metrics are simple and some complicated but you have to select metrics which fulfill your expectations. This section indicates the criteria for performance evaluation.
20	**Violation and Penalty**	It is mandatory section of SLA and depicts all penalties against each violation of SLA terms & conditions and poor performance. The penalty should be ethical, according to law and justified able. Both customer and vender should negotiate on this section.
21	**Duration**	Duration that this SLA is expected to remain in place before it is reviewed.
22	**SLA Review Procedure**	The procedure to review the SLA and the group of person involved in this review procedure.
23	**Proprietary Rights And License**	Detail description about proprietary rights and license. Every country has its own proprietary and licensing rights and in case of agreement with out of country vender, then global proprietary rights will be applicable.

S#	Area to address	Description
24	**Confidentiality**	Every organization have to make it sure its information secure and confidential during the execution and after termination of agreement.
25	**Liability**	This is important section describing liabilities of other party in case of any loss and damage of equipments, loss of customer programs or data resulting from any breakdown of, or fault in, the equipments unless such breakdown or fault is caused by the negligence or willful misconduct of the other Party or any Third Party that is appointed by the other Party.
26	**Assignment**	This is very important section that clearly defines the ways about assignment and transfer of rights. For example Neither Party shall assign or otherwise transfer for this Agreement or any of its rights and obligations there under whether in whole or in part without the prior written consent of the other.
27	**Sub-Contracts**	Policy about sub-contract with any third party by both sides like vender shall not, without the prior written consent of the customer, enter into any subcontract with any person for the performance of any part of this Agreement.
28	**Notices**	The mechanism about issuing notice regarding termination of SLA, Violation of SLA and etc. It also specify about ways to receive and deliver notice to other party like though special messenger, courier, email and postal service.
29	**Law**	The laws imply on this Service Level Agreement.
30	**Disputes**	It specifies how to deal with disputes, if any dispute which may arise between the parties concerning this Agreement. Moreover, it also determines which forum will be used to sue against other party.
31	**Technical Considerations**	This section describes any technical considerations which are essential to be written in this document like service catalog.
32	**Notes & Comments**	Any Notes and comments regarding software maintenance SLA in outsource contract.

9. COMMENTS AND CONCLUSION

The Service Level Agreement is best way to improve organization performance and efficiency. In outsourcing context, customer advice outsource vendor to give guarantees for quality of service which are mentioned in service level agreements and if any party failed then accordingly penalty can be imposed upon accordingly. Service level agreements for outsourcing are very beneficial for customer and vender. Performance Metrics are used to measure the performance of service level agreement and maximize control over service objectives.

This paper describes key areas of service level agreement, common pitfall and its avoidance techniques, steps to design service level agreement and key components of software maintenance designing service level agreement in outsource contract. Although there are several SLA design templates are available and already research work in designing of SLA has already been done but by taking advantage of this paper proposed SLA for software maintenance and recommendations, the customer / vender can gain greater values from its service level agreements investment and create better software maintenance service level agreements in outsource contract in future.

Further theory and practice research is needed in future.

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THE IMPACT OF BRAIN DRAIN ON THE ECONOMY OF PAKISTAN

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ABSTRACT

With the liberalization of emigration laws and the declining economic conditions, brain drain is becoming a massive phenomenon in Pakistan. This study aims at determining the impact of brain drain on the economy of the country. The data collected from the Bureau of Emigration and Overseas Employment, World Development Indicators and from the annual reports issued by State Bank of Pakistan covers the period 1980-2010. Simple linear regression analysis has been performed using Annual Outflow of Labor as the independent variable and Workers' Remittances, Foreign Direct Investment, Imports, Gross National Income per Capita, Private Consumption, Private Savings, And Gross Domestic Product as the dependent variables. The study reveals that annual outflow of labor has a positive impact on all seven of the dependent variables, indicating that, contrary to common belief; brain drain carries positive implications for the national economy.

1. INTRODUCTION

Brain Drain refers to the movement of skilled workers and intellectuals from one country to another, for acquiring better standards of living and higher income. This movement of skilled emigrants can occur between countries at different stages of development. But the problem is more serious for less developed countries because of their low level of development and small stock of human capital. Brain Drain is also termed as the emigration of skilled workers or outflow of labor (Rapoport, 2002).

With the liberalization of emigration laws and the declining economic conditions, "brain drain" is becoming a massive phenomenon in Pakistan that needs to be addressed in order to save the long term economic growth of the country. Pakistan being the 6th most populous country in the world has seen an increasing trend of its work force migrating to other countries. According to the Pakistan Overseas Employment Corporation during the last 30 years nearly 360000 professionals including doctors, scientists, engineers and teachers have migrated to other countries for acquiring better living standards and this human capital flight has increased radically in the recent years (Bureau of Emigration and Overseas Employment, Pakistan, 2011). In Pakistan the major determinants of Brain Drain are unemployment, income incentive, weak institutions, poverty and academic reasons. Over the years, Brain Drain has contributed both positively and negatively towards the development of the country. Workers' Remittances as a result of outflow of labor can improve a country's creditworthiness and thereby increase its access to international credit markets for financing infrastructure and other developmental projects. In Pakistan remittances are an important source of foreign

exchange earnings after exports of manufactured goods and have shown tremendous growth in the recent years. This is so because the migration of Pakistani workers has been on a rising trend, due to which the flow of workers' remittances have quadrupled during the last decade (Kock and Sun, 2011). Furthermore, Foreign Direct Investment is also considered to be a positive impact of Brain Drain. Migration serves as a channel of information transfer across international borders and thus through the presence of Foreign Direct Investment, contribute to the integration of their home country with the global economy. In addition to this, remittances received as a result of outflow of labor also leads to increase in income of the households of the emigrants which further leads to increase in private consumption and private savings. Contrary to these benefits, Brain Drain has an interesting yet important impact on the economy of Pakistan, in the form of imports. The remittances received are not only in the form of money but also include gifts that mainly consist of imported goods. In Pakistan huge amount of remittances (nearly \$ 4 billion) are made by migrants to their home country on imports, which serves as a leakage in the economy (Khan, Khattak, Bakhtiar, Nawab, Rahim and Ali, 2007).

2. PROBLEM STATEMENT

What is the impact of Brain Drain on the Economy of Pakistan?

3. OBJECTIVES OF THE STUDY

In accordance with the problem statement, the main objective of the study is to analyze the impact of Brain Drain on Workers' Remittances, FDI, Imports, GNI per Capita, Private Consumption, Private Savings and GDP of Pakistan for the period 1980 to 2010.

4. METHODOLOGY AND MODEL SPECIFICATION

This study mainly deals with determining the impact of Brain Drain on Workers' Remittances, FDI, Imports, GNI per Capita, Private Consumption, Private Savings and GDP of Pakistan. Secondary data has been collected for Pakistan from 1980-2010. Therefore the following seven simple linear models were estimated using Ordinary Least Square method. The statistical package which was used to interpret the results in this study was SPSS.

$$WR = \beta_0 + \beta_1 AOL + \varepsilon_i \tag{1}$$

$$FDI = \beta_0 + \beta_1 AOL + \varepsilon_i \tag{2}$$

$$GNI = \beta_0 + \beta_1 AOL + \varepsilon_i \tag{3}$$

$$IM = \beta_0 + \beta_1 AOL + \varepsilon_i \tag{4}$$

$$PC = \beta_0 + \beta_1 AOL + \varepsilon_i \tag{5}$$

$$PS = \beta_0 + \beta_1 AOL + \varepsilon_i \tag{6}$$

$$GDP = \beta_0 + \beta_1 AOL + \varepsilon_i \tag{7}$$

where AOL is Annual Outflow of Labor, WR is Workers' Remittances, FDI is Foreign Direct Investment, GNI is Gross National Income per Capita, IM is imports, PC is Private Consumption, PS is Private Savings, GDP is Gross Domestic Product, β_0 is the Intercept

term, β_1 is the regression co-efficient with respect to Annual Outflow of Labor and ε is the stochastic error term.

5. RESULTS AND INTERPRETATIONS

The estimated regression lines are:

i)   WR = -6.693+ 0.018AOL

      Adjusted R^2=0.728   F statistic= (81.495)*
      Durbin Watson=0.658

ii) FDI = -1069.438+ 0.013AOL

      Adjusted R^2= 0.674   F statistic= (62.902)*
      Durbin Watson=0.621

iii) GNI = 726.963+ 0.005AOL

      Adjusted R^2=0.605   F statistic = (46.989)*
      Durbin Watson=0.342

iv) IM = -2142.921+ 0.096AOL

      Adjusted R^2= 0.803   F statistic = (123.366)*
      Durbin Watson= 0.638

v)   PC = -1863225.928+ 28.621AOL

      Adjusted R^2= 0.739   F statistic = (85.899)*
      Durbin Watson=0.747

vi) PS = -273666.739+ 4.762AOL

      Adjusted R^2=0.710   F statistic = (74.434)*
      Durbin Watson= 0.551

vii)   GDP = -2172315.057+ 35.599 AOL

      Adjusted R^2= 0.740   F statistic = (86.490)*
      Durbin Watson= 0.677

(* Shows that statistical significance at 1% level)

The results show that annual outflow of labor has a positive impact on all seven dependent variables. Equation (i) shows that an increase of 1 percent in the annual outflow of labor is expected to add 0.018 percent to the workers' remittances. Equation (ii) shows that an increase of 1 percent in the annual outflow of labor is expected to add 0.013 percent to the FDI. Equation (iii) shows that an increase of 1 percent in the annual outflow of labor is expected to add 0.005 percent to the GNI per Capita. Equation (iv) shows that an increase of 1 percent in the annual outflow of labor is expected to add 0.096 percent to the imports. Equation (v) shows that an increase of 1 percent in the annual outflow of labor is expected to add 28.621 percent to the private consumption. Equation (vi) shows that an increase of 1 percent in the annual outflow of labor is

expected to add 4.762 percent to the private savings. Equation (vii) shows that an increase of 1 percent in the annual outflow of labor is expected to add 35.599 percent to the GDP.

6. CONCLUSION

The study has attempted to analyze the impact of Brain Drain on the economy of Pakistan over the period 1980-2010 through human capital flight. Ordinary Least Square method has been used to analyze the impact of annual outflow of labor on different macro economic variables. The results show that annual outflow of labor has significant impact on all the dependent variables namely workers' remittances, FDI, GNI per capita, private consumption, private savings, imports and GDP. Except for imports that serve as a leakage in the economy, all the other variables are increasing in a positive manner due to Brain Drain. An increase in workers' remittances facilitates foreign exchange reserves in the country that boosts national economy and develops confidence in the foreign and local investors that further helps in generating economic activities for the prosperity of the country. Moreover the increase in foreign direct investment has an important impact on investment whether they are in the form of debt flows or equities. It is clear from the above results that brain drain has a positive impact on GNI, private consumption, private savings and gross domestic product that signifies economic growth in the country. Although imports serve as a leakage in the economy, this leakage is offset by the injection of remittances and foreign direct investment in the economy of Pakistan. Hence the results indicate that brain drain carries positive implications for the Pakistan's economy.

7. RECOMMENDATIONS

- Social, political and economic infrastructures have to be revamped in order to retain the human capital.
- Workers' remittances should be used efficiently in order to achieve economic growth.
- The issues of terrorism within the region must be dealt seriously and strict actions should be taken in order to maintain peace and tranquility. The efforts of the government to maintain law and order within the country must be increased in order to provide incentive to the emigrants to return home with increased skill that would contribute to the economic growth.
- Documentation and migrant information system should be developed and steps should be taken to record the data of migrants returning back home.

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SIMULTANEITY OF PER CAPITA GDP AND EMPLOYMENT IN RELATION WITH TRADE LIBERALIZATION, LABOR DEMAND ELASTICITY AND INVESTMENT TO GDP RATIO IN PAKISTAN

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ABSTRACT

Per capita GDP and Employment are two of the most prominent indicators of economic growth. This study uses a simultaneous equation model to analyze the relationship of each of these two variables with trade liberalization, labor demand elasticity and investment to GDP ratio. Secondary data for employment, per capita GDP, trade to GDP ratio, average monthly real wages and investment to GDP ratio has been taken for the years 1990 to 2010. The model is estimated by using the technique of Indirect Least Squares (ILS). The findings of the study highlight a significant negative impact of trade liberalization and labor demand elasticity on employment. Also, trade liberalization has been found to affect per capita GDP in a negative manner whereas investment to GDP ratio has a positive relationship with per capita GDP. On the basis of these results, this study reveals that there exists a significant positive relationship between per capita GDP and Employment.

1. INTRODUCTION

Pakistan's recent trade liberalization reforms have been honored by many international financial institutions. According to the World Bank, Pakistan's trade regime is the most open one in South Asia. Pakistan follows the lowest tariff rates as compared to the other two large economies of South Asia, i.e. India and Bangladesh. Unlike many developing countries, Pakistan did not hesitate in opening up its agricultural sector.

However, trade liberalization has not been able to generate employment in Pakistan. The reason for that lies in the fact that through trade liberalization, imports of Pakistan have risen as compared to the exports. Greater imports hinder domestic production and thus will lead to unemployment (Malik, 2011).
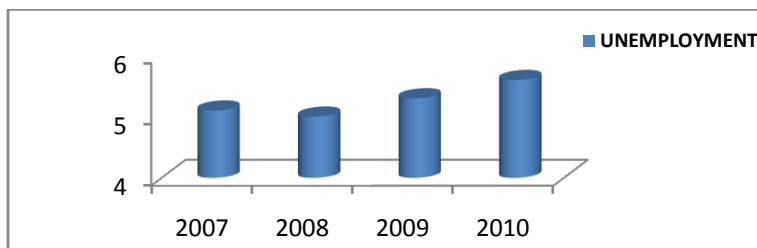
Figure 1.1 Rate of Unemployment
Source: World Bank Development Indicators, 2011.

The above figure illustrates the rate of unemployment in Pakistan. There has been persistent unemployment in Pakistan. This can be attributed to trade liberalization because as our economy is becoming more open, employment opportunities are decreasing due to lesser domestic production.

Table 1.1: Exports and Imports of Pakistan

Year	Exports (Million)	Imports (Million)
2007	16976.2	30539.7
2008	19052.3	39965.5
2009	17688.0	34822.1
2010	19290.0	34710.0

Source: Handbook of Statistics on Pakistan Economy, 2011.

Imports are increasing over the years while exports are falling. Since independence, Pakistan's trade balance is always in negative, giving rise to a trade deficit.

2. OBJECTIVES OF THE STUDY

The objectives of the study are as follows:

- To analyze the simultaneity between per capita GDP and employment in relation with trade liberalization, labor demand elasticity and investment to GDP ratio in Pakistan.
- To determine labor demand elasticity in Pakistan.

3. METHODOLOGY

Secondary data for two models for employment and per capita GDP has been taken for the analysis. The data collected for employment, per capita GDP, trade to GDP ratio, average monthly real wages and investment to GDP ratio is from 1990 to 2010. The natural log of all the variables has been taken. Simultaneous equation model has been constructed and estimated by using the technique of Indirect Least Square (ILS), which is applied when the equations under consideration are exactly identified. The estimates of reduced form equations have been obtained by Multiple Regression.

On the basis of the estimates of reduced form equations, the estimates of structural form equations have been obtained and a regression line has been constructed.

The data sets have been collected from Economic Surveys of Pakistan, for various issues, Federal Bureau of Statistics CBR Yearbook (various issues) Labor Force Survey of Pakistan and State Bank of Pakistan.

In order to run the regression analysis, natural log transformation has been made for the simultaneous equation model.

$$\ln Y_1 = \alpha_0 + \alpha_2 \ln Y_2 + \gamma_1 \ln X_1 + \gamma_2 \ln X_2 + \varepsilon_1 \tag{1}$$

$$\ln Y_2 = \beta_0 + \beta_1 \ln Y_1 + \delta_1 \ln X_1 + \delta_3 \ln X_3 + \varepsilon_2 \tag{2}$$

Here Y_1 and Y_2 are employment and per capita GDP respectively. X_1, X_2 and X_3 represent trade to GDP ratio, average monthly real wages and investment to GDP ratio respectively.

4. RESULTS AND INTERPRETATIONS

Indirect Least-Squares Method (ILS)

As both the equations are exactly identified, ILS can be applied to estimate their coefficients.

The Procedure of ILS

Stage I:

The first stage of ILS includes the construction of reduced form equations and estimating the values of reduced form parameters.

The reduced form equations obtained are as follows:

$$Y_1 = \square_{10} + \square_{11} X_1 + \square_{12} X_2 + \square_{13} X_3 + V_1 \tag{3}$$

$$Y_2 = \square_{20} + \square_{21} X_1 + \square_{22} X_2 + \square_{23} X_3 + V_2 \tag{4}$$

Stage II:

OLS has been applied for estimating the reduced form equations.

The R square value for both the models is 0.977 and 0.951 respectively and both the models are statistically significant.

Both the reduced form equations are given as follows:

$$\hat{Y}_1 = 1.200 + 0.091 Y_2 + 0.125 X_1 + 0.397 X_2$$

$$\hat{Y}_2 = 1.783 + 0.283 Y_1 + 0.106 X_1 + 0.155 X_3$$

Stage III:

From the numerical estimates of reduced form coefficients, the estimates of structural form equations can be calculated and thus we get the estimated regression model as:

$$\hat{Y}_1 = 0.806 + 0.221\, Y_2 - 0.096\, X_1 - 1.654\, X_2$$

$$\hat{Y}_2 = 0.765 + 0.848\, Y_1 - 0.366\, X_1 + 0.460\, X_3$$

The results indicate a negative significant impact of trade liberalization on employment level, which is in accordance with the study conducted by Malik (2011). When trade liberalization causes a decline in the employment opportunities of Pakistan, the immediate consequence is a reduction in per capita GDP. This reduction occurs because domestic production decreases with the fall in employment and hence fewer goods will be produced. As per capita GDP shows the value of goods produced per person in the country, as employment falls per capita GDP also falls in Pakistan.

According to the second equation, trade liberalization has a negative but significant impact on per capita GDP keeping the effect of other variables as constant, which is in conformation with the study conducted by Yasmin, et al. (2006). The endogenous variables, employment level and per capita GDP are inter linked to each other. If employment increases by one percent, per capita GDP increases by 0.848 percent keeping the effect of trade liberalization and investment to GDP ratio as constant.

Average monthly real wages have a negative significant relationship with employment level. This result is in accordance with the labor demand theory which states

an inverse relationship between real wages and employment level. Gross investment to GDP ratio has a positive relationship with per capita GDP, which is in accordance with the study conducted by Yasmin et al. (2006).

From the above regression line, the labor demand elasticity comes out to be

$$E_w = -1.654$$

The sign of labor demand elasticity calculated above is negative indicating the inverse direction of movement between trade liberalization and labor demand. The closer the value of elasticity coefficient to 1, higher is the elasticity. Here its magnitude is 1.654, ignoring the sign, indicating a relatively elastic behavior of employment level.

5. CONCLUSION AND RECOMMENDATIONS

After analyzing the simultaneity of employment and per capita GDP, it is concluded that both play a significant role in the economy of Pakistan. Trade to GDP ratio, wage rate and investment to GDP ratio has shown significant effect on employment and per capita GDP. For Pakistan, labor demand elasticity came out to be negative and highly elastic. Thus, indicating that a large increase in wage rate, will lead to a large decrease in employment. This result is in accordance with the labor demand theory which indicates an inverse relationship between wage rate and employment. As wage rates increase, firms will not want to hire more workers and therefore there would be a fall in employment level.

The following recommendations are made, based on the results obtained from the study:
- As Pakistan is a labor abundant country, it should export more labor-intensive products thus increasing employment opportunities.
- Government should improve law and order situation in the country, so that domestic investors are encouraged to invest in the country.
- Pakistan should adopt strategic approach in order to increase its exports. This requires single country exhibitions, trade shows, seminars and food festivals to increase the demand of these products in the international market.
- In order to increase the exports, the government of Pakistan should encourage the participation of private sector in trade fairs and exhibitions.
- Uninterrupted power supply to the major exporting industries of Pakistan.

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TOURISM AND ITS IMPACT ON THE GDP OF PAKISTAN

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ABSTRACT

This study has been carried out to explore the impact of Tourism on the GDP of Pakistan. The independent variables used in this study are International Tourist Arrivals, International Tourism Receipts and International Tourism Expenditures, the effects of which on the GDP of Pakistan have been investigated. Multiple linear regression analysis has been employed for empirical investigation of the data taken for the period of 1995-2010 for all the variables under consideration. The results demonstrate a positive impact of International Tourist Arrivals and International Tourism Receipts on the GDP of Pakistan. On the other hand, the data analysis reveals a negative impact of International Tourism Expenditures on the same dependent variable. The study confirms the contribution of tourism activity in the economic growth of Pakistan and suggests solutions for removing discrepancies in the tourism sector for its optimal functioning.

1. INTRODUCTION

Tourism in the whole world has been going on in different forms since ancient times. With the passage of time and deepening diversification, this sector has evolved as one of the fastest growing economic sectors in the world.

Pakistan is a home of diverse cultures, people and beautiful landscapes. The unique beauty of scenic, historical and cultural places in Pakistan invites tourists from all over the world and makes them endeavor the blessing of God to the country. Pakistan receives more than 500,000 international tourists annually. However, the volume of domestic tourism being practiced in Pakistan is fairly large as compared to the extent of international tourism (UNWTO, 2011). Different types of tourism practiced in Pakistan include leisure tourism, adventure tourism, ecotourism or environmental tourism, religious or pilgrimage tourism, heritage tourism and sports tourism. Being a developing country and rich in tourist attraction spots, Pakistan has great potential for a thriving tourism industry. It offers local people opportunities to engage in the tourism industry and make a living for themselves. Most importantly, if given due attention ,tourism would help Pakistan grow as a whole due to the influx of foreign people, foreign exchange and a more positive balance of trade.

The potentially developed tourism is, both theoretically and practically, considered as an essential factor leading to economic growth in any nation. The problem of low or nil contribution of tourism arises when this sector is ignored and not utilized to its full potential. The presence of certain risky factors, like lack of infrastructure and facilities, insecurity, poor law and order, political instability, inflation, natural disasters, role of

local and foreign media, low marketing and low environmental standards, affects the tourism adversely hampering possible tourism and economic growth in Pakistan (Kazmi, 2008). Despite all these problems, tourism in Pakistan managed to provide futile results in the form of positive contributions to the GDP in 2011. In the same year, the total contribution of the tourism sector to GDP of Pakistan was 7.3% of which direct contribution was 3.2%. Similarly, in 2011, this sector provided 2.7% of the total employment in the country. Seeing the progressiveness of this sector, the private and public investment in 2011 was 8.3% of the total investment in the economy. Finally, this sector generated 4.4% of the total exports in 2011 (WTTC Economic Impact, 2012).

2. OBJECTIVE OF THE STUDY

The objective of this study is to determine the impact of tourism on the economic growth of Pakistan. Specifically, we are interested in ascertaining the impact of international tourist arrivals, international tourism receipts and international tourism expenditures on the GDP of Pakistan.

3. METHODOLOGY

The variables selected for this study are GDP, number of international tourist arrivals (ITA), international tourism receipts (ITR) and international tourism expenditures (ITE). Particularly, the annual GDP measures the economic growth of a country, whereas, the three independent variables chosen are the most essential representing variables for tourism sector regarding their impact on economic growth.

A causal and analytical study is conducted to explore the dependence of GDP on the variables related to the tourism sector in Pakistan. Specifically, in this study, the method of multiple linear regressions is employed. Whereas, the software used to run regressions is Statistical Package for Social Sciences (SPSS).

The data used for this purpose is of secondary nature and has been taken from the web link of World Bank (WB) i.e. from WDI. The unit of measurement for all data is current U.S. million dollars.

In order to run multiple linear regressions by using GDP as dependent variable and ITA, ITR and ITE as explanatory variables, the following regression equation is made to be estimated later on.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon \tag{1}$$

where,

Y	Dependent variable i.e. GDP of Pakistan.
X_1, X_2 and X_3	Independent Variables ITA, ITR & ITE, respectively.
β_0	Intercept of the regression line.
β_1	Slope or regression coefficient with respect to X_1 or ITA.
β_2	Slope or regression coefficient with respect to X_2 or ITR.
β_3	Slope or regression coefficient with respect to X_3 or ITE.
ϵ	Error term of the model.

4. RESULTS & INTERPRETATIONS

After the discussion of methodology and regression analysis, the numerical results obtained are analyzed in the following section.

In order to run regressions, the utilized data fulfills the four most important assumptions of Classical Linear Regression Model (CLRM). Durbin-Watson and Variance Inflation Factor (VIF) tests were conducted to check autocorrelation and multicollinearity in the data. Whereas, Normal P-Plot and Scatter Diagram were drawn to examine normality and heteroscedasticity of the data values. On the basis of the results of these tests and diagrams, it is observed that the data for dependent variable is normally distributed, the values of independent variables exhibit no multicollinearity, and the model formed is free from autocorrelation and heteroscedasticity.

To check the overall significance of the formed regression model, F-Test is conducted by forming null and alternative hypotheses.

H_0: GDP of Pakistan does not depend significantly on number of ITA, ITR and ITE

H_1: GDP of Pakistan depends significantly on number of ITA, ITR and ITE.

The value of F-statistic gained is 85.536 with a p value of 0.000. Since, the p value is even less than the minimum significance level of 0.01, H_0 is rejected. It means that there exists a strong significant impact of ITA, ITR and ITE on GDP of Pakistan. Hence, this particular multiple linear regression model is highly significant at overall significance level of 100%.

Similarly, the calculated value of Coefficient of Determination R^2 is 0.977. It means that 97.7% of the variation in dependent variable (GDP) is explained by all the predictor variables (ITA, ITR & ITE) of the model. This value is high and confirms the goodness of fit of the model. Also the value of adjusted R^2 is 0.944 or 94%. It is a high value and predicting a good model too.

In order to check the individual significance of each independent variable the t-test is used. The t-statistic and p value for ITA comes out to be 1.876 (approximately equal to 2) and 0.101. ITR has 4.864 and 0.000 as its t-statistic and p value. However, the t-statistic for ITE is -2.346 with a p value of 0.037. With the help of these values it is confirmed that all the explanatory variables have a significant impact on the GDP of Pakistan.

Using the results obtained after regression analysis, the estimated regression line is as follows:

$$\hat{Y} = -64674.674 + 61118.053X_1 + 205.992X_2 - 18.159X_3 \tag{2}$$

The value of intercept i.e. β_0 is -64674.674 and the estimated value of β_1 is 61118.053. β_1 tells that a change of one million in number of ITA will lead to a change

of 61118.053 million current US dollars in the GDP of Pakistan, while keeping all other independent variables constant.

Likewise, the estimated value of β_2 is 205.992. This value indicates a increase (decrease) of one million current US dollars in ITR will increase (decrease) the GDP by 205.992 million current US dollars, while keeping the effects of ITA and ITE constant.

Lastly, the estimated value for β_3 is -18.159. It indicates a one million current US dollars increase (decrease) in ITE will decrease (decrease) the GDP by the amount of 18.159 million current US dollars, while keeping the effects of ITA and ITR constant.

5. CONCLUSION & RECOMMENDATIONS

The study concludes that tourism has a substantial impact on the GDP growth of Pakistan in terms of ITA, ITR and ITE. Predominantly, on one hand, GDP of Pakistan responds directly or positively to ITA and ITR, and on the other hand, it responds inversely or negatively to ITE.

ITA has a positive impact because increase in the number of tourist arrivals will increase exports, thus, reducing current account deficit and accelerating economic growth in Pakistan (Malik et al., 2010). The study also shows ITR as a powerful factor affecting GDP because it represents the earnings that Pakistan receives in the form of foreign exchange brought by foreigner tourists as payments while tourism activity in the Pakistan. The inflow of such money increases foreign reserves of the host country. Despite the positive relation exhibited by ITA and ITR, ITE shows a relatively negative causal relationship with GDP of Pakistan. Pragmatically, this result gained truly depicts the direction of the association between ITE and GDP. Theoretically, ITE represents the income or money spending by the nationals of Pakistan abroad. This money spent for tourism expenditure is actually a part of Pakistan's national income, so the expenditure shows the outflow of foreign exchange reserves causing a direct reduction in GDP of Pakistan.

The analysis further reveals that in any particular economy international tourism affects growth through several channels beyond the direct revenues from receipts. It is also confirmed that for economic progress in a developing country like Pakistan; tourism development plays a considerable role by injecting new money in tourism as well as other linked sectors. Similarly, tourism also helps Pakistan to progress socially by strengthening the ties between Pakistan and her trading partners.

Following are some recommendations proposed in the light of undertaken study:

- The study justifies the requirement of government intervention to work hand in hand with private sector, while encouraging and increasing tourism demand with providing and nurturing the development of tourism input supply.
- Provision of meaningful security for lives of tourists should be the priority of government.

- The print and electronic media can help to attract tourists by displaying a positive image of Pakistan and devising different advertisement strategies showing what an area has to offer to promote tourism in the country.
- There should be a proper measuring system in the national accounts of Pakistan to determine, assess and evaluate the extent of tourism, its variables and its related outcomes in the economy.
- Tourism works as a complementary sector creating affects on other sectors too. Therefore, sound, practical and careful policies should be chalked out to get overall maximum benefits.

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IMPACT OF INTRA-SAARC TRADE ON THE ECONOMIC GROWTH OF INDIA AND PAKISTAN

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ABSTRACT

Economic integration through regionalism has been a very common phenomenon in today's world and SAARC is one such trading bloc that is working towards creating economic integration in the region but it has still not achieved its desired goals. The economic cooperation between the two nuclear powers of South Asia, i.e. India and Pakistan, can reap tremendous benefits for the people of the region. This study has been conducted to analyze the impact of the intra-SAARC trade on the economic growth of Pakistan and India which are two of the most influential member states of SAARC. The data has been collected from the year-book of International Direction of Trade Statistics, IMF, and covers the time period 1990 to 2011. Simple linear regression modeling has been employed using intra-SAARC trade as the independent variable and the GDP of the two countries as the dependent variable. The results of the study indicate that there is a significant positive relationship between intra-SAARC trade and the GDP of the two South Asian economies.

1. INTRODUCTION

SAARC stands for South Asian Association for Regional Cooperation. South Asian countries formed an organization in 1985 to improve the living conditions of the people of South Asia by cooperating in different sectors like agriculture, transport, energy, trade and technology etc. It comprises of seven South Asian economies namely Pakistan, India, Nepal, Bhutan, Srilanka, Bangladesh and Maldives. These economies face common challenges of mass poverty, unemployment, lack of adequate health care and illiteracy so the formation of SAARC was indeed a very good step to eradicate some of the basic problems that are faced by these countries within the region.

The economic performance of all SAARC countries in general and that of India and Pakistan in particular has been gloomy over the past few decades. SAARC has not been successful in achieving its primary goals that could work for the betterment of South Asia. It is a sheer failure as India and Pakistan, two nuclear states, face bilateral disputes. As a result of such disputes, the two countries spend most of their budget on their defense that requires cut back on education and other developmental sectors. They fail to understand that education is the backbone of a developed and civilized society. These nations should resolve their conflicts through negotiations to reduce defense budget and give due importance to the education and other developmental sectors (Zaman, 2011).

We live in an inter-connected world, and this is especially so in South Asia becausethese countries are bound by history and geography. We share common borders,

whether on land or sea. What happens in one country necessarily affects the other. If we act with wisdom and sagacity, we all stand to benefit because no country can prosper in isolation.

This study also focuses on the world-wide competitiveness rankings of India and Pakistan to evaluate their economic performance. The figure 1.1 shows that their performance on the basis of institutions, infrastructure, macro-economic stability, higher education and training, market efficiency, technological readiness and innovation is not satisfactory.
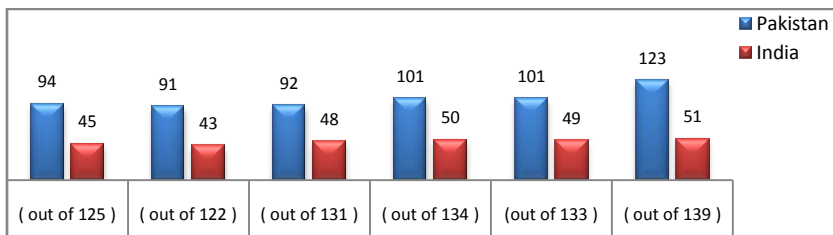
Figure 1.1: Competitiveness Rankings of India and Pakistan
Source: Annual Global Competitiveness Report of World Economic Forum, 2011.

The graph shows that both India and Pakistan has lost their worldwide positions on the basis of their economic performance in institutions, infrastructure and other pillars of GCI.

As a result of low competitiveness performance and inter-state differences, SAARC is failing to create meaningful identity as a regional bloc because of border disputes indicating that economics is dominated by politics in this region.

SAARC countries have a very bright opportunity to grow regionally by increasing intra- regional trade because South Asia has a tremendous potential for growth and so is the case with India and Pakistan. It's been over 20 years that SAARC came into existence for the purpose of overcoming common problems of South Asian economies. Unfortunately, SAARC has achieved its very few objectives. Intra-regional trade among the SAARC nations is very low when compared with other regional trade blocs of the world as the trade performance of South Asia has been gloomy. Pakistan and India's global exports and global imports exceed its intra-SAARC exports and imports (Sabur, 2003).

Pakistan and India can improve their economic performance by increasing intra-regional trade through economic cooperation. SAARC's success depends on the economic decisions taken by the two powerful nations in the region so SAARC has still a long way ahead to promote growth, security, prosperity and peace in the region if decisions are taken to benefit the region as a whole.

2. PROBLEM STATEMENT

What is the impact of intra-SAARC trade on the economic growth of the two- South Asian economies namely India and Pakistan?

3. OBJECTIVES OF THE STUDY

In accordance with the problem statement, one of the main objectives of this study is to see the impact of intra-SAARC trade on the Gross Domestic Product of India and Pakistan.

4. METHODOLOGY AND MODEL SPECIFICATION

This study makes use of the ordinary least squares (OLS) method to estimate the parameters of linear regression model. The statistical package used to interpret the results in this study was SPSS. The following functional form of the regression equation was used for the two models:

Model 1 for India:
$Y_1 = \alpha_{10} + \alpha_{11} X + u_1$, where Y_1 = GDP of India, α_{10} = intercept term, α_{11} = regression coefficient with respect to intra-SAARC trade, X = Intra-SAARC trade and u_1 = stochastic error term which includes all other factors which may affect the GDP of the country.

Model 2 for Pakistan:
$Y_2 = \alpha_{20} + \alpha_{21} X + u_2$, where Y_2 = GDP of Pakistan, α_{20} = intercept term, α_{21} = regression coefficient with respect to intra-SAARC trade, X = Intra-SAARC trade and u_2 = stochastic error term

Hypothesis Formulation

H_0: GDP of India and Pakistan does not depend significantly on intra-SAARC trade and H_1: GDP of India and Pakistan depends significantly on the intra-SAARC trade.

5. RESULTS AND INTERPRETATIONS

The assumptions of the linear regression were fulfilled to derive a simple linear regression line for both models. Shapiro Wilk's test was used to check the normality of the data, Durbin Watson test was used to check auto-correlation and scatter plot of residual and the predicted values were drawn to check heteroscedasticity in the two models. Since all the assumptions were satisfied so it was possible to run a regression analysis for this study.

The correlation coefficients between the intra-SAARC trade and the GDP of India and Pakistan was found to be 0.971 and 0.967 depicting a very strong positive correlation between the dependent and the independent variables taken for the study. Therefore, intra-SAARC trade could be used as a predictor variable in both models.

In case of model 1, the value of the adjusted R^2 was found to be 0.939 and in case of model 2 it was 0.933 indicating that 93.9% of the variation in the GDP of India is explained by the intra-SAARC trade and 93.3% of the variation in the GDP of Pakistan is explained by the intra-SAARC trade in the region. The following regression lines were obtained in case of model 1 and 2; $Y_1 = 477.109 + 493.079X$, $Y_2 = 123.689 + 51.674X$, respectively, where X = intra-SAARC trade, Y_1 = GDP of India and Y_2 = GDP of Pakistan.

The results for model 1 and 2 were highly significant indicating that one unit increase in the intra-SAARC trade would lead to a 493.079 units increase in the GDP of India and one unit increase in the intra-SAARC trade would lead to 51.674 units increase in the GDP of Pakistan. The models are reliable for prediction purpose as all assumptions of linear regression were fulfilled.

6. CONCLUSION

The results of the study revealed that intra-SAARC trade had a very strong positive impact on the GDP of the two member states of SAARC i.e. India and Pakistan but it had a more significant impact on the GDP of India than that of Pakistan. The study concludes that greater the variation in the GDP of the two economies explained by the intra-SAARC trade, greater will be the benefits of increasing intra-regional trade. Since the results were significant, the two South Asian economies can reap tremendous benefits by enhancing intra-regional trade. The reason behind it is the dependence of the Gross Domestic product on the intra-SAARC trade and the benefits such as reduced transportation costs, regional markets and free trade agreements. Thus, the SAARC economies especially India and Pakistan must enhance the intra-regional trade in order to meet all the possible objectives of SAARC.

7. RECOMMENDATIONS

This study suggests the following few things that SAARC must consider in order to become an effective regional block:
- Pakistan and India should work on overcoming the bilateral disputes that makes it a major reason of the disfunctioning of SAARC so they must resolve their disputes peacefully.
- Although, India and Pakistan have increased the intra-SAARC trade over the past few decades but they should enhance regional trade to a volume that can benefit the region as a whole and a comparative study of SAARC and EU should be done so that SAARC economies can learn a lesson from EU.

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FACTORS INFLUENCING BUSINESS COMPETITIVENESS
OF SAARC ECONOMIES

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ABSTRACT

Business Competitiveness is the level of productivity that a country maintains during a specific period. With the rapidly growing competition among the world economies Business Competitiveness has become a massive issue. This study has been conducted to determine the crucial factors influencing Business Competitiveness of SAARC economies. Simple linear regression is employed for the time period (2006-2011) in order to establish links between four pillars of the Global Competitiveness Index i.e. Higher Education and Training index, Technological Readiness index, Innovation index and Business Sophistication index taken as variables. The data is collected from Global Competiveness Report 2006-2011. The empirical results suggest that all of these four indicators are significant determinants of Business Competitiveness. As such, the inconsistent performance of SAARC economies in the four indices mainly results in their weak productive base and lack of trade diversification.

1. INTRODUCTION

South Asian Association for Regional Cooperation (SAARC) was established in 1985 in the first SAARC Summit. Its member states include India, Pakistan, Sri Lanka known as Non-Least Developed Contracting States (NLDCSs) whereas Afghanistan, Bangladesh, Bhutan, Maldives, Nepal are categorized as Least Developed Contracting States (LDCSs). The emphasis is on development of nations socially, culturally, economically, and boosting the business productivity so as to establish South Asia as an autonomous economic bloc. SAARC nations not only share border but also exhibit common problems such as low literacy rates, lack of technological agility and innovative development. 65 percent of South Asian trade potential has not yet been exploited which is been attributed to the narrow productive base resulting in lack of trade diversification impacting the economy's growing Business Competitiveness.

Business Competitiveness is taken in terms of the level of productivity that a country can attain and maintains during a specific period. Productivity is influenced by many factors that mutually condition one another. It is mainly a combination of higher education and training that creates technological agility leading to innovative development. Innovative development leads the firms to adopt new operational strategies resulting in efficient utilization of scarce resources constituting sophisticated business environment for optimal productivity. This leads nations to the road of trade enhancement and sustainable economic growth.

Economy's growing competitiveness is taken in terms of trade diversification. It is a reflector of broadening of productive base of a country with the businesses getting more efficient by improving fundamentals. SAARC nations are lagging behind as total trade potential has not been exploited. The main reason is lagging behind in terms of the factors that enhance the productivity leading to "Business Competitiveness". Global Competitiveness Report (2006-2011) indicates that Bangladesh's rank had fallen from 99th to 108th. Nepal's productive ranking also detoriated from 110th to 125th position in 2011. Pakistan's rank has fallen from 91st in 2006 to 118th in 2011. India's rank has also fallen from 43rd to 56th position. Only Sir Lankan economy has shown improvement in its productive ranking from 79th to 52nd in 2011.

In short, optimal utilization of scarce resources leads to productive or business competitiveness by ensuring employment opportunities, increasing income and improving the standard of living of the people. To achieve Global Competitiveness, nations first have to achieve Business Competitiveness.

2. PROBLEM STATEMENT

What are the factors crucial in determining Business Competitiveness of SAARC nations?

3. OBJECTIVE OF THE STUDY

To evaluate the factors determining the Business Competitiveness of SAARC nations by following pillars of Global Competitiveness Index for the period 2006 – 2011.

4. METHODOLOGY AND MODEL SPECIFICATION

The study deals with factors influencing Business Competitiveness of SAARC economies. Therefore, Ordinary Least Square is employed for the time period (2006-2011) in order to establish links between four pillars of the Global Competitiveness Index taken as variables. The statistical package used to interpret results in this study was SPSS.

$$Y = \alpha_o + \beta_1 X_1 + \mu_1 \tag{1}$$

$$X_1 = \alpha_1 + \beta_2 X_2 + \mu_2 \tag{2}$$

$$X_2 = \alpha_2 + \beta_3 X_3 + \mu_3 \tag{3}$$

$$X_3 = \alpha_3 + \beta_4 X_4 + \mu_4 \tag{4}$$

In model 1, Business Competitiveness Index Y is independent and Business Sophistication Index X_1 is dependent variable. In model 2, Business Sophistication Index X_1 is independent variable and X_2 is Innovation Index which is dependent variable in model 2. Similarly Innovation Index X_2 is independent and Technological readiness X_3 is dependent variable in model 3. In model 4, Technological readiness X_3 is independent and Higher Education and training index is dependent variable. β_1, β_2, β_3, β_4 are regression co-efficient with respect to X_1, X_2, X_3, X_4. α_o, α_1, α_2, $\alpha 3$ are the intercept and μ_1, μ_2, μ_3, μ_4, are stochastic error terms.

5. RESULTS AND INTERPRETATIONS

The estimated results of the above models for the years 2010 and 2011 are shown below:

$Y= 1.020 + 0.740\ X_1$ (1, 2010)
Adjusted R^2=0.806 F statistic= (17.668)* 0.025
T- Ratio=4.203 Durbin-Watson=1.964

$X_1= 0.854 + 0.964\ X_2$ (2, 2010)
Adjusted R^2=0.990 F statistic= (386.995)* 0.000
T- Ratio=19.672 Durbin-Watson=1.990

$X_2= -1.777+1.651\ X_3$ (3, 2010)
Adjusted R^2=0.978   F statistic= (134.757)* 0.007
T- Ratio=11.608   Durbin-Watson=1.719

$X_3= 1.183+0.544\ X_4$ (4, 2010)
Adjusted R^2=0.939 F statistic= (46.987)* 0.021
T- Ratio=6.855 Durbin-Watson=1.681

$Y= 1.314+0.666X_1$ (5, 2011)
Adjusted R^2=0.798 F statistic= (16.805)* 0.025
T- Ratio=4.099 Durbin-Watson=1.817

$X_1= 0.939+0.966X_2$ (6, 2011)
Adjusted R^2=0.912 F statistic= (42.540)* 0.007
T- Ratio=6.522 Durbin-Watson=2.147

$X_2= 0.014+1.448X_3$ (7, 2011)
Adjusted R^2=0.831 F statistic= (15.754)* 0.031
T- Ratio=4.969 Durbin-Watson=2.028

$X_3= 1.163+0.565X_4$ (8, 2011)
Adjusted R^2=0.967 F statistic= (119.138)* 0.002
T- Ratio=10.915203 Durbin-Watson=2.310

(* Shows significance of the model at a specific p value)

The results show that four variable are significant determinants of Business Competitiveness. Equation (1) shows that an increase of 1 unit in the BSUF is expected to add 0.740 units to the BCOM of a country. Equation (2) shows that an increase of 1 unit in the INOV is expected to add 0.964 units to the BSUF of a country. Equation (3) shows that an increase of 1 unit in the TECH is expected to add 1.651 units to the INOV of a country. The equation (4) shows that an increase of 1 unit in the HEDU is expected to add 0.544 units to the TECH of a country. Equation (5) shows that an increase of 1 unit in the BSUF is expected to add 0.666 units to the BCOM of a country. Equation (6) shows that an increase of 1 unit in the INOV is expected to add 0.966 units to the BSUF of a country. Equation (7) shows that an increase of 1 unit in the TECH is expected to add 1.448 units to the INOV. Equation (8) shows that an increase of 1 unit in the HEDU is expected to add 0.565 units to the TECH of a country.

Over the years 2006-2009 for model 1, an increase of 1 unit in the Business sophistication is adding 0.62 units to the Business Competitiveness for 2006, 0.59 units for 2007, 0.56 units for 2008, 0.60 units for 2009. For model 2, 1.06 units increase in Business sophistication is due to a unit increase in the Innovation for 2006, 1.000 units for 2007, 1.130 units for 2008, 1.110 units for 2009. For model 3, a unit increase in technological readiness increases innovation by 1.410 units for 2006, 1.635 units for 2007, 1.248 units for 2008 and 0.977 units for 2009. Whereas, for model 4 a unit increase in higher education causes technological readiness to increase by 0.49 units in 2006, 0.432 units in 2007, 0.570 in 2008 and 1.098 in 2009.

6. CONCLUSION

The results show that the variables are positively related to Business competitiveness and are significant factors of productive growth. Competitiveness laggards like Pakistan, Nepal and Bangladesh pose a serious challenge to the overall performance of the SAARC. Over the years (2006-2011) nations failed to maintain consistency in the Business sophistication and innovation exhibiting weakness of the economy in technological agility and education sector. This is the reason of fall in Business Competitiveness ranking of SAARC nations. Only Sir Lankan economy has shown improvement in its productive ranking from 79th in 2006 to 52nd in 2011. It has shown significant improvement in four indices over the years but the ratio of improvement is 1:4 out of 5 countries. The crux of this study is as a result of low productive performance and interstate differences SAARC is failing to boost the intra and inter regional trade to the potential level that exist in the region.

7. RECOMMENDATIONS

- Business Competitiveness demands effective pattern of investment in the education sector to strengthen the human capital that would in turn improve their ability to innovate and adapt new technology.
- SAARC economies should fully embrace the competitiveness framework introduced by World Economic Forum, to be competitive in business as well as globally.
- Business Competitiveness problems should be discussed and raised in SAARC annual meetings and member countries should actively participate in the meetings.

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IMPACT OF TELECOMMUNICATION ON THE ECONOMY OF PAKISTAN

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ABSTRACT

This study assesses the impact of telecommunication on the economic growth of Pakistan. Secondary data has been analyzed in order to examine the impact of telecommunication on the economy taking Gross Domestic Product, Investment and Foreign Trade as dependent variables and Telecom Penetration (Mobile and Landline) as the independent variable. Ordinary Least Squares (OLS) technique has been employed to test the dependence of these macro-economic variables on telecommunication for the period 2003-2010. The results of OLS regression highlight a positive effect of Telecom Penetration on GDP, Investment and Foreign Trade. The study suggests that the telecommunication sector plays a vital role in fostering the economic growth of Pakistan.

1. INTRODUCTION

Telecommunication sector of Pakistan has emerged as one of the fastest growing sectors of the economy in Asia, especially in the preceding decade. After realizing the benefits of deregulation and privatization of telecom sector in North America and Europe; the government of Pakistan also moved from a state-owned, monopolized sector towards a deregulated, private one. This became the basis of the emergence of new operators in telecommunication sector which led to competition in the market, lower tariffs, high telecom penetration, extensive network coverage and innovative products and expanding subscriptions for fixed line, cellular mobile and broadband (Bhatti, 2009).

The developing telecommunication sector is a major contributor in the development of Pakistan. It has made its highest ever contribution to the national exchequer in FY 2011 of about Rs. 117 billion. It has also been the major source of attracting foreign direct investments in the country, mounting to US$ 79 million in FY 2011. Telecom advancement has also led to the rise of imports of cellular mobile handsets to US$ 218 million in 2011. Therefore, the telecom sector has is contributing to the economic prosperity of the country (Khan, 2012).

2. PROBLEM STATEMENT

What is the impact of telecommunication on the economy of Pakistan?

3. OBJECTIVES OF THE STUDY

The aim of the study is to evaluate and analyze the impact of telecommunication on Gross Domestic Product, Foreign Trade and Investment of Pakistan for the period 2003-2010.

4. RESEARCH METHODOLOGY AND MODEL SPECIFICATION

Time series data analysis is administered to evaluate the effect of telecommunication sector on GDP, Foreign Trade and Investment. Secondary data for Pakistan for eight years from 2003-2008 is gathered. The following three simple linear models are constructed:

Model 1: $GDP = \alpha_0 + \alpha_1 T + \square t$ (1)

Model 2: $FT = \beta_0 + \beta_1 T + \square\, t$ (2)

Model 3: $I = \delta_0 + \delta_1 T + \square\, t$ (3)

where

GDP = gross domestic product of Pakistan

α_0 = the intercept term shows the total GDP when telecommunication is not taken into account

α_1 = slope coefficient which shows the change in GDP due to a unit change in telecom penetration

FT   = sum of total foreign imports and exports of Pakistan

β_0 = the intercept term shows the total foreign trade when the telecommunication is not taken into account

β_2 = slope coefficient which shows the change in trade due to a unit change in telecom penetration

I   = gross fixed investment of Pakistan

δ_0 = the intercept term shows the gross fixed investment when telecommunication is not taken into account

δ_1 = slope coefficient which shows the change in investment due to a unit change in telecom penetration

T   = total mobile and landline penetration of Pakistan

$\square\, t$ = stochastic error term.

5. DATA ANALYSIS AND INTERPRETATION

Ordinary Least Squares (OLS) method is used for analyzing the impact of telecom penetration on gross domestic product, foreign trade, and gross fixed investments. A variety of tests such as Durbin-Watson test for autocorrelation, test for heteroscedasticity and one sample Kolmogorov Smirnov test for normality are used to satisfy the assumptions of OLS.

From the results, autocorrelation is detected in all the three models. Therefore, Orcutt-Cochrane Remedial Method is applied to remove autocorrelation from the models. Autocorrelation is removed from Model 2 after one round, from Model 1 after two

rounds and from Model 3 after three rounds of Orcutt-Cochrane Remedial Method. The results and interpretation of OLS estimation of all three models are shown below:

Model 1:

The estimated regression line for Model 1 is

$$\hat{GDP} = 41492.239 + 257.093T \text{ (i)}$$

R square = 0.987   Adjusted R square = 0.984
F statistic = 300.989 (0.000)   DW Statistic = 1.821

The above equation shows that the dependent variable which is GDP in this particular model has a direct positive relationship with telecom penetration T. The model shows that the constant intercept term for the model is 41492.239, which indicates that GDP is positive with the value of 41492.239 units in case of telecom penetration taken as zero. The model also shows that the value of the slope is 257.093 indicating that an increase of 1 unit in the telecom penetration is expected to add 257.093 units to the total GDP keeping all other variables affecting GDP as constant. The value of 0.987 of R^2 in this model shows that 98.7 percent of the variation in the dependent variable i.e. GDP, is explained by the given independent variable i.e. total telecom penetration.

Model 2:

The estimated regression line for Model 2 is

$$\hat{FT} = 15.427 + 0.610T \text{ (ii)}$$

R square = 0.949   Adjusted R square = 0.939
F statistic = 93.896 (0.000)   DW Statistic = 1.781

The above equation shows that the dependent variable which is foreign trade in this particular model has a direct positive relationship with telecom penetration T. The model shows that the constant intercept term for the model is 15.427, which indicates that foreign trade is positive with the value of 15.427 units in case of telecom penetration taken as zero. The model also shows that the value of slope is 0.610 indicating that an increase of 1 unit in telecom penetration is expected to add 0.610 units to the total foreign trade keeping all other variables affecting trade as constant. The value of 0.949 of R^2 in this model shows that 94.9 percent of the variation in the dependent variable i.e. foreign trade is explained by the given independent variable i.e. total telecom penetration.

Model 3:

The estimated regression line for Model 4 is:

$$\hat{I} = 628229.567 + 24019.021T \text{ (iii)}$$

R square = 0.985   Adjusted R square = 0.980
F statistic = 193.625 (0.001)   DW Statistic = 1.526

The above equation shows that the dependent variable which is investment in this particular model has a direct positive relationship with telecom penetration T. The model

shows that the constant intercept term for the model is 628229.567, which indicates that the investment is positive with the value of 628229.567 units in case of telecom penetration taken as zero. The model also shows that the slope is 24019.021 indicating that an increase of 1 unit in the telecom penetration is expected to add 24019.021 units to the total investment keeping all other variables affecting investment as constant. The value of 0.985 of R^2 in this model shows that 98.5 percent of the variation in the dependent variable i.e. investment is explained by the given independent variable i.e. total telecom penetration.

6. CONCLUSION

The results of regression analysis show a positive effect of telecom penetration on the economic growth measured by GDP, investment, and trade.

Telecommunication sector of Pakistan is playing an important role in the growth of foreign imports and exports by providing easy access to foreign markets and new ways of marketing the products. Similarly, telecommunication is also contributing to the gross fixed investment of Pakistan by attracting more domestic and foreign investments. Hence, by affecting trade and investment, telecommunication is playing an indirect but important role in economic growth and development of Pakistan. The direct impact of telecommunication was also studied in the regression model of GDP and telecom penetration. Results show that with rising telecom penetration, the GDP of Pakistan has also enhanced leading to the economic growth. Hence, the study concludes that telecommunication has a strong positive effect on the economy of Pakistan.

7. RECOMMENDATIONS

- The government of Pakistan should take steps, such as improving security conditions of the country, to increase local and foreign investments in telecommunication sector in order to enhance its growth further.

- Telecommunication infrastructure for cellular mobile and internet should be developed in far flung areas of Baluchistan and Khyber Pakhtoon Khwah for the economic growth of those areas as well.

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DECISION RULE OF ACCEPTANCE USING IMPROVED
TWO-STAGE PLANS FOR THE WEIBULL DISTRIBUTION

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ABSTRACT

This paper considers a new attribute two-stage group sampling plan for time-truncated life tests. The design parameters of the proposed plan are determined using the two-point approach, in which the producer's and consumer's risks are satisfied simultaneously at the acceptable reliability level and the lot tolerance reliability level, respectively. The case of the Weibull distribution is described to illustrate the procedure that can be used when the quality level is expressed by a multiple of the specified mean life. Tables are constructed for various combinations of group size, quality level and shape parameter. The advantage of the proposed plan is established by comparison with the existing two-stage group sampling plan in terms of the average sample number.

KEYWORDS

Acceptance sampling; two-stage sampling; producer's risk; consumer's risk.

1. INTRODUCTION

Quality control analysis is generally divided into two parts, one is control charts and other is acceptance sampling plans. The first one is used to improve the quality of the product during the manufacturing process, whereas the second part is used to test the final items for possible acceptance or rejection. Acceptance sampling is a major tool for the inspection of products. Single acceptance sampling is widely used for that purpose. A double sampling plan is used when we cannot reach the decision on the basis of the first sample. This plan has advantages over the single sampling plan in terms of operating characteristics and the average sample number (ASN). For more details, one may refer to Aslam and Jun (2009a) and Aslam et al. (2009b).

Attributes single sampling plans have been proposed for a variety of life distributions by many authors. See, for example, Goode and Kao (1961) for Weibull distribution, Gupta and Groll (1961) for gamma distributions, Gupta (1962) for normal and log-normal distributions, Tsai and Wu (2006) for a generalized Rayleigh distribution, Kantam et al. (2001) for the log-logistic distribution, and Balakrishnan et al. (2007) for a generalized Birnbaum-Saunders distribution, Aslam and Kantam (2008) and Aslam et al. (2009c) for the generalized exponential distribution.

In single sampling, the items are usually tested one by one. However, in practice, testers are available which are used to test a number of items simultaneously. Single and double group sampling using sudden life testing is introduced by Jun et al. (2006) for the Weibull distribution. Single group sampling is considered more efficient than the single acceptance sampling in terms of cost and time to reach the final decision about the submitted lot. Recently, a single group acceptance sampling plan (GASP) for the truncated life test was proposed by Aslam and Jun (2009d) for the inverse Rayleigh and log-logistic distributions using the single-point (only consumer's risk) approach. Aslam and Jun (2009e) and Aslam et al. (2009) proposed group sampling plans for Weibull and gamma distributions by considering the producer's and consumer's risks at a time.

Aslam et al. (2009) proposed two-stage group acceptance sampling plans for Weibull distributed items by considering the producer's and consumer's risks at the same time, and assuming that the shape parameter of this distribution is known. They provided extensive tables and discuss the advantages of the two-stage group acceptance sampling plans over the single-stage group acceptance sampling plans based on truncated life tests. Further, they explained the results when the shape parameter of the Weibull distribution is known.

The producer and consumer want that acceptance sampling plans that are efficient in saving the cost and time of the experiment. In life test experiment, these two factors are directly attached to the sample size. So the purpose of this paper is to propose an improved two-stage group sampling plan in terms of the sample size. We use the two-point approach when designing the proposed plan. Two cases are considered, one of which is the case where the acceptable quality level and the lot tolerance quality level are expressed by the unreliability, and the other is the case where the quality levels are expressed by the mean ratio to the specified life under the Weibull distribution. The rest of the paper is organized as follows: The proposed group sampling plan is given in Section 2. The design of the proposed plan is described in Section 3. The Weibull case is considered in Section 4. The advantages of the proposed plan are pointed out in Section 5. Several concluding remarks are given in Section 6

2. PROPOSED TWO-STAGE GROUP ACCEPTANCE SAMPLING PLAN

We propose the following two-stage group sampling plan for a time-truncated experiment when using the type of testers with the group size of r:

1. (First stage) Draw a first random sample of size n_1 from a lot, allocate r items to each of g_1 groups (or testers) so that $n_1 = rg_1$ and put them on test for the duration of t_0. Accept the lot if the number of failures from each of k_1 groups is less than or equal to c_{1a} ($k_1 \leq g_1$). Truncate the test and reject the lot as soon as the number of failures from each of all g_1 groups is larger than to c_{1r} before t_0. Otherwise, go to the second stage.

2. (Second stage) Draw a second random sample of size n_2 from a lot, allocate r items to each of g_2 groups so that $n_2 = rg_2$ and put them on test for time t_0. Accept the lot if the number of failures from each of k_2 groups is less than or equal to c_{2a} ($k_2 \leq g_2$). Otherwise, truncate the test and reject the lot.

The proposed plan is characterized by seven design parameters, namely k_1, k_2, g_1, g_2, c_{1a}, c_{1r} and c_{2a}. The lot acceptance probability at the first stage under the proposed two-stage sampling plan is given by

$$P_a^{(1)} = \sum_{j=k}^{g_1} \binom{g_1}{j} Q_a^{\,j}(1-Q_a)^{g_1-j} \tag{1}$$

where Q_a is the probability that c_{1a} or less failures are observed in a group, which can be expressed by

$$Q_a = \sum_{i=0}^{c_{1a}} \binom{r}{i} p^i (1-p)^{r-i} \tag{2}$$

Here, p is the probability that an item fails by time t_0, which is given by

$$p = F(t_0) \tag{3}$$

where F is the cumulative distribution function (cdf) of the underlying life time distribution.

The lot rejection probability at the first stage is given by

$$P_r^{(1)} = (Q_r)^{g_1} \tag{4}$$

where Q_r, the probability of rejection at the first stage, is given by

$$Q_r = 1 - \sum_{i=0}^{c_{1r}} \binom{r}{i} p^i (1-p)^{r-i} \tag{5}$$

Now, the lot acceptance probability from the second stage is

$$P_a^{(2)} = (1 - P_a^{(1)} - P_r^{(1)}) \sum_{j=k}^{g_2} \binom{g_2}{j} Q_{2a}^j (1-Q_{2a})^{g_2-j} \tag{6}$$

where Q_{2a} is the probability that c_{2a} or less failures are observed in a group at the second stage, which is defined by

$$Q_{2a} = \sum_{i=0}^{c_{2a}} \binom{r}{i} p^i (1-p)^{r-i} \tag{7}$$

Therefore, the lot acceptance probability for the proposed two-stage group sampling plan is given by

$$L(p) = P_a^{(1)} + P_a^{(2)} \tag{8}$$

It is very important to note that the proposed two stage plan is different from the two stage plan proposed by Aslam et al. (2010). Note that the proposed plan does not reduces to two stage plan given by Aslam et al. (2010) if $k_1 = g_1$ and $k_2 = g_2$. Note also that the rejection probabilities at the first stage in proposed two stage acceptance sampling plan are different from the two stage acceptance sampling plan given in the literature.

3. DESIGN PARAMETERS INDEXED BY ARL AND LTRL
AS UNRELIABILITY

A plan is considered as good if it minimizes the risks involved in the acceptance sampling scheme. The probability of rejecting a good lot is called the producer's risk, say α, and that of accepting a bad lot is called the consumer's risk, say β. We adopt the two-point method to determine the design parameters of the plan such that the lot acceptance probability must satisfy the specified producer's and consumer's risks simultaneously (see, e.g., Fertig and Mann, 1980). The producer wants the probability of accepting should be at least $1-\alpha$ at the acceptable quality level (AQL), say p_0, and the consumer desires the probability of acceptance should be at most β at the lot tolerance quality level (LTQL), p_1, say. We will find the design parameters that satisfy the following inequalities simultaneously.

$$L(p_0) = P_a^{(1)} + P_a^{(2)} \geq 1-\alpha \tag{9}$$

$$L(p_1) = P_a^{(1)} + P_a^{(2)} \leq \beta \tag{10}$$

The quantities Q_a, Q_r and Q_{2a} will be determined from Eqs. (2), (5) and (7), respectively, corresponding to the AQL and the LTQL.

The design parameters satisfying Eqs. (9) and (10) may not be unique so we will find the parameters by minimizing the average sample number (ASN). We prefer the ASN under the LTQL to AQL as in Balamurali et al. (2005) and Jun et al. (2006). The ASN under the LTQL is given by

$$ASN(p_1) = rg_1 + rg_2(1 - P_a^{(1)} - P_r^{(1)}) \tag{11}$$

Therefore, the optimization problem to be considered is as follows:

$$\text{Minimize } ASN(p_1) = rg_1 + rg_2(1 - P_a^{(1)} - P_r^{(1)}) \tag{12a}$$

subject to

$$L(p_0) = P_a^{(1)} + P_a^{(2)} \geq 1-\alpha \tag{12b}$$

$$L(p_1) = P_a^{(1)} + P_a^{(2)} \leq \beta \tag{12c}$$

$$k_1 \leq g_1 \tag{12d}$$

$$k_2 \leq g_2 \tag{12e}$$

$$c_{1a} \leq c_{1r} \tag{12f}$$

The design parameters were determined using selected values of the AQL and the LTQL and placed in Table 1 when the group size is 5 and in Table 2 when $r = 10$. It is assumed that $\alpha = 0.05$ and $\beta = 0.10$ throughout this paper unless specified otherwise. The ASN of the proposed two-stage acceptance plan as well as the probability of acceptance

$L(p_0)$ were reported in these tables. R program is available with the authors upon request. R algorithm is given in the end of Section 4.

Table 1:
Proposed Two-stage Group Sampling Plans Indexed by AQL and LTQL (r = 5)

(AQL)	(LTQL)	k_1	g_1	c_{1a}	c_{1r}	k_2	g_2	c_{2a}	ASN	$L(p_0)$
0.005	0.100	8	9	0	0	10	11	0	97	0.9994
	0.150	5	6	0	5	6	7	0	63	0.9998
0.01	0.100	8	9	0	0	10	11	0	97	0.9932
	0.200	4	5	0	0	4	5	0	46	0.9995
	0.300	2	3	0	0	3	4	0	22	0.9998
0.050	0.250	3	4	0	0	3	5	0	35	0.9798
	0.500	1	2	0	1	1	2	0	13	0.9969
0.100	0.500	1	2	0	1	2	3	1	14	0.9903

Table 2:
Proposed Two-stage Group Sampling Plans Indexed by AQL and LTQL (r = 10)

(AQL)	(LTQL)	k_1	g_1	c_{1a}	c_{1r}	k_2	g_2	c_{2a}	ASN	$L(p_0)$
0.005	0.100	4	5	0	9	5	6	0	107	0.9998
	0.150	3	4	0	0	3	4	0	62	0.9998
0.01	0.100	4	5	0	0	4	4	0	91	0.9943
	0.200	2	3	0	0	2	5	0	43	0.9991
	0.300	1	2	0	0	3	4	0	20	0.9909
0.050	0.250	2	4	0	0	1	3	0	46	0.9641
	0.500	1	2	1	1	3	17	4	20	0.9926
0.100	0.500	1	2	1	2	1	2	1	22	0.9906

From these two tables, we observe the following characteristics for the sampling plans.

1) When the AQL is fixed and the LTQL is large, all the design parameters and the ASN become small. In the same situation, there is no specific trend in $L(p_0)$.

2) When the AQL is fixed and the LTRL is large, there is no specific trend in $L(p_0)$.

3) The proposed two-stage plan is better than the single-stage plan for all values of AQL and LTQL.

Example 1:

Suppose that an experimenter wants to adopt the proposed group sampling plan when making a decision of accepting or rejecting the submitted lots of products. Multi-item testers with group size of 5 are used for the test. It would like to keep the consumer's risk below 10 percent if the unreliability is 0.200, whereas the producer's risk should be less than 5 percent when the unreliability is as low as 0.010. It is seen from Table 1 that

$k_1, g_1, c_{1a}, c_{1r}, k_2, g_2, c_{2a} = 4, 5, 0, 0, 4, 5, 0$ for the proposed group sampling plan, which

is implemented as follows: Take a sample of 20 items from a lot and allocate 5 items to each of 5 groups. Accept the lot if there are no failures in 4 groups out of 5 groups and reject the lot if more than 0 failures occur in the any of 4 groups. Otherwise, go to the

second stage. Draw a random sample of size 20 from a lot and allocate 5 items to each of 5 groups. Accept the lot if no failure occurs in 4 out of 5 groups, otherwise reject the lot.

