ISBN 978-969-8858-20-9

Vol. 31

PROCEEDINGS

15th International Conference on Statistical Sciences:

Theme:
Advances in Statistics, Economic Growth, Challenges and Development

March 9-11, 2017

at

Shah Abdul Latif University Khairpur

Khairpur Mir's, Sindh (Pakistan)

JOINTLY ORGANIZED BY

Shah Abdul Latif University Khairpur
(DEPARTMENT OF STATISTICS)
Khairpur 66020, Mir's, Sindh, Pakistan
URL: http://salu.edu.pk

Islamic Countries Society of Statistical Sciences
44-A, Civic Centre, Sabzazar, Multan Road, Lahore (Pakistan)
URL: http://www.isoss.net

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Editor

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 1-4

USING CLUSTER ANALYSIS TO GROUP DIVISIONS OF THE PUNJAB PROVINCE ACCORDING TO THEIR HEALTH SERVICES IN 2014

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ABSTRACT

The heath services in Punjab province were not classified in terms of clusters of recommendation domains. The aim of this paper is to make classification of health services provided to divisions of Punjab province. Cluster analysis through hierarchical method was used as statistical method for clustering health services in form of recommendation domains throughout the whole Punjab province, in order to assists in formulating sound and appropriate health plans for each cluster (domain). Real data from the Punjab development Statistics of 2015 are used, and the variables considered are 29 variables. The key result is that, according to Health services the divsions of the Punjab province are grouped into four similar clusters by one of SPSS solutions from 2 to 4, first cluster includes Bahawalpur, Faisalabad, Rawalpindi, Sahiwal and Sargodha division, followed by D.G. Khan and Multan, Gujranwala construct the third cluster and the last cluster is Lahore division. The paper recommends that, method of clustering of domains must be applied to the lower levels of the divisions.

KEYWORDS

Cluster analysis; Hierarchical method; Divisions; Recommendation domains.

1. INTRODUCTION

In this paper, the focus is on nine divisions of the Punjab province namely; namely Bahawalpur, D.G. Khan, Faisalabad, Gujranwala, Lahore, Multan, Rawalpindi, Sahiwal and Sargodha division. This paper wishes to pinpoint to the use of cluster analysis in recognizing the characteristics of individual territorial divisions and in grouping these divisions into homogenous groups. The main hypothesis of the study is that, using the methods of cluster analysis singles out those divisions that represent a more suitable subject matter of regional policy instruments and measures (Ivan, 2005). The highlighting of regional variation of the distribution of various phenomena is the most important proper planning requirements and has a direct impact in depicting the comprehensive development and regional development policies (Mozamel, 2015).

2. LITERATURE REVIEW

2.1 Cluster Analysis (CA)

Cluster analysis is the art of finding groups in data. Basically, one wants to form groups in such way that objects in the same group are similar to each other, whereas objects in different groups are as dissimilar as possible (Kaufma. Rousseeuw, 1967). The aim of cluster analysis is to cluster a given set of data or objects into cluster (subsets, groups, classes). This clustering should have the following properties: Homogeneity within the cluster and heterogeneity between clusters (Höppner et al. 1999).

2.2 Hierarchical Clustering Technique

It is one of the traditional methods of cluster analysis. In (HCT) the data is not divided into cluster in a single step, we need instead to the successive stages, it is divided into Agglomerative Methods (AM) and Divisive Methods (DM). The (AM) begins with a series of successive mergers of the n units, which turn into groups, while (DM) divides **n** units sequentially to precise divisions. In the group average method, which will be used in this paper and it is one of (AM), the distance between each two clusters is defined as un average distance between each pair of items (Mozamel, 2016).

3. METHOD AND MATERIALS

Cluster analysis through hierarchical method and agglomerative method through the average linkage (between groups) were used to study the variables. The data were taken from Punjab government statistics of 2015, 29 variables were analyzed by SPSS program with solution from two to four clusters.

Table 1
Cluster Membership

Case	4 Clusters	3 Clusters	2 Clusters
1. Bahawalpur	1	1	1
2. D.G. Khan	2	1	1
3. Faisalabad	1	1	1
4. Gujranwala	3	2	1
5. Lahore	4	3	2
6. Multan	2	1	1
7. Rawalpindi	1	1	1
8. Sahiwal	1	1	1
9. Sargodha	1	1	1

Source: The Researcher from SPSS

The range of solution in SPSS is determined from 2 to 4, so, the results distribution is shown in the three columns in table 4. In case of clustering to two groups, the first group contains all divisions except Lahore, which comprises the second group. In clustering to three groups, the first group includes seven divisions, the second group covers Gujranwala and the third group includes only Lahore. In case of clustering to four groups, the first group involves five divisions, the second group contains D.G. Khan and Multan, the third group comprises Gujranwala, the last group involves only Lahore.

4. DISCUSSION

From table 1 the first solution shows that Lahore is the best in health services with reference to the other divisions. In the second solution all divisions are homogeneous, the best are Lahore and Gujranwala respectively. In third solution Bahawalpur, Faisalabad, Rawalpindi, Sahiwal and Sargodha division construct the first cluster, which weaker with regard to health services, followed by D.G, khan and Multan, Gujranwala comes third and Lahore is the optimum.

5. RECOMMENDATIONS

1. Updating of Punjab province and divisions Database to allow a greater number of variables for the researchers.
2. Classification of domains using method of cluster analysis ought to be applied in the different lower levels of the Punjab government.
3. Promotion of health services in the weaker divisions namely Gujranwala, Faisalabad, Rawalpindi, Sahiwal and Sargodha.

6. CONCLUSION

By implementing cluster analysis the nine divisions are grouped into three styles of health services using solution from 2 – 4 in SPSS program. In the first style the divisions are grouped in to two clusters, in the second style they are grouped in to three clusters and in the third style they are grouped into four clusters.

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EXAMINING FOR GRANGER CAUSALITY BETWEEN CLIMATE VARIABLES OF DIFFERENT CITIES OF PAKISTAN

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ABSTRACT

Now a days change in climate is an interesting factor in the world. The change in temperature has disturbed climate all over the world. This ultimately effects all other activities. This study focused on causal relationship of precipitation (R_f) with other parameters i.e. minimum temperature (T_{min}), maximum temperature $\left(T_{max}\right)$, relative humidity $\left(R_h\right)$, wind speed $\left(W_s\right)$, and atmospheric pressure $\left(A_p\right)$ for the mean monthly data between 1990 and 2015 for four cities: Karachi, Lahore, Peshawar and Quetta. Stationarity as a prerequisite for causal relationship of parameters were checked through Phillips-Perron test. Correlation between precipitations with other parameters is checked and finally Granger causality test is applied for the required causal relationship between precipitations with other parameters. The result shows that all the parameters are stationary at level and there exists causal relationship between precipitation and other variables, which is an indication of dependent amongst the variables i-e precipitation is the function of the other variables.

KEYWORDS

Climate variables, correlation, Stationarity, PP test and Granger causality test.

INTRODUCTION

Climate may be defined as general weather condition which occurs in the air above the earth, like precipitation, temperature, wind speed, relative humidity and atmospheric pressure etc. The change in temperature has disturbed climate all over the world. This climate change, ultimately effects all other activities. This study analyzes and measures the causal relationship of the precipitation with other parameters i.e. minimum temperature, maximum temperature, relative humidity, wind speed and atmospheric pressure for four cities: Karachi, Lahore, Peshawar and Quetta. The dependence of precipitation on temperature has been discussed a lot by many researchers Habiyaaremye Gabriel 2010, Buishand and Brandsma 1999, Wilby and Wigely 1997, but argue that temperature alone does not give a sufficient effect on the precipitation. Various climatic variables have relationship between them, this relationship may be observed through correlation and Granger causality test. In correlation change in the relationship between two variables or their interdependence is measured. Here we discuss about the change of

one variable causing the change in other variable. In time series data, if event A occurs before event B then we say that A is causing B. These ideas can be generated through the Granger causality test, *X* (Granger causes) *Y* if past value of *X* can help to explain *Y*. Stationarity as a prerequisite for causal relationship of parameters were checked through Phillips-Perron test.