4. DESIGN PARAMETERS FOR A WEIBULL DISTRIBUTION

The producers are always interested to improve the quality level of their products. The quality level can be expressed in terms of ratio of the unknown true average life to the specified average life, μ / μ_0, say. In this situation, the AQL and the LTQL can be expressed as multiples of the specified life. To develop the acceptance, the underlying lifetime distribution should be known. We consider as a lifetime model the Weibull distribution with given shape parameter m and unknown scale parameter λ. The probability of non-conforming or unreliability can be expressed with the cdf of the Weibull distribution, which is given by

$$F(t) = 1 - \exp(-(t / \lambda)^m), \ t \geq 0 \tag{12}$$

The mean life of the Weibull distributed items is given by

$$\mu = (\lambda / m)\Gamma(1 / m) \tag{13}$$

Then, the unreliability at time t_0 can be obtained from (3) as

$$p = 1 - \exp(-b^m (t_0 / \mu)^m) \tag{14}$$

where
$$b = \Gamma(1 / m) / m$$

It would be convenient to specify the termination time t_0 as a multiple of the specified life μ_0. That is, we will consider that $t_0 = a\mu_0$ for a constant a. In this way, the unreliability in (14) reduces to

$$p = 1 - \exp(-(ab)^m (\mu / \mu_0)^{-m}) \tag{15}$$

If r_0 is the AQL, expressed as the mean ratio at the producer's risk, and r_1 is the corresponding LTQL at the consumer's risk, the design parameters should be obtained by solving the following two inequalities:

$$L(p_0 \mid \mu / \mu_0 = r_0) \geq 1 - \alpha \tag{16}$$

$$L(p_1 \mid \mu / \mu_0 = r_1) \leq \beta \tag{17}$$

We will consider $r_1 = 1$ because the acceptance of a lot should indicate a true mean life greater than the specified life. We will assume various ratios for r_0.

Tables 3-6 show the optimal design parameters for the Weibull distribution with shape parameter m (= 1, 2), experiment termination ratios (a=0.5, 1.0) number of testers r (= 5, 10). The ASN and the probability of acceptance at p_0 for the two-stage group plans are also provided in these tables.

Table 3: Proposed two-stage group sampling plan when *r* = 10 and *a* = 0.5
for the Weibull distribution with *m* = 1

β	μ/μ_0	k_1	g_1	c_{1a}	c_{1r}	k_2	g_2	c_{2a}	ASN	$L(p_0)$
	2	2	4	2	2	4	5	3	60	0.9513
	4	1	2	1	2	2	3	3	27	0.9892
0.25	6	1	2	1	1	1	2	6	20	0.9648
	8	1	2	1	1	3	4	5	20	0.9857
	10	1	2	1	1	1	2	6	20	0.9932
	2	3	4	0	2	5	8	3	83	0.9510
	4	2	3	0	1	1	2	2	33	0.9552
0.10	6	1	2	1	1	1	4	9	20	0.9648
	8	1	2	1	1	2	3	5	20	0.9857
	10	1	2	1	1	3	4	9	20	0.9932
	2	3	4	2	2	5	12	2	103	0.9551
	4	2	3	1	1	3	4	2	35	0.9511
0.05	6	1	2	0	1	2	3	3	23	0.9647
	8	1	2	0	1	2	4	1	23	0.9845
	10	1	2	0	1	1	2	1	22	0.9922
	2	3	4	0	2	13	19	3	143	0.9545
	4	2	3	0	1	4	6	2	39	0.9501
0.01	6	2	3	1	1	3	4	1	35	0.9795
	8	2	3	1	1	1	2	0	33	0.9901
	10	2	3	1	1	2	4	1	35	0.9940

Note: the dash (-) shows that the parametric values are found to be high and (↑) shows the same values and shows the values does not satisfy the conditions.

Table 4: Proposed two-stage group sampling plan when *r* = 10 and *a* = 1.0
for the Weibull distribution with *m* = 1

β	μ/μ_0	k_1	g_1	c_{1a}	c_{1r}	k_2	g_2	c_{2a}	ASN	$L(p_0)$
	2	1	2	0	5	1	2	5	30	0.9525
	4	1	2	4	4	14	17	10	20	0.9976
0.25	6	1	2	2	2	3	19	0	20	0.9643
	8	1	2	2	2	16	19	9	20	0.9893
	10	1	2	2	2	4	20	6	20	0.9961
	2	2	3	4	4	2	6	4	46	0.9515
	4	1	2	3	3	1	2	8	20	0.9740
0.10	6	1	2	2	2	8	20	0	20	0.9643
	8	1	2	2	2	4	19	4	20	0.9893
	10	1	2	2	2	18	19	5	20	0.9961
	2	2	3	4	4	6	10	5	57	0.9541
	4	1	2	2	3	1	2	3	21	0.9708
0.05	6	1	2	2	2	8	18	3	20	0.9643
	8	1	2	2	2	17	20	3	20	0.9893
	10	1	2	2	2	10	18	1	20	0.9961
	2	2	3	1	4	7	12	5	68	0.9516
	4	1	2	1	3	3	4	5	23	0.9737
0.01	6	1	2	1	2	1	2	5	20	0.9643
	8	1	2	1	2	1	3	0	20	0.9531
	10	1	2	1	2	1	2	2	20	0.9959

Table 5: Proposed two-stage group sampling plan when *r* = 5 and *a* = 0.5
for the Exponential distribution with *m* = 2

β	μ/μ_0	k_1	g_1	c_{1a}	c_{1r}	k_2	g_2	c_{2a}	ASN	$L(p_0)$
	2	3	4	0	2	5	7	0	50	0.9625
	4	3	4	0	0	5	6	0	41	0.9991
0.25	6	3	4	0	0	4	5	0	38	0.9999
	8	3	4	0	0	4	5	0	38	0.9999
	10	3	4	0	0	4	5	0	37	0.9999
	2	4	5	0	0	9	13	0	79	0.9600
	4	4	5	0	0	5	6	0	50	0.9986
0.10	6	4	5	0	0	5	6	0	50	0.9999
	8	4	5	0	1	5	6	0	53	0.9999
	10	4	5	0	0	5	6	0	50	0.9999
	2	6	7	0	0	8	12	0	92	0.9536
	4	6	7	0	0	5	6	0	63	0.9972
0.05	6	5	6	0	0	6	7	0	62	0.9999
	8	5	6	0	0	6	7	0	62	0.9999
	10	5	6	0	0	5	6	0	63	0.9999
	4	7	8	0	2	8	9	0	85	0.9925
0.01	6	7	8	0	0	8	9	0	84	0.9996
	8	8	9	0	0	7	8	0	84	0.9999
	10	8	9	0	0	7	8	0	84	0.9999

Table 6: Proposed two-stage group sampling plan when *r* = 5 and *a* = 1.0
for the Exponential distribution with *m* = 2

β	μ/μ_0	k_1	g_1	c_{1a}	c_{1r}	k_2	g_2	c_{2a}	ASN	$L(p_0)$
	2	1	2	0	1	1	2	2	12	0.9514
	4	1	2	0	0	6	8	1	10	0.9526
0.25	6	1	2	0	0	7	19	0	10	0.9893
	8	1	2	0	0	16	17	4	10	0.9965
	10	1	2	0	0	3	5	3	10	0.9985
	2	1	2	0	1	2	6	1	16	0.9512
	4	1	2	0	0	15	16	1	10	0.9526
0.10	6	1	2	0	0	9	20	4	10	0.9893
	8	1	2	0	0	16	19	0	10	0.9965
	10	1	2	0	0	5	13	3	10	0.9985
	2	1	2	0	1	7	9	2	20	0.9502
	4	1	2	0	0	14	18	4	10	0.9893
0.05	6	1	2	0	0	11	17	5	10	0.9893
	8	1	2	0	0	8	11	3	10	0.9965
	10	1	2	0	0	5	9	5	10	0.9985
	2	2	3	0	1	4	7	1	27	0.9589
	4	2	3	0	0	1	2	0	16	0.9844
0.01	6	2	3	0	0	1	2	0	16	0.9985
	8	2	3	0	0	1	2	0	16	0.9997
	10	2	3	0	0	2	3	0	16	0.9999

Table 7: Proposed two-stage group sampling plan when *r* = 10 and *a* = 0.5 for the Weibull distribution with *m* = 2

β	μ/μ_0	k_1	g_1	c_{1a}	c_{1r}	k_2	g_2	c_{2a}	ASN	$L(p_0)$
	2	2	4	0	0	1	3	0	51	0.9690
	4	2	3	0	0	1	3	0	39	0.9984
0.25	6	2	3	0	0	1	5	0	46	0.9999
	8	2	3	0	0	1	2	0	36	0.9999
	10	2	3	0	0	3	6	1	49	0.9999
	2	3	4	0	0	8	10	1	84	0.9581
	4	2	3	0	0	2	4	0	42	0.9983
0.10	6	2	3	0	0	2	4	0	42	0.9998
	8	2	3	0	0	3	5	0	46	0.9999
	10	2	3	0	0	3	5	0	46	0.9999
	2	3	4	0	0	3	8	0	76	0.9568
	4	3	4	0	0	4	5	0	62	0.9927
0.05	6	3	5	0	0	3	4	0	70	0.9999
	8	5	6	0	0	2	3	0	78	0.9999
	10	3	4	0	0	4	7	0	71	0.9999
	2	4	5	0	0	8	9	0	80	0.9808
	4	3	4	0	0	8	9	0	80	0.9808
0.01	6	3	4	0	0	4	6	0	82	0.9998
	8	4	5	0	0	4	6	0	82	0.9999
	10	3	4	0	0	7	8	0	76	0.9999

Table 8: Proposed two-stage group sampling plan when *r* = 10 and *a* = 1.0 for the Weibull distribution with *m* = 2

β	μ/μ_0	k_1	g_1	c_{1a}	c_{1r}	k_2	g_2	c_{2a}	ASN	$L(p_0)$
	2	1	2	3	3	3	4	10	20	0.9926
	4	1	2	1	1	10	19	2	20	0.9936
0.25	6	1	2	1	1	14	20	3	20	0.9997
	8	1	2	1	1	13	18	6	20	0.9999
	10	1	2	1	1	2	18	2	20	0.9999
	2	1	2	2	3	1	3	1	24	0.9825
	4	1	2	1	1	6	19	5	20	0.9936
0.10	6	1	2	1	1	10	20	4	20	0.9997
	8	1	2	1	1	12	19	6	20	0.9999
	10	1	2	1	1	14	19	8	20	0.9999
	2	1	2	1	3	1	2	2	24	0.9727
	4	1	2	1	1	2	18	1	20	0.9936
0.05	6	1	2	1	1	9	19	3	20	0.9997
	8	1	2	1	1	9	20	4	20	0.9999
	10	1	2	1	1	14	20	0	20	0.9999
	2	2	3	2	2	1	3	2	33	0.9803
	4	1	2	0	1	1	2	4	20	0.9936
0.01	6	1	2	0	0	5	6	3	20	0.9867
	8	1	2	0	0	5	6	3	20	0.9867
	10	1	2	0	0	4	11	10	20	0.9943

We noted that as mean ratio is increased, the values of the design parameters are decreased and this trend is true for all the shape parameter values of the Weibull distribution. The ASN is decreased for the same values of a and r when the shape parameter is increased.

Algorithm:
Basic Steps Involved In Simulating Average Sample Number (ASN)

Input:
Let D be the data, obtained from "ns" number of bootstrap samples from the given range.
- for *(i in 1:J)* {
- Create "ns" number of bootstrap samples at each iteration.
- Computation of various probabilities involved in the estimation of ASN.
- Measurement and selection of LP0 & LP1, where (LP0 $\geq$ 0.95 and LP1 $\leq$ β).
- Measurement and selection of ASN that is minimum i.e. ASN_min.
- ASN[i] = ASN_min
- *}*

Output:
$ASN^* = minimum\ (ASN)$; it provides minimum value of ASN against the selected parameters.

For the simulation of Average Sample Number (ASN), **R** software has been implemented. We have chosen 10000 bootstrap samples at each iteration and repeated this process for 1000 times to search for the minimum value of ASN.

5. ADVANTAGES OF THE IMPROVED PLAN

The advantage of the proposed plan over the existing plan given in Aslam et al. (2010) is made in terms of ASN in Table 7. For comparison purposes, we considered the same values of the termination ratio, numbers of testers, quality levels and shape parameters. The ASN of the original two-stage plan is borrowed from Aslam et al. (2010).

Table 9:
Comparison of ASNs between Two Group Sampling Plans
with r=5 for Weibull having m=2

β	$\mu/\mu_0 = r_2$	a=0.5		a=1.0	
		Existing Plan	Proposed Plan	Existing Plan	Proposed Plan
0.25	2	60	50	14	12
	4	15	41	7	10
0.10	2	114	79	20	10
	4	21	50	7	16
0.05	2	678	92	26	20
	4	25	63	7	10
0.01	2	931	-	125	27
	4	143	85	16	16

(-) shows that plan parameters do not exist

We can see from Table 7 that there is no specific pattern in the probability of acceptance and design parameters of both plans. We can compare two plans in terms of ASN; see, for example, Aslam et al. (2010). It is clear that the plan proposed in this paper provides smaller ASN than the existing plan. So, the proposed plan is better than the existing plan in reducing the ASN for specified values of the quality level and existing plan provides less ASN at high quality level only.

The advantage of the proposed two stage plan over the single stage plan given in Aslam et al. (2010) is made in terms of ASN. For comparison purpose, we considered the same values of number of testers, AQL and LQL. It is clear from Table 10 the ASN from the proposed plan is less than the existing single stage sampling plan. existing plan provides less ASN at high quality level only. So, the proposed plan us better than single stage plan in term of ASN.

Table 10:
Comparison of proposed two stage Sampling Plans and Existing Single Stage Sampling Plan

p_0	p_1	r=5		r=10	
		Existing Plan	Proposed Plan (ASN)	Proposed Plan	Proposed Plan (ASN)
0.005	0.100	40	97	50	107
	0.150	30	63	40	62
0.01	0.100	140	97	80	91
	0.200	25	46	30	43
0.05	0.250	50	35	40	46
	0.500	10	13	10	20

Example 2

Suppose that a manufacturer adopts a two-stage group sampling plan for lot acceptance of a product when the lifetime of this product follows the Weibull distribution with shape parameter 2, the test duration is 500 hours, and 5 items can be equipped in each tester and tested simultaneously. The manufacturer wants to determine the number of groups (testers) needed for each stage of the sampling plan if consumer's risk of acceptance is 10% and consumer risk is 5%. If $\mu/\mu_0 = 2$, then from Table 5, the required plan is $k_1, g_1, c_{1a}, c_{1r}, k_2, g_2, c_{2a} = 3, 4, 0, 2, 5, 7, 0$. According to the proposed plan the experimenter needs a sample of size 20 from the first stage and accept the lot if no failure occurs before 500 hours in 3 groups out of 4 groups. Otherwise, go to the second stage. Draw a random sample of size 35 from a lot and allocate 5 items to each of 7 groups. Accept the lot if no failure occurs in 5 out of 7 groups, otherwise reject the lot. At the same values of μ/μ_0, shape parameter, the existing plan provides $g_1, c_1, g_2, c_2 = 8, 7, 1, 2$. So according to exist plan we need 40 items at the first stage and 35 items on the second stage. Further it requires number of failures from all the groups to reach the same decision about the submitted product as in proposed plan.

6. CONCLUSION

A new two-stage group acceptance sampling plan is developed by considering the AQL and LTQL. The design parameters are found satisfying the producer's and the consumer's risks at the same time. A case is considered in which the underlying distribution is Weibull with known shape parameter. It is found that the proposed plan is better than the existing plan. Propose plan is more flexible than the existing plans to reach on the same decision. To design the plan, the mean life of the Weibull distribution is used as the quality parameter of the product. Even in skewed distribution such as the Weibull distribution median perform better than the mean. But, in practice, experimenter would like to use the mean life as the quality parameter instead of the median life because mean is best indicator of central measure than the mean. However, the proposed plan is flexible and can be used for the median life easily. For future research, one can take another statistical distribution such as the generalized exponential distribution, gamma etc. to develop the improved group plans using the mean and median life at the same time.

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WAVELET CHARACTERIZATION OF PARAMETRIC TURBULENCE IN THE IONOSPHERE OF PAKISTAN

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ABSTRACT

The influence of solar activity on daily communication system of ionosphere turbulence such as perturbation concentration and parcel velocity were analyzed in upper and lower atmospheric layers of ionosphere by spectral analysis, Fast Fourier Transformation and wavelet transformation in one dimension Haar 5-Leves for approximation and detail. Wavelet analysis is a method in mathematical analysis to study non stationary data. Wavelet analysis is used to quantify both signal of variability non stationary unification between ionospheric turbulence and solar activity. We presented these result under changing ionosphere condition.

KEYWORDS

Ionosphere; Turbulence; Perturbation; Spectral analysis; Wavelet; Haar; Fast Fourier Transformation.

INTRODUCTION

The continue flow of solar wind interacts with geomagnetic field to create the magnetosphere. Ionosphere is part of magnetosphere, and is created by the ionization of neutral particles. Ionospheric perturbations can be enumerated and life time fluctuations could be estimate and we can investigate that monitoring of this important region of our globe is as critical for progress in the future as it was in the past. Turbulence means unstable and disorderly movements and the ultimate result of current of air, water and moving with circular motion against the main current, it is rotatory motion state of different size and shape. Earth is unique but not independent. Earth science and space science are linked together in a particular in a particularly close and natural way. The ionosphere lies in the region of upper atmosphere of the earth. The ionosphere changes greatly from hour to hour, day to day, day to night and so on. The structure of the ionosphere differs from altitude to different layers. Primarily, this difference is in the density of ions and electrons. At the base of ionosphere, about 80 Km above the surface of the earth, there are about 10^4 ions in each cubic cm. In comparison, at this point in the atmosphere these are about 2×10^{16} non ionized or neutral particles per cubic cm , thus at 80 Km above the earth, the ratio of neutral particles to ions is about 2×10^{12} : 1. The influences of solar activity on ionosphere such as plasma turbulence were analyzed by

spectral analysis and wavelet analysis one dimensional, Wavelet Characterization of Parametric Turbulence in the Ionosphereic region at Pakistan Air Space by powerful multi-resolution technique. This is a new approach of exploring, ionospheric turbulence at Pakistan air space. [2-11]

MATERIAL AND METHODS

The study was based on the analysis of ionosphere turbulence at Pakistan air space such as perturbation concentration and parcel velocity were analyzed in upper and lower atmospheric layers of ionosphere by spectral analysis, Fast Fourier Transformation and wavelet transformation one dimension Haar 5-Levels for approximation and detail. [2]

COMPUTATION OF TURBULENCE FLUX

We have computed Turbulence flux Ionospheric layer at Pakistan region, by using perturbed electron concentration ($N^{/}$), and perturbed parcel velocity ($V^{/}$). Turbulent flow varies randomly with time at a location as shown in Fig. 1, thus turbulent flow is unsteady. The product $\overline{U^{/}N^{/}}$ represents the transport of Kinematics Turbulent Flux:

$$\overline{U^{/}N^{/}} = \frac{U_E^{/}N_E^{/} + U_{F2}^{/}N_{F2}^{/}}{2}$$

$U_{F2}^{/}$ An Instantaneous and Local Perturbation Scalar Velocity content

$U_E^{/}$ An Instantaneous and Local Perturbation Scalar Velocity content

$N_{F2}^{/}$ An Instantaneous and Local Perturbation Plasma Concentration content

$N_E^{/}$ An Instantaneous and Local Perturbation Plasma Concentration content [14]

RESULTS AND DISCUSSION
SPECTRAL ANALYSIS

The ionospheric turbulence clearly appears to follow a cyclical pattern as shown in Fig. 1, the line spectrum on periodogram Fig. 2 constructed to identify the randomness in the ionospheric turbulence due to influences of solar activity and other factors. Fig. 3. Illustrate graph spectral density and frequency for ionospheric turbulence F_2 data. Fig. 4, Predicted value vs. observed value of fitting model for turbulence and electron concentration at ionospheric F_2 layer. It is in fact descried a seasonality among the time series event. The purpose of spectral analysis is to present Ionosphere that examines data series in term of their frequency content. Frequency domain analysis has become much more central to our ability to decode the cause and effect of upper atmospheric changes. Fast Fourier Transformation techniques further aided the application of frequency domain analysis in upper atmosphere. Review the result shows the five largest peak frequencies are five largest periodogram peaks. The large peaks in which identified the periodogram. The number of observations 365, number of observations after padding: 364 and Transformations: Mean = 2.4458 subtracted. Largest peak frequencies corresponding value

Value:	Frequency
41.21:	0.0055
19.28:	0.0330
07.64:	0.0302
05.19:	0.0357
04.01:	0.3510

FOURIER ANALYSIS

We have presented the fundamental idea of the signals then explain for ionospheric turbulence as shown in table 1, Fig. 3, Frequencies, Periods, coefficient Cosine, coefficient Sin, Periodogram, Density, Hamming Weight. A goal of time series analysis in the frequency domain is to reliably separate, periodic oscillation from the random and a-periodic fluctuations. Fourier analysis is one of the famous methods for identifying periodic components. Fourier series, $f(t) = \overline{f(t)} + \sum_P [A_P Cos(\omega_P t) + B_P Sin(\omega_P t)]$ In

which $\overline{f(t)}$ is the mean value of the function, A_P, B_P are the coefficient of trigonometric function of Sin ($(\omega_P t)$ and Cos ($\omega_P t$) and the angular frequency ω_P are integer p = 1, 2, 3, ….., multiples of the total length of the time series. This concept is central to spectral analysis techniques, the collection of Fourier coefficient having amplitude A_P, B_P, form a period –gram.

WAVELET ANALYSIS

We consider ionospheric turbulence data at Pakistan air space is presented in detail in Fig. 5 and descriptive statistics shows in Table: 2 and 3 histogram and cumulative histogram of ionospheric turbulence at Pakistan air space in Fig. 6, 7, and 8. Wavelets are a recently developed signal processing tool enabling the analysis on several timescale of the local properties of complex signals that can present non stationary zone. [1]

Wavelet analysis is a powerful tool to find the mode of variations and also to study how it varies with time by decomposing time series into frequency space. Finally, powerful multi resolution with respect to Haar carried out. This is decomposes actual signal into different signals to be analyzed into principal and residual part. By mathematical

$$s = A_j + \sum_j D_j$$

S, A_j and D_j are the signal, principal part j level and residual part j level the relation between principal and residual part j level by mathematical

$$A_{j-1} = A_j + D_j$$

The wavelet used in the study by Haar of the level 5, the dyadic scale is $a = 2^j$ for level 5 the resolution is given by 1/a, or 2^{-j}.

In order to next approach the cyclic study maximum and minimum values, we carried out PMT analysis of ionospheric turbulence, Fig. 1 the variation is presented different resolution at level 5 of Haar in the detailed and approximated part the cyclic variation is also present at different level in the detailed and approximated part at the lowest resolution several peaks appear. The Haar analyzing wavelet in continuous time series function

$$\{1; 0 \leq t < \frac{1}{2}t_0$$

$$f(t) = \{-1; \frac{1}{2}t_0 \leq t < t_0$$

$$\{0; otherwise,$$

The corresponding wavelet in discrete time series

$$\{1; n = 1$$

$$h_n = h_0\{-1; n = 2$$

$$\{0; otherwise$$

The quantity $h_0 = 1/\sqrt{2}$ is a constant [2] Fig. 5 shows wavelet 1-D analysis Haar of ionospheric turbulence at Pakistan air space, present data is non linear there is local minimum in this period show a positive skewness of frequency histogram

$$s = a_5 + d_5 + d_4 + d_3 + d_2 + d_1$$

The advance and more recent analysis of ionospheric turbulence at Pakistan air space, between 1 to 365 components data point. We have constructed model ionospheric turbulence data. We used wavelet transformation becomes a common method of analyzing plasma turbulence.[1, 2]

CONCLUSION

In this paper we proposed an idea to find out the hidden prediction in different parameters quantifying for the ionospheric turbulent were analyzed in upper and lower atmospheric layers of ionosphere by spectral analysis, FFT and wavelet transformation one dimension Haar 5-Levels for approximation and detail at Pakistan air space. The more precise results obtained using hypothesis test for randomness and normality in data. These results might be useful in astrophysical plasma especially in upper atmospheric plasma. [12, 13]

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Plot of variable: turb*10^12
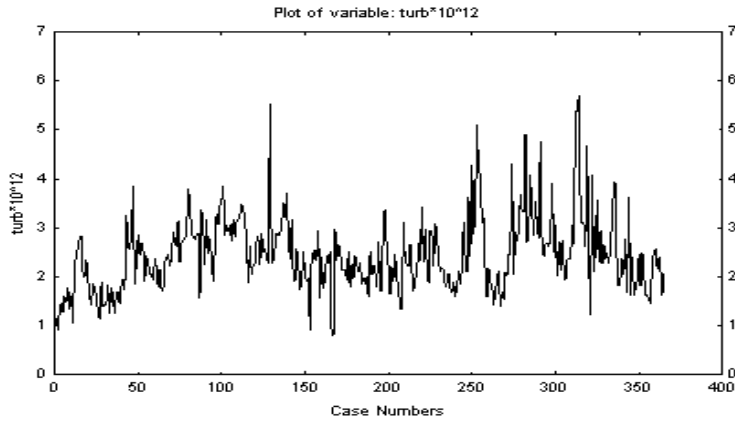
Fig.1: The graph of time series for ionospheric turbulence F_2 layer.
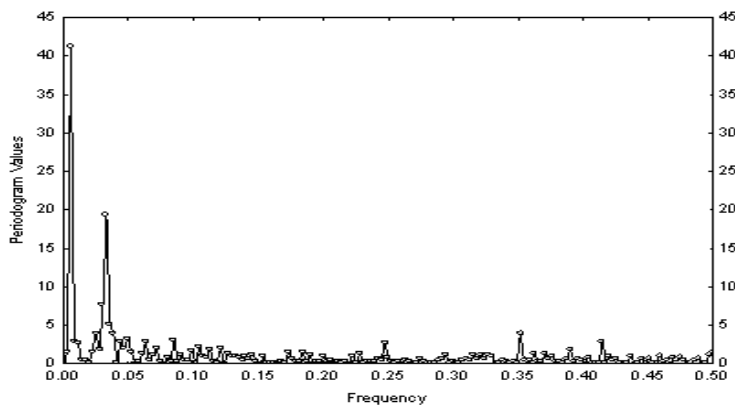
Fig.2:   Illustrate graph periodogram value and frequency for
           ionospheric turbulence F_2 data
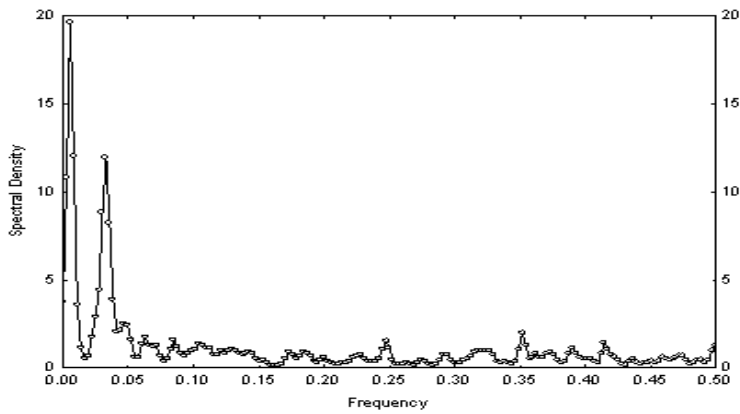
Fig.3: Illustrate graph spectral density and frequency for ionospheric
         turbulence F2 data

Fig.4. Predicted value vs. observed value of fitting model for turbulence
         and electron concentration at ionospheric F2 layer.

(a)

Histogram

(b)
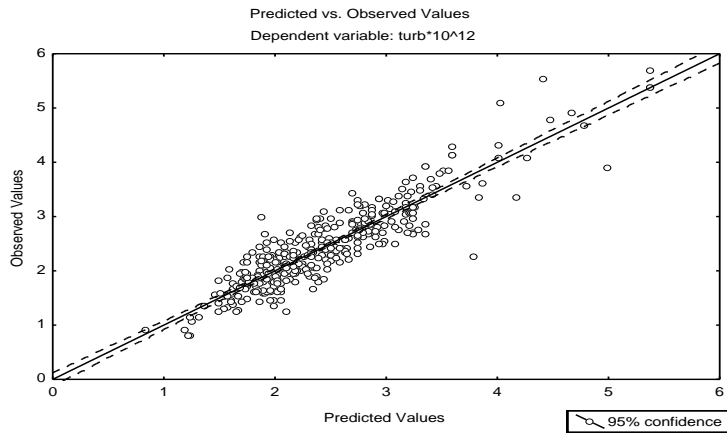
Cumulative histogram
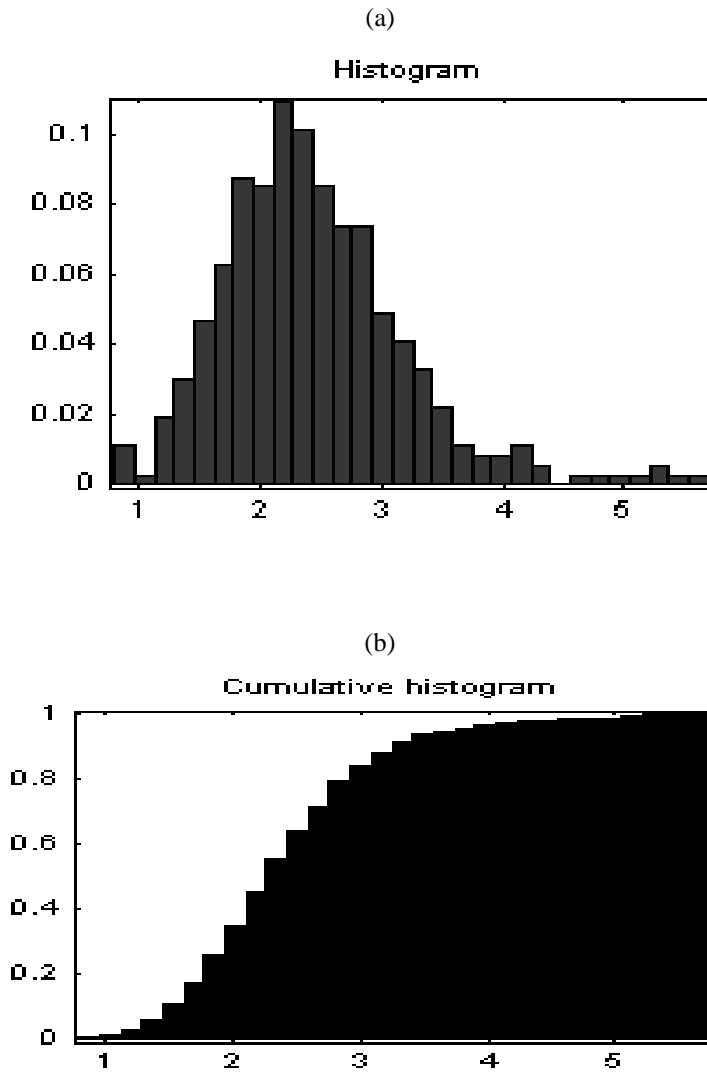
Fig.5: Descriptive statistics, (a) histogram and (b) cumulative histogram
        of ionospheric turbulence at Pakistan air space
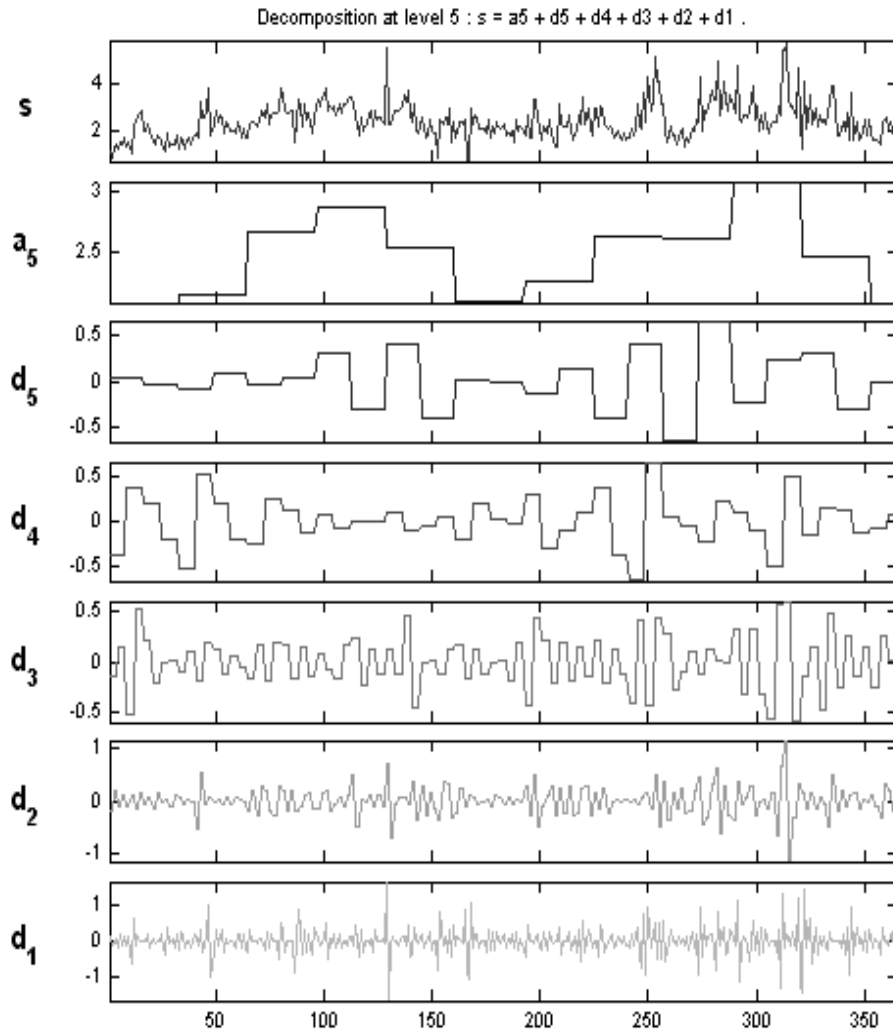
Fig.6: Decomposed at level 5, $s = a_5 + d_5 + d_4 + d_3 + d_2 + d_1$
of ionospheric turbulence at Pakistan air space
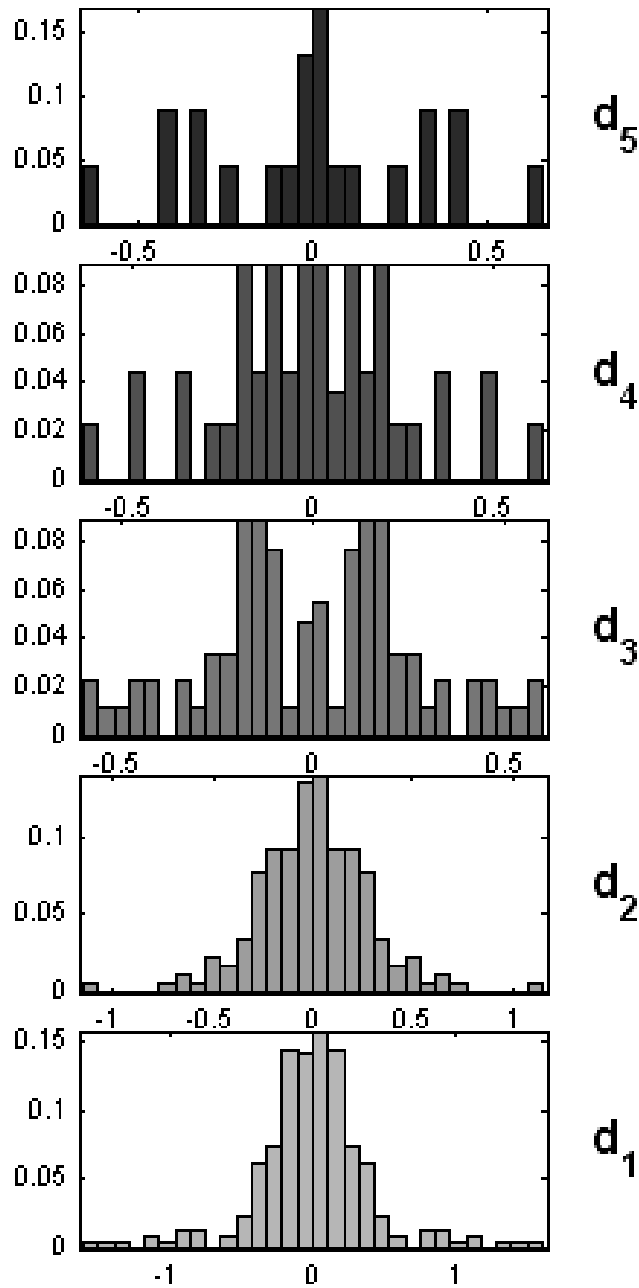
Fig.7:   Reconstructed details analyzed at level 5, residual part of
         ionospheric turbulence at Pakistan air space
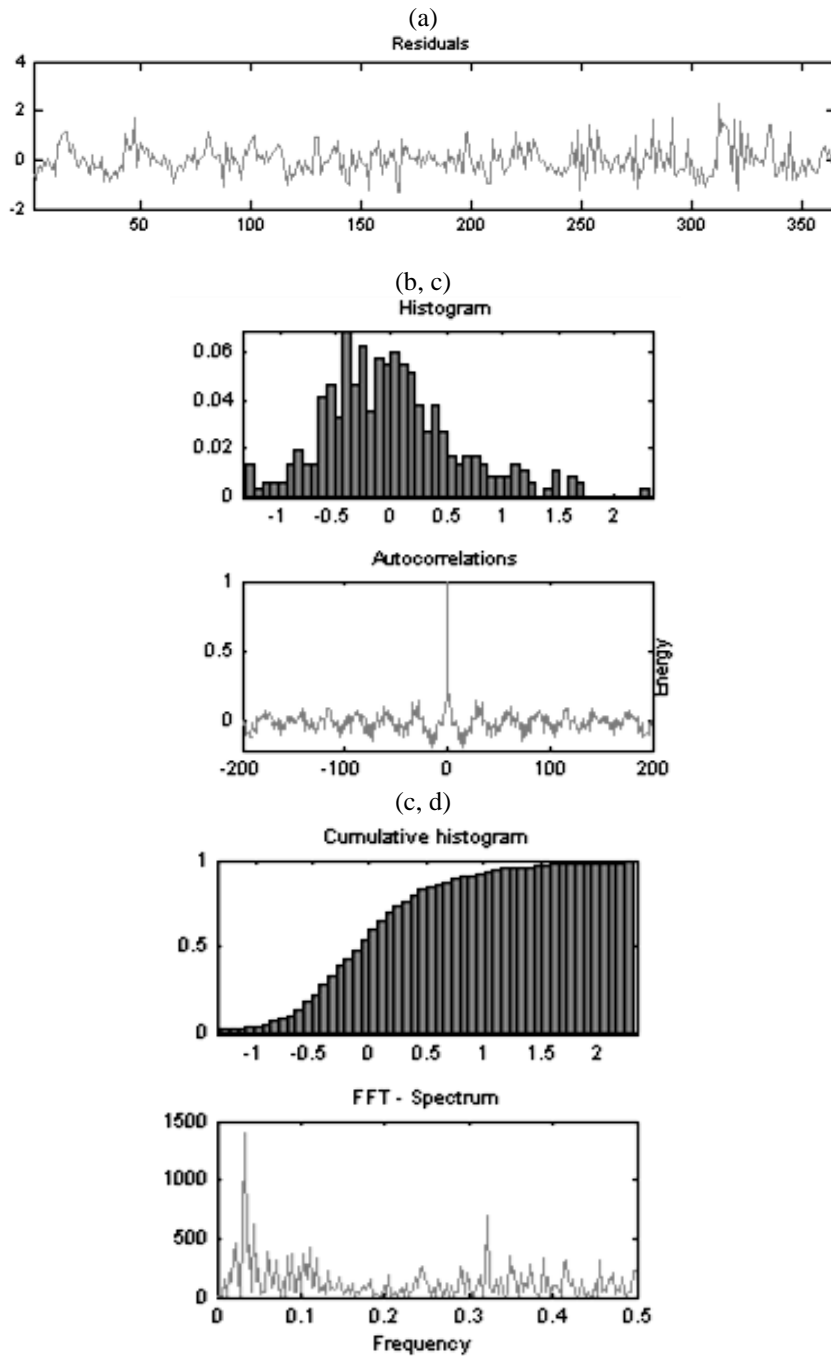
Fig. 8: The graph of residual, autocorrelation, FFT-spectrum, histogram, cumulative
histogram wavelet analysis for ionospheric turbulence F2 layer.

Table 1:
Illustrate for ionospheric turbulence as frequency (F), period (P), coefficient cosine and coefficient of sine, periodogram (Pgm), and density (Den)

F:	P:	Cos:	Sin:	Pgm:
0.000		0.003	0.000	0.002
0.002	364	-0.045	0.082	1.607
0.005	182	-0.437	-0.186	41.20
0.008	121	-0.088	-0.092	2.987
0.010	091	-0.071	0.099	2.734

Table 2:
Descriptive statistics, quantities of ionospheric turbulence at Pakistan air space

Mean:	2.5
Median:	2.3
Mode:	2.2
Max:	5.7
Min:	0.8
Range:	4.9
St. Dev.:	0.7
Median AD:	0.5
Mean AD:	0.6

Table 3:
Descriptive statistics, residual quantities of ionospheric turbulence at Pakistan air space

Mean:	-0.0005
Median:	-0.0519
Mode:	-0.4000
Max:	2.3060
Min:	-1.3000
Range:	3.6050
St. Dev.:	0.6000
Median AD:	0.3387
Mean AD:	0.4500

ON THE INVERSE GAUSSIAN RECORD VALUES
AND ASSOCIATED INFERENCE

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ABSTRACT

In this paper we discuss the record values arising from inverse Gaussian IG μ, λ distribution. We compute the means, variances and co-variances of the upper record values from the Inverse Gaussian. In the next phase, this paper proposes to determine the best linear unbiased estimators (BLUEs) for the mean parameter and shape parameter of inverse Gaussian distribution. We construct confidence intervals for the mean and shape parameters by using BLUEs. Finally, we discuss prediction for the future records.

1. INTRODUCTION

Record values and associated inference are of great interest and importance in several real-life data involving weather, economic, athletic and sports. Record values appear in many statistical applications. Chandler (1952) introduced the statistical study of record values. Ahsanullah (1980) has been discussed the BLUEs and prediction for the future record values from two parameter exponential distribution. Same work in this context has been done as, Balakrishnan and Chan (1993) for Rayleigh and Weibull distribution, on normal record values by Balakrishnan and Chan (1998), Balakrishnan, Ahsanullah and Chan (1995) for the logistic distribution, Dunsmore (1983), Sultan (2000) for uniform distribution, Sultan and Moshref (2000) from the generalized Pareto distribution, Sultan et al. (2002) for generalized power function distribution, Sultan et al. (2008) from Gamma distribution with three parameter, Sultan (2010) for the Inverse Weibull lifetime model.

If $X_1, X_2, \cdots$ be identically and independently distributed random variables with cumulative distribution function F(x). Let $Y_n = \max(\min)$ $X_1, \cdots X_n$, $n \geq 1$. We say that X_j is an upper (lower) record value of $X_n, n \geq 1$, if $Y_j > < Y_{j-1}, j > 1$. By definition, X_1 is an upper record value as well as lower record value. One can go from lower records to upper records by replacing the original sequence of random variables by $-X_j, j \geq 1$ or if P $X_j > 0$ by $1/X_i, i \geq$. We will say that the upper record values as record values unless it is mentioned.

Schrodinger (1915) gave the first passage time distribution of Brownian motion with positive drift. Tweedie (1957) suggested the name "inverse Gaussian" because of the inverse relationship between cumulant generating function of the first passage time distribution and that of the normal distribution. Folks and Chhikara (1978) gave e

detailed review of the characteristics properties and application of inverse Gaussian distribution. The two parameter family of inverse Gaussian distributions is considered a population model for a wide range of applications. The probability function is given by;

$$f\left(x;\mu,\lambda\right)=\sqrt{\frac{\lambda}{2\pi x^3}}\exp\left[-\lambda\left(x-\mu\right)^2\big/2\mu^2 x\right], \qquad x>0,\mu>0,\lambda>0$$

where μ is scale parameter and λ/μ is shape parameter and its cdf;

$$F\left(x\right)=\Phi\left[-\sqrt{\lambda/x}\left(1-x/\mu\right)\right]+e^{2\lambda/\mu}\Phi\left[-\sqrt{\lambda/x}\left(1+x\right)\right]$$

where $\Phi\left(z\right)$ is the cdf of standard normal distribution function;

$$\Phi\left(z\right)=\frac{1}{2}\left[1+erf\left(\frac{z}{\sqrt{2}}\right)\right]$$

Inverse Gaussian distribution has a wide range of applications, such as, biology, medicine, economics, reliability and life testing. Areas of applications have also been included length of strikes, sanatorium stays, employee service time, equipment functional life-span etc. IGD has been used to describe the time for a pool to empty, to model the time to failure of a device, to model the distribution of strikes. But no work has been done on inverse Gaussian distribution from record values. In next section we will discuss inverse Gaussian distribution on upper record values.

2. INVERSE GAUSSIAN RECORD VALUES (IGR)

Let $X_{U\,1},X_{U\,2},\cdots,X_{U\,n}$ denote the upper record values arising from a sequence of X_i of i.i.d $IG\left(\mu,\lambda\right)$ variables. Then the probability density function of the nth upper record value $X_{U\,n}$ is given by;

$$f_n\left(x\right)=\frac{1}{\Gamma n}\left[-\ln\left(\Phi\left(a_x\right)-e^{2\lambda/\mu}\Phi\left(b_x\right)\right)\right]^{n-1}f\left(x\right), \quad 0\leq x\leq\infty,\ n=1,2,\cdots \quad (2.1)$$

where

$$a_x=\sqrt{\frac{\lambda}{x}}\left(1-\frac{x}{\mu}\right), \qquad b_x=-\sqrt{\frac{\lambda}{x}}\left(1+\frac{x}{\mu}\right)$$

$$f\left(x\right)=\sqrt{\frac{\lambda}{2\pi x^3}}\exp\left[-\lambda\left(x-\mu\right)^2\big/2\mu^2 x\right], \quad x>0, \quad \lambda>0,\mu>0 \qquad (2.2)$$

and the joint pdf of $X_{U\,n}$ and $X_{U\,m}$ is given by;

$$f_{m,n}\left(x,y\right)=\frac{1}{\Gamma m\,\Gamma\left(n-m\right)}\left[-\ln\left(\Phi\left(a_x\right)-e^{2\lambda/\mu}\Phi\left(b_x\right)\right)\right]^{m-1}$$

$$\left[\ln\left(\Phi\left(a_x\right)-e^{2\lambda/\mu}\Phi\left(b_x\right)\right)-\ln\left(\Phi\left(a_x\right)-e^{2\lambda/\mu}\Phi\left(b_x\right)\right)\right]^{n-m-1}$$

$$f\left(x\right)f\left(y\right)\Big/\left[\Phi\left(a_x\right)-e^{2\lambda/\mu}\Phi\left(b_x\right)\right] \qquad (2.3)$$

If $X_{U\,1}, X_{U\,2}, \cdots, X_{U\,n}$ be the first n upper record values from the inverse Gaussian distribution. Then the single and double moments of the upper record values are given by;

$$\mu_n^r = E\left(X^r\right) = \int_0^\infty x^r f_n(x)\,dx \qquad , r = 0,1,2,3\cdots \tag{2.4}$$

and

$$\mu_{m,n}^{r,s} = \int_0^\infty \int_0^y x^r y^r f_{m,n}(x,y)\,dxdy \qquad , 0 \leq x \leq y \leq \infty, \tag{2.5}$$

$r = 0,1,2,\cdots$

$s = 0,1,2,\cdots$

$m = 1,2,3\cdots$

$m < n,$

where $f_n(x)$ and $f_{m,n}(x,y)$ are given in eq. (2.1) and (2.3) respectively. For n=1 it becomes inverse Gaussian distribution. For n or m greater than one it is difficult to derive exact formulas for the single and double moments, so we can use some numerical techniques to find these moments for different values of n and m. In section 3, we will discuss the BLUEs of scale and shape parameters of IGR, based on these numerical results of moments.

3. BLUES AND INFERENCE

If $Y_{U\,1}, Y_{U\,2}, \cdots, Y_{U\,n}$ be the first n upper record values from the $IG(\mu, \lambda)$. BLUEs of μ and λ can be derived by using the following least squares approach as, (Balakrishnan and Cohen, 1991)

$$\mu^* = \left[\frac{\mu'\Sigma^{-1}\mu 1'\Sigma^{-1} - \mu'\Sigma^{-1}1\mu'\Sigma^{-1}}{\mu'\Sigma^{-1}\mu\ 1'\Sigma^{-1}1 - \left(\mu'\Sigma^{-1}1\right)^2} \right] Y = \sum_{i=1}^n a_i Y_{U\,i} \tag{3.1}$$

$$\lambda^* = \left[\frac{1'\Sigma^{-1}1\mu'\Sigma^{-1} - 1'\Sigma^{-1}\mu 1'\Sigma^{-1}}{\mu'\Sigma^{-1}\mu\ 1'\Sigma^{-1}1 - \left(\mu'\Sigma^{-1}1\right)^2} \right] Y = \sum_{i=1}^n b_i Y_{U\,i} \tag{3.2}$$

with

$Y = \left(Y_{U\,1}, Y_{U\,2}, \cdots, Y_{U\,n}\right)',$

$\mu = \left(\mu_1, \mu_2, \cdots, \mu_n\right)',$

$1 = \underbrace{\left(1, 1, \cdots, 1\right)}_{n}',$

and

$$\Sigma = \begin{bmatrix} \sigma_{i,j} \end{bmatrix}, \qquad \sigma_{i,j} = Cov\begin{pmatrix} X_{U_i}, X_{U_j} \end{pmatrix}, \qquad 1 \le i, \quad j \le n$$

Now the variances and covariance of these BLUEs can be computed as given in (Balakrishnan and Cohen, 1991) as,

$$Var\left(\mu^*\right) = \sigma^2 \left[\frac{\mu' \Sigma^{-1} \mu}{\mu' \Sigma^{-1} \mu \left(1' \Sigma^{-1} 1\right) - \left(\mu' \Sigma^{-1} 1\right)^2} \right] = \sigma^2 V_1 \tag{3.3}$$

$$Var\left(\lambda^*\right) = \sigma^2 \left[\frac{1' \Sigma^{-1} 1}{\mu' \Sigma^{-1} \mu \left(1' \Sigma^{-1} 1\right) - \left(\mu' \Sigma^{-1} 1\right)^2} \right] = \sigma^2 V_2 \tag{3.4}$$

$$Cov\left(\mu^*, \lambda^*\right) = \sigma^2 \left[\frac{-\mu' \Sigma^{-1} 1}{\mu' \Sigma^{-1} \mu \left(1' \Sigma^{-1} 1\right) - \left(\mu' \Sigma^{-1} 1\right)^2} \right] = \sigma^2 V_3 \tag{3.5}$$

We can determine the coefficients of BLUEs of μ and λ from (3.1) and (3.2). Then variances and covariance of the BLUEs (μ^* and λ^*) can be computed by using the means, variances and covariance derived numerically. After doing these, we will move to CI. In section 4, we will construct confidence intervals (CI).

4. CONFIDENCE INTERVALS

We can construct the confidence intervals for the shape and scale parameter through pivotal quantities given in (Sultan, 2008) as,

$$R_1 = \frac{\mu^* - \mu}{\sigma \sqrt{V_1}}, \qquad R_2 = \frac{\lambda^* - \lambda}{\sigma \sqrt{V_2}}, \qquad R_3 = \frac{\mu^* - \mu}{\sigma^* \sqrt{V_1}} \tag{4.1}$$

Confidence interval for μ (when σ is known) is

$$P\left[\mu^* - \sigma \sqrt{V_1} R_1 \left(1 - \alpha/2\right) \le \mu \le \mu^* - \sigma \sqrt{V_1} R_1 \left(\alpha/2\right)\right] = 1 - \alpha. \tag{4.2}$$

Confidence intervals for μ (when σ is unknown) is

$$P\left[\mu^* - \sigma^* \sqrt{V_1} R_3 \left(1 - \alpha/2\right) \le \mu \le \mu^* - \sigma^* \sqrt{V_1} R_3 \left(\alpha/2\right)\right] = 1 - \alpha. \tag{4.3}$$

Confidence intervals for λ

$$P\left[\lambda^* - \sigma \sqrt{V_2} R_2 \left(1 - \alpha/2\right) \le \lambda \le \lambda^* - \sigma \sqrt{V_2} R_2 \left(\alpha/2\right)\right] = 1 - \alpha. \tag{4.4}$$

After constructing CI for BLUEs, in next section, we will discuss on IGR used to predict the future records. To predict the next future record we will use the BLUEs that has been derived in previous section.

5. PREDICTION FOR FUTURE RECORDS

If we have upper records $Y_{U\,1}, Y_{U\,2}, \cdots, Y_{U\,n-1}$ have been observed from inverse Gaussian distribution, our interest is to predict the next upper record $Y_{U\,n}$. So the best linear unbiased predicted value of the next record can be (Balakrishnan and Cohen, 1991),

$$Y^*_{U\,n} = \mu^* + \lambda^* \mu_n \qquad (5.1)$$

where the BLUEs μ^* and λ^* are based on the first (n-1) records and μ_n is the nth moment of the record values.

By taking an example with size n-1, n= 3,….., 7 we generate different records from the inverse Gaussian distribution. By using the BLUEs coefficients in (3.1) and (3.2) we can estimate μ^* and λ^*. Then by using these BLUEs and the single moment derived from (2.4) we can derive the point prediction for the nth record from (5.1).

6. CONCLUSION

In this paper we consider the upper record values from the inverse Gaussian distribution. For n and m greater than one it is difficult to derive exact formulas for single and double moments of inverse Gaussian upper record values, so some numerical techniques will be used to find these moments. Based on these numerical results we will compute BLUEs of IGR and we will compute variances and covariances of these BLUEs. Then we construct confidence intervals for the shape and scale parameter of IGR. We suggested techniques how we can find the future prediction. By using the BLUEs and numeric results of single derived from (2.4) we will find the next record. No work has been done on inverse Gaussian record values now we are working on it. Hopefully this will be a great contribution in the area of record values theory.

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IMPACT OF WORKING CAPITAL MANAGEMENT ON PAKISTANI FIRM'S PERFORMANCE

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ABSTRACT

Purpose – The objective of the study is to empirically examine the impact of working capital management on Pakistani manufacturing firm's performance.

Design/methodology/approach – The study uses a sample of randomly selected companies from three manufacturing sectors i.e. consumer goods, chemical and construction & material for the period of five years ranging from 2006 to 2010. The correlation and regression analysis are used to analyze the impact of working capital management on firms performance.

Findings – The study examines the impact of different components of working capital management on firm's performance. The study finds that the average collection period of accounts receivables, inventory conversion period and cash conversion cycle have strong negative relationship with firm's performance while the current ratio has positive relationship with operating profit. The study also finds that the firm size and current assets to total assets ratio has significant positive relationship with firm's performance.

Originality/value – Traditionally the corporate finance literature was focused on long term decision making. But the recent trends show the interest of researcher in working capital management. Most of the studies on working capital management are with reference to developed economies like USA but fewer are with reference to developing economies like Pakistan. This study will contribute to the literature by analyzing the impact of working capital management on firm's performance of Pakistani manufacturing firms and by validating the results of previous studies.

Implication – Findings indicates that finance manager can improve the firm performance by focusing on each component of working capital. They can improve the firm performance by reducing accounts receivable period, inventory conversion period and cash conversion cycle. In this study of Pakistani manufacturing firms we find average collection period as most crucial component of working capital. So the manger can add value to the firm by fastening the account receivable conversion period.

KEYWORDS

1. INTRODUCTION

Working capital management is the management of current assets like cash, cash equivalents, account receivables & stock in trade and current liabilities. It involves the decision how much to invest in current assets while keeping in mind principal of tradeoff between risk and return. It is similar to management of liquidity and theoretically it has negative relationship with firm's profitability. There are two dimensions to examine working capital management; static and dynamic view (Moss and Stine, 1993). Static view uses traditional liquidity ratios like current ratio as measure of firm's liquidity and it focuses on the firm's liquidity at certain given time. Dynamic view uses the cash conversion cycle to measures firm's liquidity which focuses at the firm's ongoing liquidity from its operations. It is the more popular yard stick of working capital.

Keown *et al.* (2003) describes cash conversion cycle as the sum of average collection period plus number of day's sales in inventory minus average payment period of accounts payable. It is negatively related to the firm's profitability. When the cash conversion cycle declines the firm's profitability increases because this is due to efficient utilization of working capital and vice versa. A shorter cash conversion cycle shows that firm is speeding its receivables and slowing down its payments and also managing its inventory efficiently. It can be reduced by shortening the inventory period through rapid manufacturing and selling of products or by reducing the collection period through speeding up the collection or stretching the payments by slowing down payments.

Efficient working capital management is speeding the cash inflows (receivables), slowing down the payments (payables) and by expediting inventory turnover. The firm with optimal level of working capital leads toward profitability and maximize the share holder's value. On the other hand investment in finished goods inventory and relaxed trade credit terms leads towards higher sales. Investment in the inventory saves the firm from the risk of stock out. Increased trade credit period encourage sales by allowing customer to judge quality before payment (Deloof and Jegers, 1996). While stretching payment period (accounts payables) allows the firm to confirm the quality of the purchased products. Cash conversion cycle is commonly used measure of working capital management which was introduced by (Richardes and Laughlin, 1980).

Working capital management plays a significant role in profitability of the firms because it has direct effect on the firm's profitability and liquidity (Rehman and Nasr, 2007) and has indirect effect on the firm's value (Gentry *et al.*, 1990). For maximization of the share holders value the overall corporate strategy must include element of efficient working capital management (Ganesan, 2007). So that efficient working capital management could maximize the share holder's value by increasing the free cash flows (Afza and Nazir, 2007). Vijayakumar (2011) recommends that companies can gain the competitive advantage by efficient management of working capital. So every component of working capital like cash, account receivable and stocks plays pivotal role in profitability of any company.

This study uses both dynamic and static views as measure of working capital management. Purpose of this study is to investigate the effect of working capital management on the profitability in Karachi Stock Exchange (KSE) listed companies. The study covers a sample of 32 randomly selected companies for the period of five years

covering 2006-2010. These companies have been chosen from three manufacturing sectors i.e. material & construction, chemical and consumer goods so as to maximize the generalizability of the study.

2. LITERATURE REVIEW

Many studies have been conducted on the relationship between working capital management and firm's profitability. Different researchers have analyzed the impact working capital management on profitability in different backgrounds and found that efficient working capital management has direct impact on profitability of firm. Following are some of the relevant studies that have focused on measuring the same relationship.

Most of the researchers have empirically proved that cash conversion cycle has significant negative relationship with the firm's profitability (Shine and Soenen, 1998; Eljelly, 2004; Padachi, 2006; Rehman and Nasr, 2007; Vishnani and Shah, 2007; Garcia-Teruel and Martinez-Solano, 2007: Uyar, 2009; Talha *et al.*, 2010 and Raheman *et al.,* 2010).

Deloff (2003) uses correlation and regression analyses and conclude that there is significant negative relationship between the average receivable collection period, days of sales in inventory & average payment period and firm's profitability (operating income). The author recommends that manager of the firm can create value for company shareholders by speeding the receivable collection period and inventory days up to reasonable extent.

Samiloglu and Demirgunes (2008) provide the empirical evidence about the working capital management impact on the profitability of firms in Turkey. Their results show the average collection period of accounts receivable, average conversion period of inventory and leverage affect the corporate profitability negatively but the sale growth has positive effect. They also conclude that cash conversion cycle, firm size and its total assets are statically insignificance.

Mohamad and Saad (2010) analyze the impact of working capital management on the market valuation and firms' profitability. The correlation and multiple regression tests are used to analyze this impact. A sample of 172 firms for a period of 2003-2007 listed on Bursa Malaysia is used for the study and the results show there is negative relationship between working capital management (measured as cash conversion cycle, current assets to current liability ratio and current liabilities to total asset ratios) and profitability (measured as Tobin Q as a proxy for market value, return on total asset ROA and return on equity ROI). They also reported positive and significant relationship between current assets to total assets ratio and firms performance.

Gill *et al*. (2010) examine the effect of working capital management on the profitability of the firms. This study uses a sample of 88 manufacturing firms listed on New York stock exchange for a period of 2005-2007. They find that the working capital management (as measured through cash conversion cycle) has significant impact on profitability of the firm. They further report that collection period of accounts receivables is most crucial component among other components of cash conversion cycle. They

report no significant relationship between the number of days inventory and firms profitability. Based upon the empirical evidences they suggest that the managers can generate value for share holders by accelerating the collection period.

Hayajneh and Yassin (2011) study the relationship between working capital management and the firm's performance by using correlation and regression test and find that there is significant negative relationship between average collection period of account receivables, inventory conversion period & over all cash conversion cycle and firm's performance. They report that there is positive relationship between sales size and growth of sales with firm's profitability. They suggest that the manager can manage the working capital efficiently by reducing number of day's accounts receivable through accelerating collection secondly by reducing the processing time of raw material conversion into finished product. So this will lead to reduction in cash conversion cycle.

Nobanee *et al.* (2011) examine the impact of cash conversion cycle on profitability of the firm. The study uses the generalized method of moment system estimation and a sample of 2123 Japanese non financial firms listed at Tokyo stock exchange for the time period of 1990-2004. The results show negative relationship between the cash conversion cycle and the firms return on equity except for consumer goods and service sector.

Ebben and Johnson (2011) analyze the impact of cash conversion cycle on the liquidity, invested capital and firm's performance using a sample of 833 small US retail firms. Empirical results show that cash conversion cycle has positive relationship with invested capital and it has negative relationship with firm's performance and its liquidity.

Generally the literature shows that managing working capital efficiently leads to higher profitability. It supports that working capital components have negative relationship with firm's profitability and firm's size has positive relationship with its profitability. This research will provide the empirical evidence of relationship between working capital management and firm's profitability from Pakistani manufacturing firms from three different industrial sectors.

3. THEORETICAL FRAMEWORK

Working capital investment is one of the important areas of corporate finance because the decision of working capital investment has a significant impact on the profitability and liquidity of the companies (Shine and Soenen, 1998). The firm has to make the decision about working capital management strategies based on the principal of trade off. There are two strategies about the working capital management one is aggressive and other is conservative. If the firm decides to invest less in the working capital this leads to higher profitability and the firm also has to face the greater risk in case of shortage in the stock, reduction in sale due to trade credit terms and losing the opportunity of discount from suppliers on early payments (Wang, 2002). The traditional belief of investing too much in working capital components leads to low profitability and high liquidity. But according to traditional belief high investment in inventory will save the firm from shortage and also from price fluctuations (Blinder and Maccini, 1991). Allowing the trade credit can increase the sale of firm's products at low demand period (Emery, 1987) through permitting the customers to ensure the quality and quantity of the products (Smith, 1987) that leads to long term relationship building with customers (Ng *et al.*,

1999). According to traditional belief paying earlier to supplier the firm can take benefit from early payment discount and also build the credit worthiness. But all these advantage are offset by decease in firm's profitability due to larger investment in current assets.

To check the efficiency of working capital management one of the popular measure is cash conversion cycle. Generally literature recommends the cash conversion cycle should be as low as possible. Because shorter cash conversion cycle indicates firm is managing its cash flows efficiently. The positive outcome of cash conversion cycle indicates the need for borrowing for that specific number of days and waits for cash receiving from its customers. The negative amount of cash conversion cycle indicates the total number of days a company receive cash from customers before it has to make payment to its suppliers. So the negative results of cash conversion cycle show that firm does not need external financing to pay its suppliers (Hutchison *et al*., 2007).

This study will analyze the relationship between working capital management and firm's profitability. Here working capital management is measured through cash conversion cycle, current assets to total assets ratio and current ratio while firm's profitability is manifested by operating profit and gross profit. The study also examines the impact of each component of working capital on the firm's profitability and its components are average collection period, average payment period and average conversion period of inventory. At the same time two control variables have been introduced i.e. financial leverage and firm size.
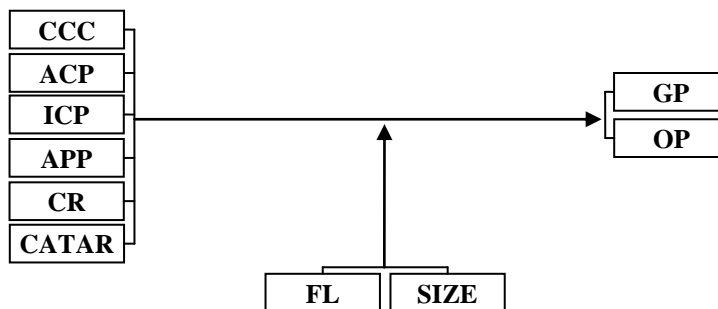
Figure 1: Model of the study

where CCC: cash conversion cycle; ACP: average collection period; ICP: inventory conversion period; APP: average payment period; CR: current ratio; CATAR: current assets to total assets ratio; FL: financial leverage; SIZE: firm size; GP: gross profit; OP: operating profit.

4. METHODOLOGY

4.1 Data and sample

To empirically examine the impact of working capital management on firm performance the study uses a sample of 32 manufacturing firms listed at Karachi Stock Exchange (KSE). These firms are from three different sectors i.e. construction & material, chemical and consumer goods. To analyze the impact this study uses five years data ranging from 2006-2010. Data is extracted from the publically available annual

reports of the firms. The annual reports are accessed through firm's websites. Firms in Financial sector are excluded from the sample because of their different nature of business.

4.2 Variables

The variables used in this study are supported by the literature and theory their explanation is given in the table 1.