DATA AND METHODOLOGY

Data Presentation: Mean monthly data of different metrological parameters like precipitation, minimum & maximum temperature, and wind speed, relative humidity and atmospheric pressure for 26 years (1990-2015) for four cities: Karachi, Lahore, Peshawar and Quetta were obtained from Computerized Data Processing Centre (CDPC) of Pakistan Metrological department (PMD) Karachi.

Table 1
Descriptive Statistics of the Different Parameters for four Cities

Parameters	KARACHI				LAHORE			
	Minimum	Maximum	Mean	St.Dev.	Minimum	Maximum	Mean	St.Dev.
R_f	0.00	270.40	17.01	38.34	0.00	640.00	55.64	85.57
$T_{\min}$	9.00	29.80	21.45	5.90	5.60	29.20	19.15	7.33
$T_{\max}$	24.40	37.70	32.39	3.15	15.20	41.80	30.47	6.83
R_h	24.00	78.00	48.02	14.07	17.00	76.00	45.97	13.19
W_s	1.50	14.10	7.70	2.57	0.10	5.20	2.34	1.16
A_p	996.40	1018.00	1007.30	6.03	999.30	1018.10	1005.94	7.67
	PESHAWAR				QUETTA			
R_f	0.00	490.00	46.07	51.95	0.00	167.60	20.47	32.74
$T_{\min}$	2.50	28.00	16.49	8.11	-5.60	23.10	8.91	8.26
$T_{\max}$	15.50	42.70	29.55	7.42	6.70	37.90	25.27	8.92
R_h	20.00	69.70	46.02	10.12	8.00	72.00	30.82	12.48
W_s	0.00	15.30	6.09	3.44	1.80	21.40	13.20	3.34
A_p	968.50	1018.80	1005.71	8.85	1364.00	1590.20	1471.22	46.78

Metrological variables and their Notations:

R_f :	Precipitation	(mm)	_k = Karachi
$T_{\min}$:	Minimum Temperature	(^{0}C)	_L = Lahore
$T_{\max}$:	Maximum Temperature	(^{0}C)	_P = Peshawar
R_h :	Relative humidity	(at 1200 UTC %)	_Q = Quetta
W_s :	Wind speed	(at 1200 UTC, knots)	
A_p :	Atmospheric pressure	(at 1200 UTC, hpa/gpm)	

METHODOLOGY

i) Correlation:

A useful measure of joint or mutual variation in two variables (covariance = σ_{xy}), the covariance must be standardized so that it is unit free pure number, correlation may be denoted by *r*. correlation is a measure of closeness of linear relationship between two variables, Karl Pearson's coefficient of correlation for any two parameters *x* and *y* is

$$r = \frac{covariance\ of\ x\ and\ y}{(standard\ deviation\ of\ x).(standard\ deviation\ of\ y)}$$

or

$$r = \frac{Sxy}{Sx.Sy}.$$

Table 2
Showing the Correlation between Precipitation with other Variables for four Cities

Parameters	Karachi	Lahore	Peshawar	Quetta
	Correlation	Correlation	Correlation	Correlation
R_f & $T_{\min}$	0.251	0.422	0.034	-0.351
R_f & $T_{\max}$	0.008	0.239	-0.104	-0.510
R_f & R_h	0.055	0.063	0.154	-0.217
R_f & W_s	0.454	0.516	0.309	0.674
R_f & A_p	-0.356	-0.464	-0.003	0.291

ii) Stationarity Test:

The important part of time series work is stationarity, the series y_t is stationary if its first and second moment are time invariant: $E(y_t) = \mu, Var(y_t) = \sigma_0^2$ and $Cov(y_t - y_{t-j}) = \sigma_j^2$. The stationarity of variables can be checked by Phillip Perron test.

iii) Phillip Perron Test:

As first differences of most macroeconomic time series variables are serially correlated, the PP-test does not requires to specify the serial correlation of the first difference under the null, PP-test also does not requires that the $\varepsilon's$ are conditionally homoscedastic. The problem of specifying p, AR-order are avoided in PP-test.