Table 1: Description of the variables

Symbol	Variable	Description
Dependent variables		
OP	Operating Profit	Net profit + tax + finance cost
GP	Gross Profit	Sales - cost of goods sold
Independent variables		
ACP	Average Collection Period	(Account receivable/sales)*365
ICP	Inventory Conversion Period	(Inventory/cost of goods sold)*365
APP	Average Payment Period	(Accounts payables/cost of goods sold)*365
CCC	Cash Conversion Cycle	ACP+ICP-APP
CR	Current Ratio	Current assets/current liabilities
CATAR	Current Assets to Total Assets Ratio	Current assets/total assets
Control variables		
FL	Financial Leverage	Total debt/total assets
SIZE	Firm Size	Natural logarithm of total assets

4.3 Hypothesis

The purpose of this study is to examine the relationship between working capital management and performance of Pakistani manufacturing firms. Testable hypothesis of this study are:

Table 2: Hypothesis

No	Hypothesis
H_1	Companies with lower cash conversion cycle tend to have improved firm performance
H_2	Companies with lower collection period tend to have improved firm performance
H_3	Companies with lower inventory conversion period tend to have improved firm performance
H_4	Companies with higher accounts payable payment period tend to have improved firm performance

4.5 Model Specification

First the study applies Pearson correlation analysis to check the association among all dependent, independent and control variables. Secondly multiple regression analysis is used to examine the impact of working capital management on firm's profitability. So following eight regression models are developed:

Table 3: Regression Model

No	Regression Model
1	$OP_{i,t} = \beta_0 + \beta_1 (ACP)_{i,t} + \beta_2 (CR)_{i,t} + \beta_3 (CATAR)_{i,t} + \beta_4 (FL)_{i,t} + \beta_5 (SIZE)_{i,t} + e_{i,t}$
2	$GP_{i,t} = \beta_0 + \beta_1 (ACP)_{i,t} + \beta_2 (CR)_{i,t} + \beta_3 (CATAR)_{i,t} + \beta_4 (FL)_{i,t} + \beta_5 (SIZE)_{i,t} + e_{i,t}$
3	$OP_{i,t} = \beta_0 + \beta_1 (APP)_{i,t} + \beta_2 (CR)_{i,t} + \beta_3 (CATAR)_{i,t} + \beta_4 (FL)_{i,t} + \beta_5 (SIZE)_{i,t} + e_{i,t}$
4	$GP_{i,t} = \beta_0 + \beta_1 (APP)_{i,t} + \beta_2 (CR)_{i,t} + \beta_3 (CATAR)_{i,t} + \beta_4 (FL)_{i,t} + \beta_5 (SIZE)_{i,t} + e_{i,t}$
5	$OP_{i,t} = \beta_0 + \beta_1 (ICP)_{i,t} + \beta_2 (CR)_{i,t} + \beta_3 (CATAR)_{i,t} + \beta_4 (FL)_{i,t} + \beta_5 (SIZE)_{i,t} + e_{i,t}$
6	$GP_{i,t} = \beta_0 + \beta_1 (ICP)_{i,t} + \beta_2 (CR)_{i,t} + \beta_3 (CATAR)_{i,t} + \beta_4 (FL)_{i,t} + \beta_5 (SIZE)_{i,t} + e_{i,t}$
7	$OP_{i,t} = \beta_0 + \beta_1 (CCC)_{i,t} + \beta_2 (CR)_{i,t} + \beta_3 (CATAR)_{i,t} + \beta_4 (FL)_{i,t} + \beta_5 (SIZE)_{i,t} + e_{i,t}$
8	$GP_{i,t} = \beta_0 + \beta_1 (CCC)_{i,t} + \beta_2 (CR)_{i,t} + \beta_3 (CATAR)_{i,t} + \beta_4 (FL)_{i,t} + \beta_5 (SIZE)_{i,t} + e_{i,t}$

where

$(OP)_{i,t}$: operating profit of firm i in year t.

$(GP)_{i,t}$: gross profit of firm i in year t.

$(CCC)_{i,t}$: cash conversion cycle of firm i in year t.

$(ACP)_{i,t}$: average collection period of firm i in year t.

$(APP)_{i,t}$: average payment period of firm i in year t.

$(ICP)_{i,t}$: inventory conversion period of firm i in year t.

$(CR)_{i,t}$: current ratio of firm i in year t.

$(CATAR)_{i,t}$: current assets to total assets ratio of firm i in year t.

$(FL)_{i,t}$: financial leverage of firm i in year t.

$(SIZE)_{i,t}$: log natural of assets of firm i in year t.

5. RESULTS AND DISCUSSION

*5.1 Descriptive An*alysis

Table 4: Descriptive statistics of 160 firm's years of 32 companies for 2006-2010

Variable	Minimum	Maximum	Mean	Std. Deviation
ACP	0	93.54	21.88	20.89
APP	0.34	1141.73	38.3	104.14
ICP	1.28	377.24	69	59.11
CCC	-764.49	333.28	52.17	98.02
CR	0.17	7.13	1.14	0.73
CATAR	0.06	0.80	0.36	0.18
OP	-651	17400	1625.1	2979.99
GP	-515	20600	2252.7	3725.15
FL	.05	.94	0.63	0.18
SIZE	8.2	10.87	9.77	0.61

The descriptive analysis represents the minimum, maximum, average and standard deviation of the variables used in the study. The mean of the average collection period is 21.8 days with the standard deviation of 20.8 days. The minimum average payment period is 0.34 days and maximum average payment period are 1141.7 days. The mean value of average payment period is 38.3 days with standard deviation of 104.1 days. The

average inventory conversion period of the firms is 69 days with 59 days of standard deviation. The cash conversion cycle used to check the efficiency of working capital management has minimum value of -764.4 days and maximum of 333.2 days. The average cash conversion cycle is 52.1 days with 98 days of standard deviation. It shows that on average firms need external financing for 52.1 days. The average of operating profit of the firms used as a sample is 1625.1 million with standard deviation of 2979.99 million rupees. It shows that value of operating profitability can deviate from average to both sides by 2979.99 million rupees.

5.2 Pearson's Correlation Coefficient analysis

The study uses Pearson's correlation analysis to check the association between working capital components and firms performance. The table 2 shows the results of correlation coefficient between the variables.

Table 5: Correlation Matrix

	ACP	APP	ICP	CCC	CR	CATAR	OP	GP	FL	SIZE
ACP	1									
APP	-.140	1								
ICP	.313**	.532**	1							
CCC	.552**	-.770**	.107	1						
CR	-.103	-.096	-.011	.073	1					
CATAR	.433**	-.198*	.396**	.542**	.340**	1				
OP	**-.254****	-.025	**-.275****	**-.194***	**.227****	.056	1			
GP	**-.246****	-.041	**-.297****	**-.188***	**.061**	.088	.881**	1		
FL	.196*	.104	.147	.020	-.467**	-.144	-.064	-.069	1	
SIZE	**-.350****	.048	**-.434****	**-.388****	-.024	-.441**	**.565****	**.555****	.053	1

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

The correlation coefficient between ACP & OP is -0.254 and ACP & GP is -0.246 at 1% level of significance. That shows the firms with higher collection period will tend to exhibit low profitability. The correlation coefficient between ICP & OP is -0.275 and -0.297 between ICP and GP at significance level of 1%. It indicates that the firms whose inventory conversion period is low will enjoy higher profitability. The CCC shows negative coefficient of -0.194 and -0.188 with OP and GP respectively at 5% level of significance. That indicates the firms can increase the profitability by reducing the cash conversion cycle. The correlation coefficient also shows that firm size has positive and significant relationship with firm performance. The correlation coefficient reveals that SIZE has significant negative relationship with ICP, ACP and CCC. It indicates that larger firms are more efficient in working capital management as compared to smaller firms.

5.3 Regression analysis

Regression analysis has been applied to examine the impact of working capital management on firm's performance. To draw conclusion using regression analysis its certain assumption must be true (Berry, 1993). Before running the regression models these assumptions were checked. Linearity assumption was checked through scatter

diagram. Normality assumption was verified through normal probability plots of the residuals which show the data is normal. To diagnose the first order autocorrelation Durbin Watson (D-W) test was applied. Its value ranges from 1.63 to 1.95 which is closer to 2 in all regression models (see table 6). It shows regression model is appropriate. High correlation problem between the independent variables is verified through correlation matrix. The correlation coefficient between different independent variables remains below 0.8 which is below the harmful limit. The problem of multicollinearity is verified through variance inflationary factor (VIF) which remained below 2 in all models and tolerance which remained above 0.5. These results show there is no problem of multicollinearity between independent variables. So the regression analysis is suitable for this study and we don't need to go for weighted or generalized least square.

After checking the regression assumptions regression models are conducted on 160 firm's-years. The results of regression models are shown in table 6. The R^2 of regression models are (.476, .497, .450, .457, .463, .482, .466, and .482) which indicates that (47.6%, 49.7%, 45 %, 45.7%, 46.3%, 48.2%, 46.6%, and 48.2%) variation in dependent variable is explained by independent variables. The value of R^2 is much better than the study by the Rehman and Nasr (2007) who reported 32.8%, 33%, 32% and 30%.

Operating Profit = - 33630 – 27.38 (ACP) + 436.60 (CR)
$\qquad$ + 6720 (CATAR) + 5659 (FL) + 3331 (SIZE) + $e_{i,t}$ (Model 1)

Gross Profit =   - 42310 – 43.18 (ACP) – 815.80 (CR)
$\qquad$ + 11390 (CATAR) -821.70 (FL) + 4379 (SIZE) + $e_{i,t}$ (Model 2)

The results of the regression models 1 and 2 show negative coefficient for ACP at 99% level of confidence. It shows that average collection period has negative impact on operating profit and gross profit. It implies that companies can improve their profitability by reducing their collection period. Based on the regression results H_2 is accepted. Our results are aligned with Deloof, 2003; Raheman and Nasr, 2007; Gill *et al*. 2010; and Hayajneh and Yassin, 2011 who also reported inverse relationship between firms profitability and average collection period. The coefficient of current ratio is positive but insignificant in model 1 but it is negative and significant at 95% level of confidence in model 2. It indicates that current ratio has negative relationship with gross profit. It implies that companies can improve their profitability by reducing their current ratio. These results of current study confirm the results of previous studies conducted by Shine and Soenen, 1998; Raheman and Nasr, 2007; Talha *et al.*, 2010; and Mohamad and Saad, 2010. However, Sayuddzaman (2006) find positive relationship with firm performance. The coefficient of CATAR is positive and highly significant at 99% level of confidence in both models 1 and 2. It shows that companies having higher current assets to total assets ratio have higher profitability. So our results are consistent with the findings of Afza and Nazir, 2007; Raheman and Nasr, 2007; Nazir and Afza, 2009 and Mohamad and Saad, 2010. The regression results also show firm size has positive impact on firm's performance measured through OP and GP. It means larger firms are more profitable as compared to smaller firms. The study by Garcia-Teruel and Martinez-Solano, (2007) and Raheman and Nasr, (2007) also reported positive relationship between firm's size and profitability. The coefficient of financial leverage is positive in model 1 and negative in

model 2 but it is insignificant in both models. It shows that firm performance has no significant relationship with financial leverage.

Operating Profit = - 35210 + .51 (APP) + 569.10 (CR) + 5379 (CATAR)
                   + 103.70 (FL) + 3494 (SIZE) + ei,t   (Model 3)

Gross Profit =   - 44730 + .37 (APP) -605.90 (CR) + 9219 (CATAR)
                   - 1535 (FL) + 4633 (SIZE) + ei,t   (Model 4)

In regression model 3 and 4 the average collection period is replaced by average payment period and other variables remained same. The coefficient of APP is positive but it is insignificant in both models. The study sample represents no significant association between firm performance and average payment period so H_4 is rejected. The current study findings are consistent with the finding of Gill *et al.* (2011) who also reported positive and insignificant relationship between APP and firm's performance. The other variables are also significant in these models as in model 1 and 2. The coefficients of CATAR and SIZE represent positive and significant impact on firm performance as measured by GP and OP. The coefficient of CR represents negative and significant impact on GP at 90% level of confidence. But the coefficient of current ratio is positive and significant at 90% level of confidence in model 3. It shows increase in current ratio will lead to increase in firm performance measured through operating profit. The study by Sayuddzaman, (2006) also reports positive relationship between firm performance and current ratio. As in model 1 and 2 FL coefficient is insignificant in these models.

Operating Profit = - 33090 - 6.67 (ICP) + 554.80 (CR) + 5958 (CATAR)
                   + 434 (FL) + 3285 (SIZE) + ei,t   (Model 5)

Gross Profit =   - 41080 – 11.71 (ICP) – 632.10 (CR) + 10300 (CATAR)
                   -974.60 (FL) + 4271 (SIZE) + ei,t   (Model 6)

The coefficient of ICP is negative and significant at *p* value of 0.000 in model 5 and 6. It means inventory conversion period has negative and significant impact on GP and OP. which implies that companies can improve their profitability by shortening the inventory conversion period so H_3 is accepted. The study results confirm the findings of Garcia-Teruel and Martinez-Solano, 2007; Raheman and Nasr, 2007 and Hayajneh and Yassin, 2011. The other variables are also significant as they were in previous models. Firm size and current assets to total assets ratio are linked positively and significantly with firm's performance. The current ratio has negative and significant impact on gross profit and has positive relationship with operating profit. The regression results of model 5 and 6 also indicate insignificant relationship of financial leverage with firm's performance.

Operating Profit = - 34090 - 4.76 (CCC) + 526.20 (CR) + 6601 (CATAR)
                   + 253.60 (FL) + 3357 (SIZE) + ei,t   (Model 7)

Gross Profit =   - 43070 – 7.27 (CCC) – 672.30 (CR) + 11130 (CATAR)
                   - 1321 (FL) + 4427 (SIZE) + ei,t   (Model 8)

The coefficient of cash conversion cycle is negative and significant in both models. The CCC coefficient is significant at 95% level of confidence in model 7 and it is significant at 99% level of confidence in model 8. So coefficient results shows that the cash conversion cycle has negative relationship with firm's performance measured through gross profit and operating profit so H_1 is accepted. It implies that companies can improve their performance through shortening the cash conversion cycle. The result of the study confirms the findings of (Lancaster and Stevens, 1996; Shine and Soenen, 1998; Lazaridis and Tryfonidis, 2006; Garcia-Teruel and Martinez-Solano, 2007; Raheman and Nasr, 2007; Uyar, 2009; Mohamad and Saad, 2010; Gill *et al.*, 2010; Hayajneh and Yassin, 2011; Ebben and Johnson, 2011 and Nobanee *et al*., 2011). Other variables show same results as in previous models. The CATAR and SIZE has positive and significant impact on firm's performance. The coefficient of CR is negative and significant at 10% in model 8 but it is positive in model 7. It implies that firms with higher current ratio have to face low gross profit. These results are consistent with findings of (Shine and Soenen, 1998; Raheman and Nasr, 2007; Talha *et al.*, 2010; and Mohamad and Saad, 2010) who also reported negative relationship between CR and firm's performance. The results also show that current ratio has positive and significant relationship with firm's performance measured through operating profit. So this finding is consistent with the Sayuddzaman, (2006) who also reported same relationship. The FL also remained insignificant in these models.

6. CONCLUSION

The purpose of this study is to empirically examine the impact of working capital management on firm's performance. The study uses both static and dynamic views of working capital. Based on the results it can be concluded that companies can improve their performance by reducing collection period of accounts receivables, inventory conversion period and cash conversion cycle. The current ratio has positive relationship with operating profit and it has negative impact on gross profit. The firm size has positive relationship with firm performance. That indicates bigger firm enjoys more profit as compared to smaller firm. The results also show the current assets to total assets ratio has positive and significant impact on firm performance. It implies that firm performance will increase with the increase in current assets to total assets ratio. The study does not find significant relationship between financial leverage and firm performance. Overall the results are constant with findings of (Lancaster and Stevens, 1996; Shine and Soenen, 1998; Lazaridis and Tryfonidis, 2006; Garcia-Teruel and Martinez-Solano, 2007; Raheman and Nasr, 2007; Uyar, 2009; Mohamad and Saad, 2010; Gill *et al,* 2010; Hayajneh and Yassin, 2011; Ebben and Johnson, 2011 and Nobanee *et al.,* 2011).

Based on the results we suggest that the companies can distinguish themselves by managing the working capital efficiently. Results show the importance of working capital management because it has significant impact on firm's performance. The study recommends that financial mangers of Pakistani manufacturing firms can improve the performance of firms by reducing the collection period, fastening the inventory conversion period and decreasing the cash conversion cycle. So the mangers should efficiently manage the current assets and current liabilities. This study adds to the literature and it also validates the previous studies.

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Table 6: Results of the regression models 1-8

(32 firms from chemical, consumer goods and construction and material, 2006-2010: 160 firm year observations)

(Operating profit as dependent variable in Model 1,3,5,7 Gross Profit as dependent variable in Model 2, 4, 6, 8)

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
Constant	-33630***	-42310***	-35210***	-44730***	-33090***	-41080***	-34090***	-43070***
ACP(β)	-27.38***	-43.18***	-	-	-	-	-	-
APP(β)	-	-	0.51	0.37	-	-	-	-
ICP(β)	-	-	-	-	-6.67*	-11.71***	-	-
CCC(β)	-	-	-	-	-	-	-4.76**	-7.27***
CR(β)	436.60	-815.80**	569.10*	-605.90*	554.80*	-632.10*	526.20*	-672.30*
CATAR(β)	6720***	11390***	5379***	9219***	5958***	10300***	6601***	11130***
SIZE(β)	3331***	4379***	3494***	4633***	3285***	4271***	3357***	4427***
FL(β)	565.90	-821.70	103.70	-1535	434	-974.60	253.60	-13210
R²	0.476	0.497	0.450	0.457	0.463	0.482	0.466	0.482
Adjusted R²	.459	0.481	0.433	0.439	0.445	0.465	0.449	0.464
F value	27.946***	30.493***	25.241***	25.906***	26.526***	28.612***	26.922***	28.612***
D-W	1.95	1.65	1.92	1.64	1.89	1.63	1.92	1.65

ACP   average collection period;
APP   average payment period;
ICP   inventory conversion period;
CCC   cash conversion period;
CR   current ratio;
CATAR   current assets to total assets ratio;
SIZE   natural logarithm of total assets;
FL   financial leverage
***   at 1% level of significance;
**   at 5% level of significance;
*   at 10% level of significance. Beta coefficients of the variable are in millions.

THE IMPACT OF FOREIGN DIRECT INVESTMENT, EXPORTS AND REMITTANCES ON ECONOMIC GROWTH OF PAKISTAN

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ABSTRACT

This study has been undertaken to analyze the impact of Foreign Direct Investment, exports and remittances on economic growth of Pakistan. Secondary annual data for the period of 1986 to 2009 has been used for running multiple regression and partial correlation analysis. Results of the multiple regression analysis show that FDI, exports and remittances have a positive and significant effect on the GDP of Pakistan. Pakistan's capacity to progress depends on its ability to attract Foreign Direct Investment, and in boosting up its exports and remittances. FDI is the most important variable with reference to GDP growth in Pakistan. The results of partial correlation analysis have shown a moderate correlation between GDP, FDI and exports.

1. INTRODUCTION

For last two decades world has experienced rapid growth in economies. The importance of FDI in world GDP has become more considerable. The reason for the importance of FDI is the expansion of foreign ownership and the opportunity presented by FDI to developing countries. Developing countries like Pakistan usually face problem regarding saving investments creating a gap. FDI is the major tool filling up this gap by increasing production employment creation, transferring advanced technology and enhancing competition. This kind of steps encourages the liberty in policies of developing countries to attract FDI inflows (Hassan, 2009). Pakistan started market- based reforms in 1990s. This included factors like incentives for foreign investors, easing foreign exchange controls, credit facility, tariff reduction and lots of other. In the same tenure polices were improved for business environment, restriction on capital outflows and inflows were lifted. Gradually foreign investors were allowed to hold 100 percent of equity in projects related industries with repairable facilities. Moving towards the capital transactions, foreign borrowing was allowed to some extent (PIDE Working Papers 2011: 67).

When Pakistan was formed in 1947, the economy was totally relying in agriculture sector. Agricultural products were the only exports. For economy to be grown, diversification was required. So industries were installed. Many ways were created for export diversification and enhancement. Pakistan started appearing in international trade conferences .The export promotion bureau and export promotion council were organized to provide help and information to the potential foreign buyers (Arif, 2009). Pakistan's merchandise exports grew to 20.2 billion dollars in 2010-11 as compared to last year, 15.9 billion dollars. This shows the 27.8 percent of increase. Whereas the biggest share of

the year was of textile sector and food group, 61.8 percent and 18.1 percent respectively (Economic survey of Pakistan, 2010-11).

Remittances of Pakistan crossed the one billion dollar mark in a single month while March 2011 and remained over the one billion for second consecutive month in April 2011, which has boosted optimism about workers' remittances to cross the 11 billion dollars this year. Pakistan Remittance Initiative (PRI) has removed many irritants and inventiveness routing of remittances through former Channel (Economic survey of Pakistan, 2010-11). As compare to 2009, worker's remittances were totaled as 9.1 billion dollars in July-April (2010-2011) against 7.3 billion dollars. This shows that worker remittances have been increased by 1.8 billion dollars; thus improving 23.8 percent balance of the amount in the current account in July-April (2010-2011) (Economic survey of Pakistan, 2010-11).

Problem Statement

How FDI, exports and remittances affect the economic growth of Pakistan?

2. OBJECTIVES OF THE STUDY

The objectives of the study are to determine the effect of FDI, exports and remittances on GDP growth in Pakistan; to examine the correlation between dependent and independent variables and to explore the most important variable in explaining the variation in GDP growth of Pakistan.

3. METHODOLOGY

An analytical study is conducted to explore the dependence of GDP on FDI, exports and remittances in Pakistan. Specifically, in this study, the methods of multiple linear regressions and partial correlation analysis are employed with the help of hypothesis testing procedures, for analyzing the dependence of one variable on other variables. The tool or software used to run regressions is Statistical Package for Social Sciences (SPSS).

Data Description

The data used for this purpose is of secondary nature and has been taken from the web link of World Bank (WB) i.e. World Development Indicators (WDI) for the period of 1986-2009.

Model Specification

Based on the previous work on this topic the following growth model is established. The equation for regression analysis is given by:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + U_t. \tag{1}$$

where:

Y_t = GDP.

X_1 = Foreign Direct Investment.

X_2 =Exports.

X_3= Remittances.

β_0= Intercept term.

β_1= Regression coefficient with respect to foreign direct investment.

β_2= Regression coefficient with respect to exports.

β_3= Regression coefficient with respect to remittances.

μ_t= Stochastic error term.

4. RESULTS & INTERPRETATIONS

After the discussion of methodology and regression analysis done with the help of SPSS software, the numerical results obtained are analyzed in the following section.

The F-test is used to check the overall significance of the model. The hypothesis formed to check the overall significance of the regression model is:

H_0: GDP of Pakistan does not significantly depend on FDI, exports and remittances.

H_1: GDP of Pakistan significantly depends on FDI, exports and remittances.

From the SPSS output, the value of F-statistic gained is 232.665 with a p value of 0.000. Since, the p value is even less than the minimum significance level of 0.01, so H_0 is rejected. It means that there exists a strong significant impact of FDI, exports and remittances on GDP of Pakistan. Hence, this particular multiple linear regression model is highly significant at overall significance level of 100%.

From the SPSS output, model summary, the calculated value of R^2 is 0.942. As the value of R^2 should be in percentage terms, so out of 100%, 94.2% of the variation in dependent variable (GDP of Pakistan) is explained by all the predictor variables (FDI, exports & remittances) of the model. As this value is high, it also confirms the goodness of fit of the model.

Table 4.6: Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	6.475E10	8.159E9		7.936	.000		
	LFDI	27.701	5.809	.667	4.768	.003	.492	2.034
	LEX	.093	.421	.034	2.220	.001	.495	2.529
	LREM	11.029	3.393	.448	3.250	.017	.507	1.972

a. Dependent Variable: LGDP

From the table 4.6, the values of unstandardized coefficients for FDI, exports and remittances are 27.701, 0.093 and 11.029 respectively. Among all these independent variables, FDI and remittances are the most influential variables for affecting the GDP, followed by exports causing minimum influence of all.

In order to check the individual significance of each independent variable for the dependent variable, the t-test is used. It can be easily analyzed with the help of hypotheses testing procedure. The t-statistic for Exports is 2.220 at significance level 0.001, remittances are 3.250 at significance level 0.017 and FDI is 4.768 at significance level 0.003. As the significance level is lesser that 0.05. The null hypothesis will be rejected and it can be concluded that there exists a positive correlation between dependent and independent variables.

Estimated Regression Line

$$Y_t = 27.701X_{1t} + 0.093X_{2t} + 11.029X_{3t} \tag{2}$$

The value of the coefficient of FDI gives the ratio of the change in GDP. The value is 27.701, which tells that percentage increase in FDI will lead to an increase in GDP by 27.701 percent holding all other independent variables constant. And the value of the coefficient of exports gives the ratio of the change in GDP. The value is 0.093, which tells that percentage increase in exports will lead to an increase in GDP by 0.093 percent holding all other independent variables constant. And the value of the coefficient of remittances gives the ratio of the change in GDP. The value is 11.029, which tells that percentage increase in remittances will lead to an increase in GDP by 11.029 percent holding all other independent variables constant.

Partial correlation test is used to reject or accept the null hypothesis. These partial correlation tests will indicate whether the macroeconomic variables like FDI and exports are strongly correlated with GDP or not, when the effect of remittances has been removed.

a) Hypothesis 1

    H_0:   FDI and GDP are not partially correlated, keeping the effect of remittances as constant.

    H_1:   FDI and GDP are partially correlated, keeping the effect of remittances as constant.

The value of the partial correlation coefficient between GDP and FDI after removing the effect of remittances is positive i.e. 0.533.This indicates a positive partial correlation between GDP and FDI. The significance level is 0.009, indicating that the results of the partial correlation between the two variables are statistically significant. Therefore, the null hypothesis is rejected and the alternative hypothesis is accepted. The closer the value of correlation coefficient is to 1, the degree of association between the two variables is strong. The degree of correlation between GDP and FDI is moderate.

b) Hypothesis 2

    H_0:   Exports and GDP are not partially correlated, keeping the effect of remittances as constant.

    H_1:   Exports and GDP are partially correlated, keeping the effect of remittances as constant.

The value of the partial correlation coefficient between GDP and exports after removing the effect of remittances is positive 0.588.This indicates a positive partial correlation between GDP and exports. The significance level is 0.003, indicating that the results of the partial correlation between the two variables is statistically significant. Therefore, the null hypothesis is rejected and the alternative hypothesis is accepted. The degree of correlation between GDP and exports is moderate.

5. CONCLUSION AND RECOMMENDATIONS

It has been found through the regression analysis that all the selected important variables are statistically significant in terms of their impact on the GDP of Pakistan. As illustrated in the regression line, FDI and remittances are the most important variables, which are positively contributing in Pakistan's GDP. It means that increase in both will lead to an increase in GDP or vice versa. However, exports have a limited impact on the economy of Pakistan.

These findings are consistent with the results of the previous researches conducted either on Pakistan or worldwide. According to the research, FDI is the most prominent factor for determining GDP. FDI increases inflow of technologies and improved infrastructure which contributes to economic growth, which is in accordance with the study of Iqbal, Muhammad and Sher (2010), and Mihalis (2008). Boosting up exports also has a significant positive effect on GDP of Pakistan.

However, the regression coefficient with respect to exports shows a weak impact. The results obtained are in confirmation with previous findings by Mohanity (2007) and Afzal (2006). Rahman (2009) also explained that international trade is an engine of economic growth at international level and one of the major reasons for the rapid growth is the excellent export performance. Results also show that 1 percent increase in remittances will cause 11.029 percent rise in GDP. Remittances increase the standard of living of a country because there exist a positive and significant relationship between remittances and GDP. The results are in harmony with Halkides (2008) and Prasad (2005).

FDI is the most significant variable, which needs to be encouraged for assuring that the country achieves and sustains economic growth. Furthermore, growth process increases due to remittances of migrants, who are skilled, comparatively educated and belong to urban areas. Attracting FDI and encouraging investment can increase domestic production. Therefore, the country's exports can be increased, thus leading to economic prosperity. The analysis further reveals that for economic progress in a developing country like Pakistan; the efforts of government to maintain law and order within the country must be increased so as to attract more foreign investors within the country.

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COLOR PREFERENCE AND ITS EFFECT ON PERSONALITY

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ABSTRACT

The research focuses on the effect of color preference on the personality traits. To assess the views of the respondents, a sample of 150 students was selected with respect to the socioeconomic classes. The respondents selected black as their favorite color and almost equal proportion liked dark and light color. Pearson Chi square test was used to test the association. It was determined that preference of color has only significant association with Aggression-Hostility while the favorite color was found highly significant with personality traits. T-test was applied to test the difference between two groups (i.e. gender and preference of color).

1. INTRODUCTION

Color gives objects and surfaces, a personality and aura that goes beyond our senses and invoke specific emotional responses. Different shades and hues affect us differently and can work on our subconscious to bring us peace, anxiety or inspiration. Color preferences are the tendency for an individual or a group to prefer some colors over others, including a favorite color. In general, people have a connection with certain colors due to their experiences with objects of those colors.

In contemporary psychology, the "Big Five" factors (or Five Factor Model; FFM) of personality are five broad domains or dimensions of personality that are used to describe human personality has emerged as a robust model for understanding the relationship between personality and various academic behaviors. The Big Five factors are openness, conscientiousness, extraversion, agreeableness, and neuroticism (Costa & McCrae, 1992).

The color preferences are not necessarily fixed and they may vary even within a given day depending on mood or with growing age and life experiences. Subjects with similar color preference may have similar personality traits, the subjects' psychological reactions/ attitudes to the primary colors (blue, red, yellow and green) reflect their basic psychological needs. When, for example, someone does not like red, he/she unconsciously reflects anxiety (Luscher, 1971). The introverts, are preoccupied with their own thoughts and emotions, so they choose a social environment that allows them to achieve a medium level of the optimal excitement. The effects of color on excitement have shown that there is a correlation between color preferences and personality traits (Eysenck, 1967). Similar personality traits may be reflected through diverse color preference patterns, e.g., the high creativity might be associated with the yellow or red color preference at the first place, and the blue color preference at the last place (Lange and Rentfrow, 2007).

2. RATIONALE OF THE STUDY

Choosing the color of own choice plays a vital role in developing personality and helps in decision making as to which color you choose in your surroundings. This research will be carried out in order to examine the effect of color preference on personality traits, gender, age and different income groups.

3. OBJECTIVES

The aims and objectives of the study are to identify whether there is any significant difference in color preference with respect to gender, age. Determine the significant difference between the personality traits with respect to preference of color. Determine whether there is an association between Favorite color and the personality traits.

4. RESEARCH METHODOLOGY

The target population is divided according to three economic classes. A sample of 150 students have been selected from Kinnaird College and Forman Christian College, Queen mary college, MAO college, Govt. Fatima Jinnah College, and Govt. college Ravi road.A cross sectional study has been conducted among the students of Colleges. A structured questionnaire has been used as a tool for collecting data, which consists of Zuckerman-Kuhlman Personality Questionnaire and some basic questions about color. SPSS 16 and MS Excel have been used as data analysis tool.

5. RESULTS AND INTERPRETATIONS

Data has been analyzed using statistical techniques. In this univariate analysis 50% of the respondents are females and 50% of the respondents are males. Majority (54%) of the respondents are from age 18-20 and majority of the respondents (37%) have monthly income between 30,000 to 49,000. 47.33% prefer light colors while 52.67% prefer dark colors. The following bar chart shows that 33% selected Black and 16% selected Red as their favorite color.

Figure 5.1 favorite colors of the respondents

In bivariate analysis, Pearson Chi-sq. of association was used to explore the association of age, gender, income and personality traits with the preference of color and favorite color respectively. The obtained results shows that preference of color has an association with only Aggression where as there is insignificant relationship between preference of color with Neuroticism, Activity, Sociability and Impulsive sensation, age, gender and monthly income respectively (i.e. P-value > 0.05). Favorite color has an association with Aggression, Neuroticism, Activity, Sociability and Impulsive sensation, gender and monthly income respectively whereas insignificant relationship was observed with age (i.e. P-value > 0.05).
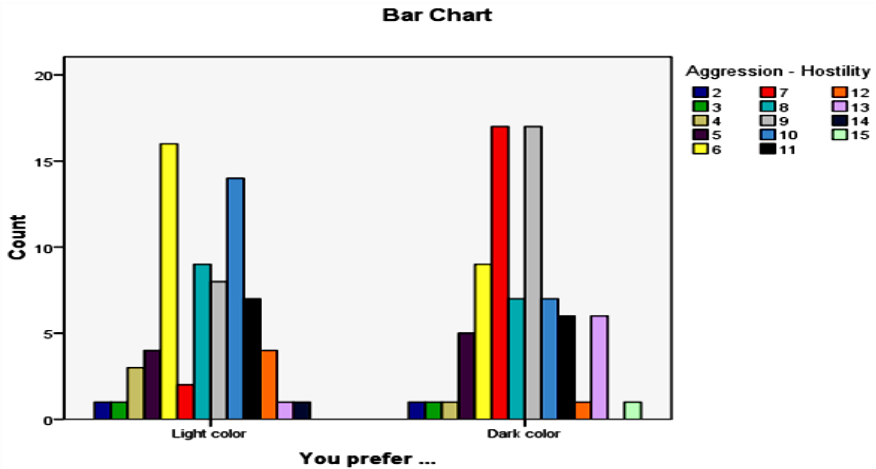
Figure 5.2.1
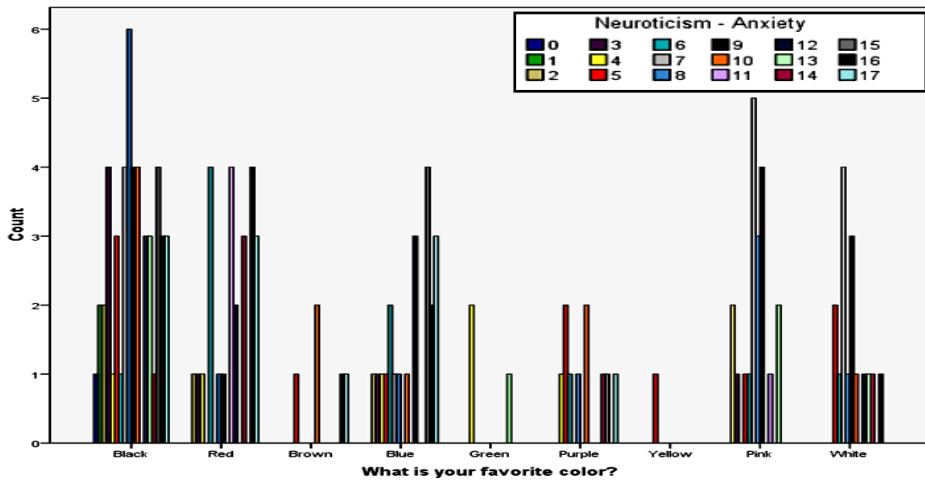
Figure 5.2.2
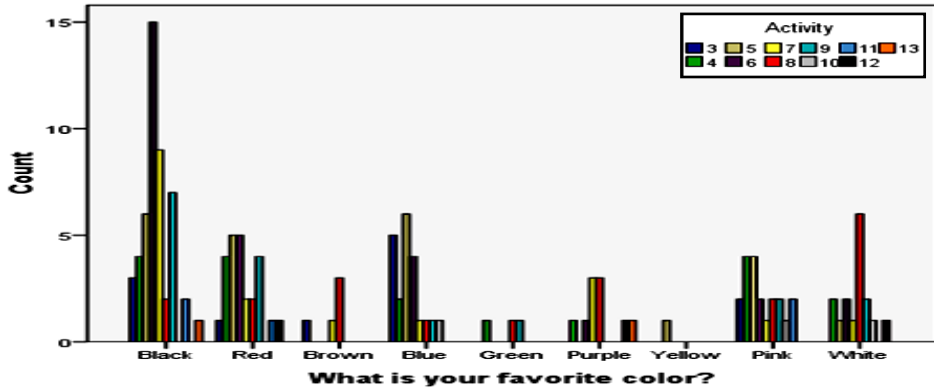
Figure 5.2.3
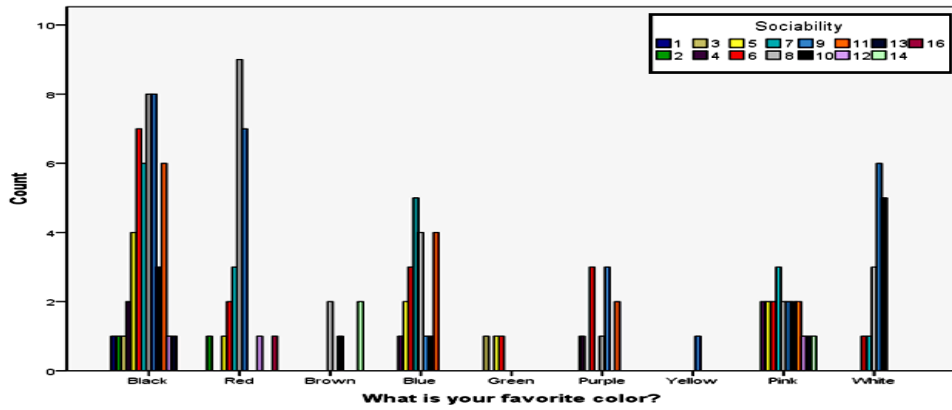
Figure 5.2.4
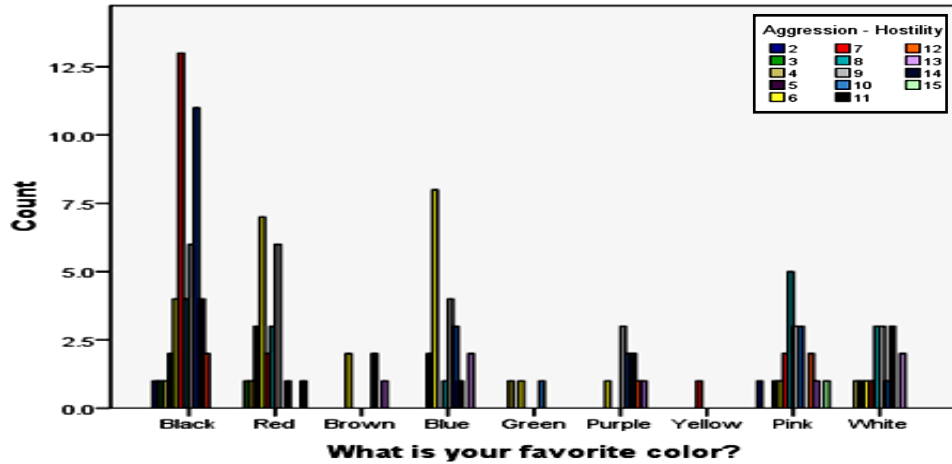
Figure 5.2.5
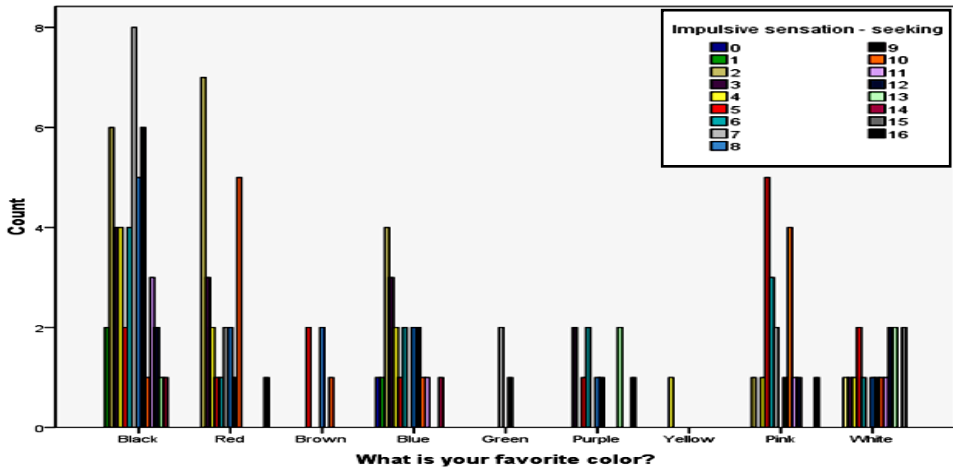
Figure 5.2.6

Figure 5.2.7

Bar Chart
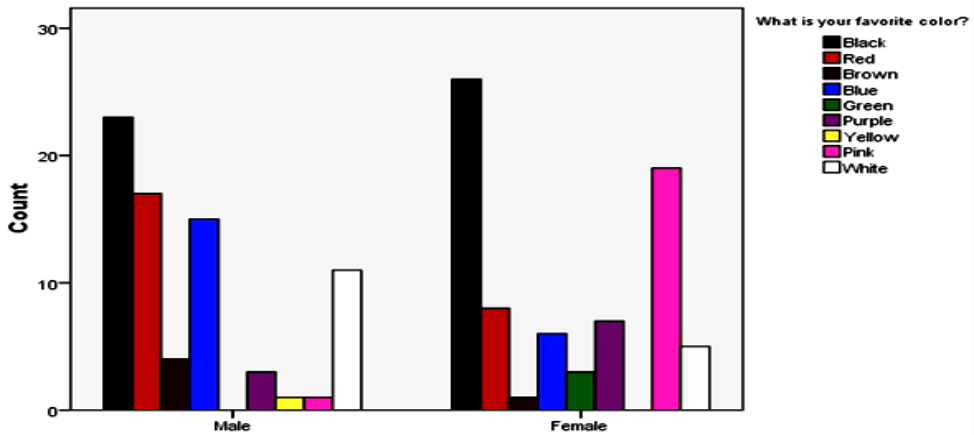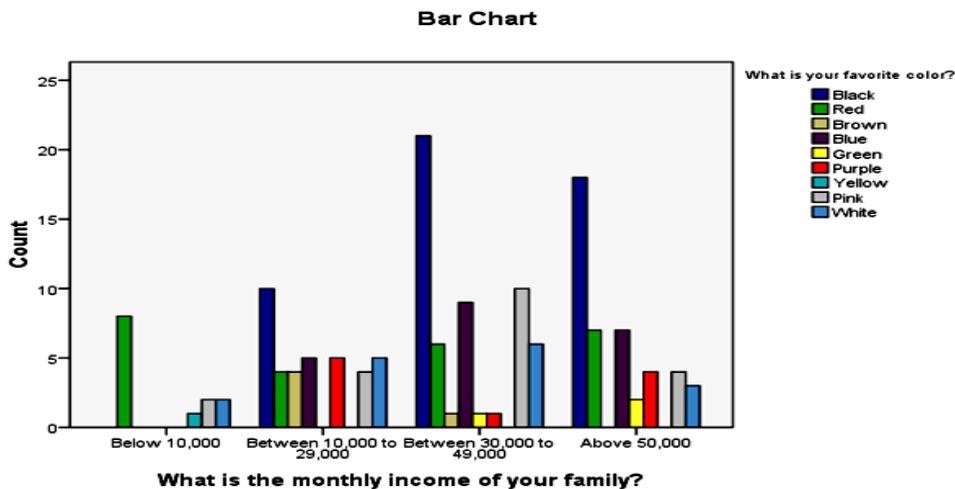
Figure 5.2.8

From the figure 5.2.1 it can be observed that the respondents who choose light color score low on Aggression while who choose dark color score high on Aggression. From the figure 5.2.2, it can be observed that the respondents who choose red followed by black as their favorite color has high Anxiety score while the respondents who have chosen pink and white color have scored low on anxiety. From the figure 5.2.3, it is observed that the respondents who have chosen pink and blue scored low on the activity while those who selected black as their favorite color scored high on activity scale. From the figure 5.2.4, it is observed that the respondents who have chosen black scored high on dimensions of sociability scale while who selected white scored low on sociability scale. From the figure 5.2.5, it is observed that the respondents who have chosen black followed by red and blue scored high on aggression- hostility score while those who selected pink followed by white scored moderately on aggression- hostility. From the figure 5.2.6, it is observed that the respondents who have chosen black followed by red and blue scored low on the dimensions of impulsive sensation while those who selected white and pink as their favorite color scored high and have tendency to act impulsively. From the figure 5.2.7, it can be clearly observed that male selected black followed by red and blue as their favorite color while female selected black followed by pink as their favorite color. From the figure 5.2.8, it can be observed that majority of the respondents of income class between 30,000 to 49,000 and above 50,000 selected black as their favorite color respectively.

Table 5.3
T-Test of Personality Traits with Respect to Preference of Color and Gender

Personality Traits	t-test (color)	Sig.(2-tailed)	t-test (sex)	Sig.(2-tailed)
Neuroticism – Anxiety	-1.073	0.285	9.046	0
Activity	-0.874	0.384	-2.344	0.02
Sociability	-1.279	0.203	0.622	0.535
Aggression - Hostility	-0.255	0.799	-1.237	0.218
Impulsive sensation	-0.609	0.543	-1.818	0.071

The table 5.3 represents tests of the difference between the two groups. The column labeled Sig. (2-tailed) displays a probability from the t distribution with 148 degrees of freedom. It can be observed that there is no significant difference in the personality traits with respect to the preference of color (p-value > 0.05). It can be observed that Neuroticism – Anxiety and Activity are significantly different respect to the gender (p-value < 0.05).

6. CONCLUSION

The research is carried out to statistically analyze the relationship of color preference on the personality traits. A sample of 150 students from different colleges, which were divided according to the socioeconomic classes, was used. In Univariate analysis, the respondents selected black as their favorite color followed by red. 47% prefer light color while 53% pre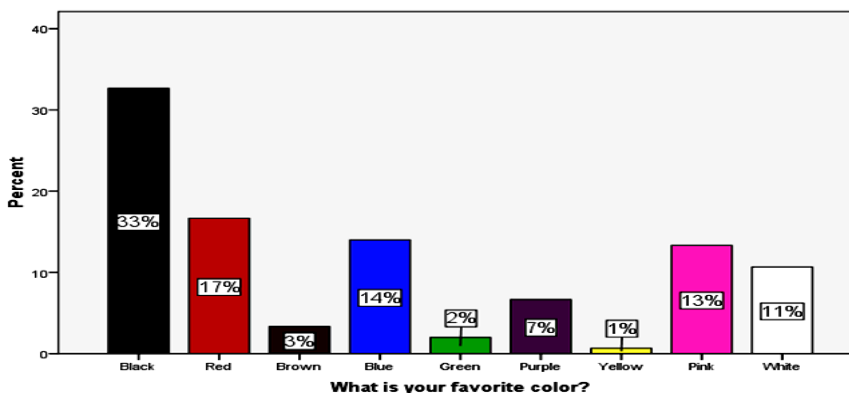fer dark color. In bivariate analysis, Pearson Chi-sq. of association was used to explore the association of age, gender, income and personality traits with the preference of color. There was no association found between age and preference of color. Similarly insignificant association was observed between gender and preference of color. The respondents who selected light color score low on Aggression. The association of favorite color comes out to be highly significant with all the personality traits. Using T-test, It was obtained that no significant difference was found among the personality traits with respect to preference of color and Neuroticism and Activity are significantly different with respect to gender respectively.

7. RECOMMENDATION

- For the future research work, it can be recommended to find the relation of color preference with emotions and moods.
- It can also include exploring the influence of color using hue, saturation; brightness etc. on the personality.
- Moreover, an experiment can be conducted among students by using different color of walls or environment and investigate various factors of the personality.

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INFORMATION SECURITY METRICS FRAMEWORK
FOR CLOUD COMPUTING

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ABSTRACT

Cloud computing has recently emerged as new computing paradigm which basically aims to provide customized, reliable, dynamic services over the internet. Cost and security are influential issues to deploy cloud computing in large enterprise. Privacy and security are very important issues in terms of user trust and legal compliance. Information Security (IS) metrics are best tool used to measure the efficiency, performance, effectiveness and impact of the security constraints. It is very hard issue to get maximum benefits from Information security metrics in cloud computing.

The aim of this paper is to discuss security issues of cloud computing, and propose basic building blocks of information security metrics framework for cloud computing. This framework helps cloud users to create information security metrics, analyze cloud threats, processing on cloud threats to mitigate them and threat assessment.

1. INTRODUCTION

Cloud computing is becoming more attractive as it provides all computing services on your desktop or somewhere inside your company's network. Cloud computing services are provided by the another company and can be acceessed over the intenet. Cloud users can use these services without knowing how these resources are being managed and thier location.

Cloud computing has several constratints which are bareer in cloud deployment. The major constraint is security. Till todate, lot of efforts has been made to improve security in cloud computing but still cloud computing (especially public cloud) is insecure. İnformation security threats often influence to the corporate processes and operations directly.

To handle these challenges, different information security frameworks, Information security standards and guides are exited and it is very hard and careful process to select one of the best framework and standard based on their security requirements.

Information security metrics is best tool to measure the security of the cloud service provider e.g. how secure is cloud service provider or is it deployed comprehensive security measure?

Cloud security threats can be drived from metrics and solve through threat modeling techniques. Mostly, these techniques are particularly for networks but not specific for cloud computing.

This paper is organized into 04 sections. The first section is about cloud computing definations, brief about information security and metrics. The following section will discuss cloud security issues. Then the section three will present state of the art related with security metrics and threat modeling and the next section will proposebasic building block of information security metrics framework for cloud computing. The last section is about conclusion and future work.

2. CLOUD SECURITY ISSUES

Cloud computing has several advantages over traditional computing which makes it better and powerful solution for cloud users. In public and private cloud, senstitive data and critical appliations are shared in cloud environment. Several security concerns are arises that need to be addressed.

This section describes about cloud security issues, and cloud security threats when considering cloud computing. Some of the cloud computing issues faced by the cloud customers:

- Data Location
- Data Theft
- Data loss
- Data Integrity
- Privacy Issues
- Regulatory Requirements
- Diaster Revovery / Business Continuity Plan

2.1 Data Location
Cloud networks span over continents, countries and regions, the physical location of the data are spread accross several geographical areas. The cloud user has no physical access over the data even does not know about its location. The cloud service provider (CSP) also does not reveal where all data are stored. Some time, the CSP store one customer data on differenct locations or countries [11]. Therefore, different country privacy laws are applied one one customer data. The cloud user should require from CSP to store and process data in a specific jourdictions and strictly obey privacy rules of those juridications [1] [2].

2.2 Data Theft
Most of cloud service providers offer the services which they are lease from other service provider (external). The cloud users does not know that's going behind the scene and they are just supposing that it is their service provider's service [2]. There is a high possibility that the external service provider attacked by malicious users and customer data may stolen.

2.3 Data Loss
It is very serious and dangerous problem in cloud computing. If the vender shutdown its business due to some legal obligations or some sort of financial problems then cloud user data will be loss or might be misuse [3][1].

2.4 Data Integrity

In cloud computing, anyone from any location can access the data. Cloud does not differentiate between common data and sensitive data [6]. It is necessary for the CSP to ensure the integrity by making their system capable to check over the cloud data from any illegal modification [10]. To overcome data integrity security issue, Third Party Auditor assistance must be use [13].

2.5 Privacy Issues

The cloud users store their sensitive data on CSP site and its vender's responsibility to secure information from other operators and intruders. The CSP should implement multiple level password and code words protection to grant access on sensitive data and implement current privacy laws [4]. The cloud user should to thoroughly read privacy issues before using cloud computing [5].

2.6 Regulatory Requirements

The Cloud venders often claim that they have implemented all security measures but in reality it is not [8]. The cloud user does not know about the exact security measures taken by the CSP. Many organizations in USA, Canada, or the European Union have implemented regulatory requirements (e.g. ISO/IEC 27002, ITIL and COBIT). The cloud user must ensure that his / her CSP meet these requirements [2] [7].

2.7 Dıaster Revovery

As mentioned above, the cloud user does not know about the data physical locations. These all physical locations faces natural and unnatural disaster threats like fire, storm, flood, earth kicks and loss of power [9]. To mitigate these issues, backup / copies of data on multiple countries is recommended [2].

2.8 Cloud Securıty Threats

Cloud computing faces same security threats that are currently found in the existing networks (LAN, WAN, Intranet). These threats and risks came in cloud computing in various forms. Some of the network issues occur in cloud computing are listed below [6] [8].

- Denial of Service
- Man in the Middle
- Net Sniffing
- Port Scanning
- SQL Injection Attack
- Flooding attack

Cloud Security Alliance (CSA) in 2010 did a research on cloud computing threats and identified top 10 cloud threats [12] in which 05 major 05 threats are given below:

- Unknown Risk Profile
- Shared Technology Vulnerabilities
- Malicious Insider
- Abuse and Nefarious Use of Cloud Computing
- Account. Service & Traffic Hijacking

Moreover, the cloud faces XML Signature Element Wrapping, browser security and Cloud Malware Injection Attack threats [6].

3. STATE OF THE ART

In cloud computing, information security metrics provide a useful and practical way to measuring information security. Different security metrics were proposed to enhance the security of the organization. Wayne et al. [14] has identified technical security metrics for the operators of control systems. Inigo et al. [15] proposed resource level metrics for specifying fine grain guarantees on CPU performance. Mark [16] discussed the possibility of creating of meaningful security metrics for communication system and proved that it is not possible to measure trust in an absolute sense. Patrick [17] discussed operational level security metrics and described how operational management can take benefits from security metrics.

Moreover, various taxonomies and state of the art for security metrics were written to explore existing security metrics and their merits and demerits. Reijo [18] [19] and [20] surveyed existing security metrics and proposed security metrics taxonomy for ICT product Research and Development (R&D). These taxonomies can be used to enhance the composition of feasible security metrics. Rostyslav et al. [21] examined current state of the art information security measurement and practical issues concerning the subject matter. SANS [22] defined seven key steps which could be use as guide in the process of establishing security metrics program.

The U.S National Institute of Information Standards and Technology (NIST) [23] presented its security metrics taxonomy namely NIST SP 800-26 and NIST SP 800-55. NIST divided security metrics into three categories (management, technical and operational) and seventeen sub-categories.

Vaughn et al. [25] propose taxonomy for information assurance metrics and divided the same into two distinct categories (organizational security metrics and metrics for Technical Target of Assessment). Seddigh et al. [24] introduce information assurance metrics taxonomy for IT Network assessment. This taxonomy has three categories (security, Quality of Service (QoS) and availability) and further considered technical, organization and operational metrics under these three metrics.

Threat Modeling is a technique used to identify, analyse and mitigate threats. Ebenezer et al. [26] proposed a goal oriented approach for Security Threat Modeling and applied approach by Modeling and analysing security threats of an online banking system. SANS [27] defined the processes to create and use threat model and discussed the significance of application security at design time.

Jesus et al. [28] used scenario driven approach to develop common security metrics framework for cloud. This proposed framework takes Risk-driven Security Metrics as input; analyze threats, defined security requirements and policy formalization. Finally the security is evaluated by using quantitative security levels. The author introduces the scenario based common processes to handle cloud security threats. These processes are resembles to security threats modeling processing like Microsoft Security Development Lifecycle (SDL Threat Modeling). The authors used "Risk-Driven Security

Metrics" approach as input without its detail about it and used threat analysis phase to identify security risks instead of describing analysis techniques. Authors used ENISA guide as input for next step i.e. Security requirement in his proposed framework. Moreover, the authors do not mentioned the detail of each phase.

4. SECURITY METRICS FRAMEWORK

Many organizations have inadequate or sometimes have no experience to develop information security metrics, identify threats for their process control system. To help them in preparing a suitable set of security metrics, identify threats and mitigation threat, common framework for developing and integrated set of processes control information security metrics and threats is proposed for cloud computing. The proposed information security metrics framework for cloud computing is shown in Figure 1. The information security metrics framework has four major stages:

1. Metrics Preparation
2. Threat Identification and Analysis
3. Threat Processing
4. Application

1. Metrics Preparation

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                          ┌──────────────────┐
          ┌──────────────│   Web Services    │
          │               └──────────────────┘
   ┌──────▼────────┐      ┌──────────────────┐
   │ Data Warehouse │◄────│    Questioner     │
   └───────┬────────┘      └──────────────────┘
          │
   ┌──────▼──────────────────────────────────────┐
   │ Collect and Aggregate Information Security Data │
   └──────┬──────────────────────────────────────┘
          │
   ┌──────▼──────────────────────────────┐
   │  Drive Information Security Metrics   │
   └──────────────────────────────────────┘
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2. Threat Identification and Analysis

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   ┌──────────────────────────────────────┐
   │  Cloud Threat Elicitation from Metrics │
   └──────────────────┬───────────────────┘
                      │
   ┌──────────────────▼───────────────────┐
   │         Analyse Cloud Threat          │
   └──────────────────────────────────────┘
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3. Threat Processing

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                                    ┌──────────────────────┐
                              ┌────►│ Security requirements │
                              │     └──────────────────────┘
   ┌──────────────────────┐   │     ┌──────────────────────┐
   │ Threat Solving Activities ├───┼────►│   Security policy     │
   └──────────┬───────────┘   │     └──────────────────────┘
              │               │     ┌──────────────────────┐
              │               ├────►│  Tools and techniques │
              │               │     └──────────────────────┘
              │               │     ┌──────────────────────┐
              │               └────►│   Cost assessment     │
              │                     └──────────────────────┘
   ┌──────────▼────────────────────┐
   │   Define Threat Severity Level  │
   └─────────────────────────────────┘
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4. Application

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   ┌────────────────────────────────────────────────────────┐
   │ Decision Makers Evaluate Security before Taking Action  │
   └────────────────────────────────────────────────────────┘
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Fig. 1: Framework for Information Security Metrics for Cloud Computing

4.1 Metrics Preparation

The IS metrics preparation phase involves information security metrics development team to develop useful information security metrics. Metrics preparation phase consist of two major steps:

1. Collect and Aggregate Information Security Data
2. Drive Information Security Metrics

4.1.1 Collect and Aggregate Information Security Data

This is first step in which data regarding information security will be obtained through different techniques like web services, questioners and data warehouse.

4.1.1.1 Web Services

Multiple web services are available that gather information security related information from different sources. The same information can then be made visible to users by using various visualization tools. To make information meaningful, information is linked to Key Performance Indicators (KPIs), IS standards and metrics. [44].

Security Information Management (SIM) also referred as Security Event Management (SEM) tools that report on information security data collected from a number of sources. This data can be normalized, aggregated, correlated and archived from various data stores [44].

4.1.1.2 Questionnaire

Information Security metrics framework uses questionnaire technique to configure desire performance levels of KPIs (minimum level of performance and desired level of performance) and metrics. Questionnaire can be base on SAN'S audit checklist and internal and external audit results [44].

4.1.2 Drive Information Security Metrics

The output of the section 1 is information security related data that abstracted from various sources and stored in data warehouse. KPIs have various metrics associated with it and used to measure the effectiveness of security control. The data warehouse stores metrics information like description, weighting value, desired value, actual value and minimum acceptable value for the metrics [44].

There are internationally accepted frameworks, standards and guides are available for guidance in metrics development. IT Infrastructure Library (ITIL) and Control Objectives for Information and related Technology (CobiT) are renowned frameworks. International Organization for Standardization (ISO) has ISO/IEC 27002 information security and control standards which also can be used to drive information security metrics. At present, ISO/IEC WD 27018 and ISO/IEC WD TS 27017 ISO information security standards are under development. SAN'S has also published information security metrics guide which is very helpful for cloud users to drive information security metrics.

4.2 Threat Identification and Analysis

The 2nd Phase of this proposed framework is about threat elicitation and analysis. In this phase, the threats are identified from information security metrics and different techniques like threat tree are applied to analyze the threat.

4.2.1 Cloud Threat Elicitation from Metrics

After developing information security metrics, the next step is to elicit cloud security threats from metrics. Different approaches has already been in practice for threat elicitation like STRIDE model and categorized threat list. Furthermore, Myagmar et al. [27] and Ebenezer et al. [26] described methods for threat elicitation.

The STRIDE classified the threats as Spoofing, Tampering, Repudiation, Denial of service, and Elevation of privilege can be used to elicit security threats [26].

Mark et al. [30] presented threat model based on based on threat metrics and discussed various metrics sources.

4.2.2 Analyse Cloud Threat

This is most critical section of the security metrics framework in which cloud threats are critically analyzed. Threat analysis goals are to discover what, why, when, where, why and how threat attack on the system and what are the security risks of a cloud system. This step is very important for threat solving activities and adequate security mechanism [28]. It is important for threat analyst to understand the threats that exist in the environment.

At present, several threat analysis, assessment and modeling methods are existed to guide users about this section [31].

Threat tree, attack tree and some same type of methodologies are used to identify threat type, represent threat and analyze threat [31]. In tree analysis, top-down approach is used to determine viable threat vectors.

Jones [32] presented threat assessment methodology in which threat agents are identified and categorized to analyze their capabilities. The motivational factors of the agents are also examined. Vulnerability Instantiation Methodology (VIM) is a two-stage method that uses vulnerabilities and their relationships to identify and analyze threats.

Threat assessment models are also used to analyze threats like Operationally Critical Threat, Asset, and Vulnerability Evaluation (OCTAVE) [34] framework was designed to identify and manage security risks. It helps organization to identify information assets, threats to those assets and vulnerabilities that may expose those assets to the threats. Similarly Amenaza IT Threat Tree Modeling System [35] was developed to model threats in hierarchy trees. It calculates risk of the threat and impact of the threat to get risk value.

European Network and Information Security Agency (ENISA) [33] has made assessment of the security risks and benefits of cloud computing. Moreover it provided security guidance for potential and existing users of cloud computing.

4.3 Threat Processing

After Analysis of threat, this phase is defined different activities that help cloud users to process on identified IS threats. This phase is very critical & technical and required due concentration of threat solving team.

4.3.1 Threat Solving Activities

This is very important section of this proposed framework. Threat solving activities are divided into of four (4) sub activities which are as under:

- Security requirements
- Security policy
- Tools and techniques
- Cost assessment

4.3.1.1 Security Requirements

Requirement statement is consisting of goals that must be fulfilled in order to mitigate threat [29]. To solve the cloud threat, it is essential to define security requirements. This section identify about the action(s) to be taken in order to minimize the probability of a particular threat or risk [28].

For example, the threat is *"Attacker uses DoS attacks to reduce availability of the system"*. The attack tree that used in previous section analyzed that this threat can be either flooding the network interface or filling up the available disk space [35]. The requirement needs to mitigating said threat could be *"The system shall not allow any user to successfully use DoS attack to reduce availability of the system"* [35].

ENISA [33] made assessment of cloud computing risks and its associated requirements to mitigate cloud threats. Cloud Security Alliance (CSA) [37] [38] white papers (Security Guidance for Critical Areas of Focus in Cloud Computing V3.0 and Cloud Control Matrix) are also very helpful in defining security requirements.

4.3.1.2 Security Policy

A security policy is also called as set of security requirements mentioned in previous section. A security policy is defined against each security requirements. For example, the National Institute of Standards and Technology (NIST) provide a guidelines and recommendations to users to securing public Web servers that "*all traffic between the Internet and Web server*" should be controlled and that "*all inbound traffic to the Web server except traffic which is required, such as TCP ports 80 (HTTP) and/or 443 (HTTPS)*" should be denied.

4.3.1.3 Tools and Techniques

Analysis of tools and techniques is a process that uses to find out solution of threat. Information for tools and techniques can be obtained from different sources like websites, news feeds, IRC message boards, security forums, groups and intelligent activities [39].

It is fact that all cloud management tools not work equally for all cloud providers and not allows customers to manage their internal and external security as single unit [40].

Examples of Tools are HyTrust's virtual management appliance; it provides server configuration templates, VMWARE vSphere security configuration assessment against industry frameworks. CohesiveFT sells cloud security tools (VPN-Cubed virtual firewall and router), and management tools to build VM templates and monitoring of management tasks [40].

4.3.1.4 Define Threat Severity Level

After analysis of tools and techniques for cloud threats, it is essential to define threats severity levels. The use of severity levels always plays an important by informing the peoples about the event should trigger within time frame. Cloud Security Alliance (CSA) [41] in its white paper (Top threats to cloud computing V1.0) identified top 10 cloud

security threats with their impacts and given recommendation against each threat. Symantec.cloud [43] defined four (4) threat levels for cloud.

In general, threat severities levels can be classified into five levels based on their severity [43] [42]. The highest severity level poses the most serious threat to cloud security. The following Table-1 defines five severity levels for cloud security threats.

Table-1: Definition of Severity Levels

Severity	Level	Description
1	Minimal	In minimal, the intruder can just get information about the user by performing passive attack like eavesdropping and session hijacking. Continuous monitoring is required.
2	Medium	The hacker or intruder can get sensitive information about the users and there is no discernible network activity.
3	Serious	This level is developed when the intruder may able to get partial access of resources or able to get read access of specific files. Serious level condition is fulfilled when malicious code reaches moderate risk rating.
4	Critical	Critical situation arise when intruder gain control on host or have full access to read the files or malicious code reaches sever risk rating. Increased monitoring is required.
5	Urgent	It is highest severity level in which intruder can easily gain full control on the host. The intruder gain entire network control with read and write access. Top priority should give to urgent level.

5. APPLICATION

The last activity of this framework focus on the use of the security metrics and threat severity levels by the decision makers. They evaluate the security and take suitable actions. In this last phase, IS metrics, cloud security threats and severity levels are accessed by the decision makers. Risk assessment may be used to provide additional information to support this last activity.

Security assessment is continuous process. Information Security metrics development, threat identification and assessment activities never end. It is recommended that upgrade metrics as time to time new threats and vulnerabilities came into control system.

5. COMMENTS AND CONCLUSION

Cloud Computing is itself a virtualization of resources. These resources can be accessed with minimal mangement and without requiremnet of having knowledge of system that deliver it. Cloud has lot of benefits (like scalability, pay as per use, share resource in on place and less management) over traditional computing which makes it famous and powerful.

Cloud Service Providers claims that they meet full security requirements but in reality it is not. To evaluate performance and effectivness of cloud service provider's deployed security as per terms and conditions laid down in Service Level Agreement (SLA),

Information Security metrics are best tools. There are several techniques to develop security metrics in which one is used in this paper proposed framework.

Clouds faces various security issues and same kind of threat and vulnerabilities like traditionals networks. To mitigate these threats, attack tree, threat tree and similar tree-based threat modeling methodologies can be used.