The Phillips and Perron (Biometrika, 1988) suggested non parametric transformations of τ statistics from the original DF regressions.

Let y_t has the AR(p) form

$$y_t = a_1 y_{t-1} + - - - + a_\rho y_{t-\rho} + \varepsilon_t$$

This model can be written as

$$y_t = py_{t-1} + \delta_1 \Delta y_{t-1} - - - + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t$$

Here

$\rho = a_1 + \cdots + a_\rho$
$\delta_1 = -(a_2 + \cdots + a_\rho)$
$\delta_2 = -(a_3 + \cdots + a_\rho)$
... ...
$\delta_\rho = -a_p$

The unit root null hypothesis will be $H_0 : p = 0$

Let our model is

$y_t = py_{t-1} + \varepsilon_t$ $\varepsilon_t \sim I(0)$ with mean zero

PP-process is
Regress y_t on y_{t-1},
compute,
Modify τ to get Z
Under null hypothesis, Z's asymptotic distribution is the DF distribution for τ.

Table 3
Showing Stationarity at level I(0) through Phillip Perron Test

Variable	Karachi		Lahore		Peshawar		Quetta	
	Adj.t-stat	Prob.*	Adj.t-stat	Prob.*	Adj.t-stat	Prob.*	Adj.t-stat	Prob.*
R_f	-13.71	0.00	-11.50	0.00	-3.05	0.03	-12.68	0.08
T_{min}	-2.99	0.04	-2.69	0.08	-2.75	0.07	-5.97	0.00
T_{max}	-8.73	0.00	-3.74	0.00	-3.17	0.02	-3.06	0.03
R_h	-3.89	0.00	-8.07	0.00	-9.02	0.00	-6.93	0.00
W_s	-5.00	0.00	-6.25	0.00	-4.52	0.00	-8.46	0.00
A_p	-3.16	0.02	-3.05	0.03	-3.66	0.01	-4.30	0.00

iii) Granger Causality Test:

Causality test is defined as to measure the ability of one variable to predict (and therefore cause) the other. If two variables y_t and x_t, affect each other by the distributed lags then their relationship can be measured through VAR model. The possible four cases of relationship may be, y_t causes x_t, x_t causes. y_t bi directional relationship and two variables are independent. Granger (1969) defined causality test as, a variable y_t is said to be Granger cause x_t, if x_t can be predicted with greater accuracy by using past values of the y_t rather than not using such past values, all other terms remaining unchanged.

The Granger causality test for two stationary variables y_t and x_t is

$$X_t = a_1 + \sum_{i=1}^{n}(\alpha_i y_{t-i}) + \sum_{j=1}^{m}(\beta_j x_{t-j}) + \mu_{1t} \ldots\ldots\ldots\ldots 1$$

$$y_t = a_2 + \sum_{i=1}^{n}(\lambda_i y_{t-i}) + \sum_{j=1}^{m}(\delta_j x_{t-j}) + \mu_{2t} \ldots\ldots\ldots\ldots 2$$

Granger causality mean the lagged of one variable influence to other variable significantly.

Table 4
Showing the Correlation of Precipitations and Wind Speed City Wise

Parameter	Karachi	Lahore	Peshawar	Quetta
Rf_K	**Rf_K**	**Rf_L**	**Rf_P**	**Rf_Q**
	1.000	0.385	0.242	0.039
Rf_P	**Rf_K**	**Rf_L**	**Rf_P**	**Rf_Q**
	0.242	0.288	1.000	0.293
Ws_K	**Ws_K**	**Ws_L**	**WS_P**	**Ws_Q**
	1.000	0.545	0.785	0.594
Ws_P	**Ws_K**	**Ws_L**	**WS_P**	**Ws_Q**
	0.785	0.785	1.000	0.621