The proposed informaton security metrics framework for cloud computing helps the organizations to develop cloud security metrics, identify threats from metrics, analysis and mitigate threat. The proposed framework is flexible, open and technology-agnostic and able to be extended through new addition of new security metrics or as per organizations specific requirements.

As future work, Several research challenges has been identified with the proposed informatoin security metrics framework for cloud computing particularly with the quantititive evaluation of security threats. Moreover, it is also under consideration to make different scenarios that support this proposed framework and develop security evaluation techniques for better decison about security threats.

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BAYESIAN PARAMETER ESTIMATION OF HYBRID CENSORED POWER FUNCTION DISTRIBUTION UNDER DIFFERENT LOSS FUNCTIONS

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ABSTRACT

In this study, parameter of hybrid censored power function distribution is estimated with Bayesian method using noninformative and informative priors. A comparison of estimated parameters is made on the basis of bias and posterior risks under Square Error Loss Function (SELF), Quadratic Square Error Loss Function (QSELF), Weighted Error Loss Function (WELF) and Precautionary Error Loss Function (PELF) to explore the best prior type and loss function for parameter estimation. Simulation based results show that the estimator with informative prior has minimum bias and minimum risks as compared to the noninformative prior based estimator. Among the four loss function, the best results are found with the usage of SELF.

KEYWORDS

Noninformative priors, Informative Priors, Loss Functions

1. INTRODUCTION

Power function distribution is frequently used to study the electrical component reliability. Ahsanullah (1996, 1997), Cramer (2003) and Al-Hussaini (2004) presented the estimators of parameters of the uniform and power function distribution respectively. Saran and Pandey (2004) derived linear unbiased estimates of the parameters of a power function distribution based on k-th record values. Kang and Jung (2008) estimate the power function distribution based on Type-II censored sample. Saleem, Aslam and Economouc (2010) worked on mixture of power function and compared the results of Bayes estimates under type-I right censored data.

Censoring is an unavoidable feature of the life time data in life-testing and reliability studies, because experimenter may not always obtain complete information on failure times for all experimental units (See Ebrahimi, 1990, 1992). Data obtained from such experiments are called censored data. In this study we used hybrid censoring scheme which is the mixture of Type- I and Type-II censoring schemes (See Kundu, 2007). Under this scheme, the test is terminated when a pre-chosen number (r) out of n items has failed or when a pre-determined time, T, on test has been reached. The lifetimes of the sample units are independent and identically distributed (i.i.d.) random variables. It is also assumed that the failed items are not replaced.

2. MODEL DESCRIPTION

The power function distribution has standard probability density function and cumulative distribution function as follows:

$$f(x) = \alpha x^{\alpha-1} \quad \alpha > 0; \qquad 0 \le x \le 1 \tag{1}$$

and

$$F(x) = x^{\alpha} \quad \alpha > 0; \qquad 0 \le x \le 1 \tag{2}$$

where α is a shape parameter.

The power function distribution is also the distribution of the inverse of a Pareto distribution and it is a special case of the Beta distribution $(\lambda, 1)$. The power function distribution is a uniform distribution when $\lambda=1$, and the density function is decreasing (increasing) if $0 < \lambda < 1$.

2.1. Hybrid Censored Likelihood Function

In this section we provide the MLEs of the unknown parameters. The likelihood function for the hybrid censored data may be written as

$$L(\theta) = \frac{n!}{(n-D^*)!} \left[\prod_{i=1}^{D^*} f(x_i, \theta) \right] \left[1 - F(T^*, \theta) \right]^{n-D^*} \tag{3}$$

$T^* = \min(T, x)$, and D^* denoted The number of units that would fail before the time T^* (see Kundu, 2007).

Using (1) and (2) in (3), the result becomes

$$L(\alpha) \propto \frac{n!}{(n-D^*)!} \alpha^{D^*} e^{(\alpha-1)C_1} \sum_{j=0}^{n-D^*} \binom{n-D^*}{j} (-1)^{n-D^*-j} \left(T^*\right)^{\alpha(n-D^*-j)} \tag{4}$$

where

$$C_1 = \sum_{i=1}^{D^*} \ln x_i$$

3. BAYESIAN ESTIMATION ASSUMING UNINFORMATIVE PRIORS

The non-informative prior (Uniform prior) of α is given by:

$$\lambda(\alpha) = \alpha^{-1} \qquad\qquad \alpha > 0 \tag{5}$$

Bayesian analysis with non-informative priors is very common in circumstances when no prior information is available. Uniform prior is one of the most broadly used non informative priors, introduced and used by Bayes (1763) while working on the parameter estimation of a particular distribution.

Based on the (4) and (5), the posterior density function of α under hybrid censored sample will be:

$$f(\alpha \mid \mathbf{x}) = \frac{\alpha^{D^*-1} \sum_{j=0}^{n-D^*} \binom{n-D^*}{j} (-1)^{n-D^*-j} e^{-\alpha\{-(n-D^*-j)\ln T^* - C_1\}}}{\Psi_1} \qquad \alpha > 0 \tag{6}$$

where the normalized constant can be obtained as

$$\Psi_1 = \Gamma D^* \sum_{j=0}^{n-D^*} \frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*}}$$

3.1 Expressions for the Bayes Estimator and Posterior Risk under Square Error Loss Function

$$\hat{\alpha}_s = E(\alpha|\mathbf{x}) = \frac{1}{\Psi_1}\Gamma(D^*+1)\sum_{j=o}^{n-D^*} \frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*+1}} \tag{7}$$

$$R(\hat{\alpha}_s) = Var(\alpha^2|\mathbf{x}) = \frac{1}{\Psi_1}\Gamma(D^*+2)\sum_{j=0}^{n-D^*} \frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*+2}}$$
$$-\left[\frac{1}{\Psi_1}\Gamma(D^*+1)\sum_{j=o}^{n-D^*} \frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*+1}}\right]^2 \tag{8}$$

3.2 Bayes Estimator and Posterior Risk under Quadratic Square Error Loss Function

$$\hat{\alpha}_q = \frac{\dfrac{1}{\Psi_1}\Gamma(D^*-1)\sum_{j=o}^{n-D^*} \dfrac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*-1}}}{\dfrac{1}{\Psi_1}\Gamma(D^*-2)\sum_{j=o}^{n-D^*} \dfrac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*-2}}} \tag{9}$$

$$R(\hat{\alpha}_q) = 1 - \frac{\left[\dfrac{1}{\Psi_1}\Gamma(D^*-1)\sum_{j=o}^{n-D^*} \dfrac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*-1}}\right]^2}{\dfrac{1}{\Psi_1}\Gamma(D^*-2)\sum_{j=o}^{n-D^*} \dfrac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*-2}}} \tag{10}$$

3.3 Bayes Estimator and Posterior Risk under Weighted Loss Function

$$\hat{\alpha}_w = \cfrac{1}{\cfrac{1}{\Psi_1}\Gamma(D^*-1)\sum_{j=o}^{n-D^*}\cfrac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*-1}}} \tag{11}$$

$$R(\hat{\alpha}_w) = \frac{1}{\Psi_1}\Gamma(D^*+1)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*+1}}$$

$$- \cfrac{1}{\cfrac{1}{\Psi_1}\Gamma(D^*-1)\sum_{j=o}^{n-D^*}\cfrac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*-1}}} \tag{12}$$

3.4 Bayes Estimator and Posterior Risk under Precautionary Loss Function

$$\hat{\alpha}_p = \sqrt{\frac{1}{\Psi_1}\Gamma(D^*+2)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*+2}}} \tag{13}$$

$$R(\hat{\alpha}_p) = 2\left(\sqrt{\frac{1}{\Psi_1}\Gamma(D^*+2)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*+2}}} \right. $$
$$\left. - \frac{1}{\psi_1}\Gamma(D^*+1)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* - C_1\}^{D^*+1}}\right) \tag{14}$$

4. BAYESIAN ESTIMATION ASSUMING INFORMATIVE PRIORS

The two-parameter gamma distribution, widely used in reliability and survival analysis as an informative prior, has the following probability density function.

$$f\left(\alpha|k,d\right) = \frac{d^k}{\Gamma k}\alpha^{k-1}e^{-d\alpha} \; ; \; k>0 \;\; d>0 \tag{15}$$

Based on the (4) and (15), the posterior density function of α under hybrid censored sample will be:

$$f(\alpha|\mathbf{x}) = \frac{\alpha^{D^*+k-1}\left[\sum_{j=0}^{n-D^*}\binom{n-D^*}{j}(-1)^{n-D^*-j}e^{-\alpha\{-(n-D^*-j)\ln T^*+C_2\}}\right]}{\Psi_2} \tag{16}$$

Let

$$C_2 = d - C_1$$

where normalized constant is equal to

$$\Psi_2 = \Gamma(D^*+k)\sum_{j=0}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^*+C_2\}^{D^*+k}}$$

4.1 Expressions for Bayes Estimator and Posterior Risk under Square Error Loss Functions

$$\hat{\alpha}_s = \frac{1}{\Psi_2}\Gamma(D^*+k+1)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^*+C_2\}^{D^*+k+1}} \tag{17}$$

$$R(\hat{\alpha}_s) = \frac{1}{\Psi_2}\Gamma D^*+k+2\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^*+C_2\}^{D^*+k+2}}$$
$$-\left[\frac{1}{\Psi_2}\Gamma(D^*+k+1)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^*+C_2\}^{D^*+k+1}}\right]^2 \tag{18}$$

4.2 Bayes Estimator and Posterior Risk under Quadratic Square Error Loss Function

$$\hat{\alpha}_q = \frac{\frac{1}{\Psi_2}\Gamma(D^*+k-1)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^*+C_2\}^{D^*+k-1}}}{\frac{1}{\Psi_2}\Gamma(D^*+k-2)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^*+C_2\}^{D^*+k-2}}} \tag{19}$$

$$R(\hat{\alpha}_q) = 1 - \frac{(\frac{1}{\Psi_2}\Gamma(D^*+k-1)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^*+C_2\}^{D^*+k-1}})^2}{\frac{1}{\Psi_2}\Gamma(D^*+k-2)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^*+C_2\}^{D^*+k-2}}} \tag{20}$$

4.3 Bayes Estimator and Posterior Risk under Weighted Loss Function

$$\hat{\alpha}_w = \cfrac{1}{\cfrac{1}{\Psi_2}\Gamma(D^*+k-1)\sum_{j=o}^{n-D^*}\cfrac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* + C_2\}^{D^*+k-1}}} \tag{21}$$

$$R(\hat{\alpha}_w) = \frac{1}{\Psi_2}\Gamma(D^*+k+1)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* + C_2\}^{D^*+k+1}}$$
$$- \cfrac{1}{\cfrac{1}{\Psi_2}\Gamma(D^*+k-1)\sum_{j=o}^{n-D^*}\cfrac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* + C_2\}^{D^*+k-1}}} \tag{22}$$

4.4 Bayes Estimator and Posterior Risk under Precautionary Loss Function

$$\hat{\alpha}_p = \sqrt{\frac{1}{\Psi_2}\Gamma(D^*+k+2)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* + C_2\}^{D^*+k+2}}} \tag{23}$$

$$R(\hat{\alpha}_p) = 2\left(\sqrt{\frac{1}{\Psi_2}\Gamma(D^*+k+2)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* + C_2\}^{D^*+k+2}}}\right. $$
$$\left. - \frac{1}{\Psi_2}\Gamma(D^*+k+1)\sum_{j=o}^{n-D^*}\frac{\binom{n-D^*}{j}(-1)^{n-D^*-j}}{\{-(n-D^*-j)\ln T^* + C_2\}^{D^*+k+1}}\right) \tag{24}$$

5. SIMULATION STUDY

The behavior of the estimates of parameter regarding their minimum Bias and risks under different loss function using different censoring conditions for informative and non informative priors is studied using simulation using MatLab 7.

5.1 Simulation Study of Bayes Estimators and Risks Assuming Uniform Prior under Different Loss function.

The samples are simulated with the parameter and censoring conditions as Alpha (α) = 5, Number of Censored Observation (R): 10.0 to 15.0 and the Censoring Time (T): 0.5 to 0.9. The Brief summary of the estimates along with censoring conditions under different loss function are shown in the Table 4.1. and Table 4.2 from the all possible combination of R (with increment of 1) and T (with the increment of 0.1) with the objectives (1) to explore the censoring conditions at which different loss function provide

estimates with minimum bias and minimum risk and (2) to explore the loss function which best results from the four studies loss functions.

These results shows that the censoring conditions (15, 0.5), (15, 0.7), (15, 0.8), (15, 0.9) in square error loss function, the censoring conditions (13, 0.5), (13, 0.6), (13, 0.7), (12, 0.5) in Quadratic Square error loss function, the censoring condition (14, 0.9), (14, 0.8), (14, 0.5), (14, 0.7) in weighted loss function, and the censoring condition (15, 0.7), (15, 0.8), (15, 0.6), (15, 0.5) in precautionary loss function are the combinations where we achieve estimates with minimum bias and with minimum risk from the rest of combinations of (R, T).

Table 4.1:
Bayes Estimates and Posterior Risks under Different Loss Function
for Different Censoring Conditions for Uniform Prior

	Size (r)	Censoring Time (t)	No. of censoring objects (D)	Estimate	Posterior Risk
SELF	15.0	0.5	7	1.9928	0.0209
	15.0	*0.7*	*12*	*2.0000*	*0.0203*
	15.0	0.8	13	1.9982	0.0206
	15.0	0.9	15	1.9873	0.0206
QSELF	12.0	0.5	8	1.9447	0.0720
	13.0	0.5	8	2.0473	0.0718
	13.0	0.6	7	2.0617	0.0716
	13.0	*0.9*	*11*	*2.0171*	*0.0714*
WLF	*14.0*	*0.9*	*14*	*2.0074*	*0.0382*
	14.0	0.8	13	2.0228	0.0380
	14.0	0.5	11	2.0231	0.0382
	14.0	0.7	11	2.0286	0.0378
PLF	*15.0*	*0.7*	*12*	*1.9364*	*0.0350*
	15.0	0.8	13	1.9347	0.0352
	15.0	0.6	13	1.9208	0.0355
	15.0	0.5	7	1.9293	0.0355

The best combinations from all four loss functions are presented in bold form. We can see the censoring condition (15, 0.7) and the unbiased estimate(2.0000) with the minimum risk (0.0203) in SELF, the censoring condition (13, 0.9) and the estimate with the minimum bias (2.0171) and the risk (0.0714) in QSELF, the censoring condition (14, 0.9) and the estimate with the minimum bias (2.0074) and the minimum risk (0.0382) in WLF and in the last the censoring condition (15, 0.7) and the Bayes estimate (1.9364) and the minimum risk(0.0350) in PLF, are the combinations which provides estimates with the minimum bias and the minimum posterior risk from the rest of the combination shown in Table 4.1. By analyzing all four best combinations, is can be seen that the SELF provide best results at the combination (15, 0.7) with the Bayes estimate 2.0000 and the minimum risk 0.0203.

5.2 Simulation Study of Bayes Estimators and Risks Assuming Gamma Prior under Different Loss function.

For this simulation, the censoring conditions are as Alpha of Power function distribution (α) = 5, Number of Censored Observation (R): 10.0 to 15.0, the Censoring Time (T): 0.5 to 0.9. Shape parameter of Gamma prior (k): 0 to 3 and Scale parameter of Gamma prior(d): 0 to 3.

Table 4.2:
Bayes Estimates and Posterior Risks under Different Loss Function
for Different Censoring Conditions for Gamma Prior

	Size (r)	Censoring Time (t)	Hyper parameter		No. of censoring objects (D)	Estimator	Posterior Risk
			k	d			
SELF	11.0	0.5	0	3	10	2.0047	0.0193
	14.0	0.7	0	2	11	2.0030	0.0202
	15.0	*0.6*	*1*	*2*	*11*	*2.0021*	*0.0188*
	15.0	0.9	1	2	15	1.9988	0.0190
QSELF	10.0	0.8	0	1	14	1.9969	0.0714
	12.0	0.5	1	1	11	1.9966	0.0670
	12.0	*0.5*	*2*	*3*	*10*	*2.0002*	*0.0628*
	12.0	0.7	2	3	11	2.0020	0.0626
WLF	11.0	0.8	0	1	11	1.9965	0.0379
	13.0	0.7	1	3	12	1.9997	0.0354
	14.0	0.7	1	2	14	2.0008	0.0355
	15.0	*0.6*	*2*	*2*	*13*	*2.0024*	*0.0333*
PLF	15.0	0.8	0	1	14	2.0055	0.0337
	15.0	*0.9*	*1*	*3*	*14*	*2.0041*	*0.0315*
	15.0	0.9	0	1	14	2.0018	0.0339
	15.0	0.5	1	3	10	1.9940	0.0317

These results shows that the combination of censoring conditions and the hyper parameter in the format of (R, T, k, d) which provide the estimates with minimum bias and minimum risk from the rest of the combinations are (11, 0.5,0, 3) , (14, 0.7,0, 2), (15, 0.6, 1, 2), (15, 0.9, 1, 2) respectively in square error loss function, in QSELF, the best combinations are (10, 0.8, 0, 1), (12, 0.5, 1 , 1), (12, 0.5, 2, 3), (12, 0.7), 2, 3) respectively, in WELF, the combinations are (11, 0.8, 0, 1), (13, 0.7, 1, 3), (14, 0.7, 1, 2), (15, 0.6, 2, 2) and finally in WELF, the combinations are (15, 0.8, 0, 1), (15, 0.9, 1, 3), (15, 0.9, 0, 1), (15, 0.5, 1, 3) respectively

The best combinations from all four loss functions are shown in the table in bold form. By analyzing these combinations, it is clear that the performance of the Bayes estimates in case of informative prior is better under SELF.

5.3 Comparison of Bayes estimators and Posterior Risks for Noninformative and Informative Prior under Different Loss Functions

To make decision that which of the Bayesian analysis (either using uniform or Gamma prior) produces better results, the Bayes's estimates and the posterior risks at some selected combinations under different loss functions are presented in Table 4.5. The comparison is made on the basis of minimum Posterior Risks. The results clearly shows that the Gamma prior provides best results under all loss function as compared to the uniform prior as the posterior risk of Bayes estimates with gamma have minimum posterior risks as compared to Bayes estimates with uniform prior on all loss functions. Further it can also be seen that the Bayesian Analysis with Gamma prior provide best results under SELF as the Bayes estimates are with minimum posterior risks as compared to the remaining loss functions.

Table 4.1:
Comparison of Bayes Estimator and Posterior Risk of Non Informative
and Informative Prior under Different Loss Functions

	Uniform Prior					Gamma Prior						
	Size	Censoring Time	No. of censoring objects	Estimator	Posterior Risk		Size	Censoring Time	Hyper parameter		No. of censoring objects	Estimator	Posterior Risk
									k	d			
	(r)	(t)	(D)				(r)	(t)			(D)		
SELF	15.0	0.5	7	1.9928	0.0209	SELF	15.0	0.5	1	2	11	2.0171	0.0184
	15.0	*0.7*	*12*	*2.0000*	*0.0203*		15.0	0.7	0	1	13	2.0586	0.0191
	15.0	0.8	13	1.9982	0.0206		*15.0*	*0.8*	*1*	*2*	*15*	*2.0030*	*0.0185*
	15.0	0.9	15	1.9873	0.0206		15.0	0.9	1	2	15	1.9988	0.0190
QSELF	13.0	0.5	8	2.0473	0.0718	QSELF	13.0	0.5	0	0	9	2.0435	0.0718
	13.0	0.6	7	2.0617	0.0716		13.0	0.6	0	0	10	2.0329	0.0716
	13.0	*0.7*	*10*	*2.0233*	*0.0715*		13.0	0.7	0	0	12	2.0230	0.0715
	12.0	0.5	8	1.9447	0.0720		*12.0*	*0.5*	*2*	*3*	*10*	*2.0002*	*0.0628*
WELF	14.0	0.9	14	2.0074	0.0382	WELF	14.0	0.9	1	2	14	2.0245	0.0350
	14.0	0.8	13	2.0228	0.0380		14.0	0.8	0	0	13	2.0053	0.0384
	14.0	0.5	11	2.0231	0.0382		14.0	0.5	0	0	12	2.0331	0.0380
	14.0	*0.7*	*11*	*2.0286*	*0.0378*		*14.0*	*0.7*	*1*	*2*	*14*	*2.0008*	*0.0355*
PELF	*15.0*	*0.7*	*12*	*1.9364*	*0.0350*	PELF	15.0	0.7	0	1	10	1.9915	0.0340
	15.0	0.8	13	1.9347	0.0352		*15.0*	*0.8*	*0*	*1*	*14*	*2.0055*	*0.0337*
	15.0	0.6	13	1.9208	0.0355		15.0	0.6	0	1	11	2.0131	0.0338
	15.0	0.5	7	1.9293	0.0355		15.0	0.5	1	3	13	1.9915	0.0340

6. CONCLUSION

We consider the Bayesian analysis of the power function distribution using hybrid censored data. We use four different types of loss functions assuming informative and non-informative priors. On the basis of simulation study, the behavior of Bayes estimates and Posterior risks is observed under all loss function. If we compare the Bayes estimates and the posterior risk for informative and non-informative prior under four loss functions, we conclude that informative prior gives us best result then non informative prior. Among all the loss functions square error loss function has the more unbiased Bayes estimates of parameter with the minimum risk for incase of both informative and non-informative prior. The optimum combination with the uniform prior is (15.0, 0.7) under square error loss function with the Bayes estimate (2.0000) and the posterior risk (0.0203). Similarly the optimum combination for the gamma prior is (15.0, 0.6, 1, 2) under square error loss function with the Bayes estimate (2.0021) and the posterior risk (0.0188).

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A MODIFIED GENERALIZED CLASS OF ESTIMATORS IN TWO PHASE SAMPLING USING MULTI-AUXILIARY VARIABLES CONSIDERING THE NON-RESPONSE ON STUDY VARIABLE

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ABSTRACT

In this paper we have proposed a general class of estimators in two phase sampling using multi-auxiliary variables for estimating population mean of study variable considering the non-response on only study variable. Special cases of suggested class have also been deduced. The expressions of bias and mean square error have also been derived. Mathematical and empirical comparison has been made for comparing the efficiency of proposed estimators.

KEY WORDS

Multi-Auxiliary Variables; Non-response; Two Phase Sampling; general class of estimators.

1. INTRODUCTION

The most common method of data collection in survey research is sending the questionnaire through mail. The reason may be the minimum cost involved in this method. But this method has a major disadvantage that, a large rate of non response may occur which may result in an unknown bias at any assumption because of the fact that the estimate based only on responding units is representative of the both responding and non responding units.

Personal interview is another method of data collection which generally may result in a complete response, but the cost involved in personal interviews is much higher than the mail questionnaire method. We may conclude from the above discussion that the advantage of one method is the disadvantage of other and vice versa. Hansen and Hurwitz (1946) combined the advantages of both procedures. They considered aproblem to determine the number of mail questionnaires along with the number of personal interviews to take in following up non-response to the mail questionnaire in order to attain the required precision at minimum cost.

We assume that the population of size N is divided into two groups, one group consisting of the units who respond called response class and the other group consisting

of the units who do not respond is called non-response class. Let N_1 and N_2 be the sizes of responding class and non responding class of population respectively. A simple random sample of size n_1 is selected from population of size N, out of which n_{11} units respond and n_{12} do not. We can say that n_{11} is the sample selected from N_1 units and n_{12} be the number of units selected from N_2 units. Let r be the size of the sample selected from n_{12} where $r = \dfrac{n_{12}}{k}$, $k > 1$. Let $\bar{y}_1$ and $\bar{y}_{2r}$ denote the sample means based on n_{11} and r units respectively.

Hansen and Hurwitz (1946) proposed the following estimator to estimate the population mean $\bar{Y}$ based on n_{11} and r units:

$$\bar{y}_1^* = \frac{n_{11}\bar{y}_1 + n_{12}\bar{y}_{2r}}{n_1} \tag{1.1}$$

The estimator $\bar{y}_1^*$ is unbiased and has variance

$$Var\ \bar{y}_1^* = \lambda_1 S_y^2 + \theta S_{y_2}^2 \tag{1.2}$$

where $\lambda_1 = \dfrac{1}{n_1} - \dfrac{1}{N}$ and $\theta = \dfrac{W_2\ k_1 - 1}{n_1}$; $W_2 = \dfrac{N_2}{N}$, S_y^2 and $S_{y_2}^2$ be the population variances based on N and N_2 units respectively.

1.1 Two-Phase Sampling in the Presence of Non-Response at Second Phase using Single Auxiliary Variable

Single phase sampling will not be appropriate in the situation when cost of drawing a single large sample is very high. To overcome, a large sample is selected at first phase and information only on auxiliary variable is recorded. Then, a smaller second phase sample is selected and information on both study and auxiliary variables is recorded. This scheme in which sampling is done in two phases is called two-phase sampling or double sampling. The two phase sampling may cause a loss of efficiency but this loss is compensated by the use of auxiliary information.

Let a sample of size n_1 is selected by simple random sampling without replacement (SRSWOR) from a population of size N which is divided into two groups N_1 (response group) and N_2 (nonresponse group). We assume that at the first phase, all the n_1 units provide information on auxiliary variable x_{1i}, a smaller second phase sample of size n_2 is selected from first phase sampling units n_1 and information on both study and auxiliary variable is recorded. Out of n_2 units, let n_{21} units respond and n_{22} do not. We assume that n_{21} units belong to the population of response group i-e N_1 and n_{22} belong to the population of nonresponse group i-e N_2. Using the Hansen and Hurwitz (1946) approach

of sub-sampling from n_{22} non-respondents, we take a subsample of $r = \dfrac{n_{22}}{k}$, k>1 so that the r units can be re-contacted by personal interview. Let $\bar{x}_1$ be the sample mean of auxiliary variable x_1 based on the first-phase sample.

1.1.1   Modified Hansen and Hurwitz' (1946)Estimator

To compare the estimators of two-phase sampling with Hansen and Hurwitz (1946) estimator, it is modified for two phase sampling as:

$$\bar{y}_2^* = \frac{n_{21}\bar{y}_2 + n_{22}\bar{y}_{2r}}{n_2} \tag{1.3}$$

The estimator $\bar{y}_2^*$ is unbiased and has variance

$$Var\ \bar{y}_2^* = \lambda_2 S_y^2 + \theta S_{y_2}^2 \tag{1.4}$$

where $\lambda_2 = \dfrac{1}{n_2} - \dfrac{1}{N}$, $\lambda_3 = \dfrac{1}{n_2} - \dfrac{1}{n_1}$ and $\theta = \dfrac{W_2}{n_2}\dfrac{k-1}{}$; $W_2 = \dfrac{N_2}{N}$

1.1.2   Khare and Srivastava (1993, 1995) Estimators:

Khare and Srivastava (1993, 1995) proposed the following ratio, product and regression type estimators using two-phase sampling where non-response is considered at second phase.

a) **Ratio Estimator**

$$T_{R2d} = \bar{y}_2^* \left(\frac{\bar{x}_1}{\bar{x}_2} \right) \tag{1.5}$$

$$Bias\ T_{R2d}\ \approx \frac{1}{\bar{X}}\ \lambda_3\ RS_x^2 - S_{xy} \tag{1.6}$$

and

$$MSE\ T_{R2d}\ \approx \lambda_3\ S_y^2 + R^2 S_x^2\ 1-2C\ +\lambda_1 S_y^2 + \theta S_{y2}^2 \tag{1.7}$$

where $C = \dfrac{\beta}{R}$, $\beta = \dfrac{S_{xy}}{S_x^2}$, $R = \dfrac{\bar{Y}}{\bar{X}}$

b) **Product Estimator**

$$T_{P2d} = \bar{y}_2^* \left(\frac{\bar{x}_2}{\bar{x}_1} \right) \tag{1.8}$$

$$Bias\ T_{P2d}\ \approx \frac{\lambda_3}{\bar{X}} RS_x^2 \tag{1.9}$$

and

$$MSE \ T_{P2d} \ \approx \lambda_3 \ S_y^2 + R^2 S_x^2 \ 1+2C \ +\lambda_1 S_y^2 + \theta S_{y_2}^2 \tag{1.10}$$

c) Regression Estimator

$$T_{LR2d} = \bar{y}_2^* + b^{**} \ \bar{x}_1 - \bar{x}_2 \ \text{where} \ b^{**} = \frac{S_{xy}}{S_x^2}, \tag{1.11}$$

$$Bias \ T_{LR2d} \ \approx \beta_{xy} \left(\frac{N \ N-n_1}{N-1 \ N-2} \frac{\mu_{21}}{n_1 \bar{X} S_{yx}} - \frac{N \ N-n_1}{N-1 \ N-2} \frac{\mu_{21}}{n_1 \bar{X} S_{yx}} \right) \tag{1.12}$$

$$MSE \ T_{LR2d} \ \approx \lambda_3 \ 1-\rho^2 \ S_y^2 + \lambda_1 S_y^2 + \theta S_{y_2}^2 \tag{1.13}$$

Tabasum and Khan (2004) revisited the ratio estimator T_{R2d} of Khare and Srivastava (1993) and found that the cost of these estimators is less than the cost gained by Hansen and Hurwitz' (1946) estimator.

1.1.3 Singh et al. (2010) Estimators:

Singh et al. (2010) suggested the followinggeneralized version of modified exponential ratio and exponential product type estimators using two-phase sampling.

$$t_{(b)d}^{(2)} = \bar{y}_2^* \exp\left[b\left(\frac{\bar{x}_2 - \bar{x}_1}{\bar{x}_2 + \bar{x}_1} \right) \right] \tag{1.14}$$

$$Bias \ t_{(b)d}^{(2)} \ \approx \frac{b}{2\bar{X}} \lambda_3 S_{yx} + \ 1-b \ \frac{\bar{Y}}{4\bar{X}_i^2} \lambda_2 S_x^2 + \ b+1 \ \frac{\bar{Y}}{4\bar{X}_i^2} \lambda_1 S_x^2 - 2\frac{\bar{Y}}{4\bar{X}_i^2} \lambda_1 S_x^2 \tag{1.15}$$

and

$$MSE \ t_{(b)d}^{(2)} \ \approx \lambda_2 S_y^2 + \theta S_{y2}^2 + \lambda_3 \left[\left(\frac{bR^2 S_x^2}{4} \right) \ b+4C \ \right] \tag{1.16}$$

When b = -1 and 1, the expression given in (1.14) becomes Exponential Ratio Estimator and Exponential Product Estimator along with their Bias and mean square error respectively.

Quite often, we possess information on several variates and it may be considered important to make use of the whole of the available material to improve the precision. Several authors have used the multi-auxiliary characters for estimating the population mean using the known values of population means of auxiliary variables without considering the problem of non response. Some of the names among them may be Olkin (1958), Shukla (1965, 66), Raj (1965), Rao and Mudholkar (1967), Mohanty (1967, 70), Srivastava (1971), Khare and Srivastava (1980, 81), Khare (1983), Srivastava and Jhajj (1983) and Sahoo (1986).

However, it is a common practice in sample surveys that the data may not be obtained for all the units selected in a sample due to some problem of non response. Tripathi and Khare (1997) considered the simultaneous estimation of several population means for

two types of non-responses (partial and complete) using single phase sampling. Khare and Sinha (2009) proposed two classes of estimators for estimating the population mean of study character using multi-auxiliary characters in the presence of non response.

In the following section we propose a general class of estimators in two phase sampling using multi-auxiliary variables for estimating population mean of study variable, in the presence of non-response for both study and auxiliary variables at second phase.

2. PROPOSED ESTIMATORS

In this section, we will suggest a general class of estimators for two-phase sampling using multi-auxiliary variables when non response is considered only on study variable.

Let we have p auxiliary variables $x_1, x_2, ...x_p; p = p_1 + p_2 + p_3$ to estimate our study variable y. The population of size N is divided into two groups, one group consisting of the units who respond called response class and the other group consisting of the units who do not respond is called non-response class. Further, let $\bar{X}_i$ denote the known population mean of ith auxiliary variable, $\bar{x}_{1(i)}$ and $\bar{x}_{2(i)}$ denote the sample means of ith auxiliary variable from the first and second phase samples respectively.

Let the sampling errors of study and auxiliary variables are respectively

$$\bar{y}_2^* = \bar{Y} + \bar{e}_{\bar{y}_2^*} \tag{2.1}$$

$$\bar{x}_{1i} = \bar{X}_i + \bar{e}_{\bar{x}_{1i}} \tag{2.2}$$

$$\bar{x}_{2i} = \bar{X}_i + \bar{e}_{\bar{x}_{2i}} \tag{2.3}$$

Let the deviation vectors are

$$d'_{1\times p_j} = \left[d_i \right]_{1\times p_k}, \qquad \text{where } d_i = e_{\bar{x}_{2i}} - e_{\bar{x}_{1i}} \text{ and } k = 1, 2 \& 3 \tag{2.4}$$

and $\quad d^{*'}_{1\times p_j} = \left[d_i^* \right]_{1\times p_k} \qquad \text{where } d_i^* = e_{\bar{x}_{2i}}^2 - e_{\bar{x}_{1i}}^2 \tag{2.5}$

The necessary expectations are:

$$E \; \bar{e}_{\bar{y}_2^*} = E \; \bar{e}_{\bar{x}_{2i}} = E \; \bar{e}_{\bar{x}_{1i}} = 0, \tag{2.6}$$

$$E \; d_{p_k \times 1} e_{\bar{y}_2^*} = \left[\lambda_3 S_{yx_i} \right]_{p_k \times 1} = \lambda_3 S_{yx_{p_k \times 1}}, \qquad \text{where } i = 1, 2, ..., p_k \tag{2.7}$$

$$E \; d_{p_k \times 1} d'_{1\times p_k} = \left[\lambda_3 S_{x_i x_j} \right]_{p_k \times p_k} = \lambda_3 S_{xx_{p_k \times p_k}}, \quad \text{where } i, j = 1, 2, ..., p_k \tag{2.8}$$

and $\quad E \; d^*_{p_k \times 1} = \left[\lambda_3 S_{x_i}^2 \right]_{p_k \times 1} = \lambda_3 S_{x_{p_k \times 1}}^2 \tag{2.9}$

We have proposed the following general class of estimators using multi-auxiliary variables.

$$t_1^m = \left(\bar{y}_2^* + a \sum_{i=1}^{p_1} \alpha_i \; \bar{x}_{1i} - \bar{x}_{2i} \right) \left[b \prod_{i=1}^{p_2} \left(\frac{\bar{x}_{1i}}{\bar{x}_{2i}} \right)^{c\beta_i} + d \exp \sum_{i=1}^{p_3} \gamma_i \left(\frac{g \; \bar{x}_{2i} - \bar{x}_{1i}}{\bar{x}_{2i} + \bar{x}_{1i}} \right) \right] \qquad (2.10)$$

where a, b, c, d and g are suitable constants to be chosen for generating members of this class and α, β and γ are unknown constants to be minimized for obtaining the minimum mean square error of the proposed class.

Using (2.1), (2.2) and (2.3) and ignoring the third and higher order terms for each expansion of product, we get

$$t_1^m \approx \left(\bar{Y} + e_{\bar{y}_2^*} + a \sum_{i=1}^{p_1} \alpha_i \; e_{\bar{x}_{1i}} - e_{\bar{x}_{2i}} \right) \left[b \left(1 + c \sum_{i=1}^{p_2} \beta_i \left(\frac{e_{\bar{x}_{1i}} - e_{\bar{x}_{2i}}}{\bar{X}_i} + \frac{e_{\bar{x}_{2i}}^2}{\bar{X}_i^2} - \frac{e_{\bar{x}_{1i}} e_{\bar{x}_{2i}}}{\bar{X}_i^2} \right) \right) \right.$$
$$\left. + d \left(1 + \sum_{i=1}^{p_3} \gamma_i \frac{g}{2\bar{X}_i} \; e_{\bar{x}_{2i}} - e_{\bar{x}_{1i}} - \sum_{i=1}^{p_3} \gamma_i \frac{g \; e_{\bar{x}_{2i}} - e_{\bar{x}_{1i}} \; e_{\bar{x}_{2i}} + e_{\bar{x}_{1i}}}{4\bar{X}_i^2} \right) \right]$$

or

$$t_1^m \approx \left(\bar{Y} + e_{\bar{y}_2^*} + a \sum_{i=1}^{p_1} \alpha_i \; e_{\bar{x}_{1i}} - e_{\bar{x}_{2i}} \right) \left(b + bc \sum_{i=1}^{p_2} \beta_i \frac{e_{\bar{x}_{1i}} - e_{\bar{x}_{2i}}}{\bar{X}_i} + bc \sum_{i=1}^{p_2} \beta_i \frac{e_{\bar{x}_{2i}}^2}{\bar{X}_i^2} \right.$$
$$\left. - bc \sum_{i=1}^{p_2} \beta_i \frac{e_{\bar{x}_{1i}} e_{\bar{x}_{2i}}}{\bar{X}_i^2} + d + d \sum_{i=1}^{p_3} \gamma_i \frac{g}{2\bar{X}_i} \; e_{\bar{x}_{2i}} - e_{\bar{x}_{1i}} - d \sum_{i=1}^{p_3} \gamma_i \frac{g \; e_{\bar{x}_{2i}} - e_{\bar{x}_{1i}} \; e_{\bar{x}_{2i}} + e_{\bar{x}_{1i}}}{4\bar{X}_i^2} \right)$$

or

$$t_1^m \approx \left(\bar{Y} + e_{\bar{y}_2^*} + a \sum_{i=1}^{p_1} \alpha_i \; e_{\bar{x}_{1i}} - e_{\bar{x}_{2i}} \right) \left(b + d + bc \sum_{i=1}^{p_2} \beta_i \frac{e_{\bar{x}_{1i}} - e_{\bar{x}_{2i}}}{\bar{X}_i} + bc \sum_{i=1}^{p_2} \beta_i \frac{e_{\bar{x}_{2i}}^2}{\bar{X}_i^2} \right.$$
$$\left. - bc \sum_{i=1}^{p_2} \beta_i \frac{e_{\bar{x}_{1i}} e_{\bar{x}_{2i}}}{\bar{X}_i^2} + d \sum_{i=1}^{p_3} \gamma_i \frac{g}{2\bar{X}_i} \; e_{\bar{x}_{2i}} - e_{\bar{x}_{1i}} - d \sum_{i=1}^{p_3} \gamma_i \frac{g \; e_{\bar{x}_{2i}} - e_{\bar{x}_{1i}} \; e_{\bar{x}_{2i}} + e_{\bar{x}_{1i}}}{4\bar{X}_i^2} \right)$$

Simplifying and writing in matrix notation, we get

$$t_1^m \approx \bar{Y} + e_{\bar{y}_2^*} - a\alpha'_{1\times p_1} d_{p_1 \times 1} \quad 1 - bc\beta'_{1\times p_2} X_{p_2 \times p_2} d_{p_2 \times 1}$$
$$+ bc\beta'_{1\times p_2} X_{p_2 \times p_2} X_{p_2 \times p_2} e^2_{\bar{x}_{2_{p_2 \times 1}}} - bc\beta'_{1\times p_2} X_{p_2 \times p_2} X_{p_2 \times p_2} e_{\bar{x}_{2_{p_2 \times 1}}} e'_{\bar{x}_{2_{1\times p_2}}} I_{p_2 \times 1}$$
$$+ \frac{dg}{2} \gamma'_{1\times p_3} X_{p_3 \times p_3} d_{p_3 \times 1} - \frac{dg}{4} \gamma'_{1\times p_3} X_{p_3 \times p_3} X_{p_3 \times p_3} d^*_{p_3 \times 1}$$

where $X = Diagonal \left[\bar{X}_1^{-1} \quad \bar{X}_2^{-1} \quad \cdots \quad \bar{X}_{p_k}^{-1} \right]$,

$$\alpha'_{1\times p_1} = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_{p_1} \end{bmatrix},$$

$$\beta'_{1\times p_2} = \begin{bmatrix} \beta_1 & \beta_2 & \cdots & \beta_{p_2} \end{bmatrix},$$

and $\quad \gamma'_{1\times p_3} = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_{p_3} \end{bmatrix}$

Ignoring third and higher order terms for each expansion of product and simplifying, we can write the expression as

$$t_1^m - \bar{Y} \approx e_{\bar{y}_2^*} - a\alpha'_{1\times p_1}d_{p_1\times 1} - \bar{Y}bc\beta'_{1\times p_2}X_{p_2\times p_2}d_{p_2\times 1} + bc\beta'_{1\times p_2}X_{p_2\times p_2}X_{p_2\times p_2}e^2_{\bar{x}_{2_{p_2\times 1}}}$$

$$-\bar{Y}bc\beta'_{1\times p_2}X_{p_2\times p_2}X_{p_2\times p_2}e_{\bar{x}_{2_{p_2\times 1}}}e_{\bar{x}_{2_{1\times p_2}}}I_{p_2\times 1} + \bar{Y}\frac{dg}{2}\gamma'_{1\times p_3}X_{p_3\times p_3}d_{p_3\times 1}$$

$$-\bar{Y}\frac{dg}{4}\gamma'_{1\times p_3}X_{p_3\times p_3}X_{p_3\times p_3}d^*_{p_3\times 1} - bc\beta'_{1\times p_2}X_{p_2\times p_2}d_{p_2\times 1}e_{\bar{y}_2^*} + \frac{dg}{2}\gamma'_{1\times p_3}X_{p_3\times p_3}d_{p_3\times 1}e_{\bar{y}_2^*}$$

$$+abc\alpha'_{1\times p_1}d_{p_1\times 1}d'_{1\times p_2}X_{p_2\times p_2}\beta_{p_2\times 1} - \frac{adg}{2}\alpha'_{1\times p_1}d_{p_1\times 1}d'_{1\times p_3}X_{p_3\times p_3}\gamma_{p_3\times 1} \qquad (2.11)$$

Squaring and taking expectation of (2.11) and ignoring the second order terms for each expansion of product, we can write the expression as

$$E\left(t_1^m - \bar{Y}\right)^2 \approx E\left(\bar{e}_{\bar{y}_2^*} - a\alpha'_{1\times p_1}d_{p_1\times 1} - \bar{Y}bc\beta'_{1\times p_2}X_{p_2\times p_2}d_{p_2\times 1} + \frac{\bar{Y}dg}{2}\gamma'_{1\times p_3}X_{p_3\times p_3}d_{p_3\times 1}\right)^2$$

or

$$MSE\left(t_1^m\right) \approx E\left[\bar{e}_{\bar{y}_2^*} - h'_{1\times 3}H_{3\times 1}\right]^2 \qquad (2.12)$$

where $h' = \begin{bmatrix} \alpha'_{1\times p_1} & \beta'_{1\times p_2} & \gamma'_{1\times p_3} \end{bmatrix}_{1\times 3}$ and $H = \begin{bmatrix} -ad_{p_1\times 1} \\ -\bar{Y}bcX_{p_2\times p_2}d_{p_2\times 1} \\ \frac{\bar{Y}dg}{2}X_{p_3\times p_3}d_{p_3\times 1} \end{bmatrix}_{3\times 1}$

To find the optimum value of h, differentiating the equation B w. r. t h and equating to zero, we get

$$2H_{3\times 1}E\left[\bar{e}_{\bar{y}_2^*} - H'_{1\times 3}h_{3\times 1}\right] = 0$$

or

$$E\left(H_{3\times 1}\bar{e}_{\bar{y}_2^*}\right) - E\left(H_{3\times 1}H'_{1\times 3}\right)h_{3\times 1} = 0$$

or

$$\phi_{yx_{3\times 1}} - T_{x_{3\times 3}}h_{3\times 1} = 0 \qquad (2.13)$$

where

$$T_x = \lambda_3 \begin{bmatrix} a^2 S_{xx_{p_1 \times p_1}} & \bar{Y}abc S_{xx_{p_1 \times p_2}} X_{p_2 \times p_2} & -a\dfrac{\bar{Y}dg}{2} S_{xx_{p_1 \times p_3}} X_{p_3 \times p_3} \\[2ex] \bar{Y}abc X_{p_2 \times p_2} S_{xx_{p_2 \times p_1}} & \bar{Y}^2 b^2 c^2 X_{p_2 \times p_2} S_{xx_{p_2 \times p_2}} X_{p_2 \times p_2} & -\dfrac{\bar{Y}^2 bcdg}{2} X_{p_2 \times p_2} S_{xx_{p_2 \times p_3}} X_{p_3 \times p_3} \\[2ex] -\dfrac{\bar{Y}adg}{2} X_{p_3 \times p_3} S_{xx_{p_3 \times p_1}} & -\dfrac{\bar{Y}^2 bcdg}{2} X_{p_3 \times p_3} S_{xx_{p_3 \times p_2}} X_{p_2 \times p_2} & \dfrac{\bar{Y}^2 d^2 g^2}{2} X_{p_3 \times p_3} S_{xx_{p_3 \times p_3}} X_{p_3 \times p_3} \end{bmatrix}$$

and $\phi_{yx_{3 \times 1}} = \lambda_3 \begin{bmatrix} a S_{yx_{p_1 \times 1}} \\[1.5ex] \bar{Y}bc S_{yx_{p_2 \times 1}} \\[1.5ex] \bar{Y}dg X_{p_3 \times p_3} S_{yx_{p_3 \times 1}} \end{bmatrix}$

From (2.13),

$$h_{3 \times 1} = T_{x_{3 \times 3}}^{-1} \phi_{yx_{3 \times 1}} \tag{2.14}$$

Now, from (2.11), ignoring the third and higher order terms for each expansion of product, we get

$$\begin{aligned}
E\left(t_1^m - \bar{Y}\right) \approx E\Big(& e_{\bar{y}_2^*} - a\alpha'_{1 \times p_1} d_{p_1 \times 1} - \bar{Y}bc\beta'_{1 \times p_2} X_{p_2 \times p_2} d_{p_2 \times 1} + \bar{Y}bc\beta'_{1 \times p_2} X_{p_2 \times p_2} X_{p_2 \times p_2} e^2_{\bar{x}_{2_{p_2 \times 1}}} \\
& - \bar{Y}bc\beta'_{1 \times p_2} X_{p_2 \times p_2} X_{p_2 \times p_2} e'_{\bar{x}_{2_{p_2 \times 1}}} e'_{\bar{x}_{1_{p_2}}} I_{p_2 \times 1} + \bar{Y}\frac{dg}{2} \gamma'_{1 \times p_3} X_{p_3 \times p_3} d_{p_3 \times 1} \\
& - \bar{Y}\frac{dg}{4} \gamma'_{1 \times p_3} X_{p_3 \times p_3} X_{p_3 \times p_3} d^*_{p_3 \times 1} - bc\beta'_{1 \times p_2} X_{p_2 \times p_2} d_{p_2 \times 1} e_{\bar{y}_2^*} + \frac{dg}{2} \gamma'_{1 \times p_3} X_{p_3 \times p_3} d_{p_3 \times 1} e_{\bar{y}_2^*} \\
& + abc\alpha'_{1 \times p_1} d_{p_1 \times 1} d'_{1 \times p_2} X_{p_2 \times p_2} \beta_{p_2 \times 1} - \frac{adg}{2} \alpha'_{1 \times p_1} d_{p_1 \times 1} d'_{1 \times p_3} X_{p_3 \times p_3} \gamma_{p_3 \times 1} \Big)
\end{aligned}$$

Now, using the results of expectation from (2.7), (2.8) and (2.9), bias can be written as:

$$\begin{aligned}
Bias\left(t_1^m\right) \approx & \ \bar{Y}bc\lambda_2 \beta'_{1 \times p_2} X_{p_2 \times p_2} X_{p_2 \times p_2} S^2_{x_{p_2 \times 1}} \\
& - \lambda_1 \bar{Y}bc\beta'_{1 \times p_2} X_{p_2 \times p_2} S_{xx_{p_2 \times p_2}} X_{p_2 \times p_2} I_{p_2 \times 1} - \lambda_3 \bar{Y}\frac{dg}{4} \gamma'_{1 \times p_3} X_{p_3 \times p_3} X_{p_3 \times p_3} S^2_{x_{p_3 \times 1}} \\
& + \lambda_3 abc\alpha'_{1 \times p_1} S_{xx_{p_1 \times p_2}} X_{p_2 \times p_2} \beta_{p_2 \times 1} - \lambda_3 \frac{adg}{2} \alpha'_{1 \times p_1} S_{xx_{p_1 \times p_3}} X_{p_3 \times p_3} \gamma_{p_3 \times 1} \\
& - \lambda_3 bc\beta'_{1 \times p_2} X_{p_2 \times p_2} S_{yx_{p_2 \times 1}} + \lambda_3 \frac{dg}{2} \gamma'_{1 \times p_3} X_{p_3 \times p_3} S_{yx_{p_3 \times 1}}
\end{aligned}$$

The optimum values of unknowns are given in equation (2.14). By using the normal equations that are used to find the optimum values given in equation (2.14), equation (2.12) can be written as

$$MSE\ \left(t_1^m\right) \approx E\ \left(e_{\bar{y}_2^*}\ e_{\bar{y}_2^*} - h_{1\times3}^{'} H_{3\times1}\right)$$

or

$$MSE\ \left(t_1^m\right) \approx E\ \left(e_{\bar{y}_2^*}^2\right) - h_{1\times3}^{'} E\ \left(H_{3\times1} e_{\bar{y}_2^*}\right)$$

or

$$MSE\ \left(t_1^m\right) \approx \lambda_2 S_y^2 + \theta S_{y_{(2)}}^2 - h_{1\times3}^{'} \phi_{yx_{3\times1}}$$

Now, using equation (2.12), we get:

$$MSE\ \left(t_1^m\right) \approx \lambda_2 S_y^2 + \theta S_{y_{(2)}}^2 - \phi_{yx_{1\times3}}^{'} T_{x_{3\times3}}^{-1} \phi_{yx_{3\times1}}$$

3. SPECIAL CASES OF PROPOSED ESTIMATORS

In this section we propose some special cases of proposed class of estimators. These members of the class are given in the following table.

Table 1: Some existing estimators as special cases of proposed class

S#	a	b	c	d	g	p_1	p_2	p_3	Estimator	
1	0	0	0	1	0	0	0	0	$\bar{y}_2^*$	Hansen and Hurwitz (1946)
2	0	1	1	0	0	0	1	0	$\bar{y}_2^* \left(\dfrac{\bar{x}_1}{\bar{x}_2}\right)$ Ratio	Khare and Srivastava (1993, 1995)
3	0	1	-1	0	0	0	1	0	$\bar{y}_2^* \left(\dfrac{\bar{x}_2}{\bar{x}_1}\right)$ product	
4	1	0	0	1	0	1	0	0	$\bar{y}_2^* + \alpha\ \left(\bar{x}_1 - \bar{x}_2\right)$ Regression	
5	0	0	0	1	1	0	0	1	$\bar{y}_2^* \exp\left(\dfrac{\bar{x}_1 - \bar{x}_2}{\bar{x}_1 + \bar{x}_2}\right)$ Exponential-ratio	Singh et al. (2010)
6	0	0	0	1	-1	0	0	1	$\bar{y}_2^* \exp\left(\dfrac{\bar{x}_2 - \bar{x}_1}{\bar{x}_2 + \bar{x}_1}\right)$ Exponential-product	

Some more special cases of proposed class of estimators are given in the following table.

Table 2: Special Cases of Proposed Class of Estimators

S#	A	b	c	d	g	Estimator	Type
1	0	1	1	0	0	$t_{r_1}^m = \bar{y}_2^* \prod_{i=1}^{p_2}\left(\dfrac{\bar{x}_{1i}}{\bar{x}_{2i}}\right)^{\beta_i}$	Ratio
2	0	1	-1	0	0	$t_{p_1}^m = \bar{y}_2^* \prod_{i=1}^{p_2}\left(\dfrac{\bar{x}_{2i}}{\bar{x}_{1i}}\right)^{\beta_i}$	Product
3	1	0	0	1	0	$t_{lr_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)$	Regression
4	-1	0	0	1	0	$t_{d_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)$	Difference
5	0	0	0	1	1	$t_{er_1}^m = \bar{y}_2^* \exp\left[\sum_{i=1}^{p_3}\gamma_i\left(\dfrac{\bar{x}_{1i}-\bar{x}_{2i}}{\bar{x}_{1i}+\bar{x}_{2i}}\right)\right]$	Exponential ratio
6	0	0	0	1	-1	$t_{ep_1}^m = \bar{y}_2^* \exp\left[\sum_{i=1}^{p_3}\gamma_i\left(\dfrac{\bar{x}_{2i}-\bar{x}_{1i}}{\bar{x}_{2i}+\bar{x}_{1i}}\right)\right]$	Exponential Product
7	1	1	1	0	0	$t_{rcr_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)\prod_{i=1}^{p_2}\left(\dfrac{\bar{x}_{1i}}{\bar{x}_{2i}}\right)^{\beta_i}$	Regression-cum-ratio
8	1	1	-1	0	0	$t_{rcp_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)\prod_{i=1}^{p_2}\left(\dfrac{\bar{x}_{2i}}{\bar{x}_{1i}}\right)^{\beta_i}$	Regression-cum-product
9	1	0	0	1	1	$t_{rcer_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)\exp\sum_{i=1}^{p_3}\gamma_i\left(\dfrac{\bar{x}_{1i}-\bar{x}_{2i}}{\bar{x}_{1i}+\bar{x}_{2i}}\right)$	Regression-cum-exponential-ratio
10	1	0	0	1	-1	$t_{rcep_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)\exp\sum_{i=1}^{p_3}\gamma_i\left(\dfrac{\bar{x}_{1i}-\bar{x}_{2i}}{\bar{x}_{2i}+\bar{x}_{1i}}\right)$	Regression-cum-exponential-product
11	-1	1	1	0	0	$t_{dcr_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)\prod_{i=1}^{p_2}\left(\dfrac{\bar{x}_{1i}}{\bar{x}_{2i}}\right)^{\beta_i}$	Difference-cum-ratio
12	-1	1	-1	0	0	$t_{dcp_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)\prod_{i=1}^{p_2}\left(\dfrac{\bar{x}_{2i}}{\bar{x}_{1i}}\right)^{\beta_i}$	Difference-cum-product
13	-1	0	0	1	1	$t_{dcer_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)\exp\sum_{i=1}^{p_3}\gamma_i\left(\dfrac{\bar{x}_{1i}-\bar{x}_{2i}}{\bar{x}_{1i}+\bar{x}_{2i}}\right)$	Difference-cum-exponential ratio
14	-1	0	0	1	-1	$t_{dcep_1}^m = \left(\bar{y}_2^* + \sum_{i=1}^{p_1}\alpha_i\ \bar{x}_{1i} - \bar{x}_{2i}\right)\exp\sum_{i=1}^{p_3}\gamma_i\left(\dfrac{\bar{x}_{1i}-\bar{x}_{2i}}{\bar{x}_{2i}+\bar{x}_{1i}}\right)$	Difference-cum-exponential product

The following table contains the expressions of bias and MSE of above special cases.

Table-3: Bias and MSEs of Special Cases

S#	Estimator	Bias	MSE
1	$t_{r_1}^m$	$\lambda_2 bc\beta'_{1\times p_2} X_{p_2\times p_2} X_{p_2\times p_2} S^2_{x_{p_2\times 1}}$	
2	$t_{p_1}^m$	$-\lambda_1 bc\bar{Y}\beta'_{1\times p_2} X_{p_2\times p_2} S_{xx_{p_2\times p_2}} X_{p_2\times p_2} I_{p_2\times 1}$ $-\lambda_3 bc\beta'_{1\times p_2} X_{p_2\times p_2} S_{yx_{p_2\times 1}}$	
3	$t_{lr_1}^m$		$\lambda_2 S_y^2 + \theta S_{y(2)}^2 - \lambda_3 \beta'_{1\times p_2} S_{yx_{p_2\times 1}}$
4	$t_{d_1}^m$	0	where $\beta'_{1\times p_2} = S'_{yx_{1\times p_2}} S^{-1}_{xx_{p_2\times p_2}}$
5	$t_{er_1}^m$	$-\dfrac{\lambda_3 \bar{Y} de}{4} \gamma'_{1\times p_3} X_{p_3\times p_3} X_{p_3\times p_3} S^2_{x_{p_3\times 1}}$	
6	$t_{ep_1}^m$	$+\dfrac{\lambda_3 de}{2} \gamma'_{1\times p_3} X_{p_3\times p_3} S_{yx_{p_3\times 1}}$	
7	$t_{rcr_1}^m$	$\bar{Y}\lambda_2 bc\beta'_{1\times p_2} X_{p_2\times p_2} X_{p_2\times p_2} S^2_{x_{p_2\times 1}}$	
8	$t_{rcp_1}^m$	$-\lambda_1 \bar{Y}bc\beta'_{1\times p_2} X_{p_2\times p_2} S_{xx_{p_2\times p_2}} X_{p_2\times p_2} I_{p_2\times 1}$	$\lambda_2 S_y^2 + \theta S_{y(2)}^2 + \lambda_3 \ -\beta'_{1\times p_1} S_{yx_{p_1\times 1}}$
9	$t_{dcr_1}^m$	$-\lambda_3 bc\beta'_{1\times p_2} X_{p_2\times p_2} S_{yx_{p_2\times 1}}$	$-\beta'_{1\times p_1} S_{xx_{p_1\times p_2}} W^{-1}_{x_{p_2\times p_2}} S_{xx_{p_2\times p_1}} \beta_{1_{p_1\times 1}}$
10	$t_{dcp_1}^m$	$+\lambda_3 abc\alpha'_{1\times p_1} S_{xx_{p_1\times p_2}} X_{p_2\times p_2} \beta_{p_2\times 1}$	$+2S'_{yx_{1\times p_2}} W^{-1}_{x_{p_2\times p_2}} S_{xx_{p_2\times p_1}} \beta_{1_{p_1\times 1}}$
11	$t_{rcer_1}^m$	$-\lambda_3 \bar{Y} \dfrac{de}{4} \gamma'_{1\times p_3} X_{p_3\times p_3} X_{p_3\times p_3} S^2_{x_{p_3\times 1}}$	$-S'_{yx_{1\times p_2}} W^{-1}_{x_{p_2\times p_2}} S_{yx_{p_2\times 1}}$
12	$t_{rcep_1}^m$	$+\lambda_3 \dfrac{de}{2} \gamma'_{1\times p_3} X_{p_3\times p_3} S_{yx_{p_3\times 1}}$	
13	$t_{dcer_1}^m$		
14	$t_{dcep_1}^m$	$-\lambda_3 \dfrac{ade}{2} \alpha'_{1\times p_1} S_{xx_{p_1\times p_3}} X_{p_3\times p_3} \gamma_{p_3\times 1}$	

4. THEORETICAL COMPARISON FOR THE PROPOSED CLASS

In this section we compare our proposed class of estimators with Hansen and Hurwitz (1946).

The comparison of proposed class of estimators with Hansen and Hurwitz (1946) is:

$$MSE \ \bar{y}_2^* - MSE \ t_1^m = \phi'_{yx_{1\times 3}} T^{-1}_{x_{3\times 3}} \phi_{yx_{3\times 1}} > 0$$

We can observe that our proposed class of estimators is efficient than Hansen and Hurwitz (1946).

5. EMPIRICAL COMPARISON

For empirical comparison, we have used the data set of Census report of Faisalabad district (1998) Pakistan, used by Ahmad, et al. (2009b). The size of this population is $N = 283$. The first 28% of the observations are considered as non-response $N_2 = 80$ and then $N_1 = 203$. The detail of study and auxiliary variables is given in Table A.1 of Appendix A. The necessary matrices required for the calculation of bias and MSE are given in Table A.2 and Table A.3 respectively. The results of percent relative efficiency (PRE) and bias of the proposed and existing estimators for different values of n_1, n_2 and k are given in Table A.6 and Table A.7 respectively.

We have developed a general class of estimators using multi-auxiliary variables consisting fourteen types of estimators.

The bias and PRE of existing and suggested estimators are given in Table A.6 and Table A.7 respectively. From Table A.6, the PRE of six estimators (ratio, product, regression, difference, exponential, exponential ratio and exponential product) is same and similarly, of eight remaining estimators is same. The class consisting of eight estimators is more efficient because the PRE of this class is greater than the class consisting of six estimators. To check the efficiency within this class of estimators, we need to observe the behavior of bias from Table A.7. This class of eight estimators having same MSE is again split according to two classes, each consisting of four estimators. By observing the values of bias, we can conclude that the class consisting of four estimators (Regression-cum-exponential-ratio, Regression-cum-exponential-product, difference-cum-exponential-ratio and difference-cum-exponential-product) is efficient, therefore it is preferred because it has minimum bias.

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Appendix A

Table A.1: Description of Variables

Description of variables	
Y_2	Population of currently married
X_1	Population of both sexes
X_2	Population of primary but below matric
X_3	Population of matric and above
X_4	Population of 18 years old and above
X_5	Population of women 15-49 years old

Table A.2: Average of all the Variables w. r. t N and N_2

Variables	N	Mean	N_2	Mean
Y_2	283	1511.261484	80	1825.9
X_1	283	10931.4417	80	13226.025
X_2	283	1969.286219	80	2622.5625
X_3	283	754.360424	80	1048.275
X_4	283	6173.162544	80	8146.425
X_5	283	2457.681979	80	2953.7375

Table A.3: Variance-covariance matrix w. r. t N

	Y_2	X_1	X_2	X_3	X_4	X_5
Y_2	621542.7328	3963614.8	792130.418	303104.0047	2481864.876	823818.0161
X_1	3963614.842	29441849	5836183.44	2230311.546	17507723.7	6588651.038
X_2	792130.4178	5836183.4	2815406.61	624113.7475	3788276.829	1390088.258
X_3	303104.0047	2230311.5	624113.748	276331.9476	1795351.395	533963.3136
X_4	2481864.876	17507724	3788276.83	1795351.395	39560446.26	4060606.428
X_5	823818.0161	6588651	1390088.26	533963.3136	4060606.428	2196836.707

Table A.4: Variance-covariance matrix w.r.t N_2

	Y_2	X_1	X_2	X_3	X_4	X_5
Y_2	915368.5722	5470158.3	1207503.21	429915.8	3939985.41	1236879.163
X_1	5470158.306	43396500	8701020.14	3059560.347	27495437.31	8863944.5
X_2	1207503.209	8701020.1	2605859.46	788441.1218	6822705.087	1983612.909
X_3	429915.8	3059560.3	788441.122	382683.9487	3553687.451	716956.6554
X_4	3939985.41	27495437	6822705.09	3553687.451	120333017.3	6217267.278
X_5	1236879.163	8863944.5	1983612.91	716956.6554	6217267.278	2000402.069

Table A.5: Some Essential Matrices used in the analysis

$S_{x_{r\times r}}$	29441849	5836183.44	2230311.546	$S_{x_{s\times s}}$	39560446.26	4060606.428
	5836183.4	2815406.61	624113.7475		4060606.428	2196836.707
	2230311.5	624113.748	276331.9476	$S_{x_{2\,s\times s}}$	120333017.3	6217267.278
$S_{x_{2\,r\times r}}$	43396500	8701020.14	3059560.347		6217267.278	2000402.069
	8701020.1	2605859.46	788441.1218		17507723.7	6588651.038
	3059560.3	788441.122	382683.9487	$S_{x_{s\times r}}$	3788276.829	1390088.258
$S_{x_{2\,s\times r}}$	27495437.31	8863944.5	-----		1795351.395	533963.3136
	6822705.087	1983612.909	$S'_{yx_{1\times r}}$	3963614.8	792130.418	303104.0047
	3553687.451	716956.6554	$S'_{yx_{2\,1\times r}}$	5470158.3	1207503.21	429915.8
$S'_{yx_{1\times s}}$	2481864.876	823818.0161	-----	$S'_{yx_{2\,1\times s}}$	3939985.41	1236879.163

Table A.6: Percent Relative Efficiency w.r.t $\bar{y}_2^*$ of Existing and Proposed Estimators

k	n_1	n_2	T_{R1d}	T_{lr1d}	$t_{R(-1)d}^{(1)}$	$t_{r_1}^m, t_{p_1}^m, t_{l_1}^m,$ $t_{d_1}^m, t_{ep_1}^m, t_{er_1}^m$	$t_{rcr_1}^m, t_{rcp_1}^m, t_{dcr_1}^m, t_{dcp_1}^m,$ $t_{rcer_1}^m, t_{rcep_1}^m, t_{dcer_1}^m, t_{dcep_1}^m$
2	150	110	128.78	128.81	120.58	128.81	156.26
2	150	120	121.01	121.03	115.29	121.03	138.83
2	160	110	135.48	135.52	125.01	135.52	172.99
2	160	120	127.71	127.74	119.87	127.74	153.75
3	150	110	118.95	118.97	113.85	118.97	134.53
3	150	120	113.96	113.97	110.32	113.97	124.59
3	160	110	122.95	122.97	116.63	122.97	143.01
3	160	120	118.08	118.10	113.24	118.10	132.75
4	150	110	114.12	114.14	110.44	114.14	124.91
4	150	120	110.45	110.46	107.79	110.46	117.99
4	160	110	116.96	116.98	112.46	116.98	130.49
4	160	120	113.42	113.43	109.93	113.43	123.54
5	150	110	111.26	111.27	108.38	111.27	119.48
5	150	120	108.35	108.36	106.26	108.36	114.18
5	160	110	113.45	113.46	109.96	113.46	123.62
5	160	120	110.67	110.67	107.95	110.67	118.38

Table A.7: Bias of Existing and Proposed Estimators

k	n_1	n_2	T_{R1d}	$t^{(1)}_{R(-1)d}$	$t^m_{r_1}, t^m_{p_1}$	$t^m_{ep_1}, t^m_{er_1}$	$t^m_{rcr_1}, t^m_{rcp_1}, t^m_{dcr_1}, t^m_{dcp_1}$	$t^m_{rcer_1}, t^m_{rcep_1}, t^m_{dcer_1}, t^m_{dcep_1}$
2	150	110	0.02	0.012	-0.57	-0.46	0.30	-0.11
2	150	120	0.02	0.008	-0.58	-0.32	0.35	-0.07
2	160	110	0.03	0.014	-0.49	-0.54	0.22	-0.13
2	160	120	0.02	0.010	-0.50	-0.40	0.26	-0.09
3	150	110	0.02	0.012	-0.57	-0.46	0.30	-0.11
3	150	120	0.02	0.008	-0.58	-0.32	0.35	-0.07
3	160	110	0.03	0.014	-0.49	-0.54	0.22	-0.13
3	160	120	0.02	0.010	-0.50	-0.40	0.26	-0.09
4	150	110	0.02	0.012	-0.57	-0.46	0.30	-0.11
4	150	120	0.02	0.008	-0.58	-0.32	0.35	-0.07
4	160	110	0.03	0.014	-0.49	-0.54	0.22	-0.13
4	160	120	0.02	0.010	-0.50	-0.40	0.26	-0.09
5	150	110	0.02	0.012	-0.57	-0.46	0.30	-0.11
5	150	120	0.02	0.008	-0.58	-0.32	0.35	-0.07
5	160	110	0.03	0.014	-0.49	-0.54	0.22	-0.13
5	160	120	0.02	0.010	-0.50	-0.40	0.26	-0.09

FACTORS CAUSING JEALOUSY AMONG STUDENTS

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ABSTRACT

The research is conducted on "factors causing jealousy among students". The objective of the study was to find out the factors that causes jealousy. A cross sectional study using structural questionnaire was carried out. A sample size of 240 in which 50% males and 50% females was collected from Kinnaird College, Queen merry College, Government Fatima Jinnah College, Forman Christian college, MAO College and Government College Ravi Road. Pearson chi square test was used to find association between gender and different variables that are causing jealousy. Factor analysis used to find the significant factors causing jealousy. The results showed that prevalence of jealousy is significant in women. Moreover jealousy with respect to qualification, finance, lack of confidence and less parent's attention, low family income, hatred, insecurity and less trust are the main factors of jealousy.