Table 5
Pairwise Granger causality results

Karachi	Lag-1			Lag-2			Lahore	Lag-1			Lag-2		
H_0: does not Granger cause	**Obs.**	**F-Stat**	**Prob.**	**Obs.**	**F-Stat**	**Prob.**	**H_0: does not Granger cause**	**Obs.**	**F-Stat**	**Prob.**	**Obs.**	**F-Stat**	**Prob.**
T_{min} does not Granger cause R_f	311	21.708	5.E-06	310	12.441	6.E-06	T_{min} does not Granger cause R_f	311	35.570	7.E-09	310	24.857	1.E-10
R_f does not Granger cause T_{min}		0.895	0.345		1.735	0.178	R_f does not Granger cause T_{min}		0.902	0.343		29.466	2.E-12
T_{max} does not Granger cause R_f	311	7.620	0.006	310	12.065	9.E-06	T_{max} does not Granger cause R_f	311	30.349	8.E-08	310	23.122	4.E-10
R_f does not Granger cause T_{max}		1.041	0.309		9.803	7.E-05	R_f does not Granger cause T_{max}		3.685	0.056		43.834	2.E-17
R_h does not Granger cause R_f	311	31.169	5.E-08	310	16.775	1.E-07	R_h does not Granger cause R_f	311	4.958	0.026	310	40.197	3.E-16
R_f does not Granger cause R_h		3.821	0.512		4.374	0.013	R_f does not Granger cause R_h		3.471	0.063		4.928	0.008
W_s does not Granger cause R_f	311	8.796	0.003	310	5.530	0.004	W_s does not Granger cause R_f	311	4.864	0.028	310	6.836	0.001
R_f does not Granger cause W_s		5.927	0.016		8.624	0.000	R_f does not Granger cause W_s		6.054	0.014		7.843	0.001
A_p does not Granger cause R_f	311	46.999	4.E-11	310	24.629	1.E-10	A_p does not Granger cause R_f	311	75.719	2.E-16	310	44.352	1.E-17
R_f does not Granger cause A_p		18.648	2.E-05		2.610	0.075	R_f does not Granger cause A_p		15.913	8.E-05		2.952	0.054
Peshawar	**Lag-1**			**Lag-2**			**Quetta**	**Lag-1**			**Lag-2**		
H_0: does not Granger cause	**Obs.**	**F-Stat**	**Prob.**	**Obs.**	**F-Stat**	**Prob.**	**H_0: does not Granger cause**	**Obs.**	**F-Stat**	**Prob.**	**Obs.**	**F-Stat**	**Prob.**
T_{min} does not Granger cause R_f	311	1.699	0.193	310	7.158	0.000	T_{min} does not Granger cause R_f	311	58.565	3.E-13	310	29.684	2.E-12
R_f does not Granger cause T_{min}		13.549	0.000		1.277	0.280	R_f does not Granger cause T_{min}		12.648	0.000		6.760	0.001
T_{max} does not Granger cause R_f	311	2.621	0.107	310	4.167	0.016	T_{max} does not Granger cause R_f	311	66.198	1.E-14	310	32.438	2.E-13
R_f does not Granger cause T_{max}		43.455	2.E-10		41.591	1.E-16	R_f does not Granger cause T_{max}		29.246	1.E-07		11.368	2.E-05
R_h does not Granger cause R_f	311	0.481	0.489	310	0.683	0.506	R_h does not Granger cause R_f	311	25.958	6.E-07	310	12.504	6.E-06
R_f does not Granger cause R_h		14.329	0.000		12.619	5.E-06	R_f does not Granger cause R_h		8.102	0.005		5.966	0.003
W_s does not Granger cause R_f	311	3.183	0.075	310	5.469	0.005	W_s does not Granger cause R_f	311	27.862	2.E-07	310	21.701	2.E-09
R_f does not Granger cause W_s		3.779	0.053		2.151	0.118	R_f does not Granger cause W_s		14.749	0.000		19.662	9.E-09
A_p does not Granger cause R_f	311	0.262	0.609	310	3.344	0.037	A_p does not Granger cause R_f	311	31.779	4.E-08	310	20.189	6.E-09
R_f does not Granger cause A_p		10.543	0.001		5.136	0.006	R_f does not Granger cause A_p		0.495	0.482		6.929	0.001