1. INTRODUCTION

Jealousy makes people focus on the negative, which will only fuel your anxiety. When people are jealous, they often take things too personally. People who are jealous must first figure out what they hope to accomplish through their jealousy. It is a often painful emotional response that typically occurs in relationships when a third party is seen as a threat in some way. Jealousy is a secondary emotion and typically refers to the negative thoughts and feelings of insecurity, fear, and anxiety over an anticipated loss of something that the person values, particularly in reference to a human connection. Jealousy often consists of a combination of presenting emotions such as anger, sadness, and disgust. Jealousy is a familiar experience in human relationships. There are different types of jealousy. Family jealousy most commonly occurs between siblings, but can occur between cousins or any other relative. This type of jealousy is usually borne out by too much attention being given to one person over another. Jealousy among friends is one of the most commonly outspoken types of jealousy. In school-aged friends, jealousy is common when one excels at an activity, subject or socialization. Professional jealousy, also known as work jealousy, refers to the jealousy that arises between coworkers; for example, often following a promotion or any other workplace success that is not awarded to every employee. Normal jealousy is the most common form of jealousy, which usually amounts to little more than someone's "heart dropping" when they see something they wish for. Normal jealousy is often easy to conceal and cope with, but can easily make a person self-conscious and branch off into other types of jealousy. Abnormal jealousy is usually defined as a psychological disorder in which a person experiencing this type of jealousy cannot conceal it. A person with abnormal jealousy may feel insecure, and

oftentimes a person will go out of her way and harm someone else in an attempt to satisfy herself.

The following are possible root causes for jealousy. The main cause for feelings of jealousy is one's doubts about one's abilities or skills. Having a poor self-image is another cause of jealousy. If one believes that he or she looks ugly then chances are this person will be experiencing feeling of jealousy whenever he or she meets someone who looks better than him or her. One of the root causes behind jealousy is being afraid. This fear can be a fear of ending up alone, a fear of being rejected or a fear of losing something. Feelings of insecurity are the result of the two previously mentioned causes. A poor self-image and lack of self confidence can result in making one feel insecure about a relationship and this can be a strong reason that can make one jealous. Building one's self-confidence, knowing about one's points of strength and knowing for certain that one is worthy can be sufficient to eliminate any feelings of jealousy. Contrary to common beliefs, it is possible to fix one's mental self-image and to change the idea one has about his/her looks. Fixing one's mental self-image will have a great impact on one's self-confidence and will certainly reduce those feelings of jealousy. One's physical self-image plays an important role too so one should make sure that both the mental and physical self-image are adjusted. If one experiences jealousy in relationships then building one's self-confidence and fixing one's self-image are bound to remove any feelings of insecurity one might be feeling. If one feels jealous of one's colleagues at work then one might want to check the Ultimate guide to dealing with insecurity in order to better know how to get rid of those feelings of insecurity and so reduce the feelings of jealousy.

Whatever the kind of fear one is experiencing, dealing with one's fears will help a lot in one's journey for getting over jealousy; whether it's a fear of abandonment or a fear of rejection; dealing with one's fears is one basic step that helps in dealing with jealousy. Even though jealousy comes from within sometimes one needs to fix the external world along with fixing one's inner-self. One direct result of jealousy is anger. Anger not only makes one feel bad and frustrated but also it destroys one's relations with others. On one's journey to deal with jealousy, one should make sure one knows how to control and manage one's anger to avoid unwanted consequences.

2. PROBLEM STATEMENT

What are the factors that cause jealousy among students?

3. OBJECTIVE OF THE STUDY

To identify the most common type of jealousy among friends or family, explore the factors that cause jealousy and determine the comparison among gender regarding jealousy.

4. RESEARCH METHODOLOGY

The population consists of the students of Lahore. The target population is divided according to three economic classes, Kinnaird College for women, Forman Christian College Lahore, Queen Mary College, MAO College, Government Fatima Jinnah

College and Government College Ravi Road. A cross sectional study conducted among the students of colleges. A structural questionnaire has been used as a tool for collecting data. Stratified sampling design has been used to obtain the sample. A total of 240 students (subjects) given a questionnaire about jealousy and to fill in personally. Data collected on jealousy is a primary data. After collection of primary data through the questionnaire data compiled, tabulated and analyzed on spreadsheet Excel, Minitab and SPSS. Researcher applied Chi-square test of association between the variables and factor analysis to find factors.

5. RESULTS AND INTERPRETATION

H_0: prevalence of jealousy in females is less than males

H_1: prevalence of jealousy in females is greater than males

Proportion testing of jealousy among gender

Sample	X	N	Sample	p-value
1	225	240	0.937500	0.000
2	15	240	0.062500	0.000

Difference= $p(1) - p(2)$

$p(1) - p(2) = 0$ (vs. > 0): $Z = 39.60$ P-Value $= 0.000$

INTERPRETATION

As p-value is 0.000 which is less than 0.05 so we reject H_0 and accept H_1. So we conclude Prevalence of jealousy is more in females than males.

SIGNIFICANCE VARIABLES WITH GENDER

Variable1 * Variable2	p-value	Contingency coefficient
Gender * jealousy is more common among friends or family	0.00	0.205
Gender * Insecurity	0.00	0.273
Gender * Hatred	0.00	0.188
Gender * Competition	0.001	0.233
Gender * Sincerity	0.00	0.286

INTERPRETATION:

As p value is less than 0.05 so these variables are significant with gender.

SIGNIFICANCE VARIABLES WITH SOCIO ECONOMICS

Variable1 * Variable2	p-value	Contingency coefficient
Socio economics * jealousy is more common	0.038	0.7
Socio economics * jealousy is more common among friend or family	0.038	0.7
Socio economics * insecurities	0.029	0.8
Socio economics * hatred	0.004	0.7
Socio economics * competition	0.00	0.7
Socio economics * good dress	0.03	0.8
Socio economics * teacher preference	0.03	0.8
Socio economics * more qualified	0.00	0.8
Socio economics * financially strong	0.01	0.8
Socio economics * reaction	0.023	0.8

INTERPRETATION:

As p value is less than 0.05 so these variables are significant with socio economics status.

FACTOR ANALYSIS

	Component					
	1	**2**	**3**	**4**	**5**	**6**
what is your family income	.115	.410	-.470	.173	.163	-.165
your level of confidence	-.040	.566	-.202	.212	-.425	.189
do you think jealousy is more common in	-.381	.064	.058	-.028	.062	.466
according to your opinion jealousy is more common among	-.084	-.023	-.064	.719	-.057	-.003
jealousy occurs more in a relationship with less trust	-.054	.052	.082	.029	.067	-.757
do you have any feeling of insecurity	-.378	.297	.014	-.387	-.817	.202
jealousy creates hatred	.193	-.022	.727	-.752	.041	-.095
do you think jealousy creates competition among friends and family	.113	-.023	.681	.412	-.037	-.019
when you look someone more attractive than you, what will you feel	-.335	.598	.240	-.006	.092	-.172
when someone wearing good dress what will you feel	.250	-.509	.150	.110	.057	.134
do you think you are sincere with your friends	.500	-.230	.286	.471	.181	.015
do you think your friends would leave you if better someone better came along	.337	-.019	.049	.398	.316	.389
if your friends get good grades what will you feel	-.075	.092	-.103	-.026	.057	.015
how do you feel when your teacher gives more preference to your friends	.460	-.277	.110	.112	.423	.003
what will be your reaction if your friend/cousin got engaged/married earlier than you	.611	-.019	.174	-.103	-.064	.048
if in a family someone is more qualified than you, you will feel	.707	-.032	-.014	.042	.109	-.077
if in your family someone is more financially strong than you will feel	.724	-.136	.061	.050	-.062	.013
what will be your feeling when your parents give more attention to your siblings	.067	.534	-.055	-.288	.083	.224

Jealousy with respect to qualification and finance, Low level of confidence and less parent's attention, Low family income, Insecurity and Less trust are main factors of jealousy.

6. CONCLUSION

The research is about factors causing jealousy among students. The study identifies 85.8% of respondents are jealous and 14.2% of respondents are not jealous. The results

conclude that jealousy occurs due to insecurity, competition and hatred. However teacher preference to other student and lack of self confidence also play a vital role in causing jealousy. Best way in order to cope with jealousy is to avoiding the people from you are jealous. The results of proportion test shows that prevalence of jealousy is more in women than men. The bivariate analysis shows that confidence level, insecurity, competition, sincerity and hatred are associated with gender. Jealousy is more common among friends or family, insecurities, competition, wearing good dress, someone is more financially strong and different reactions due to jealousy are associated with socio economics status. Although these factors are significant with socio economics but jealousy equally present in every class to some extent. Therefore socio economics status shows low involvement with respect to jealousy. The results of proportion test shows that prevalence of jealousy is more in women than men. In multivariate analysis, factor analysis concludes that jealousy with respect to qualification and finance, low level of confidence and less parent's attention, low family income, hatred, insecurity and less trust are significant factors causes jealousy among students.

7. RECOMMENDATIONS

- This study has only been conducted on the students due to time and resource constraints. The survey should be conducted from common man of rural and urban areas, by doing so; we can improve this research as that would give a better representation of the population.
- For the further analysis it could be recommended that the variables which are extracted can be used for further multivariate, regression, analysis.

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FACTORS AFFECTING THE ACADEMIC PERFORMANCE
OF STUDENTS

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ABSTRACT

College students have many obstacles to overcome in order to achieve their optimal academic performance. The study was aimed to find the factors which are affecting the academic performance of the students. The objective of the study was to find whether socio-economic class, student attitude toward studies, family behavior and teacher's friendly behavior are the main factors affecting student academic performance. A structured questionnaire was used to collect primary data. A sample of size 150 was collected from Kinnaird College, Queen Marry College and Islamia College Cooper Road. Percentages and chi-square test were used to analyze research questions. The result of Pearson chi-square indicates that CGPA is strongly associated with family income, mothers' education, score in quizzes, access to internet and time spent on studies. It also shows that family income is associated with time spent on studies and score in quizzes. Factor analysis was applied to find some major factor and it was examined that time spent on studies, teacher's cooperation, family income, parents' qualification, family behavior, attendance, Friends Company, extracurricular activities, time management, coaching, stress and sleep are significant factors that affects student academic performance. This study concluded that student's academic performance is dependent on family income, family support, teachers' cooperation and student attitude toward studies.

1. INTRODUCTION

Academics play an important role in creating a student personality, future and his/her life. Academics helps a student to know about his/her society, culture, history, geography, morals, environment and the daily math that he/she needs in every days life. Academics help a student to prepare for the real world and opens up doors to the world around him/her.

Academic performance refers to how students deal with their studies and how they cope with or accomplish different tasks given to them by their teachers. Academic performance is the ability to study and remember facts and being able to communicate your knowledge verbally or down on paper.

In educational institutions, success is measured by academic performance, or how well a student meets the standards set out by the local government and the institution itself. As career competition grows ever fiercer in the working world, the importance of students doing well in school has caught the attention of parents, legislators and government education departments alike.

These are some factors affecting student's academic performance
- Grades
- Attendance
- Extracurricular activities
- Behavior
- Group study
- Time management
- Communication
- Socio-Economic Background
- Parents' Education
- Parents' support
- Student motivation toward studies
- Contribution of Teacher
- Peer Group Influence
- Learning Environment
- Pressure
- Stress.

2. PROBLEM STATEMENT

What are the factors that affect the academic performance of the students?

3. OBJECTIVES OF THE STUDY

The objective of the study are (a) To explore the student's behavior towards academic performance. (b) To identify how family attitude affect the student's academic performance. (c) To assess how socio-economic conditions affect the academic performance of the students. (d) To determine the contribution of teachers towards the academic performance of the students.

4. RESEARCH METHODOLOGY

The population consists of the female students of Lahore. The target population is divided according to three economic classes upper, middle and lower. And three colleges from each class were selected (Kinnaird College for women, Queen Mary College and Islamia College, Copper Road). A cross sectional study was conducted among the students of these colleges. A structural questionnaire has been used as a tool for collecting data. Stratified sampling design has been used to obtain the sample. A cross sectional study conducted among the students of different universities. A total of 150 students were selected as a sample size. After collection of primary data through the questionnaire data compiled, tabulated and analyzed on spreadsheet Excel, Minitab and SPSS. Researcher applied Chi-square test of association between the variables and factor analysis to find factors.

5. RESULTS AND INTERPRETATIONS

Table 1:
Bivariate significant results (p-value<α)

Variable1*variable2	p-values
CGPA* Family Income	.002
CGPA* Mother education	.001
CGPA* Time spend on studies	.031
CGPA* Mode of study	.000
CGPA* Score in quizzes	.004
CGPA* Activities at home	.014
CGPA* Internet at home	.003
Income* Time spend on studies	.018
Income* Score in studies	.000
Family behavior * Mode of study	.003

INTERPRETATION

As p value is less than 0.05 so these variables are significant.

Table 2:
Multivariate results of cases using factor analysis

FACTORS	NAMES
1	Time spent on studies and teachers cooperation
2	Family income and access to administrative office
3	Parents qualification
4	Attendance and low grades
5	Extracurricular activities
6	Friends company
7	Coaching
8	Stress and sleep
9	Assignment and clarity of lectures
10	Family behavior and college facilities
11	Time management
12	Type of resident

INTERPRETATION:

Table 2 shows the result of factor analysis. Factor analysis was applied to find some major factor and it was examined that time spent on studies, teacher's cooperation, family income, parents' qualification, family behavior, attendance, Friends Company, extracurricular activities, time management, coaching, stress and sleep are significant factors that affects student academic performance.

6. CONCLUSION

The outcome of the study offers an important insight into factors that affect students' performance. Academic performance is measured through the ordinal scale of grade point average (GPA). According to the results of the Univariate analysis, 91% of the respondents have a CGPA between 2.5 to 4.00, which is considered good. In bivariate analysis, Chi-square test of association and Pearson's rank correlation were applied. On the basis of the obtained findings it is concluded that there is a strong association of CGPA with family income, mothers' education, score in quizzes, and access to internet and time spent on studies. As the p-value of all these variables is less than .05 that is why we accept researcher hypothesis. It also shows a strong association of family income with time spent on studies and scores in quizzes.

In multivariate analysis, the outcomes of factors analysis revealed that the academic performance of a student is affected by time spent on studies, teacher cooperation, family income, parent's qualification, family behavior, attendance, friends company, extracurricular activities, time management, coaching, stress and sleep. The overall conclusion of this study is that student's academic performance is dependent on family income, family support, teacher's cooperation and student attitude towards study.

7. RECOMMENDATIONS

As for recommendations, for a better representation of the population the sample size selected for this study should be large. This study has only been conducted on the female students of select colleges, but there is a need to carry out this research on both male and female students of a larger number of colleges. Also due to time and resource constraints the survey has not been conducted on rural areas students, by doing so; we can improve this research as that would give a better representation of the population. It has been observed from the survey results that students are often biased in their responses and don't tell about their actual study habits, so as to create an image, which they find admirable, this tends to give distorted results. There is a need to create awareness amongst students about the importance of such studies and how they should be taken seriously so that true benefits can be reaped by these studies. Taking into account all these considerations will give more accurate results that will help the academic performance of the students in future.

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EMPATHY AND SOCIAL ANXIETY IN FICTION AND NON-FICTION READERS

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ABSTRACT

Book readers are perceived to be lacking in social skills even though this may only be true for non-fiction readers and not for fiction readers. Reading fiction has been shown to improve empathy and decrease social anxiety. The aim of the current study was to compare the levels of empathy and social anxiety of fiction and non-fiction readers. Convenient sampling was used to recruit a Sample of 50 fiction readers and 50 non-fiction readers from among the students of the University of Central Punjab, Lahore. No restrictions of religion, marital status, family set up, or socioeconomic class status was made. The questionnaires they were administered consisted of the Author Recognition Test, the Basic Empathy Scale and the Revised Cheek and Buss Shyness Scale. The Statistical Package for Social Sciences, version 17.0 was used to analyze the data. Correlation and Comparative research design were employed. The scores of both groups were compared using Independent sample t-test, which revealed no significant difference in the levels of empathy or social anxiety between fiction and non-fiction readers. However, Pearson Product Moment Correlation revealed that there is a significant negative correlation between empathy and social anxiety, which implies that people who scored high on empathy scored low on social anxiety. It is recommended that future research be conducted on a larger sample.

KEYWORDS

Empathy, Social Anxiety, Fiction, Non-fiction, Pakistani College students.

1. INTRODUCTION

Empathy involves sharing what one perceives or thinks somebody else is feeling. It can be defined from a multidimensional perspective, having cognitive and affective dimensions. Affective empathy is an emotional response to other people's situation while cognitive empathy is the ability to recognize and apprehend the thoughts and viewpoints of another person (Dovidio, 2006; Strayer & Eisenberg, 1990; Garaigordobil, 2009). As technology is advancing, people are becoming increasingly socially isolated. This indicates a decrease in social support systems which correlates negatively with social anxiety (Hampton, Sessions, & Her, 2011). Several theorists are of the viewpoint that empathy can helps in creating and developing positive social interactions and relationships (Leite et al. 2007). Previous researches prove that empathy correlates positively with positive social behaviors like consideration for others and negatively with social anxiety and negative social behaviors like antisocial behavior and withdrawal (Garaigordobil, 2009).

Social Anxiety is the anxiety that results from real or anticipated observation in social situations (Al-Ali, Singh & Smekal, 2011). It has been proved that the tendency to evade social interactions, like shy and socially anxious people do, prohibits the development of social competence and positive social behaviors (Burgess, Rubin, Cheah & Nelson, 2001).

Where Fiction is defined as something invented by the imagination or simulated or the act of feigning or creating with imagination (Merriam-Webster Dictionary, 2012). Non-fiction consists of works reporting facts (Foustas & Pinnell, 2001). Previous researches have suggested reading fiction may improve empathy and decrease social anxiety as compared to reading non-fiction (Mar, Oatley and Peterson, 2009.,Mar et al. 2005).

The aim of this study is to compare the levels of empathy and social anxiety in fiction and non-fiction readers, and to find the relationship between empathy and social anxiety. It was hypothesized that fiction readers will have more empathy and better social relationships than non-fiction readers and that there will be a negative relation between empathy and social anxiety.

2. METHODOLOGY

Participants
A total of 50 fiction readers and 50 non-fiction readers from the University of Central Punjab participated in the study. No restriction on age, sex, marital status, religion or family setup was made.

Tools
Basic Empathy Scale (BES)
Basic Empathy Scale was used to measure empathy (Jolliffe, and Farrington,2006). The scale consists of 20 items. The responses are on a five-point Likert scale ranging from 1 representing 'strongly disagree' to 5 representing 'strongly agree'. Items 1, 6, 7, 8, 13, 18, 19 and 20 are reverse scored. Scale scores range from 20 (lowest empathy) to 100 (highest empathy). The scale is a reliable and valid tool. Correlation of the BES with the Interpersonal Reactivity Index, which measures both cognitive and affective components of empathy, ranges from .43 for females to .53 for males. Reliability ranges from .79 for the cognitive scale to .85 for the affective scale (Jolliffe & Farrington, 2006).

Author Recognition Test (ART)
For classifying participants as fiction or non-fiction readers, the Author Recognition Test was used (Mar,R., Oatley, K., Hirsh, J., Paz, J., & Peterson, J. B. 2005). Participants are asked to check off from a list of names, the names of authors that they recognize. Guessing or indiscriminate checking is prevented by the use of foils or fake names that are subtracted from the total scores. The predictive validity of the ART is higher than self-report measures and equal to the daily diary approaches. The overall reliability of the test is .96 (Mar et al. 2005).

Revised Cheek and Buss Shyness Scale (RCBS)
The Cheek and Buss Shyness Scale was used to assess social anxiety and the tendency to become inhibited in social situations. The scale originally consisted of 9 items, but subsequent revisions have yielded 11 and 13-item versions. The 13 item

version was used for this research. The scoring is on a five-point Likert scale with 1 for 'very uncharacteristic' to 5 for 'very characteristic'. Items 3, 6, 9 and 12 are reverse scored. Scale scores range from 13 (lowest shyness) to 65 (highest shyness). The scale has significant reliability and validity. Internal consistency of the scale is .90; test-retest reliability is .88. Convergent validity is also high, ranging from .77 with Social Avoidance and Distress Scale and .86 with Interaction Anxiousness to .79 with Social Reticence (Robinson, Wrightsman & Andrews, 1991).

Procedure

Permission sought from the Authors of the Basic Empathy Scale, the Author Recognition Test and the Revised Cheek and Buss Shyness Scale. A formal permission was taken from the University of Central Punjab to conduct the study in their institution. Then written consent of the participants was taken and they were informed about the purpose and rationale of the study. It was explained to them that they could withdraw from the study any time that they wished to do so and that the data obtained would be used only for research purposes. The questionnaires administered, approximate time taken to fill the questionnaires was 15 minutes.

3. RESULTS

The means, standard deviations, and correlations among the variables were calculated using Statistical Package for Social Sciences, Version 17.0 and is shown in the tables 1.

Table 1
Mean Standard Deviation, Independent Sample t-test, d.f, and correlation for Empathy and Social Anxiety of Fiction and Non-fiction readers (n-100)

	Fiction/Non-fiction	Mean	SD	*t*	df r
Empathy	Fiction readers	69.28	9.99	0.023	98
	Non-fiction readers	69.32	7.27		-.229*
Social Anxiety	Fiction Readers	35.8	9.12	0.58	98
	Non-fiction readers	36.8	8.08		

Note: * p < 0.05

The above table shows that there is no significant difference between the levels of empathy and social anxiety of fiction and non-fiction readers. There is a significant negative relation between empathy and social anxiety.

4. DISCUSSION

Results of the study partially support the hypotheses of the study. Independent sample t-test (Table 1) indicated that there was no significant difference in the levels of empathy and social anxiety in fiction and non-fiction readers. Some of the reasons for this can be difference in language and culture of the sample and the authors in the Author Recognition Test. Also, the Author Recognition Test does not test the actual amount of reading a person has done but only his exposure to a certain kind of literature.

Cheon, Mathur, and Chiao (2010) are of the viewpoint that individuals from a similar cultural background are more responsive to the quality and strength of verbal and non

verbal expressions of others who share their culture, which results in a better understanding and more empathy towards someone with a similar cultural background. Culture influences a persons's perception of the feeling of oneself and of others.

Moreover, Pakistan is a collectivist society. The people of Pakistan, in general, have strong social support systems which act as a hindrance for social anxiety. Frese (2009) studied social support as a moderator of relationship between work stressors and psychological dysfunctioning. Results indicated that psychological dysfunction as a result of stress is higher when social support is lower and lower when social support is higher. So, social support acts as a buffer to stress, especially for dysfunctions such as social anxiety.

However, there was a negative relationship between social anxiety and empathy. Previous literature supports the findings of the present study. Achim and colleagues examined and compared empathy and social anxiety and found a negative correlation between them.

5. LIMITATIONS AND RECOMMENDATIONS

The sample of the present study is very small, it consists of 100 participants. The sample size needs to be increased and a more reliable sampling strategy needs to be applied so that results are more generalizable.

Furthermore, there are a limited number of authors in the ART, there might be other authors that the participants know of or read. Also, the ART measures only a person's exposure to certain text and the actual amount of reading he does. Moreover, English is a second language in Pakistan and the authors of the ART write exclusively in English. This difference in the culture and language of the authors of the ART (and the characters they write about) and the participants may also be interfering in the comprehension and effects of this literature on the participants. So, a more reliable method of classifying participants as fiction or non-fiction readers needs to be used, in addition to the ART. For example, the daily diary approaches.

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A STUDY ON FACTORS CAUSING SOCIAL ANXIETY
AMONG YOUTH OF LAHORE

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ABSTRACT

Social Anxiety is the third most common psychological problem in the world these days. Prevalence rate of Social Anxiety is more in Adolescent of age between 14 to 19 years. This research focuses on factors (Gender, Family System, Family Income, Mother and Father Education, Presence of Trauma, Personality Traits and Self Esteem) causing Social Anxiety in youth of Lahore. Multiple Sampling designs including Stratified Random Sampling, Cluster Random Sampling and Convenience Sampling have been used; a sample of 150 was selected from students of Lahore Grammar School, Crescent Model School, Queen Mary College and WAPDA School belonging to different Socio Economic status. A cross sectional study using structured questionnaire was carried out including basic information, Social Interaction Anxiety Scale by Mattick and Clarke (1998), Big Five Personality Inventory by John & Martinez (1998) and State Self Esteem Scale by Heatherton & Polivy (1991). Univariate, bivariate and multivariate analyses were carried out on the data collected. Results indicate that only 9.3% of the respondents are facing Social Anxiety. Test of association indicates that Social Anxiety is significantly associated with socio-economic status, personality traits and self-esteem, while no relationship was found between Gender and Social Anxiety. Another important factor socio-economic status is also significantly associated with personality traits, self-esteem, trauma and family system. Multiple Regression Model was developed which indicates that Social Self Esteem, Extraversion, Socio Economic Status and Conscientiousness are strong predictors of Social Anxiety in youth of Lahore.

1. INTRODUCTION

Anxiety (also called anguish or worry) is a psychological and physiological state characterized by somatic, emotional, cognitive, and behavioral components. It is the displeasing feeling of fear and concern. When anxiety becomes excessive, it may fall under the classification of an anxiety disorder (Anxiety, 2012).

Social anxiety is the fear of social situations and the interaction with other people that can automatically bring on feelings of self-consciousness, judgment, evaluation, and inferiority. Social anxiety is the fear and anxiety of being judged and evaluated negatively by other people, leading to feelings of inadequacy, embarrassment, humiliation, and depression. United States, epidemiological studies have recently fixed social anxiety disorder as the third largest psychological disorder in the country, after depression and alcoholism. It is estimated that 7-8% of the population suffers from some form of social anxiety at the present time. People with social anxiety disorder tend to feel

quite nervous or uncomfortable in social situations. They are very concerned that they will do something embarrassing or humiliating, or that others will think badly of them. These individuals are very self-conscious and constantly feel "on stage" (Richards, 1996). The Diagnostic and Statistical Manual of Mental Disorders (DSM-IV-TR, 2000) defines social anxiety (social phobia) as an intense fear of negative evaluation from others, and a chronic concern and self-doubt about one's social ability and social performance. Although the terms social phobia and social anxiety are often used interchangeably in the literature, the former more accurately refers to a diagnosed condition, whereas the latter, a milder form of social discomfort.

There are three types of social anxiety: **Performance (or non-generalized) social anxiety** disorder affects individuals only when they are performing in front of others. **Generalized social anxiety** disorder affects a person in many different social situations, from going to a party to speaking with their boss. **Avoidant personality disorder** is considered by many anxiety specialists to be the most severe form of social anxiety. It usually starts at an early age and is much more common in males (Types of Social Anxiety, n.d.).

There are some **factors that cause Social Anxiety** such as average age of social anxiety is between age 11 and 19 teenage years, Women are more likely to have social anxiety than men *(*Christian Nordqvist, 2010), Low Socio-Economic status can cause social anxiety, Children brought up in nuclear family system are more prone to social anxiety as compared to Joint Family System, Embarrassing experiences at school, humiliation for performance or body image, Lack of parental education, different personality traits: Extraversion, Agreeableness, Conscientiousness, Neuroticism, Openness to Experience and low Self Esteem can cause Social Anxiety.

There are three types of symptoms for social anxiety **1) Emotional:** Excessive self-consciousness, anxiety and extreme fear in everyday social situations, intense worry for days, weeks, or even months before an upcoming social situation. **2) Physical:** Red face or blushing, Shortness of breath, Upset stomach, nausea, Trembling or shaking, Increase heartbeat, Sweating or hot flashes, Feeling dizzy or faint. **3) Behavioral:** Avoiding social situations, Staying quiet or hiding in the background in order to escape notice and embarrassment, Drinking before social situations in order to soothe your nerves (Melinda Smith and Ellen Jaffe-Gill, 2012).

2. PROBLEM STATEMENT

What are the Factors that cause Social Anxiety among youth of Lahore?

3. OBJECTIVES OF THE STUDY

In accordance with the problem statement, the main objective of the study was to identify the Factors causing Social Anxiety among youth of Lahore and to compare these Factors Socio Economic Status and Gender wise.

4. METHODOLOGY AND MODEL SPECIFICATION

A cross sectional study is conducted among the youth consisting of both Boys and Girls from age 14 to 19. A multiple probability sampling designs including Stratified Random Sampling, Cluster Random Sampling and Convenience Sampling designs have been used accordingly. Total sample of 150 students have been taken in this research. A sub sample of size 50 has been taken from each Socio Economic Group. The primary data is collected through a structured questionnaire consisting of basic demographic questions, Social Interaction Anxiety Scale by Mattick and Clarke (1998), Big Five Personality Inventory by John & Martinez (1998) and State Self Esteem Scale by Heatherton &Polivy (1991). Data is analyzed using univariate (frequency distribution, graphs), bivariate (chi-square test of association, spearman's rank correlation) and multivariate (multiple linear regression) statistical techniques. The statistical package SPSS and Minitab are used to analyze and interpret results in this study.

$$\chi^2 = \sum_{k=1}^{n}(e_k - f_k)2/e_k \tag{1}$$

We reject H_0 if the Chi Square test statistic is large enough so that the area beyond it (under the chi-square curve with (r-1) (c-1) degrees of freedom) is less than 0.05.

$$r_s = 1 - 6 \, \Sigma d_i^2 \, / \, n(n^2 - 1) \tag{2}$$

In Spearman's Rank Correlation di is the difference in the ranks given to the two variable's values for each item of data.

$$SA = \beta_0 + \beta_1 G + \beta_2 FS + \beta_3 FI + \beta_4 FE + \beta_5 ME$$
$$+ \beta_6 WP + \beta_7 S + \beta_8 TE + \beta_9 PT + \beta_{10} SE + \varepsilon_I \tag{3}$$

where SA is Social Anxiety which is dependent variable, G is Gender, FS is Family System, FI is Family Income, FE is Father's Education, ME is Mother's Education, WP is Working Parent, S is Symptoms of Social Anxiety, TE is Traumatic Experience, PT is Personality Traits and SE is Self Esteem are independent variables. $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10}$ are Regression Coefficient.

5. RESULTS AND INTERPRETATION

Table 1: Significant variables

Independent Variables	Dependent Variable (Social Anxiety)
Socio Economic Status	(-0.172)* 0.036 (S.r.c)
Extraversion	(-0.348)*0.000 (S.r.c)
Agreeableness	(-0.218)*0.007 (S.r.c)
Conscientiousness	(-0.211)*0.010 (S.r.c)
Neuroticism	(0.321)*0.000 (S.r.c)
Openness to Experience	(-0.169)*0.039 (S.r.c)
Performance Self Esteem	(-0.273)*0.001 (S.r.c)
Social Self Esteem	(-0.259)*0.001 (S.r.c)

((Value)*P-value < 0.05 significance level (Spearman's rank correlation))

Table 1: Results show that Socio Economic Status, all five personality traits (Extraversion, Agreeableness, Conscientiousness, Neuroticism, Openness to Experience), Performance and Social Self Esteem are playing significant role in developing Social Anxiety.

Table 2: Significant variables in Male and Female

Independent Variables	Dependent Variable (Social Anxiety)	
	Male	**Female**
Father's Education	(-0.239)*0.039 (S.r.c)	
Mother's Education	(-0.273)*0.018 (S.r.c)	
Extraversion	(-0.297)*0.010 (S.r.c)	(-0.383)*0.001 (S.r.c)
Agreeableness	(-0.260)*0.024 (S.r.c)	
Conscientiousness	(-0.231)*0.046 (S.r.c)	
Neuroticism	(0.496)*0.000 (S.r.c)	
Openness to Experience		(-0.234)*0.044 (S.r.c)
Performance Self Esteem		(-0.329)*0.004 (S.r.c)
Social Self Esteem	(-0.378)*0.001 (S.r.c)	

((Value)*P-value < 0.05 significance level (Spearman's rank correlation))

Table 2: Results indicates that Mother's and Father's Education, all personality traits except Openness to Experience and Social Self Esteem are significant in Male on the other hand Extraversion, Openness to Experience and Performance Self Esteem are significant in Female.

Table 3: Significant variables in Lower Middle and Upper Classes

Independent Variables	Dependent Variable (Social Anxiety)		
	Lower Class	**Middle Class**	**Upper Class**
Traumatic Experience	(6.723)*0.039 (Fisher's Exact Test)		
Family System	(6.704)*0.028 (Fisher's Exact Test)		
Extraversion	(-0.428)*0.002 (S.r.c)	(-0.376)*0.007 (S.r.c)	
Agreeableness	(-0.330)*0.019 (S.r.c)		
Conscientiousness		(-0.284)*0.046 (S.r.c)	(-0.407)*0.003 (S.r.c)
Neuroticism		(0.331)*0.019 (S.r.c)	(0.321)*0.023 (S.r.c)
Openness to Experience			(-0.279)*0.050 (S.r.c)
Performance Self Esteem		(-0.294)*0.038 (S.r.c)	(-0.291)*0.041 (S.r.c)
Social Self Esteem			(-0.518)*0.000 (S.r.c)

((Value)*P-value < 0.05 significance level (Spearman's rank correlation))

Table 3: Results suggest that Trauma, Family System, Extraversion and Agreeableness are playing significant role in Lower Class, Extraversion, Conscientiousness, Neuroticism and Performance Self Esteem are significant in Middle

Class and Conscientiousness, Neuroticism, Openness to Experience, Performance and Social Self Esteem are significant in Upper Class.

Social Anxiety = 87.5 - 1.05 Social Self Esteem - 0.529 Extraversion
                 - 3.93 Socio Economic Status - 0.479 Conscientiousness

R-Sq=55.7%, DW-Statistics = 2.16, (F-Statistics = 32.54 P-value = 0.000)

On the average one unit increase in Social Self Esteem will decrease Social Anxiety by 1.05 units keeping all variables in the model constant, on the average one unit increase in Extraversion will decrease Social Anxiety by 0.529 units as well as on the average increase in Socio Economic Status and Conscientiousness, Social Anxiety will decrease by 3.93 and 0.479 units respectively.

6. CONCLUSION

The results of this study indicate that 14.7% of the respondents were suffering from Social Phobia and 9.3% were suffering from Social Anxiety individuals from Social Phobia were mostly from Lower Class. There was significant relationship between Social Anxiety and Socio Economic Status. Trauma and Family System (Joint, Nuclear) were associated significantly with Social Anxiety in Lower Class. Chi square test indicates that there was no association between Social Anxiety and Gender so we conclude that Male and Female are equally likely to develop Social Anxiety. In Male, Mother's and Father's education was playing significant role. All five personality traits play significant role in developing Social Anxiety. Performance and Social Self Esteem was playing significant role but Appearance Self Esteem was not associated with Social Anxiety in youth. In the end Multiple linear regression analysis suggests that Social Self Esteem, Extraversion, Socio Economic Status and Conscientiousness are strong predictors of Social Anxiety in youth of Lahore.

7. RECOMMENDATIONS

- Comparison between Socially Anxious and Not Socially Anxious children should be made and effects of Social Anxiety in their daily life should be studied.
- Conscientiousness is playing significant role in Social Anxiety in any individual so Self Consciousness Scale should be used for obtaining better results in further studies.
- Parenting Style should be used in assessing Social Anxiety in children because children whose parents are more controlling and over protective are more Socially Anxious and their Social Self Esteem is Low. Parenting style may play significant role in preventing Social Anxiety in children.
- Children should be involved in different activities such as games, drama, essay writing and small presentations where they can perform and speak in the presence of other people from the beginning of their lives so that they may not develop Social Anxiety.
- This study could also be completed with a clinical sample or a sample representative of the general population.

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DISCRETE INVERSE GAMMA DISTRIBUTION

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ABSTRACT

Like Gamma distribution the inverse Gamma distribution has also been playing a leading role in Bayesian statistics and in lifetime modeling. Due to emerging use of inverse Gamma distribution we developed discretized version of inverse Gamma distribution by adopting the discrete concentration approach so that we can be able to use it in modeling discrete lifetime data. Increasing and decreasing failure rate properties are also ensured in discrete set up. Moreover a direct link among discrete inverse Gamma, its continuous counterpart and Gamma distribution are established. Some distributional properties, the parameters estimation and a real data example are also presented.

KEY WORDS

Inverse Gamma distribution, reliability parameters, negative moments, discrete version, generating functions.

1. INTRODUCTION

In statistical science various probability models (continuous or discrete) are being used for the analysis of survival data. Although the continuous lifetime distributions are playing their role very well in the analysis of survival data yet in certain situations where lifetime can best be described through nonnegative integer valued random variables, for example life of a switch is measured by the number of strokes, life of equipment is measured by the number of cycles it completes or the number of times it is operated prior to failure, life of a weapon is measured by the number of rounds fired until failure, number of years of a married couple successfully completed, this in turn demands further development in the discrete lifetime distributions. The discrete lifetime data is generally realized from continuous set up in two common situations e.g. i) a product is scrutinized only once a time period i.e. a day, an hour and a month etc. and observation is made on the number of time periods successfully completed prior to failure of the product ii) an equipment operates in cycles and researcher observes the number of cycles successfully completed prior to failure of the device. If the observed data values are very large e.g. thousands of revolutions, cycles, blows etc. then one prefer the continuous lifetime distribution to model such a data. Reliability of the discrete or counted data is measured on the bases of success or failure which are modeled through imperfect models like binomial, negative binomial, geometric and Poisson distributions. To overcome such drawbacks the researchers use the discretization approach i.e. the discrete success-failure

data based on small and to some extent large samples are efficiently modeled through discretized version of continuous lifetime distributions.

Various discretizing approaches exist in the literature which are i) moment equalization approach ii) discrete concentration approach iii) failure rate approach (see Roy and Ghosh (2009)) iv) to use the Burr and related differential equations and convert them into their discrete counterpart with the help of difference equation method v) time discretization approach. Due to these approaches discretized distributions are finding their way into survival analysis. In this regard an initial attempt was made by Nakagawa and Osaki (1975) who discretized the Weibull distribution. Latter on a number of researchers like Szablowski (2001), Bracquemond and Gaudoin (2003), Roy (2003, 2004), Kemp (2006), Krishana and Pundir (2007), Krishna and Pundir (2009) and Jazi et al. (2010) and number of others developed and studied discretized version of lifetime distributions as well as applied them on a discrete set of data in various discipline of life like engineering, social sciences, medical sciences, and forestry etc. Classifications of discrete distribution have been made by number of researcher like Khalique (1989) and Kemp (2004). The characterization of discrete lifetime distributions is mainly based on the reliability parameters namely mean residual life functions, survival and failure functions, failure rate functions, conditional variances and reversed hazard functions. In order to develop reliability theory in discrete discipline various attempts has been initiated in multiple directions. We hereby made an attempt to develop the discrete inverse Gamma distribution. The development of discrete inverse Gamma distribution and its failure rate demonstration are discussed in section two, the generating functions, properties and the link between discrete inverse Gamma and its continuous counterpart are studied in section three and parameters' estimation and real data example is studied in section four.

2. DISCRETIZATION

2.1 Discrete Concentration

As discretization of continuous lifetime distribution is an emerging issue of discrete reliability theory, so various approaches as mentioned earlier exist in the literature. However these approaches are used by various researchers under different circumstances. An initial attempt of discretization made by Nakagawa and Osaki (1975) based on the preservation of survival function of continuous Weibull distribution. Many well known discrete distributions came into being due to this property e.g. If X follows the exponential distribution with survival function $S x = \exp -\lambda x, \lambda > 0$ and $x \geq 0.$ Then on preserving the survival function of the exponential distribution at integers, we get the probability mass function of the geometric distribution which is $p_x = q^x - q^{x+1}, 0 < q < 1$ and $x = 0,1,2,3,...$ .As there is one to one correspondence between survival function of geometric distribution and exponential distribution, so a number of researchers considered the geometric distribution as discrete exponential distribution with lack of memory property. Moreover if the survival functions of discretized distribution retain the same functional forms as its continuous counterparts then many reliability measures and class properties under series, parallel and coherent structures will remain unchanged (see Roy (2004)). In view of the above characteristics we have adopted this approach for discretizing the inverse Gamma distribution. The inverse Gamma distribution being used frequently in Bayesian statistics as

a conjugate prior and lifetime modeling (see Li et al. 2011). Its survival and failure functions are of the form

$$S(x) = P_r(X \geq x) = \frac{\gamma\left(\alpha, \frac{\beta}{x}\right)}{\Gamma(\alpha)}, \quad F(x) = P_r(X < x) = \frac{\Gamma\left(\alpha, \frac{\beta}{x}\right)}{\Gamma(\alpha)}, \qquad \beta > 0, \alpha > 0, x \geq 0,$$

where $\gamma(\alpha, x)$ and $\Gamma(\alpha, x)$ are the lower and upper incomplete gamma functions defined as $\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1} \exp(-t)\, dt$ and $\gamma(\alpha, x) = \int_0^x t^{\alpha-1} \exp(-t)\, dt$.

Its rth moment, mean and variance are $\mu'_r = \frac{\beta^r \Gamma(\alpha - r)}{\Gamma(\alpha)}$, Mean $= \frac{\beta}{\alpha - 1}$,

Variance $= \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$. Its co-efficient of skewness and kurtosis are

$$\beta_1 = \frac{16(\alpha-2)}{(\alpha-3)^2} \Rightarrow \beta_1 > 0 \text{ if } \alpha > 2, \beta_1 < 0 \text{ if } 0 < \alpha < 2 \text{ and } \beta_1 = 0 \text{ if } \alpha = 2, \beta_2 = \frac{3\alpha+15(\alpha-2)}{(\alpha-3)(\alpha-4)},$$

the $\beta_2 = 3$ if $\alpha = 2.2$.

Its hazard function is $h(x) = \dfrac{f(x)}{S(x)} = \dfrac{\beta^\alpha x^{-\alpha-1} \exp\left(-\dfrac{\beta}{x}\right)}{\gamma\left(\alpha, \dfrac{\beta}{x}\right)}.$

2.2 Discrete Inverse Gamma Distribution

The preserved survival function of discrete inverse Gamma at integers is

$$S_x = P_r(Y \geq x) = \sum_{j \geq x} p_j = \frac{1}{\Gamma(\alpha)}\left[\gamma\left(\alpha, \frac{\beta}{x}\right)\right], \beta > 0, \alpha > 0,\ x = 0,1,2,3,..,\text{where } S_0 = 1,$$

where $Y = [X]$ denote the observed discrete random variable i.e. Y is equal to the greatest integer less than or equal to X. The probability mass function of Y is

$$p_x = S_x - S_{x+1} = \frac{1}{\Gamma(\alpha)}\left\{\gamma\left(\alpha, \frac{\beta}{x}\right) - \gamma\left(\alpha, \frac{\beta}{x+1}\right)\right\}, \beta > 0, \alpha > 0,\ x = 0,1,2,.. \quad (2.2.1)$$

$$p_x = \frac{1}{\Gamma(\alpha)}\left\{\Gamma\left(\alpha, \frac{\beta}{x+1}\right) - \Gamma\left(\alpha, \frac{\beta}{x}\right)\right\}, \qquad \beta > 0, \alpha > 0,\ x = 0,1,2. \quad (2.2.2)$$
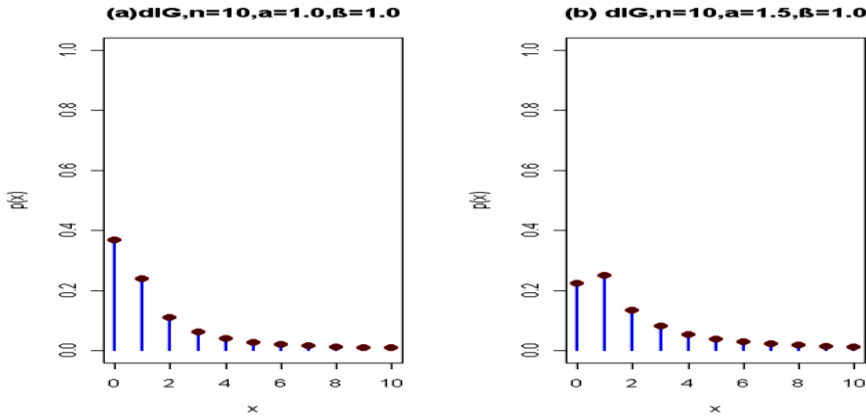
Fig. (2.2.1)

Fig. (2.2.1) shows the probability plots for discrete Inverse Gamma distribution for different values of the parameter α and β the curve is adopts the reverse J shaped.

$$\text{or } p_x = \frac{1}{\Gamma\alpha} \int_{\frac{\beta}{x+1}}^{\frac{\beta}{x}} z^{\alpha-1}\exp{-z}\,dz, \qquad \beta > 0, \alpha > 0, x = 0,1,2,.... \qquad (2.2.3)$$

Corollary:

When we take $\alpha = \frac{\nu}{2}$ and $\beta = \frac{1}{2}$ in either expression (2.2.1) or (2.2.2) or (2.2.3) we get discrete inverse chi-square distribution and for $\alpha = 1$ the expression (2.2.1) or (2.2.2) or (2.2.3) yields inverse exponential distribution of the form

$$p_x = q^{\frac{1}{x+1}} - q^{\frac{1}{x}}, \qquad 0 < q < 1, x = 0,1,2,....$$

where $q = \exp(-\beta)$.

2.3 Failure Rate Functions for discrete Inverse Gamma

The failure rate function for dIG distribution is defined as

$$h_x = 1 - \frac{\gamma\left(\alpha, \frac{\beta}{x+1}\right)}{\gamma\left(\alpha, \frac{\beta}{x}\right)}, \quad \alpha > 0, \beta > 0, x = 0,1,2,3,....\bullet$$

3. SOME PROPERTIES

Theorem 3.1:

The Inverse Gamma distribution follows increasing failure rate (IFR) and decreasing failure rate (DFR) pattern for different choices of parameters (α, β).

Proof:

$$\text{IFR if } \frac{\partial^2 \log f(x)}{\partial x^2} < 0 \text{ or } x < \frac{2\beta-1}{\alpha}, \text{ DFR if } \frac{\partial^2 \log f(x)}{\partial x^2} > 0 \text{ or } x > \frac{2\beta-1}{\alpha}.$$

This completes the proof.

Theorem 3.2:

The discrete inverse gamma distribution is discrete increasing failure rate (dIFR) when $x < \dfrac{2\beta-1}{\alpha}$ and discrete decreasing failure rate (dDFR) $x > \dfrac{2\beta-1}{\alpha}$ at integers.

Proof:

For proof see theorem 3.1 and following lemma for discrete case.

Lemma:

If X is a continuous random variable with increasing (decreasing) failure rate IFR (DFR) distribution then Y= [X] discrete increasing (decreasing) failure rate dIFR (dDFR). (for proof see Krishna and Pundir (2007))
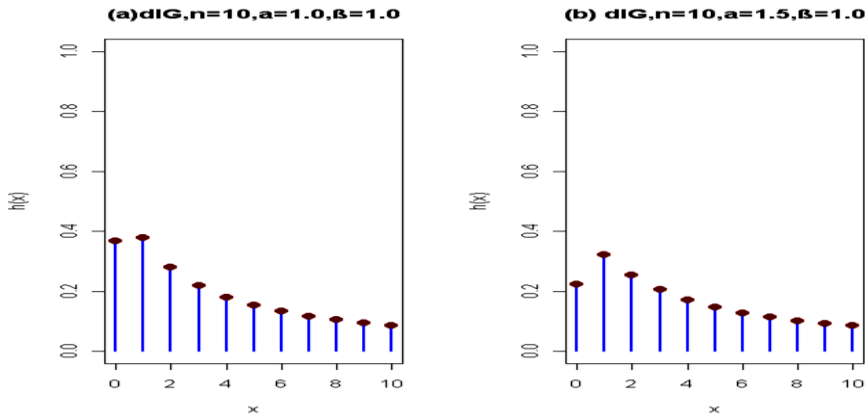
Fig. (2.2.2)

The Fig. (2.2.2) shows the failure rate probability for the product lie between zero and unity. It shows that failure rate is increasing (IFR) and decreasing (DFR) when the following inequality holds at the integers.

$x(\alpha+1) < 2\beta$ for increasing failure rate IFR ;

$x(\alpha+1) > 2\beta$ for decreasing failure rate DFR

Now for dIG the second failure rate function is defined as

$$h_x^* = ln\left(\frac{S_x}{S_{x+1}}\right) = ln\left(\frac{\gamma\left(\alpha, \dfrac{\beta}{x}\right)}{\gamma\left(\alpha, \dfrac{\beta}{x+1}\right)}\right), \ \alpha > 0, \ \beta > 0, \ x = 0,1,2,... \text{ where } S_x = \frac{\gamma\left(\alpha, \dfrac{\beta}{x}\right)}{\Gamma(\alpha)}.$$
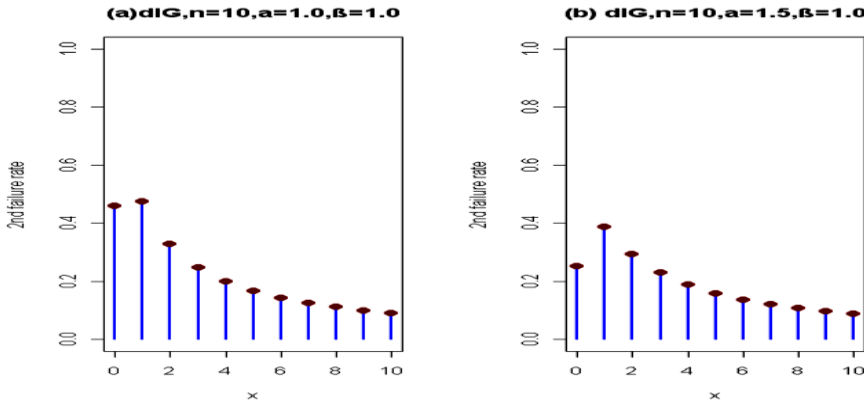
Fig(2.2.3)

Fig. (2.2.3) shows the same behavior as in case of failure function but as we have seen that the failure rate function behaves as the conditional probability with uppermost value as one whereas in second failure rate function the upper most value is not one it may be greater than one while depicting the same situation as depicted by failure function earlier in fig (2.2.3).

Theorem 3.3:

Let $Y = [X]$ be an integer valued random variable which follows the discrete Inverse Gamma distribution i.e. $Y \sim dIG\ \alpha,\beta$. Then the probability generating function for Y is

$$G\ t\ = \sum_{x=1}^{\infty} t^x - t^{x-1}\ \frac{\gamma\left(\alpha, \frac{\beta}{x}\right)}{\Gamma\ \alpha} + 1.$$

where $P_r\ Y \geq x\ = \dfrac{\gamma\left(\alpha, \frac{\beta}{x}\right)}{\Gamma\ \alpha}$ and $\gamma\left(\alpha, \frac{\beta}{x}\right)$ is the lower incomplete gamma function.

Proof:

We have $G\ t\ = E\ t^x\ = \sum_{x=0}^{\infty} t^x P\ Y = x$

where $\sum_{x=0}^{\infty} t^x P\ Y = x\ = \sum_{x=1}^{\infty} t^x P_r\ Y \geq x\ - t^{x-1} P_r\ Y \geq x\ + 1.$ So   This completes the proof.

Corollary:

If we take $t = e^{tx}$ then the moment generating function for Y is

$$G\ e^t\ = \sum_{x=1}^{\infty} e^{tx} - e^{t\ x-1}\ \frac{\gamma\left(\alpha, \frac{\beta}{x}\right)}{\Gamma\ \alpha} + 1.$$

Theorem 3.4:

Let $Y = [X]$ be an integer valued random variable which follows the discrete Inverse Gamma distribution i.e. $Y \sim dIG\ \alpha,\beta$ defined in equation (2.2.3) then its moment generating function is

$$M_Y\ t\ = \sum_{x=0}^{\infty} \sum_{j=0}^{\infty} \binom{\alpha\text{-}1}{j}\ tx\ ^jJ\left(\alpha\text{-}j; \frac{\beta}{x+1} - tx; \frac{\beta}{x} - tx\right),$$

where

$$J\left(\alpha\text{-}j; \frac{\beta}{x+1} - tx; \frac{\beta}{x} - tx\right) = \frac{1}{\Gamma\ \alpha}\left[\Gamma\left(\alpha\text{-}j, \frac{\beta}{x+1} - tx\right) - \Gamma\left(\alpha\text{-}j, \frac{\beta}{x} - tx\right)\right]$$

and $\Gamma\ \alpha,x$ is the upper incomplete gamma function.

Proof:

We have $M_Y\ t\ = E\ e^{tx}\ = \sum_{x=0}^{\infty} e^{tx}p_x,$

where $p_x = J\left(\alpha; \frac{\beta}{x+1}; \frac{\beta}{x}\right),$ $\qquad \beta > 0, \alpha > 0,\ x = 0,1,2,....$

$$M_Y\ t\ = \sum_{x=0}^{\infty} \sum_{j=0}^{\infty} \binom{\alpha\text{-}1}{j}\ tx\ ^jJ\left(\alpha\text{-}j; \frac{\beta}{x+1} - tx; \frac{\beta}{x} - tx\right).$$

This completes the proof.

Corollary:

Using equation (2.2.3) then the rth moment for discrete inverse gamma distribution is

$$\mu'_r = \frac{\beta^{\alpha}}{\Gamma\ \alpha} \sum_{x=0}^{\infty} x^r \left\{\frac{\exp\left(-\frac{\beta}{x}\right)}{x^{\alpha}} - \frac{\exp\left(-\frac{\beta}{x+1}\right)}{x+1^{\ \alpha}}\right\}.$$

Corollary:

Put $e^t = t \Rightarrow t\ \ln e\ = \ln t \Rightarrow t = \ln t$ we get probability generating function of $dIG\ \alpha,\beta$

$$M_Y\ \ln t\ = \sum_{x=0}^{\infty} \sum_{j=0}^{\infty} \binom{\alpha\text{-}1}{j}\ \ln t\ x\ ^jJ\left(\alpha\text{-}j; \frac{\beta}{x+1} - \ln t\ x; \frac{\beta}{x} - \ln t\ x\right).$$

$$M_Y\ \ln\ 1+t\ = \sum_{x=0}^{\infty} \sum_{j=0}^{\infty} \binom{\alpha\text{-}1}{j}\ \ln\ 1+t\ x\ ^jJ\left(\alpha\text{-}j; \frac{\beta}{x+1} - \ln\ 1+t\ x; \frac{\beta}{x} - \ln\ 1+t\ x\right).$$

However if we expand $\ln\ 1+t$ and place a condition that $\ln\ 1+t\ \cong t$ because "t" lies in the neighborhood of zero we see that the resulting expression will again become moment generating function of $dIG\ \alpha,\beta$ i.e.

$$M_Y \left(t \right) = \frac{1}{\Gamma\left(\alpha\right)} \sum_{x=0}^{\infty} \sum_{j=0}^{\infty} \binom{\alpha-1}{j} (tx)^{j} J\left(\alpha-j; \frac{\beta}{x+1} - tx; \frac{\beta}{x} - tx\right).$$

$$\ln\left(1+t\right) \cong t \quad \text{on neglecting higher powers of } t$$

Theorem 3.5:

Let $Y = [X]$ be an integer valued random variable which follows the discrete Inverse Gamma distribution i.e. $Y \sim dIG\left(\alpha,\beta\right)$ Then the rth for Y is

$$\mu'_r = \sum_{x=1}^{\infty} \left[x^r - \left(x-1\right)^r\right] \frac{\gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma\left(\alpha\right)},$$

where $P_r\left(Y \geq x\right) = \dfrac{\gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma\left(\alpha\right)}$ and $\gamma\left(\alpha,\frac{\beta}{x}\right)$ is the lower incomplete gamma function.

Proof:

We have $\mu'_r = \sum_{x=0}^{\infty} x^r P\left(Y = x\right)$,

$$\sum_{x=0}^{\infty} x^r P\left(Y = x\right) = \sum_{x=1}^{\infty} x^r \left[P_r\left(Y \geq x\right) - \left(x-1\right)^r P_r\left(Y \geq x\right)\right],$$

the series converges i.e.as $x \to \infty$ the tail probabilities approaches to zero.

This completes the proof.

Theorem 3.6:

If $Y = [X]$ then the first order negative moment of the random variable $Y \sim dIG\left(\alpha,\beta\right)$ is

$$E\left(Y+a\right)^{-1} = \sum_{x=0}^{\infty} \left\{ \left[\left(x+a\right)^{-1} - \left(x+a-1\right)^{-1}\right] \frac{\gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma\left(\alpha\right)}\right\} + \frac{1}{a-1}.$$

where $a > 0, \alpha > 0$ and $\beta > 0$ and $P_r\left(Y \geq x\right) = \dfrac{\gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma\left(\alpha\right)}$, for $x = 0,1,2,3,.....$

Corollary:

The sth order negative moment of the random variable $Y \sim dIG\left(\alpha,\beta\right)$ is

$$E\left(Y+a\right)^{-s} = \sum_{x=0}^{\infty} \left\{ \left[\left(x+a\right)^{-s} - \left(x+a-1\right)^{-s}\right] \frac{\gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma\left(\alpha\right)}\right\} + \frac{1}{\left(a-1\right)^{s}},$$

where $a > 0, \alpha > 0$ and $\beta > 0$.

Theorem 3.7:

The sth order negative factorial moment of the random variable $Y \sim dIG\ \alpha,\beta$

$$\mu'_{-s}\ a\ = \sum_{x=0}^{\infty} \left\{ \frac{1}{x+a_s} - \frac{1}{x+a-1_s} \right\} \frac{\gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma\ \alpha} + \frac{1}{a-1_s}.$$

where $x+a_s = x+a\ \ x+a+1\ ...\ x+a+s-1$, $a > 0, \alpha > 0$ and $\beta > 0$

and $P_r\ \ Y \geq x\ = \dfrac{\gamma\left(\alpha,\frac{\beta}{x}\right)}{\Gamma\ \alpha}$, for $x = 0,1,2,3,.....$

Theorem 3.8:

Let $Y_1 \leq Y_2 \leq Y_3 \leq ...... \leq Y_i \leq ...... \leq Y_n$ denote an order sample of size n drawn identically independently from the discrete inverse Rayleigh distribution whose distribution function is $F_{x-1} = \sum_{j=0}^{x-1} p_j = g_{\alpha,\ x-1}$, $S_x = 1 - F_{x-1}$, $0 < g_{\alpha,\ x} < 1$, $\alpha > 0, \beta > 0$, then the probability function of ith order statistics is

$$P_r\ \ Y_i\ = x\ = K_i \left\{ g_{\alpha,\ x-1}^{\ i} {}_2F_1\ -n+i,i;i+1; g_{\alpha,\ x-1}\ -\ g_{\alpha,\ x-2}^{\ i} {}_2F_1\ -n+i,i;i+1; g_{\alpha,\ x-2} \right\},$$

and the recurrence relation between i^{th} order statistics' probabilities is

$$i+1\ P_r\ \ X_{i+1}\ = x\ = i P_r\ \ X_i\ = x\ + \binom{n}{i} \left\{ g_{\alpha,\ x-2}^{\ i}\ 1-g_{\alpha,\ x-2}^{\ n-i}\ + g_{\alpha,\ x-1}^{\ i}\ 1-g_{\alpha,\ x-1}^{\ n-i} \right\},$$

where $g_{\alpha,\ x-1} = \dfrac{\Gamma\ \alpha,x-1}{\Gamma\ \alpha}$, $K_i = \dfrac{1}{i}\binom{n}{i}$ and ${}_2F_1\ \ \alpha_1,\alpha_2;\beta_1;z\ = \sum_{n=0}^{\infty} \dfrac{\alpha_{1\ n}\ \alpha_{2\ n}}{\beta_{1\ n}} \dfrac{z^n}{n!}$,

also $\Gamma\ \alpha,x$ is upper incomplete gamma function and $\Gamma\ \alpha\ = \int_0^{\infty} t^{\alpha-1} \exp\ -\alpha t\ dt$.

Proof:

By definition the probability function of ith order statistics is

$$P_r\ \ Y_i\ = x\ = P_r\ \ Y_i\ \leq x\text{-}1\ - P_r\ \ Y_i\ \leq x\text{-}2\ , \tag{3.8.1}$$

$$P_r\ \ Y_i\ \leq x\text{-}1\ = P_r\ \text{at least i of Y's are} \leq x\text{-}1\ ,$$

where $\sum_{j=i}^{n}\binom{n}{j}\ F_{x-1}^{\ j}\ 1\text{-}F_{x-1}^{\ n-j}\ = \int_0^{F_{x-1}} \dfrac{1}{B\ i,n\text{-}i+1}\ u^{i-1}\ 1-u^{\ n-i}\ du = I_{F_{x-1}}\ \ i,n\text{-}i+1\ ,$

$I_{F_{x-1}}\ \ i,n\text{-}i+1$ is the incomplete beta function. Therefore

$$P_r \left[X_i = x \right] = \frac{n!}{i! \, n\text{-}i \, !} \int_{g_{\alpha, x\text{-}2}}^{g_{\alpha, x\text{-}1}} u^{i\text{-}1} \left[1\text{-}u \right]^{n\text{-}i} du, \qquad \text{where } F_{x\text{-}1} = g_{\alpha, \, x\text{-}1} \, , F_{x\text{-}2} = g_{\alpha, \, x\text{-}2}$$

on simplifying we get

$$P_r \left[X_i = x \right] = \frac{n!}{i! \, n\text{-}i \, !} \sum_{j=0}^{n-i} \binom{n\text{-}i}{j} \frac{-1^{\,j}}{i+j} \left(g_{\alpha, \, x\text{-}1}^{\,i+j} - g_{\alpha, \, x\text{-}2}^{\,i+j} \right), \qquad (3.8.2)$$

Therefore equation (3.8.1) can be re written as

$$P_r \left[X_i = x \right] = K_i \left\{ g_{\alpha, \, x\text{-}1}^{\,i} \; {}_2F_1 \left[-n+i, i; i+1; g_{\alpha, \, x\text{-}1} \right] - g_{\alpha, \, x\text{-}2}^{\,i} \; {}_2F_1 \left[-n+i, i; i+1; g_{\alpha, \, x\text{-}2} \right] \right\},$$

This completes the proof.

As we know that the probability function of the $i+1^{\text{th}}$ order statistics is

$$P_r \left[X_{i+1} = x \right] = K_{i+1} \left\{ g_{\alpha, \, x\text{-}1}^{\,i+1} \; {}_2F_1 \left[-n+i+1, i+1; i+2; g_{\alpha, \, x\text{-}1} \right] \right.$$

$$\left. - g_{\alpha, \, x\text{-}2}^{\,i+1} \; {}_2F_1 \left[-n+i+1, i+1; i+2; g_{\alpha, \, x\text{-}2} \right] \right\},$$

where $K_{i+1} = \frac{1}{i+1} \binom{n}{i+1}$

using the Gauss' recurrence relation (see Roohi (2003))

$$c \, {}_2F_1 \left[a, b; c; z \right] - c \, {}_2F_1 \left[a, b+1; c; z \right] + az \, {}_2F_1 \left[a, b; c; z \right] = 0,$$

let $a = -n+i$, $b = i$, $c = i+1$ and $z = g_{\alpha, \, x\text{-}1}$ then

$$i+1 \, {}_2F_1 \left[-n+i, i; i+1; g_{\alpha, \, x\text{-}1} \right] - i+1 \, {}_2F_1 \left[-n+i, i+1; i+1; g_{\alpha, \, x\text{-}1} \right]$$

$$+ \, -n+i \, g_{\alpha, \, x\text{-}1} \, {}_2F_1 \left[-n+i+1, i+1; i+2; g_{\alpha, \, x\text{-}1} \right] = 0,$$

therefore

$$i+1 \, P_r \left[X_{i+1} = x \right] = i P_r \left[X_i = x \right] + \binom{n}{i} \left\{ g_{\alpha, \, x\text{-}2}^{\,i} \left[1-g_{\alpha, \, x\text{-}2} \right]^{n-i} + g_{\alpha, \, x\text{-}1}^{\,i} \left[1-g_{\alpha, \, x\text{-}1} \right]^{n-i} \right\},$$

the general expression for recurrence relation is

$$i+1 \, P_r \left[X_{i+1} = x \right] = i P_r \left[X_i = x \right] + \binom{n}{i} \left[F_{x\text{-}2} \, S_{x\text{-}1}^{\,n-i} + F_{x\text{-}1} \, S_x^{\,n-i} \right],$$

where $F_{x\text{-}1}$ and S_x are failure and survival function at x-1 and x respectively.