RESULTS

The mean monthly data of metrological parameters, i-e minimum temperature, maximum temperature, relative humidity, wind speed and atmospheric pressure for four cities: Karachi, Lahore, Peshawar and Quetta has been analysed through correlation, stationarity, Granger causality test. Correlation and bi directional causality has been checked between precipitation and other independent variables. The parameter precipitation has smaller the mean and large standard deviation (table-1) for four cities: Karachi, Lahore, Peshawar and Quetta is due to the nonappearance of precipitation for long period (zero entries). All the parameters are stationary at level or I(0) checked by Phillip Perron test (table-3). The correlation value is small for four cities but it shows that there is correlation (table-2). Further results are as: Correlation of precipitation with other variables for cities Karachi, Lahore and Peshawar is positive except atmospheric pressure for Karachi, Lahore and Peshawar and maximum temperature of Peshawar which has negative correlation. But correlation of precipitation with other parameters for Quetta is negative except wind speed and atmospheric pressure which has positive correlation. Correlation of precipitation with wind speed for all cities is higher which indicates that high wind speed provide more precipitation. The objective was to check that whether the parameters of one city correlate with the parameter of other city, the finding show that only wind speed of one city correlated with another city, where as other parameters do not correlate (table-4). There exists a causal relationship between precipitation with other parameters (table-5).

CONCLUSIONS

Weather data of different parameters: minimum temperature, maximum temperature, wind speed, relative humidity and atmospheric pressure for four cities: Karachi, Lahore, Peshawar and Quetta were analysed through correlation, stationarity and Granger causality test. Due to long period unavailability of precipitation at selected cities, precipitation has smaller the mean and high standard deviation. Minimum values of correlation indicates the presence of relationship. All the variables are stationary at level for four cities. From this analysis it is concluded that the climate of selected cities is totally different, parameters are independent however wind speed has powerful correlation which indicates that high the wind speed more will be precipitation. There subsists a causal relationship between the precipitation and other parameters for all cities which is an indication of dependent amongst the variables i-e precipitation is the function of the other variables.

POLICY IMPLICATION

From this analysis it is concluded that precipitation has disappeared a long time of period, thus to remove the shortage of water, create a small dams in the selected cities of Pakistan that can be utilized at hard time. It is need of today to maintain the weather of the country, because high rise in temperature has disturbed the weather of the world as well as Pakistan, thus it is required to maintain the optimal limit of temperature for which afforestation must be employed to get the required results in the selected area under study, on the other side it is difficult to sustain this condition

AKNOWLEDGMENT

The author is very thankful to Mr. Nadeem Faisal, Deputy Director for Director CDPC, Pakistan Metrological department Karachi, for his value able advices and providing metrological data.

The authors are also grateful to Mr. Saghir Pervaiz Ghauri, Research Department, State Bank of Pakistan, Karachi for his value able suggestions and co-operation.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 13-20

EXPLORING THE PRIMORDIAL DIMENSIONS OF STATISTICS (IN THE LIGHT OF QURAN AND SUNNAH)

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ABSTRACT

The primordial dimensions of Statistics encompass the narration of an authentic Hadith of the Prophet (saw) and all the Quranic narratives. After the completion of the Quran's revelation it is just the development stages of the numerals of the numbers. It is evident that among all the numbers mentioned in the Quran, the most outstanding number is One (1) and the second most outstanding number is Zero (0). This reality is elaborated in detail with several verses of the Quran. Mathematicians of different eras in different civilizations tried to comprehend the significance of these two numbers and their numerals. In the current paper, it is discussed that why these two number have been considered of special importance throughout the history of using the numbers and numerals. Even in the recent times the advancements of computer technology are based upon the development of software binary languages using just these two numbers. This fact also indicates the Statistical Science is not only primordial among all sciences but is also inherently involved in every branch of knowledge. These two numbers are instinctively installed in the conscience of all human beings making them superior to non-human creatures.