Theorem 3.10:

If $X \sim G(\alpha,\beta)$ then $Z = \left[\ln \left(\dfrac{1}{\beta X} \right)^{1/\beta} \right] \sim dIG(\alpha,q)$ the discrete inverse gamma

distribution with $q = \exp(-\beta)$.

Proof:

Let us consider $X \sim G(\alpha,\beta)$ with $P_r(X \geq x) = \dfrac{\Gamma(\alpha,\beta x)}{\Gamma(\alpha)}$, $\beta > 0, \alpha > 0, x \geq 0,$

where $\Gamma(\alpha,x)$ is the upper incomplete gamma function i.e. $\Gamma(\alpha,x) = \int\limits_{x}^{\infty} t^{\alpha-1} \exp(-t)\, dt.$

then $P(Z \geq z) = P \left(\left[\ln \left(\dfrac{1}{\beta X} \right)^{1/\beta} \right] \geq z \right)$, where $[.]$ denotes the integer part

By using $[X] \geq Z \Leftrightarrow X \geq Z$ (see Krishna and Pundir (2009)).

$$P(Z \geq z) = \dfrac{\gamma(\alpha,q^z)}{\Gamma(\alpha)}, \quad \vee\ z = 0,1,2,3,... \quad q = \exp(-\beta)$$

This completes the proof.

Theorem 3.11:

If $X \sim IG(\alpha,\beta)$ then $Y = \left[\ln \left(\dfrac{X}{\beta} \right)^{1/\beta} \right] \sim dIG(\alpha,q)$ the discrete inverse gamma

distribution with $q = \exp(-\beta)$.

Theorem 3.12:

If $X \sim G(\alpha,\beta)$ then $Z = \left[\exp \left(\dfrac{1}{\beta X} \right) \right] \sim dIG(\alpha,1)$ the discrete inverse gamma

distribution.

Theorem 3.13:

If $X \sim IG(\alpha,\beta)$ then $Y = \left[\exp \left(\dfrac{X}{\beta} \right) \right] \sim dIG(\alpha,1)$ the discrete inverse gamma

distribution.

Theorem 3.14:

Let X be non-negative continuous random variable and $Y_k = \left[cX^k \right]$ be an integer valued random variable where k is a positive number. Then $Y_k \sim dIG\ \alpha,\beta$ if $X \sim IG\ \alpha,\beta$

Corollary:

The above theorems also holds for discrete inverse chi-square and discrete inverse exponential distributions when we take $\alpha = \dfrac{\nu}{2}$ and $\beta = \dfrac{1}{2}$ and when $\alpha = 1$ respectively in the above theorem.

4. MAXIMUM LIKELIHOOD ESTIMATOR OF (α, β):

4.1 When α unknown:

$$\frac{\partial \ln L\ \alpha,\beta}{\partial \alpha} = -\frac{n\Gamma'\ \alpha}{\Gamma\ \alpha} + \sum_{i=1}^{n} \left\{ \frac{\Gamma'\left(\alpha, \dfrac{\beta}{x_i + 1}\right) - \Gamma'\left(\alpha, \dfrac{\beta}{x_i}\right)}{\Gamma\left(\alpha, \dfrac{\beta}{x_i + 1}\right) - \Gamma\left(\alpha, \dfrac{\beta}{x_i}\right)} \right\} = 0,$$

4.2 When β is unknown (α is known):

$$\frac{\partial \ln L\ \alpha,\beta}{\partial \beta} = \sum_{i=1}^{n} \left\{ \frac{\Gamma'^{\beta}\left(\alpha, \dfrac{\beta}{x_i + 1}\right) - \Gamma'^{\beta}\left(\alpha, \dfrac{\beta}{x_i}\right)}{\Gamma\left(\alpha, \dfrac{\beta}{x_i + 1}\right) - \Gamma\left(\alpha, \dfrac{\beta}{x_i}\right)} \right\} = 0,$$

where

$\Gamma'^{\alpha}\left(\alpha, \dfrac{\beta}{x}\right) =$ the derivate of the upper incomplete gamma function with respect to α,

$\Gamma'^{\beta}\left(\alpha, \dfrac{\beta}{x}\right) =$ the derivate of the upper incomplete gamma function with respect to β,

where $\Gamma\ \alpha,x$ is the upper incomplete gamma function i.e. $\Gamma\ \alpha,x = \int_{x}^{\infty} t^{\alpha-1} \exp\ -t\ dt.$

4.3 Steps for finding the MLEs of Discrete Inverse Gamma Distribution:

For finding the MLEs of discrete Inverse Gamma distribution following steps should be taken
  i)   Firstly find the mean of the data
  ii)   Secondly compare the sample mean with the population mean
  iii) Thirdly take this pair of the parameter which yield the relative similar moment values
  iv)   Fourthly use this pair of parameters as a guess/seed value
  v)   Fifthly use this guess value in any computational packages and find MLEs by Newton Raphson's method of successive iteration.

vi)   Sixthly use the so developed MLEs in order to test the fitness of the data

vii) Seventhly if distribution fit is good then use these MLEs for checking the asymptotic properties of MLEs.

viii) Eightly, take the seed/guess values as

   1) If 77% to 97% of the observations are $\leq$ 3 then take
      $\alpha = 0.4$ or 0.50 and $\beta = 0.30$.

   2) If less than 70% of the observations are $\leq$ 3 then take $\alpha = 0.4, \beta = 0.30$.

4.4 Example:

The following data is taken from Krishna and Pundir (2010) in which recording of carious teeth among the four deciduous molars in a sample of size 100 children of the age 10 and 11 years. It is presumed that a symmetry between left and right molars exist and only right molars are considered with a time unit of two years. The data so collected is given below with observed and expected frequencies for different distributions

Table 5-A

Total Number of carious teeth(x)	0	1	2	$\geq$3	Total	p-value (two tail)
Observed Frequency	64	17	10	9	100	
Geo. 0.5988 f_e	59.9	24	9.6	6.5	100	0.1353
Poi. 0.67 f_e	51.2	34.3	11.5	3	100	0.00001
DPD. 0.1935 f_e	68	15.60	6.20	10.20	100	0.2417
DBD. 1.292,0.2108 f_e	66	19.40	6.70	7.90	100	0.1435
DIG. 1.50,0.8449 f_e	63.920	19.949	6.609	9.521	100	0.2655
DR. 0.6650 f_e	43.70	46.3	9.50	0.50	100	6.1×10^{-15}

The above table portrays the computed expected frequencies for Poisson, Geometric, Discrete Pareto (DP), Discrete Burr (DB), Discrete Rayleigh (DR) and Discrete Inverse Gamma (DIG) distributions. We used the MLE for the calculation of probabilities and expected frequencies. The results along with p-values are also given in it.

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DATA DISSEMINATION STRATEGY OF AGRICULTURAL CENSUS ORGANIZATION

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ABSTRACT

It is viewed that during the production of official statistics much efforts, time and budget is spent on questionnaire designing, data collection, editing, data entry, data processing tabulation and report preparation as compared to data dissemination. The official statistics is the common property of a nation and it should be available to everyone. The data dissemination strategy/policy is a vehicle to carry this property and deliver it to actual owners. Great emphasis is required to be given on data dissemination not only by using the current data dissemination practices but also adopting new dimensions to pace with the demands of the users in this era of information. It is our responsibility to provide the data to our planners, policy makers, researchers etc., according to their requirements / demands as much as possible to facilitate their work as well as to make our society well aware about the up-to-date statistics of our country. Partners at all levels of society are essential for the development of an efficient and effective data dissemination system.

The strategy of Agricultural Census Organization is to deliver our produced data to every sector of the society to make them aware of the existing censuses/surveys data. In this paper data dissemination strategy of Agricultural Census Organization is discussed and few suggestions are given for its improvement. It is proposed that beside producing the huge volumes in printing form, short volumes according to the lowest possible geographical area and preferably be in the soft from may be produced. The main findings of censuses/surveys must be included in a specially prepared brochure for display on the website as well as in the form of soft /hard copy to be provided to all the data users so that their quest for other data emerges prominently. The reports of previous censuses/surveys should be digitalized. Efforts should be made to provide the data to the maximum users as well as user's opinion surveys must be arranged.

1. INTRODUCTION

Agricultural Census Organization (ACO) was established to meet out the requirements of defunct (now) Agricultural Census Act 1958 (Act No. XVI of 1958). Primarily to carry out decennial censuses of Agriculture. Subsequently, its activities were expanded by adding full-fledged decennial censuses of Livestock as well as of Agricultural Machinery. In addition, the surveys on slaughtering of animals, milk production and commercial poultry are carried out along with Livestock Census. This Organization also conducts Mouza Census quinquennially on regular basis.

2. DATA DISSEMINATION

The demand of data generated by ACO is increasing day by day not only in the form of hard copy but also in computer readable materials. The spectrum of data users is very wide and comprises of planners from the Federal / Provincial Governments and semi-government institutions / departments researchers / academia / students from the government /semi-government functionaries / NGOs, and private Agriculture marketing agencies etc. In addition, all the international agencies like FAO, UN family members and other agencies dealing with agriculture, demand data of ACO for putting in their publications as well as for use in their agricultural and rural planning, research and development pursuits. In case of users demand for any sensitive information / data, the same is supplied with the prior approval of the Federal Statistical Authority i.e. Secretary, Statistics Division.

The ACO is disseminating the data of its censuses/surveys by the following methods:

2.1 By Printing Publications

ACO publishes aggregated data as per tabulation plan at country and provincial levels in its country reports. The provincial reports contain data of the province as well as of each district of the province. The tabulation plan of each census is revised after finalization of the census questionnaire adjusting for fresh data requirements with consultation of data users. Standard format is adopted in presentation of aggregated data in the tabulations such that a maximum proportion of data users may benefit while analyzing data and making presentations in their policy planning, research and any other type of reports / papers.

At present ACO is producing the census publications in the form of Pakistan Report, Provincial Reports (one for each province), and one report each for Gilgit-Baltistan and Azad Jammu and Kashmir and the reports on different surveys separately in the form of hard copies.

The ACO has a mailing list of main data users, separately for each census. The publications are supplied to all federal /provincial agencies, institutions, departments, public universities /research entities etc., as per mailing list and to and any other such public or private entity on demand on free of cost basis. Similarly, any individual research / academia / students, etc. may get any of the publication (s) on demand from ACO, if available in the stock, without paying any price. ACO also supplies its publication (s) (hard / soft copy) to international organizations like UN Statistics Division, FAO, IMF, ECO, OIC, World Bank, Asian Development Bank, UNESCO, etc., free of charges on complimentary basis.

2.2 By Internet and CD's etc.

The country reports on the last conducted censuses are available on the internet on the web page www.pbs.gov.pk. These publications are available to data users free of cost. It provides the latest census information/data to the domestic as well as international users. In response to the e-mails the requested information/data is also provided. However in case of demand to ACO, for copy of the publication in computer readable material, the material (floppy / diskettes / CD / USB / magnetic tape) has to be provided to be by the users.

3. SUGGESTIONS FOR THE IMPROVEMENT OF
DATA DISSEMINATION STRATEGY

Here are few suggestions for the improvement of data dissemination strategy of ACO:

3.1 Presentation of Data/Information

In addition to the current practice of publications, data/ information may also be presented in the form of:-
  ➢ Main Findings of the Census/Survey (National Level)
  ➢ Main Findings of the Census/Survey (Provincial Level)
  ➢ Main Findings of the Census/Survey (One each for Gilgit Baltistan and Azad Jammu & Kashmir)
  ➢ Extract of Statistical Tables of Census/Survey for District(One for each District)
  ➢ Presentation of all publications in the form of CDs / soft copies

3.2 Digital Library

The advancement in the field of computer and internet has brought a lot of changes in the means of our lives especially to the information related matters. The library plays a key role in keeping an organization abreast with time by providing up-to- date information. It is proposed that steps may be taken to equip the current library of the organization with digital facilities as:
  ➢ The internet access facilities may be made available in the library.
  ➢ The current census publications / information should be made available in library in digital form.
  ➢ All the previous census publications / information may be digitalized and made available in the library.
  ➢ The digital publications / information from all over the world relevant   to   the objectives / targets of our organization may be collected and made available in the library.
  ➢ The services of IT experts to get the library digitalize may be acquired.
  ➢ Library of ACO may be linked with all important libraries at national and international level.

3.3 Data to be Made Available to Maximum Users

The importance of providing the quality data to the maximum numbers of users is universally accepted. Steps should be taken to get the following involved in the data dissemination / use of our data / findings.

3.3.1 Provincial Governments

As we collect the data from all the provinces by using their staff and present the information province wise as well as at national level. It is, therefore, suggested that the provincial governments may be taken on board by soliciting them to be cohort in data dissemination task.

They may be coordinated to:-
  ➢ Plan for the dissemination of censuses / surveys data in the provinces.

  ➤ Provide the space / link on their web sites of Agriculture and other related departments along with provincial Bureaus of Statistics for our censuses / surveys results.

3.3.2 District Administration

The district administrations may also be coordinated to:-
  ➤ Plan for the dissemination of our censuses / surveys results in the district.
  ➤ Provide the space / link on web page of their district for our census / survey results.

3.3.3 Universities

  ➤ All the universities having degree programs in Statistics, Economics and Agriculture may be supplied the censuses / surveys findings.
  ➤ The universities may be invited to arrange study tour of students to ACO. By this practice the students would be informed about the latest data along with its method of collection. It would help them to use it in future with confidence.
  ➤ Students may be offered data for research purposes.
  ➤ At the time of exhibitions in the universities stall of ACO may be arranged so that maximum number of students / teachers / researchers / other participants get informed about the up-to-date data collected by ACO.

3.3.4 Libraries

The publications/CD's released by ACO may be sent to all libraries of Pakistan as per their importance and scope.

The publications/CD's of:-
  ➤ Main Findings of the Census/Surveys (Provincial Level)
  ➤ Extract of Statistical Tables of Census/Surveys for each District may be sent to libraries at district level on mandatory basis, so that maximum number of people in the district may access it.

3.3.5 Agro Based Industries

All the agro based industries may be supplied the publication "Main Findings of the censuses / surveys (National Level and/or Provincial Level) according to their type of business.

3.3.6 Agriculture Related Bodies / Organizations

All the agriculture related bodies / organizations may be supplied the publication of main findings according to their scope and density of work.

4. USERS'S OPINION SURVEY

User's opinion survey should be conducted. This type of feedback not only gives insight into the future requirements and about satisfaction level of different users but also provide the information that which part of the publication is being used most, least and not at all. This information could be of vital importance for future planning. It is proposed that such type of survey should be done before finalizing the next census / survey of the same type.

A STUDY ON THE LIVES OF SLUM DWELLERS OF URBAN LAHORE

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ABSTRACT

A slum is a rundown area of the city characterized by substandard housing and other basic necessities of life. The main idea of this research is to analyze the extent of poverty amongst the slum dwellers of Urban Lahore and to explore their health and hygiene conditions. First stage Cluster random sampling is done to select six out of the eight densely populated slum areas of Urban Lahore i.e. Gulberg, Gurumangat Road, Walton Road, Askari flats, Johar Town and Saggiyan Pull. A total of 150 women were selected using exhaustive sampling and data is collected via a structured questionnaire. The results of the study show that the average income of the slum dwellers is above the poverty line of Pakistan although they are below the poverty threshold according to UN standards. Analysis of Variance (ANOVA) was applied and verified that there is association between the area the slum dwellers belong to and their average income. The infant mortality rate amongst the slum dwellers is lower than that of Pakistan however the fertility rate is quite high.

1. INTRODUCTION

Pakistan has an agro-based economy. During its earlier years, most of the population lived in rural areas. These people earned their livelihood through agricultural yield that was productive enough to satisfy all of their needs. There were no incentives for these people to move to the cities. With the passage of time, however, the trend changed. The population grew and agricultural yield proved insufficient for them to fulfill their needs. At the same time, cities were making progress even though it was slow. The rural people had no option but to find alternatives to agriculture which they thought they could seek more effectively by moving to urban areas. So, with the promise of a better future in their minds, some of the rural inhabitants started to move towards urban areas to seek better employment and to make use of the facilities that are available in the cities to live a better life. It proved fruitful for many as they found the necessities of life for which they left their homes. This led to many more people migrating to cities, but the poor economic system and lack of policy slowly eroded the returns these people would get. This migration in large numbers led to the formation of "slums", a thickly populated, run-down, squalid part of a city, inhabited by poor people (Rehman *et al.,* 2009)

"Poverty line" is defined as the national estimates of the percentage of the population falling below the poverty line are based on surveys of sub-groups, with the results weighted by the number of people in each group. Definitions of poverty vary considerably among nations. For example, rich nations generally employ more generous

standards of poverty than poor nations. Pakistan poverty line definition is Rs. 1274 per person per month for the year 2011.

Gordon, (2005) in "Indicators of Poverty & Hunger", defines absolute poverty as the absence of any two of the following eight basic needs:

- *Food:* Body Mass Index must be above 16.
- *Safe drinking water:* Water must not come from solely rivers and ponds, and must be available nearby (less than 15 minutes' walk each way).
- *Sanitation facilities:* Toilets or latrines must be accessible in or near the home.
- *Health:* Treatment must be received for serious illnesses and pregnancy.
- *Shelter:* Homes must have fewer than four people living in each room. Floors must not be made of dirt, mud, or clay.
- *Education:* Everyone must attend school or otherwise learn to read.
- *Information:* Everyone must have access to newspapers, radios, televisions, computers, or telephones at home.
- *Access to services:* This item is undefined by Gordon, but normally is used to indicate the complete panoply of education, health, legal, social, and financial services.

2. PROBLEM STATEMENT

The purpose of this study is to analyze the living standards and health indicators of the slum dwellers of urban Lahore.

3. OBJECTIVES OF THE STUDY

In accordance with the problem statement, the main objectives of the study are to analyze the poverty line and the absolute poverty line of the slum dwellers and to determine the fertility rates, and the infant mortality rates prevalent among the slum dwellers. Also to explore whether the living and health conditions of the slum dwellers of Lahore are independent of the area they are living in.

4. METHODOLOGY AND MODEL SPECIFICATION

A cross-sectional study is conducted among the women in the slums of Urban Lahore. The primary data was collected with the help of a structured questionnaire consisting of 44 close-ended and 2 open ended questions. Keeping in view the comfort level and the environment of the slum dwellers, the questions were translated into Urdu and filled out by the researcher. Several aspects such as hygiene, personal appearances and general living conditions were recorded on the basis of researcher's observations. SPSS 17 and MS Excel have been used as data analysis tool.

5. RESULTS AND INTERPRETATIONS

Data has been analyzed using statistical techniques. In this univariate analysis 88% of the respondents are married, 5% are divorced while 7% are widowed. Majority (36%) of the respondents are from age 30-39, and 97.3% respondents are illiterate. Majority (73%) of the respondents earn through scavenging while 27% make their earning by begging. Figure 5.1 shows the areas of the respondents they belonged to.
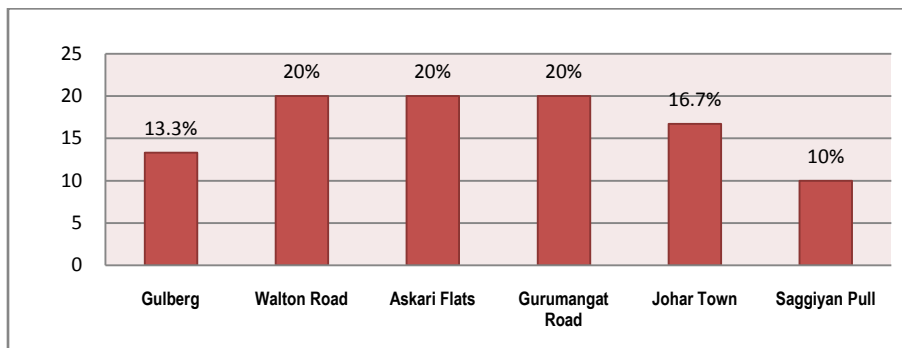
Figure 5.1: Bar Chart of Area of the Respondents

The general fertility rate in the slum dwellers comes out to be 4.65 which is much higher than the total fertility rate of Pakistan for the year 2011 i.e. 3.17. Infant Mortality rate in the slum dwellers (51.004) is much less than the Infant Mortality rate of Pakistan for the year 2011 (63.26).

In bivariate analysis, Pearson Chi-square of association was used to explore the association of area with the various health and hygiene conditions. The obtained results show that there is significant relationship between area and reason of death of slum dwellers, monthly income and other health and hygiene conditions respectively (i.e. P-value > 0.05).

Sample t-test for Poverty line according to Pakistan standards with mean equal to Rs 1274, shows that the P-value (0.000) is highly significant. Hence it is concluded that the mean income per month of the slum dwellers is much greater than Rs 1274.

One Proportion Test for Poverty threshold in the slums of Urban Lahore according to UN Standards with p=0.67 (4 out of 6 basic human needs is taken since body mass index and access to facilities could not be determined) shows that P-value is 0.000 which is highly significant. The slum dwellers of Urban Lahore thus live below the poverty threshold set according to UN standards.

In the multivariate analysis, Analysis of variance (ANOVA) is used to check the association between area of the respondents and their average incomes. Figure 5.2 shows that the P-value is equal to 0.000 which is highly significant showing that the average income per day of the slum dwellers in each area is not the same.

Figure 5.2: Analysis of Variance (ANOVA)		
Average Income per Day	**F**	**Sig. (P-Value)**
Between Groups (Combined)	**28.233**	**.000**

Multiple Comparison Test (LSD) is applied to see which slum areas are different in their income. It reveals that Askari Flats and Saggiyan Pull show insignificant results and hence have the same income. All the other areas are significantly different in their income.

6. CONCLUSION

The slum dwellers in Urban Lahore face a variety of problems. These problems include lack of education, unemployment, unclean environment and health issues. Although the average income of the slum dwellers is above the poverty line of Pakistan however it is much less than the poverty threshold according to UN standards. The slum dwellers living in posh areas of Lahore like Gulberg and Johar Town etc. have a better standard of living as compared to those living in less developed areas like Saggiyan and Askari. The infant mortality rate amongst the slum dwellers is lower than that of Pakistan however the fertility rate is quite high. Inadequate water and sanitation facility and lack of access to electricity were the main complaints of the slum dwellers and wanted the government to address them first and foremost. The solutions of their problems are possible but need excessive work to be done on the part of both the slum dwellers and the authorities. The slum dwellers are a burden on the society as long as nothing is done to resolve their issues and to help them become better citizens.

7. RECOMMENDATIONS

➢ The government should arrange programs to teach skills to the men and women of the slums so that they have a respectable source of income as compared to scavenging or begging.

➢ It was not possible to ascertain the weights and heights of the respondents so the Body Mass Index could not be calculated for the comparison of absolute poverty according to UN standards. The BMI could be calculated in future researches for better estimation of absolute poverty.

➢ This study not only provides a complete set of univariate and bivariate analysis but also provides ground for further research for comparison with the slums of other cities like Karachi etc.

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IMPACT OF BIRTH ORDER ON THE PERSONALITY
OF THE STUDENTS AGED (18-26)

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ABSTRACT

Birth order is an interesting phenomenon that attempts to explain how a child achieves the personality characteristics that he or she exhibits. The aim of the present study was to investigate the relationship between the birth order and personality traits. A sample consisting of 180 students; 90 females and 90 males with an age range of 18-26 years were recruited from the three institutions of Lahore; Punjab University, Government College University and University of Engineering and Technology. Stratified random sampling was used in the study. Analysis of variance was carried out to analyze the association between different variables. The results of this study indicate that there is no association between Birth order and Big Five Personality Traits however there is association between gender and personality traits because it was found that the extraversion and agreeableness had significance values with gender.

1. INTRODUCTION

Birth order is an interesting phenomenon that attempts to explain how a child achieves the personality characteristics that he or she exhibits. For years, the significance of an individual's birth order has been subject to many spirited debates, discussions, and research in sociology as well as other disciplines, but the complexities in these disciplines are that they are significantly difficult to determine and support. We content that a variety of influences i.e.: gender, age differences, temperament, and number of siblings, family size and other family circumstances both inside and outside of the family and specific birth order roles, contribute to individual differences in personality development. "Birth order refers to the order in which siblings are born into family. There are many exceptions to birth order, so the numerical place in the family may not correspond to the psychological birth order", Smalzer (2005). "Birth order is defined as a person's rank by age among his or her siblings", Isaaceson (2002).

"Personality literally means a 'likeable mask'." According to physiologists, personality is defined as the pattern of feelings, thinking and acting, Burger (2007).Personality is combination of an individual's inborn patterns of his thoughts, feelings, emotions and perception about himself and surroundings. There are some specific characteristics that make an individual unique and such characteristics help others to predict as to how an individual will act and respond in a given situation. Personality thus, arises from within the person and fairly remains consistent throughout life, Henry & Marquis (2005). The Five super ordinate factors have emerged, often referred to as the "Big Five" or the 5-factor model.

Extraversion: the tendency to be sociable, fun-loving, and affectionate vs. retiring, somber, and reserved.

Agreeableness: the tendency to be softhearted, trusting, and helpful vs. ruthless, suspicious, and uncooperative.

Conscientiousness: the tendency to be organized, careful, and disciplined vs. disorganized, careless, and impulsive.

Neuroticism: the tendency to be calm, secure, and self-satisfied vs. anxious, insecure, and self-pitying.

Openness to Experience: the tendency to be imaginative, independent, and interested in variety vs. practical, conforming, and interested in routine.

2. PROBLEM STATEMENT

The purpose of this study is to examine how birth order affects an individual's personality development.

3. OBJECTIVES OF THE STUDY

In accordance with the problem statement, the main objectives of the study are to analyze the impact birth order on an individual's personality and evaluate demographic effect on an individual's personality.

4. METHODOLOGY AND MODEL SPECIFICATION

A cross-sectional study is being conducted among the students of different institutes of Lahore. A sample of 180 students has been recruited from the Universities of Lahore i.e.: Punjab University, Government College University and University of Engineering and Technology. A structured questionnaire has been used as a tool for collecting data, which consists of Big Five Inventory Questionnaire and some basic questions about Birth order. SPSS 16 and MS Excel have been used as data analysis tool.

5. RESULTS AND INTERPRETATIONS

Data has been analyzed using statistical techniques. In this univariate analysis 50% of the respondents are males and 50% of the respondents are females. Majority (49.5%) of the respondents are from age 21-23 and 56.7% respondents are belong to nuclear family system and majority 33.3% of the respondents have monthly income below 30,000. Figure 5.1 shows that the 27% respondents belonged to middle order and to youngest.
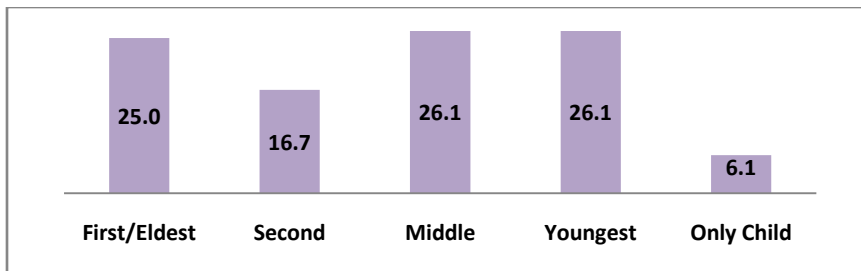
Figure 5.1: Birth order of the respondents

In bivariate analysis, Pearson Chi-square of association was used to explore the association of age, gender, income and personality traits with birth order. The obtained results show that there is insignificant relationship between birth order and age, monthly income respectively (i.e. P-value > 0.05). It is also concluded that figure 5.2.1 and 5.2.2 shows that the gender and personality traits have significant associations with extraversion and agreeableness category. Results show that females have more extraversion and agreeableness personality as compared to male.
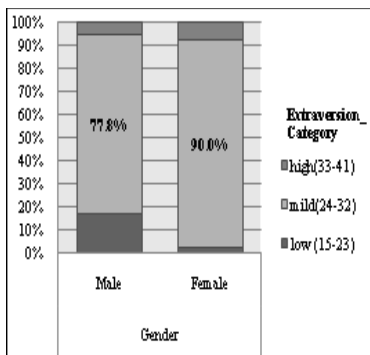
Figure 5.2.1

Figure 5.2.2

In the multivariate analysis, Analysis of variance (ANOVA) is used to check the association between the birth order and personality traits and Figure 5.3.1 shows that the p-values of all the categories are insignificant (i.e. P-value > 0.05). Thus we accept the null hypothesis and reject the null hypothesis at 5% level of significance.

Figure 5.3.1: (ANOVA) Analysis of Variance of birth order with personality traits
H0: $\mu1=\mu2=\mu3=\mu4=\mu5$
H1: At least two means are not equal

		Sum of Squares	df	Mean Square	F	Sig.
Extraversion	Between Groups	14.028	4	3.507	.303	.876
Agreeableness	Between Groups	92.787	4	23.197	.915	.456
Consciousness	Between Groups	54.900	4	13.725	.685	.603
Neuroticism	Between Groups	68.144	4	17.036	1.165	.328
Openness to experience	Between Groups	110.315	4	27.579	1.577	.182

6. CONCLUSION

The research is carried out to statistically analyze the association between the birth order and Big Five personality traits. In Univariate analysis, the majority of the female are more extraversion and agreeableness than male. From the results of bivariate analysis, it is concluded that females have more extraversion and agreeableness personality as compared to mal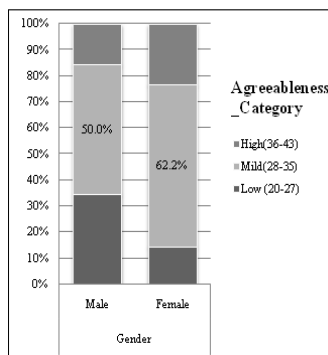e. In the multivariate analysis, (ANOVA) is used to check the association between the birth order and personality traits and Analysis of variance shows that the p-values of all the categories are insignificant thus we accept the null hypothesis and reject the null hypothesis at 5% level of significance. It is now and outdated concept where the first born or the youngest born used to get special attention from their parents. In those days families were large usually 5 to 8 kids so parents knowingly or unknowingly favor them. Now a day's however families are small 2 to 3 kids and parents are educated so equal attention is given to each child irrespective of birth order or gender. Though personality is a very complex characteristic of human beings and that the development of personality is influenced by many inborn and environmental factors, it is found that it is an individual's experiences with other members of the family and the parental treatment towards children of different birth orders that is important in the development of an individual's personality. In other words, birth order does not show an effect on the development of personality.

7. RECOMMENDATIONS

➢ It is recommended to increase the sample size and collect data from various universities so that the power and generalization of the study could be improved.
➢ Besides, researchers should take such confounding variables as sib ship size, age gap between siblings, or socioeconomic status into account if birth order study were to be conducted.
➢ In order to generalize the birth order theory, the same research should be carried out regularly in different parts of the world over a long period of time. This would give a clearer picture of the birth order effect on the development of personality and make the result a better generalization.

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INFLUENCE OF PERCEIVED STRESS ON SELF ESTEEM OF FEMALE STUDENTS OF KINNAIRD COLLEGE LAHORE

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ABSTRACT

There is a lot of stress in college students which is influencing their mental and physical status. Present study was conducted to find the relationship among self-esteem and perceived stress. It was also meant to find differences of self-esteem and perceived stress of students among different semesters. Research design was correlational. The research was based on purposive sampling technique. The sample consisted of 90 female students of Kinnaird College for women Lahore. Only bachelors' students were included. The Rosenberg self-esteem scale and Perceived stress scale were used. The Pearson product moment correlation technique and one-way ANOVA were used for data analysis. The results of Pearson correlation technique (r= -.877, p= .000) proved that there was a highly significant negative correlation between self-esteem and perceived stress. The result obtained through One-way ANOVA proved that there was difference of perceived stress of students among different semesters. It also showed that there was significant difference of self-esteem of students among different semesters. The students with high stress and low self-esteem are more in number as compare to the students with low stress and high self-esteem. This shows that students are becoming victim of stress and their self-esteem is weakened by it. The final conclusion of this study is that stress and self-esteem are highly correlated and this correlation is negative. Self-esteem can act as the remedy for the high stress level and mental health diseases caused by it, because, according to this study, the high stress level and high self-esteem are inversely related. It is important for the college administration to review policies regarding the new entrants and the outgoing semesters.

INTRODUCTION

Stress is an individual phenomenon, unique to each person and setting (Hudd et al., 2000). Stress is a condition or feeling experienced when a person perceives that personal demands and the demands made by other close people exceed the personal and social resources that individual is able to manage and deal with. People feel little stress when they have the time, experience and resources to manage a situation. It depends a lot on individual's perceptions of a condition and their ability to cope with it. (Richard, 1991). Self-esteem has been defined as an individual's evaluation of self or feelings of worthiness (Myers, 2002) or an individual's assessment of self; it is an individual's opinion regarding himself as positive or negative (Byrne & Baron, 2004). Some people perceive themselves of high worth and others may perceive themselves as worthless. A person's low self-esteem can affect their self-trust and making them not able to cope with life factors such as stress.

In this study the Faculty of Social sciences, Pure Sciences, and Humanities have been chosen as sample. The faculty of Humanities includes English Literature, Fine Arts, English Language & Linguistics, and Urdu. The faculty of Social Sciences includes Economics, Geography, International Relations, Political Science, and Psychology. The faculty of Pure Sciences includes Biochemistry, Biotechnology, Botany, Chemistry, Physics, and Zoology.

RATIONALE

The level of stress in Pakistan is very high in both genders but, females fall victim to it more easily and quickly. People of Pakistan are suffering for many social, economic, domestic, and academic problems which cause stress on high level in daily routine. The study aims at determining how much stress is influencing the self-esteem.

OBJECTIVES OF THE STUDY

- Basic aim is to determine the extent to which perceived stress affects self-esteem of female students of Kinnaird College.
- To investigate the relationship between perceived stress and self-esteem in students of Kinnaird College.
- To determine, students of which semesters have the high level of self-esteem
- To discover, students of which semesters have the high level of perceived stress.

HYPOTHESES

- H1: There is a relationship between perceived stress and self-esteem of female students.
- H2: Higher the self-esteem, lower will be the perceived stress level.
- H3: Lower the self-esteem, higher will be the Perceived stress level.
- H4: There is difference of self-esteem of students among different semesters.
- H5: There is difference of perceived stress of students among different semesters.

LITERATURE REVIEW

Reda Abouserie (1994) investigated sources and levels of stress amongst 675 second-year students at the University of Wales. Results from this study showed that students with high self-esteem are less stressed than those with low self-esteem. Self-esteem would therefore appear to have an important influence on students' stress levels. A study was conducted by Sarah K. Dixon and Sharon E. Robinson Kurpius, (2008) on the topic of depression and college stress, major concerns among undergraduates, are potentially related to self-esteem and mattering. Participants included college students (199 males and 256 females) between the ages of 18 and 23. It was found that women reported greater depression, college stress, and less self-esteem.

Following study was conducted by Subadra Panchanadeswaran and Beverly A. Dawson (2011) examined the influence of discrimination and stress on self-esteem among Dominican immigrant women. It was found that high levels of discrimination and stress were significantly associated with reduced self-esteem. Alicia D. Cast and Peter J. Burke (2002) examined marital dynamics in the first two years of marriage. During the study, it was found that self-esteem had prevented the self from experiencing stress caused by stressors like experiences and information that might otherwise cause self-distress and especially depression (Cast & Burke, 2002).

The following study used longitudinal data to observe how life events came together to form a process of stress. It took involuntary job disturbances and troubles as exemplifying life events and showed how they adversely affected self-concepts. These worse strains or stress, in turn, distorted positive concepts of self, i.e. self-esteem, and mastery. The weakened self-esteem then make person susceptible to experiencing perceived stress. (Pearlin, Menaghan, Lieberman & Mullan, 1981)

METHODOLOGY

SAMPLE

90 female students were taken from institute of Lahore namely Kinnaird College for women Lahore. The sample included those female students who had currently enrolled in their Bachelor's.

SAMPLING STRATEGY

Purposive sampling strategy was used. The faculty of Social sciences, Humanities and Pure sciences were chosen. Students were belonged to semester 2nd, 4th, 6th, and 8th. From every faculty, 3 departments were selected and 10 students from each department were the sample of the study.

INSTRUMENTS

The Rosenberg Self-Esteem Scale (Rosenberg 1965) measures self-esteem. Rosenberg (1965) reported internal consistency reliability ranging from .85 to .88 for college samples. Shin (1992) reported alpha coefficients ranging from .71 to .73. The Perceived Stress Scale (PSS) by S. Cohen (1994). It is a measure of the degree to which situations in one's life are appraised as stressful. In case of Internal reliability, Coefficient alpha is .78 (Cohen and Williamson, 1988).

RESEARCH DESIGN

Correlation research design was carried out.

PROCEDURE

Permission of the concerned authorities was taken before conducting the research. A bio data form was also attached with the questionnaires. Informed Consent of the students was also taken. It took 2 weeks to collect data.

DATA ANALYSIS

Statistical analysis was conducted on SPSS 17.0. Pearson product moment correlation technique and One-way ANOVA was used.

RESULTS

HYPOTHESIS 1: There is a relationship between perceived stress and self-esteem of female students.

Table A2: Correlation between self-esteem and perceived stress

Correlations			SES	PSS
SES	Pearson Correlation		1	-.877[**]
	Sig. (2-tailed)			.000
	N		90	90
PSS	Pearson Correlation		-.877[**]	1
	Sig. (2-tailed)		.000	
	N		90	90
**. Correlation is significant at the 0.01 level (2-tailed).				

A Pearson product-moment correlation coefficient was computed to assess the relationship between the self-esteem and perceived stress. There was a negative correlation between two variables; self-esteem and stress [r= -.877, n= 90, p= .000]. Their relationship is significant at 0.00 level.

HYPOTHESIS 2:
Higher the self-esteem, lower will be the perceived stress level.

This Person correlation shows that there was negative correlation between self-esteem and perceived stress, which means that if the value of 1 variable increases then the value of other decreases. In this case high self-esteem will cause the low perceived stress.

Table A3: Frequency and percentage of students with high self-esteem and low perceived stress

	Students with high Self-esteem and low perceived stress
Frequency	38
Percentage	42.2

The frequency of the students with high self-esteem and low stress was 38 (f=38) out of the total sample of 90 (n=90). The percentage of the students with high self-esteem and low perceived stress level was 42.2%.

HYPOTHESIS 3:
Lower the self-esteem, higher will be the perceived stress level.

This Person correlation shows that there was negative correlation between self-esteem and perceived stress, which means that if the value of 1 variable increases then the value of other decreases. In this case high perceived stress will cause the low self-esteem.

Table A4: Low self-esteem and high stress

	Students with low Self-esteem and high perceived stress
Frequency	52
Percentage	57.8

The frequency of the students with low self-esteem and high perceived stress level was 52 (f=52) out of total sample of 90. The percentage of the students with low self-esteem and high perceived stress was 57.8%.

HYPOTHESIS 4:

There is difference of self-esteem of students among different semesters

Table A5: Descriptive of difference of self-esteem of students among different semesters

	N	Mean	Std. Deviation
sem 2	18	13.00	4.627
sem 4	28	17.68	4.959
sem 6	24	14.75	6.536
sem 8	20	14.30	5.676
Total	90	15.21	5.711

Comparison of self-esteem of students among different semesters using RSE scale indicates that mean score for Semester 2 (M=13.00, SD= 4.627), and mean score of Semester 4 (M=17.68, SD=4.959) significantly differed from each other. Mean score of Semester 6 (M=14.75, SD= 6.536) and mean score of Semester 8 (M=14.30, SD=5.676) did not significantly differ from each other. However mean score for Semester 2 (M=13.00, SD= 4.627), and mean score of Semester 4 (M=17.68, SD=4.959) significantly differed from mean score of Semester 6 (M=14.75, SD= 6.536) and mean score of Semester 8 (M=14.30, SD=5.676).

Table A6: ANOVA test of difference of self-esteem of students among different semesters

ANOVA

Self-esteem scale

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	280.182	3	93.394	3.062	.032
Within Groups	2622.807	86	30.498		
Total	2902.989	89			

One way ANOVA was used to determine the difference of self-esteem of students among different semesters. There were significant differences between semesters' means as determined by one-way ANOVA, $F(3,86) = 3.062$, $P = .032$). The hypothesis was accepted.

HYPOTHESIS 5:

There is difference of perceived stress of students among different semesters

Table A7: Descriptive of differences of perceived stress of students among different semesters

	N	Mean	Std. Deviation
sem 2	18	37.00	9.935
sem 4	28	27.79	10.973
sem 6	24	33.13	12.302
sem 8	20	36.65	10.230
Total	90	33.02	11.472

Comparison of perceived stress of students among different semesters using PS scale indicated that mean score of semester 2 (M=37.00, SD=9.935), semester 4 (M=27.79, SD=10.973), semester 6 (M=33.13, SD=12.302) and semester 8 (M=36.65, SD= 10.230) significantly differed from each other.

Table A8: ANOVA test of difference of perceived stress of students among different semesters

ANOVA

Perceived stress scale

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	1316.066	3	438.689	3.628	.016
Within Groups	10397.889	86	120.906		
Total	11713.956	89			

One way ANOVA was used to determine the difference of perceived stress of students among different semesters. There were significant differences between semesters' means as determined by one-way ANOVA, $F(3,86) = 3.628$, $P = .016$). The hypothesis was accepted.

DISCUSSION

Pearson correlation had shown that there was a negative relation between the two variables, which means that if the value of one variable increases, the value of other variable decreases. The ANOVA results had shown that there was a significant difference of self-esteem and perceived stress of students among different semesters.

HYPOTHESIS 1:

There is a relationship between perceived stress and self-esteem of female students.

This hypothesis was proved. The results showed that there was a relation between self-esteem and stress at the value r= -.877 and p= .000. This means that it was a negative correlation at highly significant level. There are many studies that support the results of this study. The results of the study conducted by Reda Abouserie (1994) Sarah K. Dixon and Sharon E.R. Kurpius, (2008) on the topic of depression and college stress, major concerns among undergraduates, are potentially related to self-esteem and mattering. It was found that female students reported greater depression, college stress, and less self-esteem.

HYPOTHESIS 2:

Higher the Self-esteem, lower will be the Stress level.

It was found that the students with high self-esteem have low stress level. Person correlation results showed that there was inverse relationship between self-esteem and stress at the value r= -.877 and p= .000 According to the descriptive results of this study there are 42.2 percent of the total sample had high self-esteem and low stress. Study by Cast at el. (2002) also showed the similar results.

HYPOTHESIS 3:

Lower the self-esteem, higher will be the perceived stress level.

As it is discussed previously, Pearson correlation technique results (r= -.877 and p= .000) supported this hypothesis as well. 57.8 percent of the total sample of 90 female students had low self-esteem and high perceived stress level, which is very strong evidence in the favor of this hypothesis. There were studies conducted by Pearlin at el. in 1981 and Panchanadeswaran in 2011 which showed the similar results. High levels of stress were significantly associated with reduced self-esteem.

HYPOTHESIS 4:

There is difference of self-esteem of students among different semesters.

There were significant differences between semesters' means as determined by results, $F(3,86) = 3.062$, $P = .032$). The hypothesis is accepted because the p value was significant.

The lowest level of self-esteem is present in semester 2 and 8. However, semester 4 had highest self-esteem level. There are no previous studies to support these results. The first reason that could be the cause of low self-esteem among students of 2nd semester is that the 1st year in bachelor's is the time when students are in period of adopting, understanding, and adjusting to the systems of college and hostel. They become easily stressed and frustrated when they find things different and new. This increases their perceived stress level which in return decreases the self-esteem. In semester 8, students have to complete research of 6 credit hours as well as give exams, assignments, presentations and class tests.

HYPOTHESIS 5:

There is difference of perceived stress of students among different semesters.

There were significant differences between semesters' means as determined by one-way ANOVA, $F(3,86) = 3.628$, $P = .016$). The hypothesis is accepted.

The highest level of perceived stress was present among students of 2nd and 8th semesters. However the students of semester 4 had lower perceived stress level. As explained earlier, there are no previous studies to support these results. 2nd semester comes during the 1st year in bachelor's it is the time when students are in period of adopting, understanding, and adjusting to the systems of college and hostel.

CONCLUSION

The results show that there was a negative correlation between the two variables. The students with high stress and low self-esteem are more in number as compare to the students with low stress and high self-esteem. 42.2 percent of students are dealing very well with the stress because of their high level of self-esteem. The final conclusion of this study is that stress and self-esteem are adversely correlated. Stress and self-esteem can be decreased and increased and self-esteem can act as the remedy for the high stress level and mental health diseases caused by it.

LIMITATIONS AND RECOMMENDATIONS

The study cannot be generalized on a larger scale in Pakistan because only population of Lahore was taken as sample. No other variable's effect on self-esteem was measured.

The study was conducted in a very short span of time. The main drawback of this study is that it was conducted only for female students. There was no previous study to support the result regarding differences of self-esteem and perceived stress among different faculties and semesters. There is need to do more study in this field. The sample size should be increased.

Population of other major cities of Pakistan should be included. The sample should be taken from more than one institution. To make results more applicable, both genders should be the part of study. Standard interviews should be the part of study.

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ASSESSMENT OF PREVALENCE OF ERGONOMIC DISCOMFORT SYMPTOMS AMONG THE BRICK KILN WORKERS – A CASE STUDY OF LAHORE, PAKISTAN

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ABSTRACT

Brick kilns are one of the oldest manual material handling industries settled on the outskirts of Lahore. This study was conducted to find the detrimental ergonomic health impacts of the brick kilns on the workers of the brick kilns. The study was conducted in Wahga and Batapur areas of Lahore.

The study was carried out during the functioning period of the brick kilns from August 2011 to March 2012. Ergonomic evaluation of the workers was carried out with the help of Modified Nordic Questionnaire.

A sum of 236 individuals were interviewed and assessed for ergonomic health, out of which 165 were listed as exposed since they were the brick kiln workers. Their data was compared with 71 individuals who were grouped as control (farmers) and were not directly exposed to the environment of the brick kilns.

The brick kiln workers were under more ergonomic stress as compared to the control group and exhibited higher prevalence (P<0.005) of pain in shoulders (57%), upper (75%) and lower back (84%), knees (53%), ankles and feet (35%) in comparison to the control group.

Government of Pakistan and All Pakistan Brick Kiln Association should join hands and introduce brick manufacturing technologies such as vertical shaft brick kiln which is efficient and environmentally safe as compared to the Bull's Trench Brick Kiln. Also, the work place should be made more comfortable for the worker to reduce the ergonomic stresses.

1. INTRODUCTION

Brick is the best material for building structures in both urban and rural localities. Not only this, but with minimum maintenance, brick buildings usually survive for a long time. Brick manufacturing in many Asian countries is principally a cottage industry and for this purpose traditional kilns, which are energy-inefficient and polluting are used. In recent years, with ever-increasing urbanisation and the demand for construction materials to maintain the expansion work, brick kilns have grown both in number and size. In Pakistan alone, there are more than 10,000 brick kilns working on old technology (bull trench kiln) which used to be located in the remote areas of the country but because of escalating urbanization these brick kilns have embraced the residential areas in their polluting circle as well [1].

According to an estimate the brick kilns of Pakistan produce approximately 100 billion bricks per year [2].

Pakistan relies upon sub-standard fuel, obsolete fuel wasting units and techniques, which contribute to air pollution and emission of green house gases, thus generating negative economic and environmental impacts. In Pakistan brick production accounts for 54% of all coal consumption [3].

Studies on the environmental impacts of brick kilns in Pakistan are scarce while data on emissions from brick kilns is not available. The lack of Procedural Guidelines to analyze occupational health risks in the industries is another setback in the field of occupational health risks management in Pakistan [4].

The current situation of brick manufacturing shows that it is very labor-intensive, since bricks are generally molded with hands and dried under then sun before firing in the kiln. The firing of the bricks is mainly done in conventional set-ups such as in Bull's trench kilns (BTK). BTK is a fuel wasting and redundant technology which plays a vital part towards increasing air pollution and emission of green house gases, hence creating plentiful negative economical and environmental impacts. Beside pollution, inside this sector there are social predicaments such as bonded labor, child labor, work-place adverse living environments, unhygienic water and sanitation conditions, poor education and adverse occupational health conditions [5].

Brick manufacturing work is an age-old profession practiced all over the world. In this industry a lot of manual materials handling (MMH) tasks are performed. Although developed countries are getting benefit from the introduction of mechanization in the brick kilns, various studies still show that the workers of the brick manufacturing units suffer from musculoskeletal problems [6] [7] [8] [9].

In spite of the technological advances a great number of workers still have to perform heavy MMH jobs in the under developed countries, especially in the unorganized sectors. Data from developing countries like India show that the brick kiln workers fall in to a lot of assorted health problems due to the uneasy postures which they have to maintain while carrying heavy loads [10] [11].

Researches concluded that musculoskeletal disorders (MSDs) result from frequent truck bending, twisting and repetitive handling of several bricks at a time. In addition to this, some studies report women having a high prevalence rate of work related MSDs than of men [12] [13].

2. OBJECTIVES OF THE STUDY

The main purpose of this research study was to assess the impact of brick kilns on environment and human health. The specific objectives of the study are:

- To conduct an interview based health survey among the workers employed in various tasks of the brick kilns.
- Ergonomic evaluation of the posture and the musculoskeletal disorders (MSDs) of the workers engaged in the brick manufacturing units through questionnaire based interviews.

3. METHODOLOGY

The present study was conducted in the area of Wahga and Batapur of Lahore district of Lahore.

Figure 3.1: Map showing the sampled brick kilns

3.1 Area profile

Batapur and Wagha areas of Lahore district are situated on the Eastern outskirts of Lahore. They are the hubs of brick manufacturing units and are located approximately 5km south of Grand Trunk (GT) road. The estimated number of brick kilns in the study area is 40 and all of them employ the obsolete Bull Trench Kiln Technology. The area is neither densely populated nor well developed areas. The land is mainly used for agricultural purposes.

3.2 Population

A sum of 236 individuals were interviewed and assessed for ergonomic health, out of which 165 were listed as exposed since they were the brick kiln workers. Their data was compared with 71 individuals who were grouped as control (farmers) and were not directly exposed to the environment of the brick kilns.

Grouping of human subjects

The workers who had been working at the sampled brick kilns for more than six months were considered as 'exposed', devoid of their age, gender and job categories. The workers were put in the following major categories based on the various tasks they perform at the brick kiln.

1. Loading and unloading workers
2. Brick kiln fuel managers
3. Brick makers

For the ergonomic evaluation, all three groups were classified as high risk group, because irrespective of the job category, every worker at the brick kiln was under repetitive stress while working.

The sampling was conducted using the random sampling technique. Workers whose age was between 18- 60 years and who were working at the brick kiln site for more than 6 months were categorized into the three exposure groups mentioned above. Randomly, 13 workers were selected from each exposure group of every brick kiln. Considering the constraints of time and funds; the sample size per exposure group was determined to be 52. Supposing the loss to follow up to be approximately 10%, the final total sample size was determined to be 170.

The control group was selected from the local population involved in agriculture. Cluster sampling method was used to collect control samples from the nearby villages. People whose age were between18-60 years and belonged to the same socio-economic group as the brick kiln workers were selected.

3.3 Questionnaire

Nordic Modified Questionnaire has been used to collect the information regarding the ergonomic health effects of Lahore's brick kilns. The questionnaire included demographic information, occupational history, areas of discomfort and pain in the body. The body mass index (BMI) was also calculated by taking weight in kilograms and the height in meters squared.

3.4 Evaluation of ergonomic problems

Modified Nordic Questionnaire and Body Part Discomfort (BPD) scale was applied on these workers to identify the MSDs and the zones of discomfort in different body parts.

3.5 Data analysis

Data collected will be entered into the database and analyzed with the statistical package for social sciences (SPSS). The target sample size was calculated after establishing the confidence interval was of 95% and P (<0.05) was considered significant. The statistical significance level alpha was set to be 95. ANOVA test was employed to assess the differences in prevalence of respiratory, ergonomic and other health symptoms between the exposed and control group.

4. RESULTS

It is evident from the Table 4.1 that no significant difference was found in the age, height, Body Mass Index, average working hours between the two groups. Nevertheless, significant difference existed between their weight and working years.

Figure 4.1 and 4.2 show the prevalence of musculoskeletal disorders in the brick kiln workers. 35% of the subjects had suffered from constant neck pain for the duration of at least two months. Among those ailing, 35% were the brick moulders, 32% were from the loading and unloading group and 33% were the fuel managers

57% of the subjects complained about suffering from pain in shoulders for the duration of at least two months. Among those who suffered, 32% belonged to the brick moulding group, 34% to the loading and unloading group and 34% to the fuel managers. 27% of the subjects complained to experience pain in elbows for the duration of at least two months. Among those, 32% of the subjects belonged to the brick moulding group, 36% were from the loading and unloading group and 32% were from the fuel managing group. 31% of the subjects experienced pain in wrists and hands for the duration of at least two months. Among the sufferers, 25% belonged to the brick moulding group, 43% to the loading and unloading group and 31% to the fuel managing group.

75% of the subjects had suffered from upper back pain for the duration of at least two months. Among those who suffered 34% belonged to the brick moulding group, 33% belonged to the loading and unloading group and 33% to the fuel managing group.

84% of the subjects had suffered from pain in lower back for the duration of at least two months. Among the sufferers, 35% were the brick moulders, 33% were the loaders and unloaders of bricks and 33% were the fuel managers.

25% of the subjects complained about suffering from pain in hips and thighs for at least two months. Among the sufferers, 36% were the brick moulders, 33% were the loaders and unloaders of bricks and 31% were the fuel managers.

53% of the subjects had suffered from pain in knees for the duration of at least two months. Among those, 35% were the brick moulders, 30% were the brick loaders and unloaders and 35% were the fuel managers.

35% of the subjects had suffered from pain in ankles and feet for the duration of at least two months. Among the sufferers, 33% were the brick moulders, 30% were the loaders and unloaders of bricks and 37% were the fuel managers.

Table 4.2 shows the number of musculoskeletal discomforts experienced by the members of three exposure groups. It is apparent that among all the working groups, the pain in upper and lower back was most common. It is obvious from the statistical result showed in Table 4.3 that there was no significant difference in the prevalence of pain in neck and elbows and wrists between the control and the exposed group. There was a significantly high prevalence of pain in shoulders, upper and lower back, knees, ankles and feet among the subjects of the exposed group as compared to those of the control group.

5. CONCLUSION

Based on the observations and results of this study following conclusion has been drawn:

The study concludes that there is a strong association between the detrimental work conditions of the brick kilns and ergonomic health effects faced by the workers. Prolonged hours of tedious, repetitive work in precarious postures adopted by the brick kiln workers cause pain and discomfort in various parts of the body. However, because of their poor socio-economic conditions they have no choice but to work as much as possible to gain more money, even if they fatigued.

The environment of all the brick kilns is highly hazardous. Safety is not taken into consideration while working. There is no use of personal protective equipment and no safety measures are adopted while carrying out various chores. The workers at the brick kilns show great reluctance towards using personal protective equipment. This is indicative of their unawareness about the health and safety issues at work which is mainly due to poverty and ignorance. The managers show callous attitude towards the workers and are not bothered by their problems. The government, brick kilns owners association and other responsible authorities have been neglecting the issue of occupational health of the brick kiln workers. This is evident from the results of this study, which indicate serious occupational health disorders of ergonomic nature.

6. DISCUSSION

There was high prevalence of ergonomic disorders among the brick kiln workers because there work was more strenuous and requires additional hard work as compared to the farmers of the control group. Not only this, the diet of the brick kiln workers was not healthy as well. The brick kiln workers were barely able to obtain one full meal per day and are sometimes because of the financial constraints they are deprived of that as well.

The harsh health damaging environment of the brick kilns causes drastic damage to the health of the workers and only the toughest of them survive for long. While the farmers of the control group were mainly exposed only to dust and heat, the brick kiln workers faced multiple health threats like inhalation of gases emitting from the brick kiln, ergonomic problems, and excessive dust. The brick kiln workers are seasonal migrants as well. They work in one area for a few years and then move on. Since brick making is a seasonal work and is fully functional only six to eight months a year, it attracts the migrant labourers which are rural workers having a lean year wanting to earn some extra keep. Hence the number of working years of the worker at a brick kiln is comparatively less than that of a person involved in agricultural practices [14].

The work at brick kilns particularly involving making green bricks entails labor-intensive manual handling of heavy weights of mud and green bricks, strenuous and extended working hours in discomfited posture, tedious and repetitive work, and other risk factors which engage excessive use of hands, shoulders, back muscles and joints. Some chores are carried out in painful and cramped posture. Basra and Crawford established that brick kiln workers suffer from musculoskeletal disorders [15].

Pain in upper and lower back was the most common discomfort reported by the workers. Its justified because most of their work involves twisting of the bodies at odd angles, loading and unloading bricks from above the shoulder height and bending repeatedly to perform tasks.

The green brick moulders reported pain in the upper and lower region of the back because they had to squat for extended periods of time and sit continuously in the same uncomfortable manner while they moulded the green bricks. The loaders and unloaders of the green and baked bricks respectively had to frequently bend and undergo repetitive stress while packing and unpacking the brick kiln. The transportation of the bricks from the site of moulding and that of the baked bricks from the kiln was done manually as well. This caused stress on the lumbar region.

It was observed that most workers, regardless of their age, could not stand up quickly once they had squatted to do their daily chores. This was mainly because of the pain in the upper and lower region of the back. Similarly the fire managers of the brick kiln also squatted on the ground most of the time while they vigilantly kept the temperature of the kiln at optimum level to attain the ideal baking of the bricks. Goldsheyder et al. as the result of a survey on the musculoskeletal symptoms found that almost 77% of the workers suffered from at least one musculoskeletal disorder each year [16].

Statistical comparison of the exposed and the experimental group revealed that musculoskeletal disorders such as pain in shoulders, upper back, lower back, knees, ankles and feet were more commonly found in the exposed as compared to the control group. Various studies support the fact that workers working in the brick manufacturing units suffer from musculoskeletal disorders.

Sahu et al. states that according to OWAS and REBA methods, the postures adopted by the brick moulders have been categorized as having 'high' to 'Very High' risk levels and that of the postures adopted by the brick carriers have 'medium' to 'High' risk levels, even though complaints about the different discomfort feelings are more prevalent among the carriers [17].

This is because despite the technological improvements made by the western world in the production of bricks, in less developed countries, bricks are still made by manual handling of the materials. Carrying heavy loads and frequent bending of the trunk, holding loads away from the trunk, twisting and reaching upwards, bending at knees results in strains and sprains which if ignored can cause long term damage to the workers. The factor that the workers are not given sufficient time to recuperate or recover after a tedious task causes muscular fatigue. This is because a specific work rate is imposed on the workers and they have to complete their tasks in certain duration despite other heat stress, their physical health and any other factors involved.

Table 4.1
Demographics data of the exposed brick kiln workers and the controls (n=236)

Demographic data	Exposed workers Mean ± SD	Control workers Mean ± SD	P-Value ANOVA test
Age (years)	31.6±11.6	32.5±9.35	0.288
Weight	51.7±8.0	58±4.9	0.002
Height	4.9±0.2	5.0±0.11	0.242
Body Mass Index	19.8±2.59	21.4±2.6	0.156
Working (years)	13.6±9.1	18.12±8.2	0.001
Average working hours	9.1±3.8	9.2±2.0	0.134

NS - not statistically significant
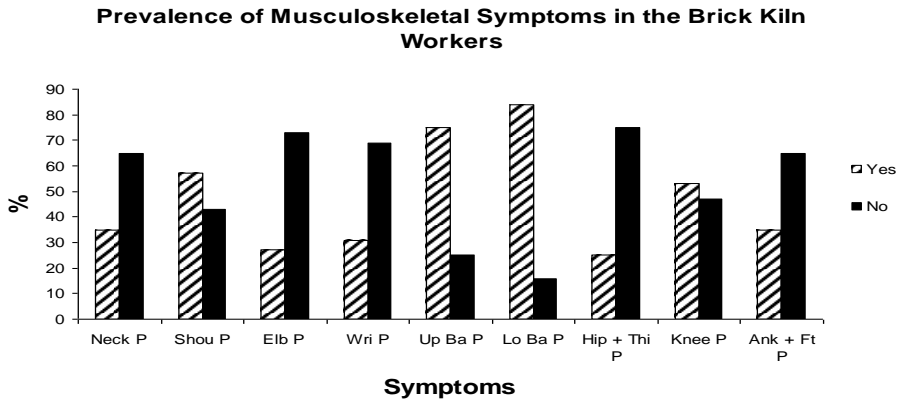
Prevalence of Musculoskeletal Symptoms in the Brick Kiln Workers

* Neck P=neck pain, Shou P=shoulders pain, Elb P=elbow pain, Wri P=wrist pain, Up Ba P=upper back pain, Lo Ba P=lower back pain, Hip + Thi P=hips and thighs pain, Knee P=knee pain, Ank + Ft P=ankle and feet pain.

Figure 4.1: **Prevalence of musculoskeletal symptoms in the brick kiln workers**
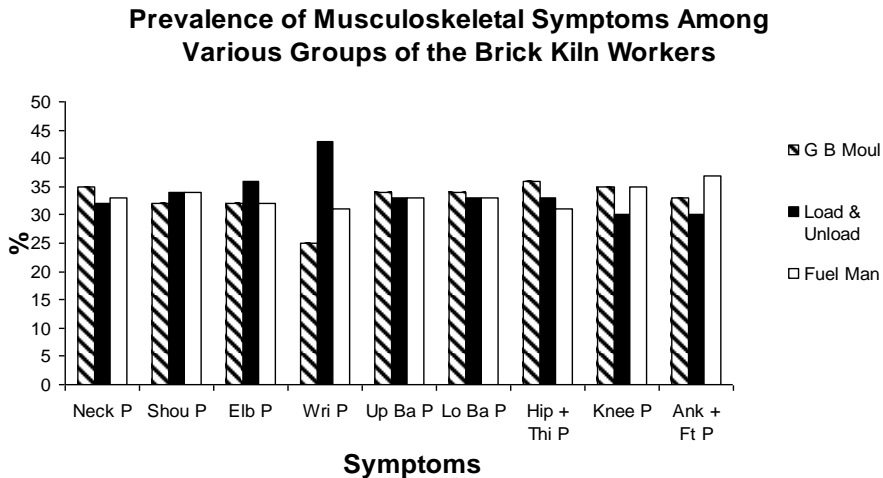
Prevalence of Musculoskeletal Symptoms Among Various Groups of the Brick Kiln Workers

* Neck P=neck pain, Shou P=shoulders pain, Elb P=elbow pain, Wri P=wrist pain, Up Ba P=upper back pain, Lo Ba P=lower back pain, Hip + Thi P=hips and thighs pain, Knee P=knee pain, Ank + Ft P=ankle and feet pain.
** G B Moul=green brick moulders, Load &unload=Loaders and unloaders of bricks, Fuel Man=Fuel managers.

Figure 4.2:   Prevalence of musculoskeletal symptoms among various groups of the brick kiln workers

Table 4.2
Number of ergonomic discomforts experienced by the workers of each group

Process/symptoms	N	S	E	W&H	UB	LB	H&T	K	A&F
Green brick moulding (55)	20	30	14	13	42	48	15	31	19
Loading and unloading of bricks (55)	18	32	16	22	41	45	14	26	17
Fuel and fire management (55)	20	32	14	16	40	45	13	31	21

*   N= neck, S=shoulders, E=Elbows, W&H= Wrists and Hands, UB= Upper back,
    LB= Lower back, H&T= Hips and thighs, K= Knees, A&F= Ankles and feet

Table 4.3
Comparison of exposed and control group for ergonomic evaluation

Pain/ Discomfort	Exposed Group Mean± SD	Control Group Mean± SD	P-value
Neck	0.3515± 0.4789	0.2394± 0.4298	0.091
Shoulders	0.5697±0.4966	0.1972±0.4007	0.000
Elbows	0.2667±0.4436	0.1972±0.4007	0.257
Wrists/Hands	0.3091±0.4635	0.2254±0.4208	0.192
Upper back	0.7455±0.4369	0.1972±0.4007	0.000
Lower back	0.8364±0.3711	0.1549±0.3644	0.000
Knees	0.5333±0.5004	0.1268±0.3351	0.000
Ankles and feet	0.3455±0.4770	0.2113±0.4111	0.040

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AN INVESTIGATION ON TYPES OF PHOBIAS EXPERIENCED
BY THE FEMALE RESIDENTS OF FOUNTAIN HOUSE

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ABSTRACT

Nowadays, even if environment is not half as sinister as it evidently was back then, fear is still a central emotion that colors our existence to the extent and phobia is the irrational fear of something. The study explored the association between age and phobias, family factors and phobias and social factors and phobias respectively. A sample of 150 female respondents including 30 cases and 120 controls is taken. Subtypes of phobias mainly differed in cases and controls dimensionally on a no phobia, mild, moderate and severe continuum. Social phobia Inventory SPIN (2000) is used to detect social phobia on a very severe, severe, moderate, mild and no phobia continuum. By applying the Pearson chi square and spearman's rank correlation, it is concluded that in cases, age is associated with Agoraphobia (fear of open places) and Acrophobia (fear of heights), marital status is associated with Agoraphobia (fear of open places) and Acrophobia (fear of heights), marital status of father is associated with social phobia, marital status of mother is associated with Social phobia and Aichmophobia (fear of sharp objects), traumatic event is associated with Mysophobia (fear of germs) and family income is associated with Acrophobia (fear of heights) and Aichmophobia (fear of sharp objects). Whereas in controls, only two results age and Agoraphobia (fear of open places) and marital status and Acrophobia (fear of heights) shows significant results. By applying multiple discriminant Analysis in cases it has been concluded that age is the best predictor in predicting Agoraphobia (fear of open places) & Acrophobia (fear of heights), marital status of father is the best predictor in predicting Nosocomeophobia (fear of hospitals) and traumatic event is the best predictor in predicting Mysophobia (fear of germs).

1. INTRODUCTION

A phobia is an intense but unrealistic fear that can interfere with the ability to socialize, work, or go about everyday life, brought on by an object, event or situation. According to the American Psychiatric Association, a phobia is an irrational and excessive fear of an object or situation. In most cases, the phobia involves a sense of fear of harm.

Phobias themselves can be divided into three specific types:

- Specific phobias (formerly called simple phobias)
- Social phobia
- Agoraphobia

Specific phobia is the fear of a particular situation or object, including anything from airplane travel to dentists. Types of Specific phobias which is used in our study are Ablutophobia (fear of bathing), Acrophobia (fear of heights), Agrizoophobia (fear of wild animals), Aichmophobia (fear of sharp objects), Mysophobia (fear of germs), Nosocomeophobia (fear of hospitals), Taphophobia (fear of graves) and Thalassophobia (fear of sea). Symptoms of specific phobias are breathlessness, excessive sweating, nausea, dry mouth, irregular heartbeat, feeling sick, shaking, losing control and anxiety attack.

Social phobia is a persistent and irrational fear of situations that may involve scrutiny or judgment by others, such as parties and other social events. This may extend to a general fear of social situations or be more specific or circumscribed, such as a fear of giving speeches or of performing (stage fright).

Agoraphobia is the intense fear of feeling trapped and having a panic attack in a public place. After an initial panic attack, the person becomes afraid of experiencing a second one. Patients literally "fear the fear," and worry incessantly about when and where the next attack may occur. As they begin to avoid the places or situations in which the panic attack occurred, their fear generalizes. Eventually the person completely avoids public places. In severe cases, people with agoraphobia can no longer leave their homes for fear of experiencing a panic attack.

2. PROBLEM STATEMENT

An investigation on types of phobias experienced by the female residents of Fountain house.

3. OBJECTIVES OF THE STUDY

(a) To explore the factors causing phobia. (b) To compare the severity of Phobias between cases and controls.(c) To explore the relationships between factors and types of phobias. (d) To test various hypotheses regarding factors causing different types of phobias.

4. METHODOLOGY

This study has been conducted among the 150 Cases and Controls females of Lahore with the respective ratio of 1:4. A cross sectional study is conducted among the cases and controls. Multiple probability sampling designs are used. Cluster, Stratified, Systematic and convenience sampling designs have been used.
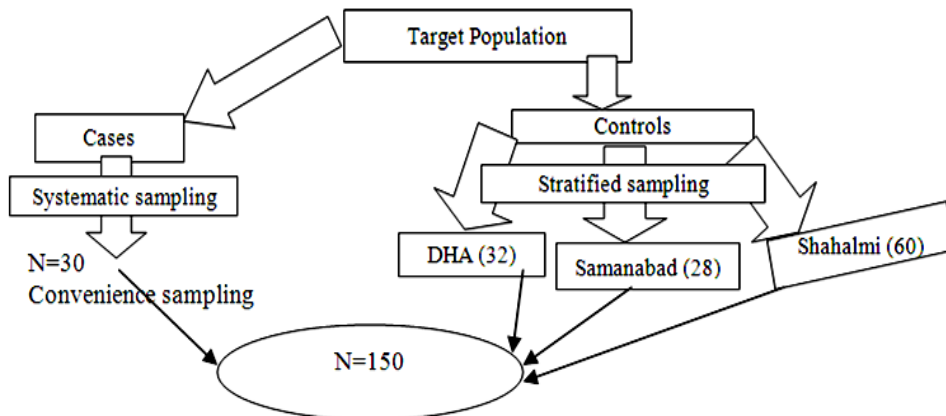
A well structured questionnaire including Social Phobia Inventory (SPIN) Connor K.M., et.al (2000) for social phobia is used. First we collected data from cases and then from controls restricted their age group and social status according to cases. Data analysis techniques include Univariate analysis, bivariate analysis (chi-square test of association and Spearman's rank correlation) and multivariate analysis (Multiple Discriminant Analysis) SPSS, MINITAB and EXCEL are used as data analysis tool.

4. RESULTS AND INTERPRETATION

Table 1: Bivariate significant results (p-value<α) of cases

Dependent Variables	Independent variables					
	Age	Marital status	Family income	Marital status of father	Marital status of mother	Traumatic event
Agoraphobia (fear of open places)	0.024	0.007				
Social phobia				0.010	0.009	
Mysophobia (fear of germs)						0.001
Acrophobia (fear of heights)	0.001	0.038	0.011			
Aichmophobia (fear of sharp objects)			0.000		0.032	

Table 2: Bivariate significant results (p-value<α) of controls

Dependent variables	Independent variables	
	Age	Marital status
Agoraphobia (fear of open places)	0.000	
Acrophobia (fear of heights)		0.004

Table 3: Common significant results of cases and controls

Variables	Cases	Controls
Age and Agoraphobia	0.024	0.000
Marital status and Acrophobia	0.038	0.004

Table 4: Multivariate results of cases using multiple discriminant analysis

Dependent Variable	Independent variables	Best predictor
Agoraphobia	Age, Marital status, Family income, Qualification, family system, Family background, Marital status of Father, Marital status of Mother, Traumatic event and age period in which you experienced any traumatic event	Age (canonical correlation = 0.696)
Social phobia		no variable is qualified
Ablutophobia		no variable is qualified
Acrophobia		Age (canonical correlation = 0.602)
Agrizoophobia		no variable is qualified
Aichmophobia		no variable is qualified
Mysophobia		Traumatic event (canonical correlation = 0.639)
Nosocomeophobia		Marital status of Father (canonical correlation=0.602)
Taphophobia		no variable is qualified
Thalassophobia		no variable is qualified

Table 1 shows the significant bivariate results of cases age and marital status are significant variable for Agoraphobia (fear of open places) and Acrophobia (fear of heights), family income is a significant variable for Acrophobia (fear of heights) and Aichmophobia (fear of sharp objects), marital status of father and marital status of mother are significant variable for Social phobia, marital status of mother is a significant variable for Aichmophobia (fear of sharp objects) and traumatic event is a significant variable for Mysophobia (fear of germs).

Table 2 shows the significant bivariate results of controls, only two results are significant i.e., age is a significant variable for Agoraphobia (fear of open places) and marital status is a significant variable for Acrophobia (fear of heights).

Table 3 shows the common significant results among cases and controls. Age and Agoraphobia (fear of open places) shows significant results and marital status and Acrophobia (fear of heights) shows significant results in common.

Table 4 shows multivariate results of cases. After applying stepwise discriminant analysis we got the best predictor variables for every dependent variable. Age is the best predictor for Agoraphobia (fear of open places) with a canonical correlation of 0.696 means that the model explains 48.5% (0.696^2) of variation of the dependent variable. Age is also the best predictor for Acrophobia (fear of heights) with a canonical correlation of 0.602 means that the model explains 36.2% (0.602^2) of variation of the dependent variable. Traumatic event is the best predictor for Mysophobia (fear of germs) with a canonical correlation of 0.639 means that the model explains 40.8% (0.639^2) of variation

of the dependent variable. Marital status of father is the best predictor for Nosocomeophobia (fear of hospitals) with a canonical correlation of 0.899 means that the model explains 80.8% (0.899^2) of variation of the dependent variable.

When we apply Discriminant analysis on Ablutophobia (fear of bathing), Aichmophobia (fear of sharp objects), Agrizoophobia (fear of wild animals), Social phobia, Taphophobia (fear of graves) and Thalassophobia (fear of sea), none of the predictor variable are qualified for this analysis. This means that not any of the predictor variables are a best predictor for these phobias.

5. CONCLUSION

From the results of Univariate analysis it is concluded that cases are suffering from moderate Agoraphobic while controls are suffering from mild Agoraphobic, cases are suffering from moderate Social phobic while controls have no Social phobia. On the whole cases are suffering from severe specific phobias while controls are suffering from mild specific phobias.

From the Bivariate analysis of both cases and controls it is concluded that there is association between age and Agoraphobia and marital status and Acrophobia.

In multivariate analysis Age shows significant results in predicting Agoraphobia and Acrophobia, Family factor (Marital status of father) shows significant results in predicting Nosocomeophobia and Social factor (traumatic event) shows significant results in predicting Mysophobia.

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DETERMINANTS OF FERTILITY
(A CASE STUDY OF DISTRICT MARDAN KHYBER PAKHTUNKHWA)

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ABSTRACT

There is abundance of literature on the factors determining the fertility level. This also includes macro or aggregate level studies to probe the phenomenon. This paper is a micro level attempt to find the factors determining fertility level. The finding reveals that literacy of the head of household (LHH), use of contraceptives (UC) and female literacy rate (FLR) are negatively related with fertility rate (FR). While female labor force participation rate (FLFP) significantly affects fertility rate (FR) with positive coefficient as revealed in other previous studies. Similarly per capita income (PCI) also has positive relation with fertility. The findings would help the policy makers regarding any policy to population control and effective use of various techniques used for decline in fertility.

1. INTRODUCTION

Population is the main focus of economics since Malthusian time. One way to study the ratio of population growth is by understanding the elements of fertility rates. The understanding of these elements could be taken as guide line for making socio- economic policies useful for poverty alleviation.

During the second half of the twentieth century, medical and health sectors advanced a lot, which reduced the mortality rate and lead to faster increase in world population. World population more than doubled growing from 2.51 billion to 6.07 billion in the era of 1950 to 2000, respectively (Hakim, 2000). This is mainly because of above average growth rate observed in less developed countries. Approximately, population growth in less developed countries has been recorded at about three times since 1950. However, efforts have been made over last few decades to control the population growth in developing and least developing countries (LDC). Developing countries like South Korea (0.9%), Indonesia (1.3%), India (1.7%), Iran (1.2%) and Bangladesh (1.7%) have somewhat controlled the population growth in last thirty years (Population Reference Bureau, 2001). This fertility decline has been made possible by progammes like Family Planning Programs sponsored either by government or by non-governmental organizations, and anti-natalist policies (UNO, 2000).

Leading Female Indicators in South Aisa

Country	Contraceptive Prevalence Rate	Female Labor Force Participation	Female Literacy Rate (+15)
India	41	38	47
Pakistan	24	24	32
Bangladesh	49	26	66
Sri Lanka	66	47	81

UN Report (2002)

Education brings socio economic changes in different societies which lead to development. Increase in population has been controlled in underdeveloped countries by adopting certain methods through education and awareness. Pakistan is facing problems in reducing population growth rate because of less female education as compared to India, Sri Lanka and Bangladesh. The role of women in economic development in Pakistan is 24% which is less than India, Bangladesh and Sri Lanka that is 38%, 26% and 47%, respectively. The high fertility rate in Pakistan is the product of low literacy rate and ineffective contraceptive prevalence methods. Similarly, the ratio of contraceptive prevalence methods in Pakistan is as less as 24% compared to 41% in India, 49% in Bangladesh and 66% in Sri Lanka.

There are many reasons by which Pakistan is behind in such developments. For instance, Pakistani society is not only strictly religious but also traditional hindrances are present. Female participation is hardly encouraged in socio economic activities and they are preferred and forced to stay within their houses. Lack of education and awareness has caused high fertility rate especially in rural areas.

Another reason of high population growth rate is the large family size. Families with large number of members are considered powerful in rural areas. Sons are given preference against daughters which sometimes increases the fertility as well as the family size.

While studying population growth of Pakistan, it is observed that Pakistan has consistently maintained high fertility and population growth in last several decades.

This study is a micro level learning being focused on a small tehsil Takht-Bhai of district Mardan in province of Khyber Pakhtoon Khwa. The selection of the tehsil is based on its highest fertility rate and population amongst other districts of province. The tehsil Takht-Bhai is located at 34°12'0N 72°1'60E with an altitude of 283 meters (931 feet) lying the south-west of the district. Its population according to 1998 census growth rate is estimated at 1.96 million in 2009. The literacy rate of the tehsil was recorded at 48.27% in 2009.

Research Objectives

- To find out the socio and economic determinants of fertility; To find out the impact of each variable on fertility in the study area; To suggest policies for reduction in fertility in the study area.

Research Hypothesis

H_0: Social and economic factors do not cause fertility
H_1: Socio and economic factors cause fertility

2. REVIEW OF LITERATURE

There is rich literate about fertility rate & their impact on socio-economic factor of developing countries. But keeping in view the objectives, the relevant literature has been reviewed & mentioned here in this paper:

Akman (2002), observed the relationship between education and fertility rate in Bangladesh. The result shows that education is an important determinant of fertility in Bangladesh. The study explains that the education in general and secondary education in particular put positive impact on reducing fertility rate and also give equal & good status to women in every walk of life. The inverse relationship between fertility & education states that if the level of education is high then the fertility rate will be low & vice versa. He suggests that education should be provided by the government to every citizen irrespective of their gender.

Fauzia et al. (1999), has conducted a survey to compare the fertility rate in urban areas and rural areas. She finds that the women in the urban areas had lower fertility than the women in the rural areas. The face behind this difference is due to the prevalence of family planning programs in urban areas. Secondly, the awareness of uses of contraceptive methods is more in urban areas as these are more accessible through government hospitals & clinics. Despite such facilities also available in rural areas, there are still some obstacle in the wide spread of family planning programs. She suggested that awareness about family planning programmes in rural areas would allow them to use different methods for reducing fertility rate. And the government should provide facilities to the rural areas & also encourage the services of private institutions which can play an important role in this respect.

M M Masih (1998) in his study finds the determinants of high fertility. He used time series data & cointegration techniques for long run & vector correction model for short run dynamics. The results suggest that female education, family planning negatively affect fertility rate in the underdevelopment economy like India.

Mohsin et al. (2003), in their of female education, marriages at lower age, ignorance of the contraceptive methods & other religious & traditional values. They use econometric techniques multiple regression model & co-integration. They suggest in their report that the government of Pakistan should focus on female education, family planning programs, encourage & create awareness regarding contraceptive methods to overcome the problem of rapidly rising population.

Shaheen et al. (2007), used cross section data to find out relationship between education & fertility & also the impact of contraceptive use of the fertility. They observed that higher will be female education the lower will be the fertility rate. The use of contraceptive is also negatively related with fertility.

Soomro G Y et al. (2000), conduct the impact of the female education & fertility implication for family planning programme in Pakistan. The data which they collected from their survey shows that the educated women have lesser children than illiterate women. They were of the view that the educated women can control their fertility even in the absence of family planning progarmme. The result suggests that the government of Pakistan should make investment in female education.

Toor IA (2007), observed that education at any level can impact the fertility rate. He uses the cross section data for the study area & finds the impact of different variables on fertility rate like school; life expectancy; education of married women; index of development; health status; female labor force participation; infant mortality rate & some religious & traditional values. He observed that fertility may be positive because of weak relationship between health facilities & infant motility rates. He also suggested that only higher education can influence the fertility rate.

3. METHODOLOGY AND DEFINITION OF VARIABLES

This part deals with the research methodology of the study "Impact of socio economic factors on fertility rate in Tehsil Takht-i-Bhai district Mardan".

3.1 Fertility Model and Definition of Variables.

Economic theories of fertility assume that parents have the number of children they do because they desire approximately that number, given the costs of birth control. This demand for more children is affected by many socio-economic factors such as the level of human capital of family members, family income and the experience of child mortality.

The main objective of the current study is to find out the significances of explanatory variables, viz. (LHH, LR….) with the dependent variable FR. For this, we have estimated the linear econometric model as

$$FR = b_0 + b_1 LHH + b_2 FLR + b_3 UC + b_4 PCI + b_5 FLFP + ui$$

where

$FR = FERTILITY\ RATE$
$LHH = LITERACY\ OF\ THE\ HOUSEHOLD\ HEAD$
$FLR = FEMALE\ LITERACY\ RATE$
$PCI = PER\ CAPITA\ INCOME$
$UC = USE\ OF\ CONTRACEPTIVES$
$FLFP = FEMALE\ LABOR\ FORCE\ PARTICIPATION$
and
$b_{i,s}$ are parameters to be estimated.

3.1.1 Education of the Head of Household and Fertility

A literate household head also having inverse impact on fertility. While being literate, he is relatively well aware of the adverse impact of the large family size.

3.1.2 Education and Fertility

Education not only important for individual but also to the society as such because it enable individual adaptable to changing conditions. Women with education favor low fertility due to several reasons. The educated mother prefers child schooling rather than

involvement in domestic or agriculture activities. Educated women also take care of her health as well as of their child. As a result of these the infant mortality rate also becomes low. Thus we can say that there is inverse relationship between fertility and female literacy rate.

3.1.3 Economic Conditions and Fertility

The relationship between economic conditions and fertility is not clear. There are two different views about income−fertility relationship. One view is that there is positive relationship between income and fertility while second view is that there is inverse relationship between income and fertility. The rationale behind this positive income-fertility relationship is that, holding everything else constant, higher income implies greater resources available to support a large family. In contrast the supporters of negative income−fertility relationship argued that most of the people with higher incomes stress upon quality of Child not on quantity and thus rise in income might lead to a reduction in fertility. Furthermore, a majority of the later studies tend to support this negative association.

3.1.4 Female Labour Force and Fertility

Female labour force participation portrays a negative relationship with the fertility rate. The literature shows that self-employment can allow women to generate income while taking care of their children and other household responsibilities. Public sector employment often involves shorter hours and the presence of child care facilities that makes it more compatible with child rearing. Most studies suggest that in underdeveloped countries the women with some job wish to have more children. However, a−priori we can't say anything about relationship between Female labor force participation and fertility rate.

3.1.5 Contraceptives and Fertility

The use of contraceptives has inverse relationship with fertility as when use of contraceptives rises, fertility will decline.

Following table explain the expected signs of different variables used in this study in relation to fertility rate in selected study area:

Variables	Expected Signs
LHH	−
FLR	−
PCI	+ / −
UC	−
FLFP	+ / −

3.2 Universe of the Study

According to objectives of the research study, a comprehensive interview schedule comprising 16 questions was developed. Considering the sensitivity of the issue in the study area, some irrelevant questions were dropped from the interview schedule and only necessary questions were asked for obtaining meaningful responses.

3.3 Sampling and Sample Size

The sample is selected from Tehsil Takht-i-Bhai district Mardan and 100 respondents were selected by simple random sampling.

3.4 Data Analysis

After data collection, tally sheets were prepared and data were further analyzed and interpreted using appropriate software (Eviews-5) widely used for statistical analysis.

4. RESULT AND DISCUSSION

The parameter estimates of the preferred econometric model are presented in Table below along with their significances as shown by the t-ratios and the Adj. R^2 of the regression. All the parameters with the exception of the intercept term which is usually added to the econometric model for computational convenience have economic meanings. The coefficients of explanatory variables are statistically significant at 5 percent level.

DEPENDENT VARIABLE: FR			
Variable	**Coefficient**	**t-Statistic**	**Prob.**
LHH	-0.174194	-3.173047	0.0038
FLR	-0.934956	-5.055045	0.0000
PCI	0.162257	2.017674	0.0068
UC	-0.713774	-2.025760	0.0074
FLFP	1.306341	2.104599	0.0380
C	4.657941	8.750563	0.0000
R-squared 0.714181			
Adjusted R-squared 0.569935			
Durbin-Watson stat 1.932871			

The coefficient of LHH, FLR and UC are negatively related with FR. It implies that a one percent change in these variables will affect the explanatory variable FR by 17%, 93% and 71% inversely and separately. In the case of PCI and FLFP, positive relation with FR is observed means a one per cent change in these variables will increase the explanatory variable FR by 16% and 130%, respectively and separately.

The Adj. R^2 provides evidence in favor of all regressors. This is a joint test of significance and is useful to know how close the estimated observations to the scattered observations are. The value of adj. R^2 is 0.71. It implies that overall 71 percent of the sample variations in the FR are explained by the selected regressors and the rest variation appears to be due to change in the factor captured by the disturbance term (e_i).

The data also do not have problem of autocorrelation as clear from the value of Durban Watson statistic above. Similarly, there is no problem of multico-linearity in the data as indicated by the value of R^2 and significant t-values.

5. **CONCLUSIONS AND POLICY RECOMMENDATIONS**

The study analyses the determinants of fertility at micro level. It binds the relationship between fertility and socio-economic determinants. The finding appears to be consistent with macro level findings which states that education is negatively related to fertility. Educated women have fewer children than uneducated i.e. when the level of education is high the fertility rate will be low and vice versa. Apart from female's education, the education of the head of the family also significantly affects the fertility rate. Other than these, the use of contraceptive prevailing measures negatively relate to fertility. On the other hand, per capita income and female labor force participation contributing significantly and positively to fertility rate in the study area.

These findings would definitely help the policy makers regarding policy to control population growth. This study being at micro level is considered to be important as micro level studies always found effective planning. While population is growing Pakistan at an alarming rate, the population planning authority in Pakistan is in process of adopting measures which could really help in controlling the issue. This study reveals that fertility decline not only requires structural change but also requires considerable awareness program in addition to other family planning **programs.**

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OFFICIAL STATISTICS:
A Historical Perspective for Good Governance

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1. INTRODUCTION

Official statistics provide a picture of a country or different phenomena through data, and images such as graph and maps. Statistical information covers different subject areas (economic, demographic, social etc.). It provides basic information for decision making, evaluations and assessments at different levels. The goal of statistical organizations is to produce relevant, objective and accurate statistics to keep users well informed and assist good policy and decision-making.

The Fundamental Principles of Official Statistics were adopted in 1992 by the United Nations Economic Commission for Europe, and subsequently endorsed as a global standard by the United Nations Statistical Commission. According to the first Principle "Official statistics provide an indispensable element in the information system of a democratic society, serving the government, the economy and the public with data about the economic, demographic, social and environmental situation"

The history of official Statistics dates back to Hindu Vedas.

Kautilya's Arthasastra, dates back to 3rd century B.C. and gives a detailed description of the conduct of agricultural, population and economic censuses and of industrial and commercial practices in the villages and towns of ancient India.

In old testaments, there are two chapters; one on Numbers (Statistics) and second on Nexus (Population). It also approximately dates back to 5000 years ago.

In the Holy Quran, the word statistics has come seven times and 'count' (عدد) numerous times. I did not search for the use of statistics during the Prophet (PBUH) and Hazrat Abu Bakar (R.A) but searched its uses in Hazrat Omar (R.A) Khilafat. He conducted surveys of people in Madina who needed help, we now call it 'Household Expenditure Survey' which was used to determine 'Social Security Allowance for the poor'. This seems to be the first scientific method of a sample survey.

He had also counted the number of persons who had special characteristics i.e. the number of literates; the number of Huffaz-ul-Quran; the number of people in specific villages and towns. Classification of people and armed forces was made by wages. He conducted population censuses in all the territories.

He maintained Registers for
- i) Armed forces
- ii) Classification of people by tribes. Similar Register system still exists in Pakistan and in the West

    iii)   Classification by language

    iv)   Zakat and Sadaqa (Charity) that includes animals and kept its register for the counting purpose.

    v)   Expenditure on wars and war booty

    vi)   Censuses

    vii)   Accounts

In another peak and glorious period of the Indian history under the great emperor Akbar's rule (around 1590 A.D.), one finds a unique compilation of administration and control through various departments of government in the masterpiece "Ain-i-Akbari" written by Abul Fazal.

In 1807, the Survey of Eastern India conducted by the Governor-in-Council, Dr. Francis Buchanan covered an area from the district of Rangpur up towards Dacca and the districts in the Eastern frontier. It covered 15 million populations. Col. Sykes established a small Department of Statistics in 1847 in the India House.

The first census in the territories of what are now Pakistani Punjab and NWFP was undertaken in 1854. In 1881, a large scale census was carried out under the Lt. Governor of Punjab. The second census was conducted in 1881.

In April 1914, a Directorate of Statistics was established and later in 1925, it merged into the Directorate of Commercial Intelligence and Statistics.

2. STATISTICAL SYSTEM AFTER INDEPENDENCE: PAKISTAN

Immediately after the independence, Central Statistical Office (CSO) was established by the Government of Pakistan in 1950. In 1972, the Central Statistical Office was upgraded to Statistics Division on the recommendation of IBRD Mission. The Division was re-organized in 1981 with Federal Bureau of Statistics (FBS). Provincial Bureaus of Statistics were established at the provincial level. Further, there are various statistical cells in Ministries at Federal and Provincial levels.

The Government of Pakistan has recently re-organized Federal Statistical System of Pakistan and changed Statistics Division into "Pakistan Bureau of Statistics" by an act. The National Statistical Council, headed by the Minister for Finance, Economic Affairs, Revenue and Statistics still exists as a policy matter that coordinates the functions of different statistical authorities at national level.

Provincial Statistical Councils had also been constituted in the provinces. The Statistics Division is the secretariat of the National Statistical Council while Provincial Bureaus of Statistics act as the secretariat of the Provincial Statistical Councils. The State Bank of Pakistan (SBP) compile International trade statistics for based on Foreign Exchange records. Pakistan is gradually improving its statistical system to meet the SDDS requirements of the IMF.

In Pakistan, the Pakistan Statistical Association, was established in 1950 under the able leadership of Late Dr. M. Ziauddin but remained defunct after his death on 1967 and unfortunately, Pakistan Statistical Association (PSA) became defunct again in 2004.

The Islamic Countries Society of Statistical Sciences (ISOSS) was established in 1985, with its headquartered in Lahore. ISOSS is active and had been organizing conferences, workshops, short courses and symposiums at various places within Pakistan and outside Pakistan. It had organized biennial Islamic Countries Conferences at various OIC member states.

3. ROLE OF STATISTICIANS IN DEVELOPING NEW STATISTICAL TOOLS AND INNOVATIVE CONCEPTS

Official Statisticians should always be on their toes to develop new tools for official Statistics. The new tools are to

- Identify new information needs
- Uncover an inherent to conservatism in statistical system and its uses
- Gauge progressive economies
- Recommend innovative attitudes in official Statistics
  (H.K. van Tuinen, 2009)

For developing innovative concepts in official statistics, on has to ask

"How much?" How many?

"What is going on?" "What are new phenomena emerging?" What is interdependence? Which processes are driving changes in the Society?

Our current question is whether our official institutions are capable of finding answers or not? How to capture an index of a better Society as statistics is for the Society! And we need to develop new statistical tools.

Official Statistics must be capable of guiding policy and law makers in taking the most relevant decisions on the
  i)   measuring public opinions and their needs
  ii)   identification of national and regional problems and solution to the problems.
  iii)   creation of a system of implementation
  iv)   creation of monitoring system
  v)   creation of accountability system
  vi)   new deliverables
  vii)   development of statistical concepts free from political pressures
  viii) indicators for policy making

The categorization of the domains of official statistics has developed some common indicators:

4. STATISTICAL THEORY RECENT DEVELOPMENTS

The classical classification of statistics and its recent developments is given in Fig.(1). The flow chart presents a comprehensive relationship among various approaches given by Zacharias et al. (2008) in Fig. (2) below:
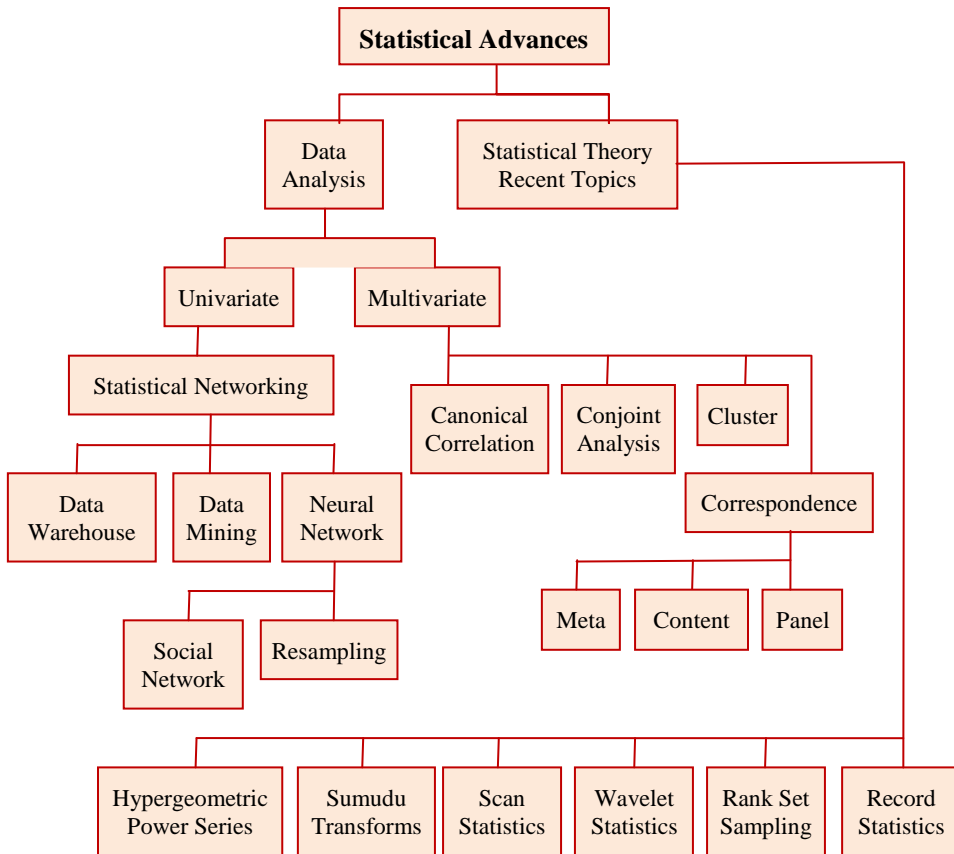
Fig. (1): Division of Statistical Theory with recent developments

Fig. (2)   Relationships among regular research approaches
[adopted from Zacharias et al. (2008)]

5. COMMON INDICATORS

1. Population statistics
2. Gender statistics
3. Employment statistics
4. Economic statistics
5. Trade statistics
6. Environment statistics
7. Energy statistics
    etc.

6. USERS OF OFFICIAL STATISTICS

The main users of official statistics are
- Governments
- Research Institute / Colleges – Universities
- Professional Statisticians
- Journals and Media
- Business
- General Public

6.1 There three main types of interests of the users:
- Business Interest
- General Interest
- Research Interest

6.2 Each one of the above main users has different needs such as
- Planning needs
- Project Monitoring needs
- Public opinion on specific issues

6.3 Producers of Official Statistics
National Offices

6.4 Data Sources
- Surveys
- Communications
- Registers
    o Records
    o Private registers
    o Statistics registers
    o Business registers
    o Agriculture stagiest

6.5 Official Statistics Presentations
- Line graphs
- Pie charts
- Tables
- Visne presentations or thematic – maps

7. QUALITY CRITERIA

- Relevance
- Impartiality
- Dissemination
- Independence
- Transparency
- Confidentially
- International Standards
  - Concepts & definitions
  - Good practices
  - IMF or SDDS (Special Data Dissemination Standard), GDDS (General Data Dissemination System)

All data collection with above quality criteria are considered as official Statistics.

8. RESEARCH AND DEVELOPMENT FOR OFFICIAL STATISTICS

- Well-documented research plan
- R&D to be associated with academia
- Funds
- Professionals dedicated to research must have at least M.Phil or Ph.D
- Joint research projects with academia
- Teaching and lecture by Statistics organization to academia and vice-versa

9. NEW METHODS OF OFFICIAL STATISTICS

1. Data Quality Control Tools
   JOS Vol. 25(2) 2009, 167

2. Methods to minimize non-sampling errors from Surveys

3. Split – Questionnaires Surveys & Multiple Phase Sampling
   JOS 25 (2) 2009, 227

4. Imputation techniques

5. New Index Number Problems
   JOS 2011 27 (4), 553

6. Estimation of Hidden Population / Masked Data / Masking Procedure
   Fuller (1993) JOS 9, 383-406

7. Micro – data

8. Random orthogonal Matrix Masking Methodology for Micro-data Release
   Int. J. Information and Computer Security (2008) 2, 86-105

9. Data Mining

10. Neural Network Statistics

11. Multiple Frame Surveys

12. Bootstraps Methods for Stratified – multistage and multiphase sampling
    JOS H. Saigo 2010, Vol 26, 193-208

13. Innovative Statistics
    JOS/Tuiven (2009) 25(4) 431 - 466

14. Web Surveys, Questionnaires, Designs

15. Analysis of Para – data

16. Para – statisticians

ESTIMATION OF FINITE POPULATION VARIANCE IN SUCCESSIVE SAMPLING IN PRESENCE OF NON RESPONSE

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ABSTRACT

In this paper a modified class of estimators is developed for the estimation of variance to deal with the problem of non-response in successive sampling over two occasions. The expressions for bias and variance of suggested class of estimators are also derived. For the estimation of variance in successive sampling in presence of non-response, no literature is available with which suggested class is to be compared. This is a first effort in successive sampling and will be helpful to handle the issue of non-response in the estimation of variance.

KEYWORD

Successive Sampling; Non Response; Bias; Mean Square Error.

1. INTRODUCTION

The role of auxiliary information in survey sampling is of great significance to improve the precision of population parameters. Due to this reason, the use of auxiliary variable which is highly correlated with study variable is of immense significance. Similarly in successive sampling the information of study variable available in past occasion is used to obtain the precise estimate of population parameter at current occasion.

Successive sampling is repeated over successive occasions, in which sample is selected by partial replacement of sample units from both occasions. This partial replacement of information causes to improve the precision of estimate at current occasion.

As characteristics of population show change over time, so single survey can only provide current feature of the population sometimes an experimenter is not only interested to estimate the change in characteristics of population but also to estimate an overall average result over all the occasions. So to deal with these challenges a survey is required to repeat over successive occasions.

In repetitive surveys sample can be selected in three different ways
i)   an independent (fresh) sample is taken at every occasion
ii) same sample is used in each occasion
iii) A portion of sample can be selected by partially rotating the sampling units i.e. sample based on matched and unmatched portion of sampling units. Matched sampling units are common at both occasions while unmatched units are taken first time at current occasion.

An independent sample on each occasion provide an efficient estimate but there are some difficulties in independent sample, e.g. if interviewer is new one then he will not get full cooperation from population and a substantial amount of time is required to develop the list of sampling units and new selection over each occasion is not economical because it takes heavy cost to contact the population.

If new selection is made for each occasion the estimate for the change in parameter is less efficient as compared with same sample is used for every occasion. To estimate the change in population parameter over successive occasions, new sample over each occasion provide precise estimate as compared to same sample over all occasions. But if a result regarding difference in average and an overall average result is required simultaneously then neither of the above cases suit, then the idea of rotation of sampling units over both occasions is the only way to deal with the problem.

2. ESTIMATION IN SUCCESSIVE SAMPLING WITHOUT NON-RESPONSE

Jessen (1942) was the pioneer in the estimation of successive sampling. He got two results, one was mean result obtained from unmatched sampling units and the other was the regression estimate using sample units of both occasions i.e. from matched portion. Then he obtained a best combined estimate by weighting the two estimates. The Jessen (1942) work was extended by Patterson (1950) and by Tikkiwal (1950, 53, 56, 64, 65, 67) and also Eckler (1955). Singh and Kathuria (1969) investigated the application of this sampling technique in the agricultural field. Hansen *et al.* (1955) and Rao and Graham (1964) have discussed rotation designs for successive sampling. Singh and Singh (1965), Singh (1968), Singh and Kathuria (1969) have also extended the work of successive sampling.

Azam (2005) developed an estimator for population variance based on matched and unmatched portion of sample on the second occasion. Singh et al. (2011) followed the estimation procedure of Jessen (1942), and proposed a class of estimators of finite population variance in successive sampling on two occasions and analyzed its properties.

Hussain (2011) extended the work of Singh et al. (2011) and proposed a general class of estimators for estimating the finite population variance in successive sampling on two occasions using multi-auxiliary variables and proved it more efficient than the class of estimators suggested by Singh et al. (2011).

3. NON-RESPONSE

When data is not obtained from some of the selected sample units then it is referred to as non-response or incompleteness of sample information from desired sample size. This in incompleteness or non-response provides mislead information and spoil the nature of population. The main sources of non-response that may cause the non-response and may affect the any particular inquiry are
i)   Respondent is not at Homes
ii) Respondent refuses to response
iii) Respondent cannot answer any way
iv) Respondent not found

As the mail survey is the most commonly used tool of data collection due to its economical nature. The main objection for this tool is that it carries too much non

response. Unlike to mail survey personal interview provide substantial complete response but it is much more costly.

Hansen and Hurwitz (1946) were the pioneer who worked on the issue of non-response. They used both techniques of data collection i.e. mail survey and face to face interview. Hansen and Hurwitz (1946) proposed a model for survey design to provide unbiased estimator of population mean or total in presence of non-response. Their pioneer model consists of the following steps:

i)   Select a sample of respondents and mail a questionnaire to all of them
ii) After the deadline is over, identify the non-response and select a sub-sample from the non-respondents;
iii) Collect data from the non-respondents in the sub-sample by interview;
iv) Combine the data from the two parts of the survey to estimate the population parameters.

Consider that a population consists of N units can be divided into two classes:

i) Those who will respond at the at the first attempt forming the response class;
ii) Those who will not respond at the at the first attempt forming the non-response class;

Assume that N_1 and N_2 are the number of units in population that belong to response and non-response class, respectively. We may regard the sample of n_1 respondents as simple random sample from response class and n_2 as a simple random sample from the non-response class. Let h_2 denote the size of the sub-sample drawn from n_2 non-respondents to be interviewed so that $n_2 = h_2 g$, where $g > 1$.

The unbiased estimator of N_1 and N_2 are

$$\hat{N}_1 = \frac{n_1}{n} N \text{ and } \hat{N}_2 = \frac{n_2}{n} N$$

The unbiased estimator of the population mean $\bar{Y}$ is given by

$$\bar{y}_{n_h} = \frac{n_1 \bar{y}_1 + n_2 \bar{y}_{h_2}}{n}$$

where $\bar{y}_{h_2}$ denote the mean of h_2 observations from the sub-sample of non-responding units.

4. ESTIMATION IN SUCCESSIVE SAMPLING WITH NON-RESPONSE

Bartholomew (1961) proposed an estimator of population mean when there is non-response among sampling units using successive sampling. Following the same sampling scheme used by Bartholomew (1961), Singh et al. (1974) proposed estimator for current population mean. Fabian and Okafor (2001) proposed estimators based upon Hansen and Hurwitz (1946) technique to treat with non-response using sampling on two occasions. They introduced two schemes of sampling and for each of the two schemes they proposed two estimators of the population mean; based on

1. Double sampling ratio estimation
2. Double sampling regression estimation

For the estimation of population mean at current occasion, Choudhary et al. (2004) focused on the problem of non-response on both the occasions in successive sampling faced during the mail surveys for the current occasion.

Singh and Sunil Kumar (2009) proposed estimator of population mean by combining (i) a double sampling multivariate product estimate from the matched portion of the sample and (ii) a simple mean based on a random sample from the unmatched portion of the sample on the second occasion, when there is non-response on both occasions.

5. NOTATION AND EXPECTATIONS

Let $U = (U_1, U_2, \ldots, U_N)$ be a finite population of N units assumed to remain unchanged on two successive time-periods. Let x_j and y_j respectively denote the values of the auxiliary variable x and study variable y $(j = 1, 2, \ldots, N)$. Note that on the previous (first) occasion, the study variable y is called the auxiliary variable x.

Let us denote $\mu_x = E\ x\ = N^{-1} \sum_{j=1}^{N} x_j$ and $\mu_y = E\ y\ = N^{-1} \sum_{j=1}^{N} y_j$ be the population means of auxiliary variable (x) at past occasion and study variable (y) at current occasion respectively.

Let us denote $\sigma_y^2 = E\ y - \mu_y\ ^2 = N^{-1} \sum_{j=1}^{N} y_j - \mu_y\ ^2$ and $\sigma_x^2 = E\ x - \mu_x\ ^2 = N^{-1} \sum_{j=1}^{N} x_j - \mu_x\ ^2$ be population variances of study variable (y) at current occasion and auxiliary variable (x) at past occasion respectively, and $\rho = \dfrac{E\ x - \mu_x\ \ y - \mu_y}{\sqrt{E\ x - \mu_x\ ^2\ E\ y - \mu_y\ ^2}}$ be the population correlation coefficient between the study variate y and the auxiliary variate x.

In successive sampling on two occasions, let the sizes of the samples drawn using simple random sampling on the first and second occasions be n_1 and n_2 respectively. While sampling on the second (current) occasion, let m units (m for matched) of the first occasion are retained and the remaining $u\ (= n_2 - m)$ units are replaced by the new units selected independently of the matched portion. The present paper utilizes the information from the first occasion on an auxiliary variable x, where the estimates of the population mean μx and population variance $\sigma^2 x$ are known, to provide an efficient estimator of the finite population variance $\sigma^2 y$ on the second (current) occasion. Let $(x_1^*, x_2^*, \ldots, x_{n_1}^*)$ be the values of the auxiliary variable x drawn by simple random sampling from the given population of N units; $(y_1, y_2, \ldots, y_m)$ be the values of the study variable y for matched portion on the second occasion; $(x_1, x_2, \ldots, x_m)$ be the values of the auxiliary variable x for matched portion on the second occasion; $y_1^{'}, y_2^{'}, \ldots, y_u^{'}$ be the values of the study variable y for the unmatched portion on the second occasion.

Let $\bar{x}^* = \dfrac{\sum_{i=1}^{n_1} \bar{x}_i^*}{n_1}$, $\bar{x} = \dfrac{\sum_i^m x_i}{m}$, $\bar{y} = \dfrac{\sum_i^m \bar{y}_i}{m}$ and $\bar{y} = \dfrac{\sum_i^u \bar{y}_i'}{u}$ be sample means of auxiliary variable at first occasion and matched portion in first occasion, of study variable of matched and unmatched portion of second occasion respectively.

Similarly, let $S_x^{*2} = \dfrac{\sum_{i=1}^{n_1} \left(x_i^* - \bar{x}^*\right)^2}{n_1 - 1}$, $S_x^2 = \dfrac{\sum_{i=1}^m \left(x_i - \bar{x}\right)^2}{m-1}$, $S_{ym}^2 = \dfrac{\sum_{i=1}^m \left(y_i - \bar{y}\right)^2}{m-1}$ and

$S_{yu}^2 = \dfrac{\sum_{i=1}^u \left(y_i' - \bar{y}'_u\right)^2}{u-1}$ be the sample variances of auxiliary variable of first occasion, auxiliary variable of matched portion in first occasion, study variable of matched portion in second occasion and study variable unmatched portion of second occasion

respectively. Let $\hat{\beta} = S_{yx} / S_x^2$ and $S_{yx} = \dfrac{\sum_{i=1}^m \left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right)}{m-1}$ be the regression coefficient and covariance between auxiliary and study variables respectively.

For simplicity, let us assume that the population size N is large as compared to sample sizes so that the finite population correction (f.p.c) terms may be ignored.

Let on the first occasion, schedule through mail are sent to n units selected by simple random sampling. We assume that at the first occasion, all the n_1 units supplied information on the auxiliary variable x. when selecting the second sample, we assume that $m = pn_1$ $(0 < p < 1)$ of the units of the second sample on the first occasion are retained for the second occasion (matched sample) and the remaining $u = nq = n_2 - m$, $(q = 1-p)$ units are replaced by a new selection from the universe of N-m left after omitting the m units. We assume that in the unmatched portion of the sample on the 2nd occasion u_1 units respond and u_2 units do not. Similarly in the matched portion m_1 units respond and m_2 units don't. Let $m_{h_2} = m_2 / \gamma$, $\gamma \geq 1$ denote the size of the subsample from the non-response class from the matched portion of the sample on the two occasions for selecting information through personal interview. Similarly, $u_{h_2} = u_2 / \gamma$, $\gamma \geq 1$ denote the size of the subsample drawn from the non-response class in the unmatched portion of the sample on 2nd occasion.

Let $\bar{x} = \dfrac{\sum_{i=1}^{n_1} x_i}{n_1}$ be the sample mean of the auxiliary variable x on the 1st occasion

based on the large sample size of n_1. Following the Hansan and Hurwitz (1946) approach,

let $\bar{x}_m'' = \dfrac{m_1 \bar{x}_{m_1} + m_2 \bar{x}_{m_{h_2}}}{m_1 + m_2}$, $\bar{x}_u'' = \dfrac{u_1 \bar{x}_{u_1} + u_2 \bar{x}_{u_{h_2}}}{u_1 + u_2}$, $\bar{y}_m'' = \dfrac{m_1 \bar{y}_{m_1} + m_2 \bar{y}_{m_{h_2}}}{m_1 + m_2}$ and

$\bar{y}_u'' = \dfrac{u_1 \bar{y}_{u_1} + u_2 \bar{y}_{u_{h_2}}}{u_1 + u_2}$ are the estimators of population means of the auxiliary variable and the study variable for the matched portion and unmatched portions respectively.

Similarly, let $S_{xm}''^{2} = \dfrac{m_1 S_{xm_1}^2 + m_2 S_{xm_{h_2}}^2}{m_1 + m_2}$, $S_{xu}''^{2} = \dfrac{u_1 S_{xu_1}^2 + m_2 S_{xm_{h_2}}^2}{u_1 + u_2}$, $S_{ym}''^{2} = \dfrac{m_1 S_{ym_1}^2 + m_2 S_{ym_{h_2}}^2}{m_1 + m_2}$

and $S_{yu}''^{2} = \dfrac{m_1 S_{yu_1}^2 + m_2 S_{yu_{h_2}}^2}{u_1 + u_2}$ are the estimators of population variances of the auxiliary variable and the study variable for the matched portion and unmatched portions respectively. Let, $S_{yx} = \dfrac{m_1 S_{yx\,m_1} + m_2 S_{yx\,m_{h2}}}{m_1 + m_2}$ be the sample covariance between

auxiliary and study variable for matched portion. Let $\bar{x}_{m_1} = \dfrac{\sum\limits_{j=i}^{m_1} x_j}{m_1}$, $\bar{x}_{m_{h_2}} = \dfrac{\sum\limits_{j=i}^{m_{h_2}} x_j}{m_{h_2}}$,

$\bar{x}_{u_1} = \dfrac{\sum\limits_{j=i}^{u_1} x_j}{u_1}$ and $\bar{x}_{u_{h_2}} = \dfrac{\sum\limits_{j=i}^{u_{h_2}} x_j}{u_{h_2}}$ are the sample means of auxiliary variable for first and

second attempt respectively in matched and unmatched portions, and

$S_{xm_1}^2 = \dfrac{\sum\limits_{j=i}^{m_1} \left(x_j - \bar{x}_{m_1}\right)^2}{m_1 - 1}$, $S_{xm_{h_2}}^2 = \dfrac{\sum\limits_{j=i}^{m_{h_2}} \left(x_j - \bar{x}_{m_{h_2}}\right)^2}{m_{h_2} - 1}$, $S_{xu_1}^2 = \dfrac{\sum\limits_{j=i}^{u_1} \left(x_j - \bar{x}_{u_1}\right)^2}{u_1 - 1}$ and

$S_{xu_{h_2}}^2 = \dfrac{\sum\limits_{j=i}^{u_{h_2}} \left(x_j - \bar{x}_{u_{h_2}}\right)^2}{u_{h_2} - 1}$ are the sample variances of auxiliary variable for first and second

attempt respectively in matched and unmatched portions. Let $\bar{y}_{m_1} = \dfrac{\sum\limits_{j=i}^{m_1} y_j}{m_1}$, $\bar{y}_{m_{h_2}} = \dfrac{\sum\limits_{j=i}^{m_{h_2}} y_j}{m_{h_2}}$

, $\bar{y}_{u_1} = \dfrac{\sum\limits_{j=i}^{u_i} y_j}{u_1}$ and $\bar{y}_{u_{h_2}} = \dfrac{\sum\limits_{j=i}^{u_{h_2}} y_j}{u_{h_2}}$ are the sample means of study variable for first and

second attempt respectively in matched and unmatched portions. Let

$S_{ym_1}^2 = \dfrac{\sum\limits_{j=i}^{m_1} \left(y_j - \bar{y}_{m_1}\right)^2}{m_1 - 1}$, $S_{ym_{h_2}}^2 = \dfrac{\sum\limits_{j=i}^{m_{h_2}} \left(y_j - \bar{y}_{m_{h_2}}\right)^2}{m_{h_2} - 1}$, $S_{yu_1}^2 = \dfrac{\sum\limits_{j=i}^{u_1} \left(y_j - \bar{y}_{u_1}\right)^2}{u_1 - 1}$ and

$$S^2_{yu_{h_2}} = \frac{\sum_{j=i}^{u_{h_2}} \left(y_j - \bar{y}_{u_{h_2}}\right)^2}{u_{h_2} - 1}$$ are the sample variances of study variable for first and second

attempt respectively in matched and unmatched portions. Let

$$S_{yx\,m_1} = \frac{\sum_{j=1}^{m_1} \left(y_j - \bar{y}_{m_1}\right)\left(x_j - \bar{x}_{m_1}\right)}{m_1 - 1} \text{ and } S_{yx\,m_{h2}} = \frac{\sum_{j=1}^{m_{h2}} \left(y_j - \bar{y}_{m_{h2}}\right)\left(x_j - \bar{x}_{m_{h2}}\right)}{m_{h2} - 1}$$

are the sample covariances of the auxiliary variable and the study variable in first and second attempt respectively for the matched portion.

Let $e_0 = \dfrac{S^2_{ym} - \sigma^2_y}{\sigma^2_y}$, $e_1 = \dfrac{S^2_{yu} - \sigma^2_y}{\sigma^2_y}$, $e_2 = \dfrac{\bar{x} - \bar{x}^*}{\bar{x}^*}$ and $e_3 = \dfrac{S^2_x - S^{*2}_x}{S^{*2}_x}$ be the relative

errors of variance of study variable in matched portion, the relative errors of variance of study variable in unmatched portion, the relative errors of mean of auxiliary variable in matched portion and the relative errors of variance of auxiliary variable in matched portion, such that

$$E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = E\left(e_3\right) = 0 \tag{4.1}$$

$$E\left(e_0^2\right) = \frac{1}{m}\left(\lambda_{40} - 1\right) + \theta\frac{1}{m}\left(\lambda_{40(2)} - 1\right) \tag{4.2}$$

$$E\left(e_1^2\right) = \frac{1}{u}\left(\lambda_{40} - 1\right) + \theta\frac{1}{u}\left(\lambda_{40(2)} - 1\right) \tag{4.3}$$

$$E(e_2^2) = \frac{n_1 - m}{n_1 m}\left(C_x^2 + \theta C_{x_{(2)}}^2\right) \tag{4.4}$$

$$E\left(e_3^2\right) = \left(\frac{1}{m} - \frac{1}{n_1}\right)\left(\lambda_{04} - 1\right) + \theta\left(\frac{1}{m} - \frac{1}{n_1}\right)\left(\lambda_{04\,2} - 1\right) \tag{4.5}$$

$$E\left(e_0 e_2\right) = \frac{n_1 - m}{n_1 m}\left[\lambda_{21}C_x + \theta\lambda_{21\,2}\,C_{x\,2}\right] \tag{4.6}$$

$$E\left(e_0 e_3\right) = \frac{n_1 - m}{n_1 m}\left[\lambda_{22} - 1 + \theta\left(\lambda_{22\,2} - 1\right)\right] \tag{4.7}$$

$$E\left(e_2 e_3\right) = \frac{n_1 - m}{n_1 m}\lambda_{03}C_x + \theta\frac{n_1 - m}{n_1 m}\lambda_{03\,2}\,C_{x\,2} \tag{4.8}$$

where $\lambda_{rs} = \dfrac{\mu_{rs}}{\sqrt{\mu^r_{20}\mu^s_{02}}}$, $\mu_{rs} = \dfrac{\sum_{j=1}^{N}\left(y_j - \mu_y\right)^r\left(x_j - \mu_x\right)^s}{N}$, $\lambda_{rs(2)} = \dfrac{\mu_{rs(2)}}{\sqrt{\mu^r_{20(2)}\mu^s_{02(2)}}}$

$$\mu_{rs(2)} = \frac{\sum_{j=1}^{N_2} \left(y_{j(2)} - \mu_{y(2)} \right)^r \left(x_{j\,2} - \mu_{x\,2} \right)^s}{N_2} \quad \text{for r, s=0,1,2,3,4.}$$

$$m = m_1 + m_2 , u = u_1 + u_2 , \theta = \frac{N_2}{N}\left(\frac{u_2}{u_{h_2}} - 1 \right) \text{and } \theta = \frac{N_2}{N}\left(\frac{m_2}{m_{h_2}} - 1 \right).$$

6. GENERAL CLASS OF ESTIMATOR

Following Jessen (1942) approach, we proposed a general class of estimators for estimating the finite population variance σ_y^2 which is weighted combination of the two estimators S_m^{*2} and S_{yu}^{*2}, where S_m^{*2} is a general class of estimators for estimating the finite population variance σ_y^2 on the matched portion of the sample consisting of m units in presence of non-response and S_{yu}^{*2} is an unbiased estimator of σ_y^2 based on unmatched units is S_{yu}^2 with variance, such that

$$S^{*2} = \phi S_m^{*2} + \left(1-\phi \right) S_{yu}^{*2} \tag{5.1}$$

where ϕ is a constant, which is determined in such a way that variance of S^{*2} is minimum. Now general class of estimators for estimating the finite population variance σ_y^2 on the matched portion of the sample consisting of m units in presence of non-response is defined as

$$S_m^{*2} = t\left(S_{ym}^{'2}, v_1^{'}, v_2^{'} \right) \tag{5.2}$$

where $v_1^{'} = \frac{\overline{x}^{'}}{\overline{x}}$ and $v_2^{'} = \frac{S_1^{'2}}{S_x^{*2}}$.

$t\left(\bullet \right)$ is a parametric function such that, $t\left(P \right) = \sigma_y^2, t_0\left(P \right) = \frac{\partial t}{\partial S_{ym}^{'2}}\bigg|_{P} = 1$ with $P = \left(\sigma_y^2,1,1 \right)$ and satisfies certain regularity conditions similar to those given in Sirivastava and Jhajj (1980). It may easily be observed that estimators

$$\hat{\sigma}_1^2 = S_{ym}^{'2} v_1^{'\alpha_1} v_2^{'\beta_1}$$

$$\hat{\sigma}_2^2 = S_{ym}^{'2}\left[1+\alpha_1\left(v_1^{'} -1 \right) +\beta_1\left(v_2^{'} -1 \right) \right]$$

$$\hat{\sigma}_3^2 = S_{ym}^{'2}\left(w_1^{'} v_1^{'\alpha_1} + w_2^{'} v_2^{'\beta_1} \right) , \qquad w_1^{'} + w_2^{'} = 1$$

$$\hat{\sigma}_4^2 = S_{ym}^{'2} v_1^{'\alpha_1}\left(2-v_2^{'\beta_1} \right)$$

$$\hat{\sigma}_5^2 = S_{ym}^{'2} v_2^{'\beta_1} \ 2 - v_1^{'\alpha_1}$$

$$\hat{\sigma}_6^2 = S_{ym}^{'2} + \alpha_1 \ v_1^{'} - 1 + \beta_1 \ v_2^{'} - 1$$

etc. all belongs to the class S_t^{*2}, where α_1, β_1 and $w_i^{'s}$ $i = 1, 2$ are suitably chosen constants. The bias and variance of the estimator, S_t^{*2}, exist since the number of possible samples is finite and it is assumed that the function is bounded. Expanding $t \ S_{ym}^{'2}, v_1^{'}, v_2^{'}$ about the point $P = \sigma_y^2, 1, 1$ by second order of Taylor's series, we obtain

$$S_m^{*2} = t \ P + S_{ym}^{'2} - \sigma_y^2 \ t_0 \ P + v_1^{'} - 1 \ t_1 \ P$$

$$+ v_2^{'} - 1 \ t_2 \ P + \frac{1}{2} \left\{ S_{ym}^{'2} - \sigma_y^2 \right.^2 \ t_{00} \ P^*$$

$$+ v_1^{'} - 1^2 \ t_{11} \ P^* + v_2^{'} - 1^2 \ t_{22} \ P^* + 2 \ S_{ym}^{'2} - \sigma_y^2 \ v_1^{'} - 1 \ t_{01} \ P^*$$

$$+ 2 \ S_{ym}^{'2} - \sigma_y^2 \ v_2^{'} - 1 \ t_{02} \ P^* + 2 \ v_1^{'} - 1 \ v_2^{'} - 1 \ t_{12} \ P^* \tag{5.3}$$

where $t_1 \ P$ and $t_2 \ P$ are first order partial derivatives of $t \ S_{ym}^{'2}, v_1^{'}, v_2^{'}$ w.r.t. $v_1^{'}$ and $v_2^{'}$ about the point $P = \sigma_y^2, 1, 1$ and $t_{i,j} \ P^*$, $i, j = 0, 1, 2$ denote second order partial derivatives, $P^* = S_{ym}^{'*2}, v_1^{'*}, v_2^{'*}$, $S_{ym}^{*2} = \sigma_y^2 + \theta^* \ S_{ym}^{'2} - \sigma_y^2$, $v_1^{'*} = 1 + \theta^* \ v_1^{'} - 1$, $v_2^{'*} = 1 + \theta^* \ v_1^{'} - 1$ and $0 < \theta^* < 1$. Noting that $t \ P = \sigma_y^2, t_0 \ P = 1 t$ and expressing (5.3) in terms of relative errors we have

$$S_m^{*2} = \sigma_y^2 + \sigma_y^2 e_0 \ 1 + e_2 t_1 \ P + e_3 t_2 \ P$$

$$+ \frac{1}{2} \ e_0^2 \ t_{00} \ P^* + e_2^2 \ t_{11} \ P^* + e_3^2 \ t_{22} \ P^*$$

$$+ 2 \ e_0 \ e_2 \ t_{01} \ P^* + 2 \ e_0 \ e_3 \ t_{02} \ P^* + 2 \ e_0 \ e_3 \ t_{12} \ P^* \tag{5.4}$$

Taking expectation of both sides of (5.4) and ignoring second order term we get

$$E \ S_m^{*2} - \sigma_y^2 = O \ m^{-1}$$

As, error terms are very small, therefore their squares and products are even smaller and are nearly zero. Hence its contribution to the mean square error will be of the order of m^{-2}. In what follows we assume that m is large enough so that the bias is assumed negligible and the variance expression are obtained up to terms of order m^{-1}, an approximation usually taken for ratio type estimators (see Srivastava, 1981).

$$E \ S_m^{*2} - \sigma_y^2 \ ^2 = E \left[\sigma_y^2 e_0 + e_2 t_1 \ P + e_3 t_2 \ P \right]^2 \tag{5.5}$$

or

$$Var\ S_m^{*2} = \sigma_y^4 E\ e_0^{\ 2} + E\ e_2^{\ 2}\ t_1^2\ P + E\ e_3^{\ 2}\ t_2^2\ P + 2\sigma_y^2 E\ e_0 e_2\ t_1\ P$$

$$+ 2\sigma_y^2 E\ e_0 e_3\ t_2\ P + 2E\ e_2 e_3\ t_1\ P\ t_2\ P \qquad (5.6)$$

Using the results of expected values of relative errors taking from (4.2) and (4.4) to (4.8) in (5.6)

$$Var\ S_m^{*2} = \sigma_y^4 \frac{1}{m}\left[\lambda_{40} - 1 + \theta\ \lambda_{40(2)} - 1 \right] + \frac{n_1 - m}{n_1 m}\left[C_x^2 + \theta C_{x_{(2)}}^2 \right]t_1^2\ P$$

$$+ \left(\frac{1}{m} - \frac{1}{n_1} \right)\left[\lambda_{04} - 1 + \theta\ \lambda_{04\ 2} - 1 \right]t_2^2\ P$$

$$+ 2\sigma_y^2 \frac{n_1 - m}{n_1 m}\left[\lambda_{21} C_x + \theta\lambda_{21\ 2}\ C_{x\ 2} \right]t_1\ P$$

$$+ 2\sigma_y^2 \frac{n_1 - m}{n_1 m}\left[\lambda_{22} - 1 + \theta\ \lambda_{22\ 2} - 1 \right]t_2\ P$$

$$+ 2\frac{n_1 - m}{n_1 m}\left[\lambda_{03} C_x + \theta\lambda_{03\ 2}\ C_{x\ 2} \right]t_1\ P\ t_2\ P \qquad (5.7)$$

or

$$Var\ S_m^{*2} = \frac{\lambda_{40} - 1}{m}\sigma_y^4 + \frac{n_1 - m}{n_1 m}V + \theta\left\{ \frac{\lambda_{40(2)} - 1}{m}\sigma_y^4 + \frac{n_1 - m}{n_1 m}V_2 \right\} \qquad (5.8)$$

where

$$V = C_x^2 t_1^2\ P + \lambda_{04} - 1\ t_2^2\ P + 2\sigma_y^2 \lambda_{21} C_x t_1\ P$$

$$+ 2\sigma_y^2\ \lambda_{22} - 1\ t_2\ P + 2\lambda_{03} C_x t_1\ P\ t_2\ P$$

and

$$V_2 = C_{x\ 2}^2\ t_1^2\ P + \lambda_{04\ 2} - 1\ t_2^2\ P + 2\sigma_y^2\lambda_{21\ 2}\ C_{x\ 2}\ t_1\ P$$

$$+ 2\sigma_y^2\ \lambda_{22\ 2} - 1\ t_2\ P + 2\lambda_{03\ 2}\ C_{x\ 2}\ t_1\ P\ t_2\ P$$

To find the optimum value of $t_1\ P$ and $t_2\ P$ that minimize the mean square error, differentiating (3.4) with respect to $t_1\ P$ and $t_2\ P$ and equating to zero, we get

$$t_1\ P = \frac{\sigma_y^2 \left[\begin{array}{cc} \lambda_{03} C_x + \theta\lambda_{03\ 2}\ C_{x\ 2} & \lambda_{22} - 1 + \theta\ \lambda_{22\ 2} - 1 \\ - \lambda_{21} C_x + \theta\lambda_{21\ 2}\ C_{x\ 2} & \lambda_{04} - 1 + \theta\ \lambda_{04\ 2} - 1 \end{array} \right]}{C_x^2 + \theta C_{x\ 2}^2\quad \lambda_{04} - 1 + \theta\ \lambda_{04\ 2} - 1\ -\ \lambda_{03} C_x + \theta\ \lambda_{03\ 2}\ C_{x\ 2}\ ^2} \qquad (5.9)$$

and

$$t_2 \quad P \ = \ \frac{\sigma_y^2 \begin{bmatrix} \lambda_{03} C_x + \theta \lambda_{03\ 2}\ C_{x\ 2} & \lambda_{21} C_x + \theta \lambda_{21\ 2}\ C_{x\ 2} \\ - C_x^2 + \theta C_{x\ 2}^2 & \lambda_{22} - 1 + \theta\ \lambda_{22\ 2}\ -1 \end{bmatrix}}{\begin{bmatrix} C_x^2 + \theta C_{x\ 2}^2 & \lambda_{04} - 1 + \theta\ \lambda_{04\ 2}\ -1 & - \lambda_{03} C_x + \theta\ \lambda_{03\ 2}\ C_{x\ 2} \end{bmatrix}^2}$$

(5.10)

An unbiased estimator of σ_y^2 based on unmatched units is S_{yu}^2 with variance, up to terms of the first order of approximation, is given by

$$Var\ S_{yu}^{*2} \ = \frac{\sigma_y^4}{u}\ \lambda_{40} - 1\ + \theta \frac{\sigma_y^4}{u}\ \lambda_{40(2)} - 1$$

(5.11)

Now apply the variance on ( )The variance of S^{*2} is given be

$$Var\ S^{*2} \ = \phi^2 V\ S_t^{*2}\ +\ 1 - \phi\ ^2 V\ S_{yu}^{*2}$$

(5.12)

The optimum value of S^{*2} is obtain by using value of ϕ that minimizes $Var\ S^{*2}$. Differentiating (5.12) with respect to ϕ and equating to zero, we get the optimum value of ϕ as

$$\phi = \frac{V\ S_{yu}^{*2}}{V\ S_{yu}^{*2} + V\ S_t^{*2}}$$

(5.13)

Putting the results of (5.8) and (5.11) in (5.13)

$$\phi = \frac{mn_1\ \lambda_{40} - 1\ + \theta\ \lambda_{40(2)} - 1}{\left\{ n_1 n_2\ \lambda_{40} - 1\ + u\ n_1 - m\ \dfrac{V}{\sigma_y^4} \right\} + \theta \left\{ n_1 n_2\ \lambda_{40(2)} - 1\ + u\ n_1 - m\ \dfrac{V\ 2}{\sigma_y^4} \right\}}$$

(5.14)

Putting the results of (5.13) in (5.12)

$$Var\ S^{*2}\Big|_{min} \ = \frac{V\ S_{yu}^{*2}\ V\ S_t^{*2}}{V\ S_{yu}^{*2} + V\ S_t^{*2}}$$

(5.15)

Putting the results of (5.8) and (5.11) in (5.15)

$$V\ S^{*2}\Big|_{min}\ = v_1 + v_2 + \phi^2 \frac{n_1 - m}{n_1 m}\ V + \theta V_2$$

(5.16)

where

$$v_1 = \left\{ \phi^2 \frac{\lambda_{40} - 1}{m} \sigma_y^4 + \phi^2 \theta \frac{\lambda_{40(2)} - 1}{m} \sigma_y^4 \right\}$$

$$\text{and } v_2 = 1 + \phi^2 - 2\phi \left\{ \frac{\sigma_y^4}{u} \ \lambda_{40} - 1 \ + \theta \frac{1}{u} \ \lambda_{40(2)} - 1 \right\}$$

The variance of $Var \ S^{*2}$ is given by using value ϕ

$$Var \ S^{*2} \ = \ \lambda_{40} - 1 \ + \theta \ \lambda_{40(2)} - 1$$

$$\sigma_y^4 \left[\frac{n_2 \ \lambda_{40} - 1 \ + uA \ + \theta \ n_2 \ \lambda_{40(2)} - 1 \ + uA_2}{n_2^2 \ \lambda_{40} - 1 \ + u^2 A \ + \theta \ n_2^2 \ \lambda_{40(2)} - 1 \ + u^2 A_2} \right] \qquad (5.17)$$

where

$$A = \frac{V}{\sigma_y^4} \ and \ A_2 \ = \frac{V_2}{\sigma_y^4}$$

or

$$Var \ S^{*2} \ = \ \frac{\left[\begin{array}{l} n_1 \ \lambda_{40} - 1 \ + \ n_1 - m \ A \\ +\theta \ n_1 \ \lambda_{40(2)} - 1 \ + \ n_1 - m \ A_2 \end{array} \right] \ \lambda_{40} - 1 \ + \theta \ \lambda_{40(2)} - 1 \ \sigma_y^4}{n_1 n_2 \ \lambda_{40} - 1 \ + u \ n_1 - m \ A \ + \theta \ n_1 n_2 \ \lambda_{40(2)} - 1 \ + u \ n_1 - m \ A_2}$$

$$(5.18)$$

When sample sizes on both occasions are same i.e. $n_1 = n_2 = n$

$$Var \ S^{*2} \ = \ \lambda_{40} - 1 \ + \theta \ \lambda_{40(2)} - 1$$

$$\sigma_y^4 \left[\frac{n \ \lambda_{40} - 1 \ + uA \ + \theta \ n \ \lambda_{40(2)} - 1 \ + uA_2}{n^2 \ \lambda_{40} - 1 \ + u^2 A \ + \theta \ n^2 \ \lambda_{40(2)} - 1 \ + u^2 A_2} \right] \qquad (5.19)$$

To find the optimum value of u that minimizes the mean square error, differentiating (5.19) with respect to u and equating to zero, we get

$$u = \frac{n}{1 + \sqrt{1 + A^*}} \qquad (5.20)$$

where

$$A^* = \frac{A + \theta A_2}{\lambda_{40} - 1 \ + \theta \ \lambda_{40(2)} - 1}$$

Putting the value of u from (5.20) in (5.19),we get

$$Var\ S^{*2}\ =\frac{\lambda_{40}-1\ +\theta\ \lambda_{40(2)}-1\ \ \sigma_y^4}{2n}\ 1+\sqrt{1+A^*}\tag{5.21}$$

If an independent sample is taken on the second occasion (i.e. when there is no matching), then $n_2=u+m$ can be $n_2=u$,then

$$S_{yu}^2=\frac{\sum_{i=1}^{u}\ y_i-\bar{y}\ ^2}{u-1}\ =\ S_{yn_2}^2=\frac{\sum_{i=1}^{n_2}\ y_i-\bar{y}\ ^2}{n_2-1}\tag{5.22}$$

and

$$V\ S_{yu}^2\ =\frac{\lambda_{40}-1}{u}\sigma_y^4=V\ S_{yn_2}^2\ =\frac{\lambda_{40}-1}{n_2}\sigma_y^4\tag{5.23}$$

Hence the resulting variance of estimator for non-response will be

$$Var\ S_{yn_2}^2\ =\frac{\sigma_y^4}{n_2}\ \lambda_{40}-1\ +\theta\frac{\sigma_y^4}{n_2}\ \lambda_{40\ 2}\ -1\tag{5.24}$$

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RELATIONSHIP OF PARENTING STYLES WITH SELF ESTEEM AND OPTIMISM AMONG ADOLESCENTS

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ABSTRACT

The study aims at investigating the relationship of parenting styles with self-esteem and optimism among adolescents. Correlational research design was used to conduct the study. Sample was selected using purposive sampling. Instruments used were Parental Authority Questionnaire, Rosenberg Self Esteem Scale and Life Orientation Test-Revised to assess parental authority, self-esteem and optimism of adolescents respectively. The statistical method used was Pearson Product Moment Correlation Coefficient. Results derived through data analysis revealed there is a significant positive correlation of Authoritative parenting style with self-esteem and optimism whereas Authoritarian parenting style was observed to have significant inverse correlation with both self-esteem and optimism. Along with this the study also showed a significant positive correlation between Permissive parenting and self-esteem however no significant correlation was observed between Permissive parenting style and optimism.