INTRODUCTION

Primordial means anything existing or developing at its very earliest stages. Some statisticians have defined it as an irreducible, indivisible and persistent position of any subject or situation. Thus, the primordial statistics may be defined as the knowledge of statistics at the earliest stages of its history.

Dr. Absar Ahmad[1] of the Quran Academy, Lahore views the Quranic concept of Life and Death to be of primordial nature.

Dr. John Wheeler[2], a well-known Physicist has termed some persistent and irreducible random distributions of quantum mechanics phenomena to be primordial. By this definition the Quranic verses are all absolutely primordial. The Scientific verses distributed in the Quran are not random but primordial and their positions cannot be changed in any way.

Following is an authentic Hadith[3] of the Prophet (saw) narrated in 'Sahih-Al-Bukhari':

كان الله ولم يَكُنْ شَىءٌ غَيْرُهُ

Allah (swt) was there while nothing else existed.

In this Hadith, the word Allah (الله) is representative of the Unique Number One (1) and the phrase (لم يَكُنْ شَيءٌ غَيْرُهُ) is representative of nothingness, emptiness or Zero (0).

The Hadith indicates that Allah existed eternally without any beginning and at one time there was nothing else other than Allah (swt). The Hadith also negates the theory of eternal universe or similar ideas proposed by some scientists[4].

The Most Outstanding Narrative of the Quran

The Quranic revelations predominantly emphasize on recognizing and having absolutely true belief in the Oneness (توحيد) of Allah (swt). In this context, it would not be wrong to say that the Quranic narrative stated in the Sura Al-Ikhlas (Chapter-112) on the Oneness of Allah (swt) is the most outstanding narrative. The very first verse of this Sura includes the most important number of primordial statistics as to be number One (1).

قُل هُوَالله اَحَد

Say Allah is One (Al-Ikhlas-112:1)

The Arabic word (اَحَد) means a Unique Number One (1).

Another verse expresses the same reality as follows:

ٱللَّهُ لَآ إِلَـهَ إِلَّا هُوَ

Allah, There is No God but He.

The Second Most Outstanding Narrative of the Quran

The second most outstanding primordial number is indicated by the following narrative of the Quran.

فَٱعْلَمْ أَنَّهُۥ لَآ إِلَـهَ إِلَّا ٱللَّهُ

So know (O! Muhammad) No deity worthy to be worshipped exists except Allah.
(Muhammad-47:19)

In this verse the word (لَا) means 'nothing' or Zero (0). By bringing the word (لَا) before (اِلَّا اللّٰهُ) the Oneness (توحيد) of Allah (swt) has been raised to the highest level of being Absolute beyond any doubt. This narrative not only leads to the second number of primordial statistics as to be Zero (0) but also provides relationship between the numbers One (1) and Zero (0). It shows that these two numbers are inseparable parts of the same reality.

The above verses indicate the following:

1. Statistics is the science that is primordial at absolute position and is of Divine nature.
2. Number One (1) and number Zero (0) are the most outstanding primordial numbers and are of Divine nature. The number One (1) is absolutely Unique and

is independent of Zero (0), while the existence of Zero (0) depends Upon number One (1). Since human beings are created out of nothingness or Zero (0) their existence depends upon Allah (swt) while He is independent of everything and this is exactly what is meant by the verses:

قُلۡ هُوَ ٱللَّهُ أَحَدٌ (١) ٱللَّهُ ٱلصَّمَدُ (٢)

Say (O Muhammad) Allah is One, All creatures need Him (and) He does not need any of His creatures. (112:1-2).

The Significance of numbers One (1) and Zero (2)

These two numbers existed even before the creation of universe. This fact indicates that the Statistical Science is not only primordial among all sciences but is also inherently involved in every branch of knowledge.