1. INTRODUCTION

The present study aimed to investigate the relation of parenting style with self-esteem and optimism among adolescents. The main purpose was to find out that to what extent parenting styles are related to levels of self-esteem and optimism in adolescents. Moreover the results of the study helped in comparing and finding out that which parenting style is favorable in terms of self-esteem and optimism among adolescents.

A parenting style is the strategy parents choose for the rearing of their children. A lot of research has been done on the importance of parenting style on various developments (Gale Encyclopedia of Education, 2002). One of the known theories of parenting style was developed by Diana Baumrind. Diana Baumrind, a well-known researcher established parenting style theory in 1966. According to Baumrind, parenting style is the four dimension classification of parenting behavior that tells how parents deal with the child's need for nurturance and how they set limits for undesirable behavior (Cheiw, 2010). Baumrind proposed that parents fall into one of three categories: authoritarian, permissive, or authoritative (Huver et al., 2009). The theory was later extended to include negligent parents. This study includes only three parenting styles which are authoritarian, authoritative and permissive.

The first style which is the Authoritarian Parenting Style is considered a strict method. According to Baumrind the Authoritarian Parents are obedience-and status-oriented, and expect their orders to be obeyed without explanation (Baumrind, 1991). He stated that Authoritarian style tends to be high on demandingness and low or without in responsiveness (Maccoby & Martin, 1983).

Second parenting style named as Authoritative Parenting Style is considered a less strict method. According to Baumrind these parents monitor and impart clear standards for their children's conduct. They are assertive but not intrusive. Their disciplinary methods are supportive, rather than punishing (Baumrind, 1991). Baumrind stated that authoritative parenting style is high in both demandingness and responsiveness (Pellerin, 2005).

Third style is Permissive Parenting Style and in this style parents are referred to as indulgent parents, they have very few demands to make of their children. According to Baumrind, permissive parents are nontraditional and lenient, they do not require mature behavior and allow considerable self-regulation, and avoid confrontation (Baumrind, 1991). Baumrind stated that permissive parenting is high on responsiveness and low on demandingness, which means that it is very rare that these parents enforce rules for their children to follow (Jr., Overbey, and Brewer, 2005).

Adolescence is the seven years between the end of childhood and the beginning of adult life and a time of transition for both children and parents. Parents who understand these phases their children are more likely to keep a positive mental attitude during this time of change. Parents who neither understand nor help the child during this phase are more likely to develop negative attitudes (Weisbard, 2007).

Self-esteem is defined as the feelings connected to the judgment one makes about his or her own worth and feelings (Berk, 2009). Self-esteem is developed during a child's developmental stages. Parental attitudes play a major role in the development of self-esteem in children. Supportive parental behavior, including encouragement and praise is the most powerful factor in the development of self-esteem in early childhood. As children get older their experiences outside home, in school, and with peers, start playing a role and influence their self-esteem (Gale Encyclopedia of Education, 2002).

Optimism is defined as having the strong expectation that, things will turn out all right in life, despite setbacks and frustrations (Goleman, 1995). The word is originally derived from the Latin word optimum, meaning "best." Scheier and Carver, (1985) defined optimism as a generalized expectancy that good, as opposed to bad, outcomes will generally occur when confronted with problems across important life domains. Abramson and colleagues (2006) found, through both self-report measures and behavioral observations, that negative parenting, aggressive behavior for undesirable events promoted negative thinking in children. To date, there is little work directly testing the notion that positive interactions and modeling from parents affects children's expectations therefore, the present study will attempts to find out that how parenting style plays a role in adolescents optimism.

2. OBJECTIVES

- To find out the relationship of Authoritative parenting style with self-esteem in adolescents.
- To check the relationship of Authoritarian parenting style with self-esteem in adolescents.
- To examine the relationship of Permissive parenting style with self-esteem in adolescents.

- To find out the relationship of Authoritative parenting style with optimism in adolescents.
- To check the relationship of Authoritarian parenting style with optimism in adolescents.
- To examine the relationship of Permissive parenting style with optimism in adolescents.

3. METHOD

Participants

Using purposive sampling a sample of 70 (N=70) adolescents, 35 females (n=35) and 35 (n=35) males between age range of 13 to 19 were taken from schools and colleges of Lahore. Students who participated in the study were from Beacon house School System, Lahore Grammar School and The Punjab School, Lahore.

Measures
Parental Authority Questionnaire (PAQ)

Parental Authority Questionnaire (PAQ) created by John R. Buri in 1989 was used to investigate that which parenting style is being practiced with the adolescent. PAQ consists of 30-items and have three subscales based on the parental authority prototypes and each subscale consists of 10 items. There are three subscales namely permissive, authoritarian and authoritative. The scores on each range are from 10 to 50. Participants responded to each item on a 5-point Likert scales ranging from strongly disagree (scored 1) to strongly agree (scored 5). PAQ has good internal consistency measured by the alpha Cronbach's coefficient that is .75 for permissive, .85 for authoritarian and .82 for authoritative scale while good stability in test-retest reliability that is .81, .86, .78 for permissive, authoritarian, and authoritative scales respectively.

Rosenberg Self-esteem Scale (RSE)

Rosenberg Self-esteem Scale created by Morris Rosenberg in 1965 was used to measure self-esteem of adolescents. RSE is used worldwide to measure feelings of self-worth. RSE has high internal reliability which is .92 and strong construct validity. Besides that, it consists of 10 items that are examined on a four-point Likert scale, from strongly agree (scored 3) to strongly disagree (scored 0). Possible total scores range from 0 to 30. The higher scores correspond to higher levels of self-esteem.

Life Orientation Test- revised (LOT-R)

In order to measure optimism the Life Orientation Test- revised was used. LOT-R is a 10-item measure of generalized dispositional optimism (versus pessimism). It was developed by Scheier and Carver in 1985 and revised in 1994 (LOT-R). The LOT-R was presented to the subjects and they were asked to read the items and identify their level of agreement: 0= strongly disagree, 1=disagree, 2= neutral, 3= agree, and 4= strongly agree. Scores range from 0 to 24 with high scores indicating greater optimism. The Cronbach's alpha for the scale is .76. Furthermore LOT-R has high internal reliability of .82.

Procedure

Institutional permissions from the principals of the three colleges were taken before initiating the data collection for the present study. A brief consent form was also attached with the questionnaires for participants to understand the topic of the study and show

their consent for participation. Participants were then provided with a set of questionnaires which included the Parental Authority Questionnaire, Rosenberg Self-esteem scale and Life Orientation Test Revised. Maximum time given to the participants for completing the questionnaires was 20 minutes. The data collection was done during the month of March 2012. The data was analyzed using Statistical Package for Social Sciences (SPSS) version 18.0 and Pearson Correlational analysis. Mean and Standard Deviation (SD) was applied as the statistics.

Statistical Analysis

A Correlation Research Design was used. The relationship of parenting styles with self-esteem and optimism was analyzed by employing Pearson Product Moment Correlation Coefficient. Means and Standard Deviation were also calculated. Analysis was carried out using Statistical Package for Social Sciences version 18. In this study the level of significance was set at 0.01.

4. RESULTS

Table 1
Pearson Product Moment Correlation Coefficient between
Parenting styles and Self-esteem. (N=70)

		Permissive parenting style	Authoritarian parenting style	Authoritative parenting style
Self esteem	Pearson Correlation	.343[**]	-.514[**]	.438[**]
	Sig. (2-tailed)	.004	.000	.000

**Correlation is significant at the 0.01 level.

Table shows that self-esteem has a significant positive correlation with Authoritative parenting style ($r = .438$, $p < 0.01$) and Permissive parenting style ($r = .343$, $p < 0.01$) and a significant inverse correlation with Authoritarian parenting style ($r = -.514$, $p < 0.01$).

Table 2
Pearson Product Moment Correlation Coefficient between
Parenting styles and Optimism. (N=70)

		Permissive parenting style	Authoritarian parenting style	Authoritative parenting style
Optimism	Pearson Correlation	.143	-.314[**]	.475[**]
	Sig. (2-tailed)	.238	.008	.000

**Correlation is significant at the 0.01 level.

Table illustrate that optimism has a significant positive correlation with Authoritative parenting style ($r = .475$, $p < 0.01$) and a significant negative correlation with Authoritarian parenting ($r = -.314$, $p < 0.01$) style. No significant correlation of optimism is examined with permissive parenting style ($r = .143$). Correlation are significant at p value = 0.01.

Table 3
Mean and Standard Deviation of Parenting Styles, Self Esteem
and Optimism of total population (N = 70).

	M	SD
Permissive style	26.16	6.480
Authoritarian style	29.99	6.435
Authoritative style	33.43	6.243
Self esteem	16.13	3.960
Optimism	14.80	4.009

* Note: M = mean, SD = standard deviation

5. DISCUSSION

The study sets out to examine the relationship of parenting style with self-esteem and optimism in adolescents and to determine that there is a difference in the level of self-esteem and optimism among the children being practiced under the three parenting style. Discrepancies were found between the hypothesis and the results. The results show that there is a significant positive relationship of Authoritative parenting style with self-esteem and optimism in adolescents. Moreover the findings also showed a significant inverse relationship of Authoritarian parenting style with self-esteem and optimism. Along with this the study also revealed that there is a significant positive relationship between Permissive parenting and self-esteem however no significant relationship was observed between Permissive parenting style and optimism.

These findings also support previous literature and are in line with Baumrind (1991) study which was conducted to examine the effects of the three parenting styles and revealed same results. These findings also support Martinez and Garcia (2007) study who conducted a study to examine the effects of the three parenting styles on children and concluded same results. Moreover Furnham and Cheng conducted a research in 2000 and concluded similar results that Authoritative parenting style has a significant positive correlation with self-esteem among adolescents as compared to Authoritarian and Permissive parenting styles. Greiss (2010), Baldwin (2007) and Jackson et al., (2005) also revealed that Authoritative Parenting style has a significant positive correlation with optimism among adolescents and Authoritarian and Permissive parenting styles have a negative correlation with optimism.

All the hypotheses were statistically analyzed using SPSS (version 18). Pearson product moment coefficient was used to examine relationship of parenting styles with self-esteem and optimism among adolescents.

6. CONCLUSION

It is an exploratory study that aims at determining the relationship of parenting styles with self-esteem and optimism among adolescents. Findings of the study suggest that parenting styles have both positive and inverse correlations with both self-esteem and optimism. Results indicated that most frequently used parenting style among adolescents was Authoritative parenting style which was positively correlated to both self-esteem and optimism. As Adolescents is a time of change for both children and parents therefore

findings of this study have very important implications that can help parents adopt such parenting styles that foster positive traits such as self-esteem and optimism. Positive parenting style will help adolescents attain high self-esteem and high optimism.

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THE RELATIONSHIP OF DEPRESSION, PERCEIVED STRESS, PERCEIVED SOCIAL SUPPORT AND BODY MASS INDEX IN COLLEGE STUDENTS

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ABSTRACT

The purpose of this survey-based study was to explore the relationship between depression, perceived stress, perceived social support, and body mass index in college students, especially those who transition from high school to college as this period is often filled with changing relationships and students have to cope with new social pressures. A sample of 100 students from Kinnaird College for Women Lahore and Lahore University of Management Sciences was taken. The participants were asked basic questions about their gender, age, weight, and height. Weight and height were required to measure Body Mass Index (BMI). The tools used were Center for Epidemiological Studies Depression Scale, Perceived Stress Scale, and Multidimensional Scale of Perceived Social Support. It was hypothesized that people who are depressed, stressed and lack social support would have higher body mass indexes than people who do not suffer from such symptoms. The results of this study showed that perceived stress and depression are positively correlated and there are negative correlations between perceived social support and depression, and perceived social support and perceived stress. Implications of this study are that more resources need to be devoted to help people adjust into college environment and college food service providers should provide healthy and easy to eat foods so that the students do not gain weight. Moreover, social psychologist need to learn more about the factors that are leading to weight gain and obesity and should work to reduce this epidemic.

1. INTRODUCTION

There has been a rapid increase in the obesity rate. In the past ten years the obesity rate has doubled and more people are overweight (Wainer, 2010). Some people use food as a coping mechanism. Weight-related studies have found that obese individuals increase their food intake as a response to negative emotions, including depression, perceived social support and stress (Arnow, Kenardy, & Abrras, 1992; Liberman, Wurtman, & Chew, 1986; Gibson, 2006).

An individual has to go through a transition period when he/she has to shift from high school to college. This period is often filled with changing relationships and students have to cope with new social pressures. In longitudinal studies, this is a period that has previously been associated with weight gain (Levitsky, Halbmaier, & Mrdjenovic, 2004). Studies have shown that students on average gain 3 to 10 pounds during their first 2 years of college. Most of this weight gain occurs during the first semester of freshman year. College is also a time of change, and the stress of school to college transition can trigger overeating. People sometimes eat in response to emotions, anxiety, homesickness,

sadness, or stress, and all of these can be part of adapting to being away at school ("Beating the Freshman 15," 2012).

According to Wainer (2010) college students are going through a very tumultuous times in their lives due to which they use different coping mechanisms and food is one of them. Many students are emotional eaters and it provides a distraction to students. Eating often provides comfort similar to the support students got from their collocated relationships before their transition to college. Therefore, it is likely that freshmen display lower social support scores as compared to senior class students. Polivy, Herman and McFarlane (1994) conclude in their studies that emotional eating is a way, even if only temporarily, to relieve distress and mask emotions one is trying to avoid. People gain weight due to emotional eating (Strien, Herman & Verheihden, 2008).

Moreover, social support is very important for College students too, as they are going through a very confused time in their lives, which makes them prime candidates for those who use food as a coping mechanism and are emotional eaters. Eating may serve as a distraction from one's worries and eating may provide comfort similar to the support students got from their collocated relationships before their transition to college. Studies show that people involve in emotional eating because it is a way, even if only temporarily, to relieve distress and mask emotions one is trying to avoid (Polivy, Herman, & McFarlane, 1994).

Oliver and Wardle (1999) conducted another study which included a sample of 212 students. Effects of perceived stress on food choice were measured and it was found that there was increased intake of "snack-type" foods in 73% respondents during the time of stress and intake of "meal-type" foods (vegetables, meat, fish and fruits) decreased due to stress. As "snack-type" foods are dense in fats and carbohydrates, they result in a higher BMI.

Faleel, et al. (2012) and Cheng (1997) also researched separately and the results of their studies showed that perceived social support and depression are related. When an individual believes that he has no or less social support it can lead to depression.

It has been found that a common warning sign of depression is changes in appetite and weight. Often low motivation for preparing food combined with low appetite leads to poor food choices and irregular eating patterns. For some people there may be increased appetite for inappropriate foods and weight gain but whatever the case is, these habits tend to intensify depression ("Understanding Food and Mood," 2007).

Studies have also shown that there is a negative correlation between peer social support, family social support, self-esteem, optimism and, depressive symptoms (Weber, Puskar & Ren, 2010).

Previous research has shown that there is a connection between what people eat and how people feel (Oliver, Wardle, & Gibson, 2000). Furthermore, Brooks, Harris, Thrall and Woods (2002) conducted a study and found that lack of healthy diet is correlated with depression. People often do not concentrate on the nutritional content of their food and often end up eating unhealthy food when their mood changes. It has also been found that people eat to celebrate their accomplishments so that they feel better.

Results of findings are mixed, for example, depressive symptoms were positively associated with health-comprising attitudes (i.e. weight concerns), depressive symptoms were negatively associated with health-comprising behaviours (i.e. eating breakfast) in a study of adolescents, but the results showed that most associations between depressive symptoms and the diet foods were not statistically significant (Fulkerson, Sherwood, Neumark-Szainer, & Story, 2004). More research must be conducted in this area to resolve this controversy.

The aim of the study was to explore the relationship between perceived stress, perceived social support, and depression as it connects to weight changes in college students who are abnormal eaters and BMI. Another aim of the current study was to investigate gender differences in perceived stress, perceived social support, depression, BMI and abnormal eating patterns. It was hypothesized that;
Perceived stress, perceived social support, depression, and BMI are correlated.

2. METHODOLOGY

Sample
A sample of 100 students (67 females and 33 males) was taken. The population participating in this research was students of Kinnaird College Lahore and Lahore University of Management Sciences enrolled in Bachelors program. The age of the participants ranged between 18 to 23 years.

Tools
Demographics
The participants were asked basic questions about their gender and age. Weight and height were asked to calculate BMI.

Center for Epidemiological Studies Depression
Center for Epidemiological Studies Depression Scale (Radloff, 1977) is a 20 item scale. It measures depressive symptoms during the previous week. The reverse scoring questions were 4, 8, 12 and 16. The scale ranges from 0 (rarely or none of the time) to 3 (most or all of the time). The reliability of this instrument is 0.91 which is considered high. The findings indicate that the CES-D is a valid and reliable measure of depressive symptoms (Roberts, 1980).

The Perceived Stress Scale
The Perceived Stress Scale (Cohen, Kamarck, & Mermelstein, 1983) was used to measure the degree to which participants view situations in their life as stressful. It is a 10 item scale and ranges from 0(never) to 4(very often). Scores can range between 0 to 40. The reverse scoring items were 4, 5, 7 and 8. The Cronbach's alpha reliability coefficients were 0.83 to 0.87. The reliability and validity of this scale is good. (Reiz, Hino & Anez, 2010).

Multidimensional Scale of Perceived Social Support
Multidimensional Scale of Perceived Social Support (Zimet, Dahlem, Zimet & Farley, 1988) was used to measure the degree of social support each participant felt he or she had. It has 12 items and the scale ranges from 1(very strongly disagree) to 7(very

strongly agree). The scores can range between 12 to 84. The Cronbach's alpha reliability of this scale is 0.91 which is considered high (Wongpakaran, Wongpakaran & Ruktrakul, 2011).

Procedure

Permission was sought from the authors of questionnaires. The participants in this study were students enrolled in bachelors program. The data was collected from Kinnaird College for Women Lahore and Lahore University of management sciences. Once the participants were given the questionnaire, they filled the informed consent form which indicated that the participants could withdraw from the study anytime they wished. All the participants were informed that the results will be used for research purposes and confidentiality will be maintained. Instructions regarding how the participants had to fill the survey were mentioned on the survey. Participants were required to report their age, gender, weight and height. The weight and height were required to calculate Body Mass Index (BMI). Center for Epidemiological Studies Depression Scale, Perceived Stress Scale and Multidimensional Scale of Perceived Social Support, in the same order, were administered to students of different departments on the campus. Time taken to fill the survey was approximately 15 minutes.

3. RESULTS

he correlation was calculated among the variables using Statistical Package for Social Sciences 17.0 version and is shown in the table 1.

Table 1
Pearson Product Moment Correlation Between Psychosocial factors and BMI

	Dep	Str	Soc	BMI
Dep	-			
PSt	0.589**	-		
PSo	-0.425**	-0.240*	-	
BMI	0.040	-0.041	-0.217*	-

Note   Dep= Depression, PSt= Perceived Stress, PSo= Perceived Social Support, BMI= Body Mass Index, *p< 0.05, **p < 0.01

The analysis showed that there is a significant correlation between perceived stress and depression, $r(100) = .589$, $p < .01$. There is a significant negative correlation between perceived social support and depression, $r(100) = -.425$, $p < .01$, perceived social support and perceived stress, $r(100) = -.240$, $p < .05$ and BMI and perceived social support, $r(100) = -.217$, $p < .05$. BMI did not significantly correlate with depression and perceived stress.

4. DISCUSSION

The results show that BMI and perceived social support are negatively correlated. Furthermore, weight-related studies have found that obese individuals increase their food intake as a response to negative emotions such as lack of perceived social support because eating may serve as a distraction from one's worries and eating may provide comfort similar to the support students got from their collocated relationships before their

transition to college (Polivy, Herman, & McFarlane, 1994; Provencher, Polivy, Wintre, Pratt, et al., 2009).

The assumptions of the present study were supported that perceived social support is negatively correlated with depression and perceived stress (Faleel, et al., 2012) whereas, perceived stress and depression are positively correlated (Cheng, 1997).

Furthermore, it was found that depression and perceived stress are not correlated with BMI. Wainer (2010) also found out that there is no significant relationship between BMI, perceived stress and depression.

5. COMMENTS AND CONCLUSION

The purpose of the current study was to examine the relationship between psychosocial factors i.e. perceived stress, perceived social support, depression and body mass index (BMI).

The results of the present study revealed that perceived social support is negatively correlated with depression, perceived stress and BMI whereas, depression and perceived stress are positively correlated.

More data should be collected for a cross sectional or stratified analysis. Further research needs to include a larger sample so that the results can be generalized. Students took survey in their own environment and the researchers had no way to control distractions, so in order to gain a better picture in future of the causal factors that influence effects psychosocial factors and eating behaviours on body mass index (BMI), more experiment in which one experimental variable is manipulated should be undertaken. This may be very difficult for ethical and practical reasons so a longitudinal design may be more realistic. Future directions for this research include performing a longitudinal analysis. A group of students should be tracked on changed in perceived stress, perceived social support, depression, other covariates and eating patterns would add more depth to the current literature. Other psychosocial factors like subjective well-being and self-efficacy along with dietary intake and eating patterns should be measured to get in-depth view of factors effecting BMI.

The implications of this study can be that more resources should be devoted to help students adjust in college. College food service providers should provide healthy and easy to eat foods that is foods that have more nutritional value, less carbohydrates and fats. Moreover, social psychologist need to learn more about the factors that are leading to weight gain and obesity and should work to reduce this epidemic.

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ON A SELF-INVERSE PROBABILITY MODEL INVOLVING
THE MACDONALD FUNCTION

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ABSTRACT

This paper presents a self-inverse density involving the Macdonald function that has been obtained by utilizing the differential equation of Habibullah et al. (2010). The positively skewed shape of the density testifies to its potential for modeling biological and/or machine life. Some properties pertaining to moments, moment generating function and mode are derived.

1. INTRODUCTION

Whereas inverted probability distribution have attracted a considerable amount of attention during the past a few decades, the property of invariance under the reciprocal transformation seems to have attracted only on little amount of attention until very recently. Habibullah et al. (2010) describe such distributions as being "Strictly Closed Under Inversion (SCUI)" and present differential equations capable of generating an unlimited number of such distributions. As well, they provide a wide variety of properties possessed by this class of distributions.

Habibullah and Saunders (2011) introduce the concept of "self inversion" as follows: A variate Y with $Y/a \sim a/Y, a \neq 1$ can be regarded as being strictly closed under inversion in the *generalized* sense, or "generalized SCUI", so that SCUI i.e. $Y \sim 1/Y$ becomes its special case. When $Y/a \sim a/Y$, Y is to be regarded as being "self-inverse at a", and a is to be regarded as the point of reciprocity (or the point of inversion). The variate U with $U \sim 1/U$ will then obviously be regarded as being self-inverse at unity.

In this paper, we utilize the differential equation of Habibullah et al. (2010) to derive a probability density that is self-inverse at unity and involves the Bessel function of the second kind called the Macdonald function. We obtain some properties of this self-inverse probability model.

2. DIFFERENTIAL EQUATION FOR GENERATING
SELF-INVERSE DISTRIBUTIONS

Habibullah et al. (2010) provide two different types of differential equations capable of generating an unlimited number of self-inverse distributions. One of them is a differential equation containing expressions of the form e^y. The exact expression is

$$\frac{d}{dy}[\ln g(y)] = \frac{b_n e^{ny} + b_{n-1} e^{n-1} {}^y + ... + b_0}{a_n e^{ny} + a_{n-1} e^{n-1} {}^y + ... + a_0}$$

(2.1)

and the set of conditions under which this differential equation yield self-inverse distributions are

i) $a_i \neq 0$ *and* $b_j \neq 0$ for some i,j, $0 \leq i , j \leq n$,

ii) $\begin{cases} \displaystyle\sum_{i=0}^{j} a_i b_{i+n-j} + a_{i+n-j} b_i = 0, \; j = 0,1,...,n-1, \\[4mm] \displaystyle\sum_{i=0}^{n} a_i b_i = 0 \end{cases}$

(2.2)

3. MACDONALD FUNCTION

Macdonald functions are modified Bessel functions of the second kind that satisfy the differential equation

$$z^2 \frac{d^2 u}{dz^2} + z \frac{du}{dz} - z^2 + v^2 \; u = 0$$

for which they are the solution that remains bounded as z tends to infinity. Assche et al. (2000) present the following integral representation of the Macdonald function:

$$K_v \; z \; = \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} \frac{e^{-z}}{\Gamma \; v+1/2} \int_0^\infty e^{-t} t^{v-1/2} \left(1 + \frac{t}{2z}\right)^{v-1/2} dt$$

$$= \frac{1}{2}\left(\frac{z}{2}\right)^v \int_0^\infty e^{-t-z^2/4t} t^{-v-1} dt$$

(3.1)

whereas its series representation is

$$K_n \; z \; = \frac{1}{2} \sum_{k=0}^{n-1} -1 {}^k \frac{n-k-1 \; !}{k!\left(\dfrac{z}{2}\right)^{n-2k}}$$

$$+ \; -1 {}^{n+1} \sum_{k=0}^{a} \frac{\left(\dfrac{z}{2}\right)^{n+2k}}{k! \; n+k \; !}\left[\ln\frac{z}{2} - \frac{1}{2}\psi \; k+1 \; - \frac{1}{2}\psi \; n+k+1 \right]$$

with

$$\psi(z) = \frac{d}{dz}\ln\Gamma \; z \; = \ln z - \sum_{k=0}^\infty \left[\frac{1}{z+k} - \ln(1+\frac{1}{z+k})\right]$$

(3.2)

4. DERIVATION OF A SELF-INVERSE PROBABILITY MODEL INVOLVING THE MACDONALD FUNCTION

In this section we utilize differential equation (2.1) for deriving a new self–inverse probability model that involves the Macdonald function. The graphical representation of the density function being unimodal and positively skewed, this probability distribution seems to possess the potential for modeling life.

Putting n=2 in (2.1), taking $a_0 = 0$, $a_1 = 1$, $a_2 = 0$, $b_0 = a$, $b_1 = 0$ and $b_2 = -a$, and applying the transformation $X = e^Y$ we obtain

$$f(x) = \frac{1}{2K_0(2a)} \left[\frac{1}{x} e^{-a\left(x+\frac{1}{x}\right)} \right], x > 0 \tag{4.1}$$

where $K_\nu(z)$ is the Macdonald function given by (3.1).

The density has only one parameter 'a' where $a > 0$.

5. PROPERTIES PERTAINING TO MOMENTS:

This section presents the derivation of the moments about the origin, moments about the mean and moment generating function of probability distribution (4.1). A recurrence relation for the moments about the origin is also obtained.

5.1 Moments about the Origin:
The rth moment about the origin is obtained as follows:

By definition,

$$\mu_r' = E(X^r) = \int_0^\infty x^r f(x)\, dx = \frac{1}{2K_0(2a)} \int_0^\infty x^{r-1} e^{-a\left(x+\frac{1}{x}\right)} dx$$

In Gradshteyn and Ryzhik (2000) in the integral labeled 3.478-4 we find the following result:

$$\int_0^\infty x^{\alpha-1} e^{-ax-bx^{-1}}\, dx = 2\left(\frac{b}{a}\right)^{\frac{\alpha}{2}} K_\alpha\ 2\sqrt{ab}\ ,\ a > 0,\, b > 0 \tag{5.1}$$

where $K_\alpha\ z$ is the Macdonald function as defined in eq. (3.1).

Using eq. (5.1), we obtain

$$\int_0^\infty x^{r-1} e^{-a\left(x+\frac{1}{x}\right)} dx = 2\left(\frac{a}{a}\right)^{\frac{r}{2}} K_r\ 2\sqrt{a.a}\ = 2K_r\ 2a\ ,\ a > 0$$

so that

$$\mu'_r = \frac{K_r\ 2a}{K_0(2a)} \tag{5.2}$$

As such, the mean of the distribution is given by:

$$\mu_1' = \frac{K_1\ 2a}{K_0(2a)}$$

5.2 Moments about the Mean:

Using the relationships between the moments about the origin and central moments, we obtain:

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2 = \frac{K_0(2a)K_2\ 2a - \left[K_1\ 2a\right]^2}{\left[K_0(2a)\right]^2} = Variance$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\ \mu_1'^3$$

$$= \frac{\left[K_0(2a)\right]^2 K_3(2a) - 3K_0(2a)K_1(2a)K_2(2a) + 2\ K_1(2a)^3}{\left[K_0(2a)\right]^3}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\ \mu_1'^2 - 3\ \mu_1'^2$$

$$= \frac{\begin{array}{c}\left[K_0(2a)\right]^2 K_4(2a) - 4K_0(2a)K_1(2a)K_3(2a) \\ +6K_2(2a)\ K_1(2a)^2 - 3K_0(2a)\ K_1(2a)^2\end{array}}{\left[K_0(2a)\right]^3}$$

5.3 Moment Generating Function

The moment-generating function is obtained as follows:

$$E(e^{tx}) = \int_0^\infty e^{tx} f\ x\ dx = \frac{1}{2K_0(2a)} \int_0^\infty x^{-1} e^{-a-t\ x - \frac{a}{x}} dx$$

Putting $\alpha = 0$ and replacing a by a-t and b by a in (5.1), we have

$$\int_0^\infty x^{-1} e^{-a-t\ x - \frac{a}{x}} dx = 2K_0\ 2\sqrt{a\ a-t}\ ,\ a > 0$$

so that

$$E(e^{tx}) = \frac{K_0 \; 2\sqrt{a} \; a - t}{K_0(2a)}$$

5.4 Recurrence Relation:

Putting the value of $K_0 \; 2a$ obtained from the expression for the rth moment in the expression for the $(r+1)$[th] moment, we obtain

$$\mu_{r+1}' = \frac{K_{r+1} \; 2a}{K_r \; 2a} \mu_r'$$

6. MODE

Following the usual procedure for the determination of the mode, the mode comes out to be

$$\hat{x} = \frac{-1 + \sqrt{1 + 4a^2}}{2a} \; .$$

7. GRAPHICAL REPRESENTATION

Fig. 7.1 shows the probability curve corresponding to eq. (4.1). The density has a nice unimodal, positively skewed shape indicating its applicability in a variety of situations, particularly in life-length studies.
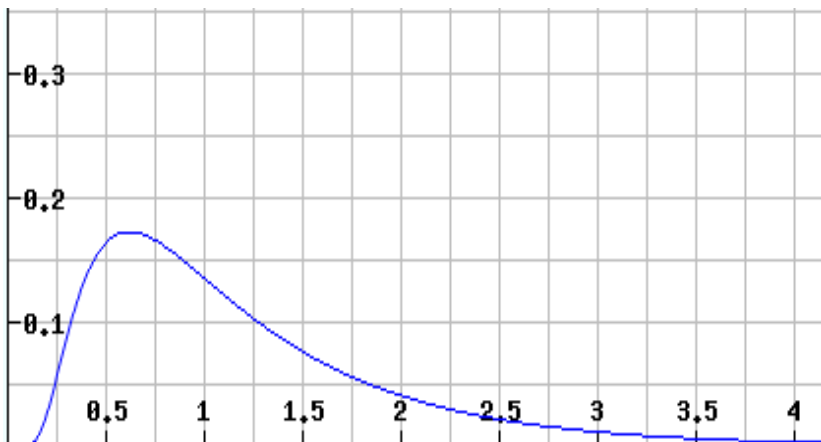
Fig 7.1

ACKNOWLEDGEMENT

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ESTIMATION OF LOCATION AND SCALE PARAMETERS OF NORMAL DISTRIBUTION BY APPROXIMATE METHODS

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INTRODUCTION

To solve the estimation problems related to the parameters of the estimates, approximate methods suggest that the efficiency of the determination of linear estimates does not seem particularly sensitive to the changes in the coefficients and may be chosen for convenience. Dowton (1966) derived linear estimates with polynomial coefficient and discussed this method for Normal and Gumbel distribution. Hirai (1967) considered the estimation of the parameters from the Rayleigh distribution by linear coefficient method and in (1972) by the Quadratic Coefficients method as a particular case of Dowton (1966) to estimate location and scale parameter from the Rayleigh distribution. Akhtar and Hirai (2007) also obtained Cubic Coefficients with four terms to estimate parameters of Rayleigh distribution. In this paper we discuss the estimation of the location and scale parameter of normal distribution by three approximate methods, linear coefficient method, Quadratic Coefficients method and Cubic Coefficients method. The methods and there use for normal distribution is explained below.

1. LINEAR COEFFICIENT METHOD

If $X_1^{(n)} \leq X_2^{(n)} \leq \ldots \leq X_n^{(n)}$ be the ordered observation from Normal distribution whose density is

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)} \; 0 \leq x \leq \infty \tag{1.1}$$

Then the method of linear coefficient can be described as below:

Let $X_1^{(n)} \leq X_2^{(n)} \leq \ldots \leq X_n^{(n)}$ be the ordered observation from Normal distribution of the form

$$F\left(\frac{x-\mu}{\sigma}\right)$$

where μ and σ are the unknown location and scale parameter respectively.

Let us make a linear transformation of the form

$$y_i^{(n)} = \frac{x_i^{(n)} - \mu}{\sigma}$$

and then $y_1^{(n)} \leq y_2^{(n)} \leq \ldots \leq y_n^{(n)}$ are the reduced ordered observation such that

$$E(Y_{(i)}^{(n)}) = \alpha_i^{(n)}$$

and $(E(Y_{(i)}^{(n)}, Y_{(j)}^{(n)}) = W_{ij}^{(n)}$ (i=1,2,3,...,n)

We have to estimate a parametric function $p = k_1\mu + k_2\sigma$ from reduced order observations. Here k_1 and k_2 are known parameters and μ and σ are the unknown parameters.

Consider the following linear combination of two statistics

$$t = \sum_{i=1}^{n} (i-1)^{(0)} x_{(i)}^{(n)} \text{ and } g = \sum_{i=1}^{n} (i-1)^{(i)} x_{(i)}^{(n)}$$

where t and g are the realization of the random variables T and G. We have to obtain an estimating function for p as u=at+bg where a and b are to be determined such that u is unbiased for p and it has minimum variance among all such u's.

So,

$$E(U) \quad = aE(T)+bE(G)$$
$$= \left(an + \frac{bn^{(2)}}{2}\right)\mu + (an\alpha_1^{(1)} + \frac{bn^2\alpha_2^{(2)}}{2})\sigma \qquad (1.2)$$

We define

$$T = \sum_{i=1}^{n} (i-1)^{(0)} Y_{(i)}^{(n)} \text{ and } G = \sum_{i=1}^{n} (i-1)^{(1)} Y_{(i)}^{(n)} \text{ as random variables.}$$

The expression $m^{(r)}$ with m and r integers is the rth factorial power of m i.e.,

$$m^{(r)} = \frac{m!}{(m-r)!}$$

U will be unbiased estimator of p if the coefficient are so chosen such that

such that $a\psi = k$

where $k = [k_1 \ k_2]$

Variance of the estimates is given as

$$Var(U) = Var(aT+bG) \qquad (1.3)$$

or

$$\text{var } (U) = \sigma^2 a\Omega a' \qquad (1.4)$$

where $a = [a \ b]$

We minimize this variance w.r.t to restrains i.e., minimize the expression

$$\chi = \sigma^2 a\Omega a' + \Phi\psi' a'$$

The minimization of χ is obtained by solutions of the equations with a $\psi = k$ yields.

$$a = k[\psi'\Omega^{-1}\psi]^{-1}\psi'\Omega^{-1}$$

and $$Var(U) = \sigma^2 k[\psi'\Omega^{-1}\psi]^{-1} k' \qquad (1.5)$$

If k = [1; 0] we get μ_2^* as an estimate of μ and similarly if k = [0; 1] we get σ_2^* as an estimate of σ. We may sometimes be interested in other values of k_1 and k_2.

where $\Phi = [\Phi_1 \ \Phi_2]$ and $\Phi_1 \ and \ \Phi_2$ are Lagrangian undetermined multipliers and

$$\psi' = \begin{bmatrix} \dfrac{n^{(1)}}{1} & \dfrac{n^{(2)}}{2} \\ \dfrac{n^{(1)}}{1}\alpha_1^{(1)} & \dfrac{n^{(2)}}{2}\alpha_2^{(2)} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} var(T) & Cov(T,G) \\ Cov(G,T) & Var(G) \end{bmatrix} = \begin{bmatrix} \Omega_{00} & \Omega_{01} \\ \Omega_{10} & \Omega_{11} \end{bmatrix}$$

$$Var(T) = \sigma^2 \ Var \ \Sigma_{i=1}^{n} (i-1)^{(0)} x_{(i)}^{(n)}$$
$$= \sigma^2 var\left(\Sigma_{i=1}^{n} y_{(i)}^{(n)}\right) = \sigma^2 \left(\Sigma_{j=1}^{n} \Sigma_{i=1}^{n} v_{ij}^{(n)}\right)$$

where

$$V_{ij}^{(n)} = w_{ij}^{n} - \alpha_i^{(n)} \alpha_j^{(n)}$$

$$Var(G) = \sigma^2 \left(\Sigma_{j=1}^{n} \Sigma_{i=1}^{n} (i-1)^{(1)} (j-1)^{(1)} v_{ij}^{(n)}\right)$$

$$Cov(T,G) = \sigma^2 \left(\Sigma_{j=1}^{n} \Sigma_{i=1}^{n} (i-1)^{(1)} (j-1)^{(0)} v_{ij}^{(n)}\right)$$

Dowton (1966) has simplified Ω and obtained the following expression

$$\Omega_{ij} = \Omega_{ji} = \frac{(i+1)(j+1)}{n^{(i+1)}} \Sigma_{s=0}^{i} b_{ij}^{(s)} (n-j-i)^{(s)} \quad i \leq j \tag{1.6}$$

$$b_{ij}^{(s)} = \frac{i!j!W_{s+j+1,s+j+1}^{(s+j+1)}}{(i-s)!(s+j-i)!s!(s+j+1)} + \frac{i!j!(j+1)^{(i+1-s)} \Sigma_{r=0}^{s-1}(-1)^r w_{j+1,j+2}^{(j+2+r)}}{(i+1-s)!(j+2+r)!(s-1-r)!}$$
$$- \frac{i!j!(i+1)^{(i+1-s)} \Sigma_{r=0}^{i}(-1)^r w_{s+j-i,s+j+1}^{(s+j-i+1+r)}}{(i+1-s)!(j-r)!(s+j-i+1+r)!} + \frac{i!j!\alpha_{i+1}^{(i+1)}[\alpha_{s+j-i}^{(s+j-i)} - \alpha_{j+1}^{(j+1)}]}{(i+1-s)!(s+j-i)!s!} \tag{1.7}$$

when s = 0 some of these terms vanish. It should be noticed that these coefficients depend only upon diagonal and next-diagonal terms of relatively small variance matrices of ordered observations and upon the expected values of the largest observations.

1.1 Estimation of parameter by Linear coefficient method for Normal distribution using order statistics

Let μ^* and σ^* be the estimates of Linear method of location and scale parameter respectively from the Normal distribution based on order statistics.
For the evaluation of the coefficients of μ^* and σ^* we have:

$$\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\theta_0}{n} & \dfrac{2\theta_1}{n^{(2)}} \\ \dfrac{\Phi_0}{n} & \dfrac{2\Phi_1}{n^{(2)}} \end{bmatrix} \tag{1.1.1}$$

Where

$$[\theta':\Phi']=\Omega^{-1}\begin{bmatrix}1 & \alpha_{(1)}^{(1)} \\ 1 & \alpha_{(2)}^{(2)}\end{bmatrix}\left[\begin{bmatrix}1 & 1 \\ \alpha_{(1)}^{(1)} & \alpha_{(2)}^{(2)}\end{bmatrix}\Omega^{-1}\begin{bmatrix}1 & \alpha_{(1)}^{(1)} \\ 1 & \alpha_{(2)}^{(2)}\end{bmatrix}\right]^{-1}$$

The coefficient of μ^* and σ^* are given as:

$a_{11} = a_1$; $a_{12} = a_1 + a_2$; $a_{13} = a_1 + 2a_2$,..., $a_{1n} = a_1 + (n-1)a_2$
$b_{11} = b_1$; $b_{12} = b_1 + b_2$; $b_{13} = b_1 + 2b_2$,.., $b_{1n} = b_1 + (n-1)b_2$

Table 1.1.1
Elements of Ω in Linear Coefficient Estimate (LCE) from the
Normal Distribution based on order statistics for $3\leq n\leq10$

n=3	0.333333 0.333333	0.333333 0.421022
n=4	0.25 0.25	0.25 0.307407
n=5	0.2 0.2	0.2 0.242582
n=6	0.166667 0.166667	0.166667 0.200480
n=7	0.142857 0.142857	0.142857 0.170884
n=8	0.125 0.125	0.125 0.148927
n=9	0.111111 0.111111	0.111111 0.131981
n=10	0.1 0.1	0.1 0.1185204

Table 1.1.2
Coefficients of Linear method μ^* for the location parameter of
Normaldistribution based on order statistics for $3\leq n\leq10$

n	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{110}
3	0.3333	0.3333	0.3333							
4	0.25	0.25	0.25	0.25						
5	0.2	0.2	0.2	0.2	0.2					
6	0.16667	0.16667	0.16667	0.16667	0.16667	0.16667				
7	0.14286	0.14286	0.14286	0.14286	0.14286	0.14286	0.14286			
8	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125		
9	0.1111	0.11111	0.11111	0.11111	0.11111	0.11111	0.11111	0.11111	0.11111	0.11111
10	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 1.1.3
Coefficients of Linear method σ* for scale parameter of
Normal distribution using order statistics for 3≤n≤10

n	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}	b_{17}	b_{18}		
3	-0.59082	0	0.59082							
4	-0.44311	-0.14771	0.14770	0.44311						
5	-0.35449	-0.177246	-0.000001	0.177244	0.354489					
6	-0.295411	-0.17725	-0.05908	0.05908	0.17725	0.295411				
7	-0.253208	-0.168805	-0.084402	0.000001	0.084404	0.168807	0.25321			
8	-0.221557	-0.158255	-0.094953	-0.031651	0.031651	0.094953	0.158255	0.221557		
9	-0.196939	-0.147704	-0.098469	-0.049234	0.000001	0.049236	0.09847	0.147706	0.196941	
10	-0.177245	-0.137857	0.098469	-0.059081	-0.019693	0.09695	0.059083	0.098471	0.137859	0.177247

Table 1.1.4
Variance covariance of μ* and σ* for the Normal distribution using
Order statistics for 3≤n≤10 .Each value should be multiplied by σ^2

	Var(μ*)	Var(σ*)
n=3	0.3333333	0.275482
n=4	0.25	0.180349
n=5	0.2	0.133775
n=6	0.166667	0.106228
n=7	0.142857	0.088049
n=8	0.125	0.075169
n=9	0.111111	0.084602
n=10	0.1	0.058132

2. QUADRATIC COEFFICIENT METHOD

In Quadratic coefficient method we use three term approximation to estimate parameters of the distribution having cumulative distribution function of the form $F(\frac{x-\mu}{\sigma})$ where μ and σ are the unknown location and scale parameter respectively.

As discussed above let us make a linear transformation of the form

$$y_i^{(n)} = \frac{x_i^{(n)} - \mu}{\sigma} \text{ (i=1,2,3,...,n)}$$

and then $y_1^{(n)} \leq y_2^{(n)} \leq ... \leq y_n^{(n)}$ are the reduced ordered observation such that $E(Y_{(i)}^{(n)}) = \alpha_i^{(n)}$ and $(E(Y_{(i)}^{(n)}, Y_{(j)}^{(n)}) = w_{ij}^{(n)}$

We have to estimate a parametric function $p = k_1\mu + k_2\sigma$ from reduced order observations .Here k_1 and k_2 are known parameters and μ and σ are the unknown parameters.

Consider the following linear combination of three statistics

$$t = \sum_{i=1}^{n} (i-1)^{(0)} x_{(i)}^{(n)}$$

$$g = \sum_{i=1}^{n} (i-1)^{(1)} x_{(i)}^{(n)}$$

$$s = \sum_{i=1}^{n} (i-1)^{(2)} x_{(i)}^{(n)}$$

where t,g and s are the realization of the random variables T , G and S.We have to obtain an estimating function for p as u=at+bg+cs where a, b and c are to be determined such that u is unbiased for p and it has minimum variance among all such u's.

So,

$E(U)$ $=aE(T)+bE(G)+cE(S)$

$$= \left(an + \frac{bn^{(2)}}{2} + \frac{cn^{(2)}}{3} \right) \mu + \left(an\alpha_1^{(1)} + \frac{bn^{(2)}\alpha_2^{(2)}}{2} + \frac{cn^{(3)}\alpha_3^{(3)}}{3} \right) \sigma \qquad (2.1)$$

We define

$$T^* = \sum_{i=1}^{n} (i-1)^{(0)} Y_{(i)}^{(n)}$$

$$G^* = \sum_{i=1}^{n} (i-1)^{(1)} Y_{(i)}^{(n)}$$

$$S^* = \sum_{i=1}^{n} (i-1)^{(2)} Y_{(i)}^{(n)} \text{ as new random variables.}$$

U will be unbiased estimator of p if the coefficient are so chosen such that

$a\psi=k$ where $k=[k_1 \, k_2]$

$Var(U)=Var(aT^*+bG^*+cS^*)$

or

var $(U) = \sigma^2 a\Omega a$, where $a=[a \, b \, c]$ $\qquad (2.2)$

and

var $(U)=\sigma^2 k[\psi'\Omega^{-1}\psi]^{-1} k$,

If $k = [1; 0]$ we get μ_2^* as an estimate of μ and similarly if $k = [0; 1]$ we get σ_2^* as an estimate of σ. We may sometimes be interested in other values of k_1 and k_2.

$$\psi' = \begin{bmatrix} \frac{n^{(1)}}{1} & \frac{n^{(2)}}{2} & \frac{n^{(3)}}{3} \\ \frac{n^{(1)}}{1}\alpha_1^{(1)} & \frac{n^{(2)}}{2}\alpha_2^{(2)} & \frac{n^{(3)}}{3}\alpha_3^{(3)} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} var(T^*) & Cov(T^*,G^*) & Cov(T^*,S^*) \\ Cov(G^*,T^*) & Var(G^*) & Cov(G^*,S^*) \\ Cov(S^*,T^*) & Cov(S^*,G^*) & var(S^*) \end{bmatrix} = \begin{bmatrix} \Omega_{00} & \Omega_{01} & \Omega_{02} \\ \Omega_{10} & \Omega_{11} & \Omega_{12} \\ \Omega_{20} & \Omega_{21} & \Omega_{22} \end{bmatrix}$$

where

$$\Omega_{ij} = \Omega_{ji} = \frac{(i+1)(j+1)}{n^{(i+1)}} \sum_{s=0}^{i} b_{ij}^{(s)} (n-j-i)^{(s)} \; i \le j \qquad (2.3)$$

$$b_{ij}^{(s)} = \frac{i!j!W_{s+j+1,s+j+1}^{(s+j+1)}}{(i-s)!(s+j-i)!s!(s+j+1)} + \frac{i!j!(j+1)^{(i+1-s)}\sum_{r=0}^{s-1}(-1)^r w_{j+1,j+2}^{(j+2+r)}}{(i+1-s)!(j+2+r)!(s-1-r)!}$$

$$- \frac{i!j!(i+1)^{(i+1-s)}\sum_{r=0}^{i}(-1)^r w_{s+j-i,s+j-i+1}^{(s+j-i+1+r)}}{(i+1-s)!(j-r)!(s+j-i+1+r)!} + \frac{i!j!\alpha_{i+1}^{(i+1)}[\alpha_{s+j-i}^{(s+j-i)}-\alpha_{j+1}^{(j+1)}]}{(i+1-s)!(s+j-i)!s!} \qquad (2.4)$$

2.1 Estimation of parameter by Quadratic coefficient method for Normal distribution using order statistics

Let μ^* and σ^* be the estimate of location and scale parameter respectively of Normal distribution for Quadratic Estimate method. For the evaluation of the coefficients of μ^* and σ^* we have

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} \frac{\theta_0}{n} & \frac{2\theta_1}{n^{(2)}} & \frac{3\theta_2}{n^{(3)}} \\ \frac{\Phi_0}{n} & \frac{2\Phi_1}{n^{(2)}} & \frac{3\Phi_2}{n^{(3)}} \end{bmatrix} \qquad (2.1.1)$$

where $[\theta' : \Phi'] = \Omega^{-1} \begin{bmatrix} 1 & \alpha_1^{(1)} \\ 1 & \alpha_2^{(2)} \\ 1 & \alpha_3^{(3)} \end{bmatrix} \left[\begin{bmatrix} 1 & 1 & 1 \\ \alpha_1^{(1)} & \alpha_2^{(2)} & \alpha_3^{(3)} \end{bmatrix} \Omega^{-1} \begin{bmatrix} 1 & \alpha_1^{(1)} \\ 1 & \alpha_2^{(2)} \\ 1 & \alpha_3^{(3)} \end{bmatrix} \right]^{-1}$

Hence the coefficient of μ^* and σ^* in Quadratic Coefficient Estimate (QCE) are given as:

$a_{11} = a_1; a_{12} = a_1 + a_2; a_{13} = a_1 + 2a_2 + 2a_3, \ldots, a_{1n} = a_1 + (n-1)a_2 + (n-1)(n-2)a_3$

$b_{11} = b_1; b_{12} = b_1 + b_2; b_{13} = b_1 + 2b_2 + 2b_3, \ldots, b_{1n} = b_1 + (n-1)b_2 + (n-1)(n-2)b_3$

Table 2.1.1
The Elements of Ω in Quadratic Coefficient Estimate (QCE) from the Normal Distribution based on order statistics for $5 \le n \le 10$

n=5	0.2	0.2	0.2
	0.2	0.242582	0.263873
	0.2	0.263873	0.303060
n=6	0.166667	0.166667	0.166667
	0.166667	0.200480	0.217386
	0.166667	0.217386	0.247834
n=7	0.142857	0.142857	0.142857
	0.142857	0.170884	0.184898
	0.142857	0.184898	0.209804
n=8	0.125	0.125	0.125
	0.125	0.148927	0.16089
	0.125	0.16089	0.181963
n=9	0.111111	0.111111	0.111111
	0.111111	0.142417	0.148948
	0.111111	0.173087	0.190486
n=10	0.1	0.1	0.1
	0.1	0.118504	0.127757
	0.1	0.127757	0.154795

Table 2.1.1
Coefficients of Quadratic Coefficient Estimate (QCE) μ* for the location
parameter of Normal distribution using order statistics for 5≤n≤8

n	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{10}
5	0.2	0.2	0.2	0.2	0.2					
6	0.16667	0.16667	0.16667	0.16667	0.16667	0.16667				
7	0.142857	0.142857	0.142857	0.142857	0.142857	0.142857	0.142857			
8	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125		
9	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111
10	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 2.1.2
Coefficients of Quadratic Coefficient Estimate (QCE) σ* for scale parameter
of Normal distribution using order statistics for 5≤n≤10.

n	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}	b_{17}	b_{18}	
5	-0.354491	-0.177246	-0.000001	0.177244	0.35449				
6	-0.295390	-0.177250	-0.059098	0.059042	0.177242	0.295430			
7	0.253226	-0.168805	-0.084286	0.000035	0.084446	0.168859	0.253270		
8	-0.221539	-0.158252	-0.094959	-0.031672	0.031645	0.094956	0.158273	0.221596	
9	-0.196979	-0.147714	-0.098457	-0.049208	0.000033	0.049266	0.098491	0.147708	0.196917
10	-0.177285	-0.137871	-0.098463	-0.059061	-0.019665	0.019725	0.059109	0.098487	0.137859

Table 2.1.3
Variance covariance of μ* and σ* for the Normal distribution using
order statistics for 5≤n≤10. Each value should be multiplied by σ²

	Var(μ*)	Var(σ*)
n=5	0.2	0.133775
n=6	0.166667	0.106228
n=7	0.142857	0.088049
n=8	0.125	0.075169
n=9	0.11111	0.065565
n=10	0.1	0.058132

3. CUBIC COEFFICIENT METHOD

A particular linear estimate with four terms approximation called Cubic Coefficient Estimate (CCE) using ordered observations is derived by using general theory of linear coefficients with polynomial coefficients given by Downton(1966). This estimate is applicable for complete samples and is shown to yield highly efficient estimators even in the case of small samples.

We extend the above mentioned methods for the Cubic Coefficient Estimate (CCE) as Consider the following linear combination of three statistics

$$t=\sum_{i=1}^{n} (i-1)^{(0)} x_{(i)}^{(n)}$$

$$g=\sum_{i=1}^{n} (i-1)^{(1)} x_{(i)}^{(n)}$$

$$s = \sum_{i=1}^{n} (i-1)^{(2)} x_{(i)}^{(n)} , \quad e = \sum_{i=1}^{n} (i-1)^{(3)} x_{(i)}^{(n)}$$

where t,g and e,s are the realization of the random variables T , G and E,S. We have to obtain an estimating function for p as u=at+bg+cs where a, b and c are to be determined such that u is unbiased for p and it has minimum variance among all such u's.

So, E(U)=aE(T)+bE(G)+cE(S)+dE(E)

$$= \left(an+\frac{bn^{(2)}}{2}+\frac{cn^{(2)}}{3}+\frac{dn^{(2)}}{4}\right)\mu+ \left(an\alpha_1^{(1)}+\frac{bn^{(2)}\alpha_2^{(2)}}{2}+\frac{cn^{(3)}\alpha_3^{(3)}}{3}+\frac{dn^{(4)}\alpha_4^{(4)}}{4}\right)\sigma \qquad (3.1)$$

We define

$$T^*=\sum_{i=1}^{n} (i-1)^{(0)} Y_{(i)}^{(n)}$$

$$G^* = \sum_{i=1}^{n} (i-1)^{(1)} Y_{(i)}^{(n)}$$

$$S^*=\sum_{i=1}^{n} (i-1)^{(2)} Y_{(i)}^{(n)} , \quad E^* = \sum_{i=1}^{n} (i-1)^{(3)} x_{(i)}^{(n)}$$

as new random variables.

U will be unbiased estimator of p if the coefficient are so chosen such that

$$a\psi=k \text{ where } k=[k_1 \ k_2]$$

$$Var(U)=Var(aT^*+bG^*+cS^*)$$

or

$$var (U)=\sigma^2 a\Omega a^{,} \text{ where } a=[a \ b \ c] \qquad (3.2)$$

and $$var (U)= \sigma^2 k[\psi'\Omega^{-1}\psi]^{-1} k^{,}$$

If k = [1; 0] we get μ_2^* as an estimate of μ and similarly if k = [0; 1] we get σ_2^* as an estimate of σ. We may sometimes be interested in other values of k_1 and k_2.

$$\Psi'= \begin{bmatrix} \frac{n^{(1)}}{1} & \frac{n^{(2)}}{2} & \frac{n^{(3)}}{3} & \frac{n^{(4)}}{4} \\ \frac{n^{(1)}}{1}\alpha_1^{(1)} & \frac{n^{(2)}}{2}\alpha_2^{(2)} & \frac{n^{(3)}}{3}\alpha_3^{(3)} & \frac{n^{(4)}}{4}\alpha_4^{(4)} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} var(T^*) & Cov(T^*,G^*) & Cov(T^*,S^*) & Cov(T^*,E^*) \\ Cov(G^*,T^*) & Var(G^*) & Cov(G^*,S^*) & Cov(G^*,E^*) \\ Cov(S^*,T^*) & Cov(S^*,G^*) & var(S^*) & Cov(S^*,E^*) \\ Cov(E^*,T^*) & Cov(E^*,G^*) & Cov(S^*,E^*) & var(E^*) \end{bmatrix} = \begin{bmatrix} \Omega_{00} & \Omega_{01} & \Omega_{02} & \Omega_{03} \\ \Omega_{10} & \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{20} & \Omega_{21} & \Omega_{22} & \Omega_{23} \\ \Omega_{30} & \Omega_{31} & \Omega_{32} & \Omega_{33} \end{bmatrix}$$

where

$$\Omega_{ij} = \Omega_{ji} = \frac{(i+1)(j+1)}{n^{(i+1)}} \sum_{s=0}^{i} b_{ij}^{(s)} (n-j-i)^{(s)} \ i \le j \qquad (3.3)$$

$$b_{ij}^{(s)} = \frac{i!j!W_{s+j+1,s+j+1}^{(s+j+1)}}{(i-s)!(s+j-i)!s!(s+j+1)} + \frac{i!j!(j+1)^{(i+1-s)}\sum_{r=0}^{s-1}(-1)^r w_{j+1,j+2}^{(j+2+r)}}{(i+1-s)!(j+2+r)!(s-1-r)!}$$

$$- \frac{i!j!(i+1)^{(i+1-s)}\sum_{r=0}^{i}(-1)^r w_{s+j-i,s+j-i+1}^{(s+j-i+1+r)}}{(i+1-s)!(j-r)!(s+j-i+1+r)!} + \frac{i!j!\alpha_{i+1}^{(i+1)}[\alpha_{s+j-i}^{(s+j-i)} - \alpha_{j+1}^{(j+1)}]}{(i+1-s)!(s+j-i)!s!} \qquad (3.4)$$

3.1 Estimation of parameter by Cubic coefficient method for Normal distribution using order statistics

Let μ^* and σ^* be the estimate of location and scale parameter respectively of Normal distribution for Cubic Estimate method. For the estimation of the coefficients of μ^* and σ^* we have

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \end{bmatrix} = \begin{bmatrix} \frac{\theta_0}{n} & \frac{2\theta_1}{n^{(2)}} & \frac{3\theta_2}{n^{(3)}} & \frac{4\theta_2}{n^{(4)}} \\ \frac{\Phi_0}{n} & \frac{2\Phi_1}{n^{(2)}} & \frac{3\Phi_2}{n^{(3)}} & \frac{4\Phi_2}{n^{(4)}} \end{bmatrix}$$

where $[\theta' : \Phi'] = \Omega^{-1} \begin{bmatrix} 1 & \alpha_1^{(1)} \\ 1 & \alpha_2^{(2)} \\ 1 & \alpha_3^{(3)} \\ 1 & \alpha_4^{(4)} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ \alpha_1^{(1)} & \alpha_2^{(2)} & \alpha_3^{(3)} & \alpha_4^{(4)} \end{bmatrix} \Omega^{-1} \begin{bmatrix} 1 & \alpha_1^{(1)} \\ 1 & \alpha_2^{(2)} \\ 1 & \alpha_3^{(3)} \\ 1 & \alpha_4^{(4)} \end{bmatrix}^{-1}$

Hence the coefficient of μ^* and σ^* in Cubic coefficient Estimate (CCE) are given as:

$a_{11} = a_1;\ a_{12} = a_1 + a_2;\ a_{13} = a_1 + 2a_2 + 2a_3, \ldots, a_{1n} = a_1 + (n-1)a_2 + (n-1)(n-2)a_3$

$b_{11} = b_1;\ b_{12} = b_1 + b_2;\ b_{13} = b_1 + 2b_2 + 2b_3, \ldots, b_{1n} = b_1 + (n-1)b_2 + (n-1)(n-2)b_3$

Table 3.1.1
The Elements of Ω in Cubic Coefficient Estimate (CCE) from the Normal Distribution using order statistics for $7 \leq n \leq 9$

n=7	0.142857	0.142857	0.142857	0.142857
	0.142857	0.170884	0.184898	0.193667
	0.142857	0.184898	0.209804	0.226843
	0.42857	0.193667	0.226843	0.251120
n=8	0.125	0.125	0.125	0.125
	0.125	0.148927	0.16089	0.168377
	0.125	0.16089	0.181963	0.196321
	0.125	0.168377	0.196321	0.216589
n=9	0.111111	0.111111	0.111111	0.111111
	0.111111	0.131981	0.142417	0.148948
	0.111111	0.142417	0.160679	0.173087
	0.111111	0.148948	0.173087	0.190486

Table 3.1.2
Coefficients of Cubic method μ* for the location parameter of
Normal distribution using order statistics for 7≤n≤9

n	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{110}
7	0.142857	0.142857	0.142857	0.142857	0.142857	0.142857	0.142857			
8	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125		
9	0.11111	0.11111	0.11111	0.11111	0.11111	0.11111	0.11111	0.11111	0.11111	

Table 3.1.3
Coefficients of Cubic method σ* for scale parameter of
Normal distribution based on order statistics for 7≤n≤9

n	b_{11}	b_{12}	b_{13}	b_{14}	b_{15}	b_{16}	b_{17}	b_{18}	b_{19}
7	-0.276958	-0.139560	-0.056244	0.000026	0.056286	0.139572	0.276920		
8	-0.245802	-0.136052	-0.065834	-0.019332	0.019270	0.065788	0.136038	0.245836	
9	-0.222112	-0.130851	-0.070188	-0.029935	-0.000096	0.030093	0.070244	0.130737	0.221760

Table 3.1.4
Variance covariance of μ* and σ* for the Normal distribution based on
order statistics for 7≤n≤9 .Each value should be multiplied by σ^2

	Var(μ*)	Var(σ*)
n=7	0.142857	0.087501
n=8	0.125	0.074625
n=9	0.111111	0.065012

4. RELATIVE EFFICIENCY

Efficiency is defined as the ratio of variances of the estimator of BLUEs to variances of the estimator to approximate methods for all choices of sample size up to order 8. These efficiency are presented in table which shows that we get high efficiency in all cases so we can replace the approximate methods with mostly used BLUE method which can't be used for large sample and when the exact variance matrix is not known.

Table 4.1
Relative Efficiency of Blue to Approximate Methods for Location Parameter

n	var(μ•) BLUE	var(μ•) Linear Coefficient Method	Efficiency %	var(μ•) Quadratic Coefficient Method	Efficiency %	var(μ•) Cubic Coefficient Method	Efficiency %
3	0.333276	0.3333					
4	0.25	0.25	100				
5	0.1999994	0.2	99.99	0.2	99.99		
6	0.1666665	0.166667	99.99	0.166667	99.99		
7	0.142857	0.142857	100	0.142857	100	0.142857	100
8	0.1250502	0.125	100	0.125	100	0.125	100

Table 4.1
Relative Efficiency of Blue to Approximate Methods for Scale Parameter

N	$\mathrm{var}(\sigma^\bullet)$ BLUE	$\mathrm{var}(\sigma^\bullet)$ Linear Coefficient Method	Efficiency %	$\mathrm{var}(\sigma^\bullet)$ Quadratic Coefficient Method	Efficiency %	$\mathrm{var}(\sigma^\bullet)$ Cubic Coefficient Method	Efficiency %
3	0.275512	0.275483	100				
4	0.180050	0.180349	99.83				
5	0.133322	0.133775	99.66	0.133775	99.66		
6	0.105690	0.106228	99.49	0.106228	99.49		
7	0.087012	0.088049	99.82	0.088049	99.82	0.087501	99.44
8	0.074748	0.075169	99.43	0.075169	99.43	0.074625	100

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