These two numbers are instinctively installed in the conscience of all human beings making them superior to non-human creatures. The significance of these two numbers in various branches of mathematics and in the development of binary software languages also illustrates them to be of unique nature.

Allah (swt) encourages the entire humanity to ponder over the Quranic narratives:

وَلَقَدۡ ضَرَبۡنَا لِلنَّاسِ فِى هَـٰذَا ٱلۡقُرۡءَانِ مِن كُلِّ مَثَلٍ لَّعَلَّهُمۡ يَتَذَكَّرُونَ

And we have certainly presented for the people in this Qur'an from all [kinds of] examples - that they might ponder. (Al-Zummar-39:27)

The point to be noted in the above verse is that Allah is inspiring all the human beings in general (not just the believers) by using the Arabic phrase: لِلنَّاسِ (for human beings).

Pondering over the Quranic narratives does the following:

- It triggers the inspirational power.
- It enhances instinctive, intuitional and intellectual power.
- It may provide clues for innovative dimensions in your research.
- This primordial dimensions are worth exploring and rewarding.
- Exploring and adding a primordial dimension in your research may provide potential to improve your research qualitatively as well as quantitatively.

Based on research in this area I have written a book titled '*The Quranic Perspective of Primordial Statistics*' which will be published by the end of March, 2017, Insha Allah.

NUMBERS, NUMERALS & DIGITS

Number: A number is a just a count or measurement that is perceived as an idea in our minds. We can show our idea of number with our fingers or by some other technique.

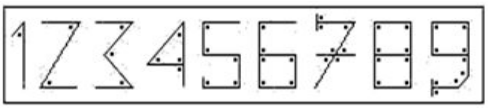
Numeral: When a number is written in the form of an established symbol it is then called a numeral. Thus "4" is a numeral of number "four".

Role of Al-Khwarizmi

Muhammad Ibn Mūsā al-Khwārizmī (780 – 850 CE) was a Persian mathematician at the House of Wisdom in Baghdad who designed the basic 10 numerals (0 to 9) based on geometrical angles. The glyph of Zero had no angle, so its shape was designed as a small

circle. He stated in his book that if no number appears in the place of tens in a calculation, a little circle should be used "to keep the rows". This circle was later called ‘ṣifr’ or Zero.

The following are the original glyphs of the numerals designed by Al-Khwarizmi. Each angle is identified by a dot.



These numbers were later modified in Maghreb (western) part of the Arab until they reached the present forms in which we use them now.

Reality of the Number One (1)

The word (اَحد) in the first verse of Sura Ikhlas has different connotative meaning than the word (اَحد) in the fourth verse. The (اَحد) in the first versere presents the Absolute One Who is (ٱلۡحَیُّ ٱلۡقَیُّومُ) while the word (اَحد) in the fourth versere presents the mortal (اَحد) who’s existence depends upon the Absolute One (1)of the first verse as elaborated in the following verses:

يَسۡـَٔلُهُۥ مَن فِی ٱلسَّمَـٰوَ ٰتِ وَٱلۡأَرۡضِ كُلَّ يَوۡمٍ هُوَ فِی شَأۡنٍ (٢٩)

Whoever is within the heavens and earth asks Him (for his needs); every day He is in state of dignity and glory. (Al-Rehman-55:29)

كُلُّ شَیۡءٍ هَالِكٌ إِلَّا وَجۡهَهُۥ

Everything will be destroyed except His Face. (Al-Qasas-28:88)

The Absolute (اَحد), the Absolute (صَمَد) and the Absolute (كُفُوًا) is the Creator of everything including the ordinary number Ones (1) like human beings. How the Absolute Number One (1) and the created number ones (1) can be translated into statistical concepts is not known and requires the attention of statisticians to explain this reality.

Realities of Life and Death

Dr. Absar Ahmad[1] pointed out that the Arabic word (الحیات = life) may be considered as illustrative of number One (1) and the Arabic word (ٱلموت = death) as illustrative of the number Zero (0).Consider the following verse:

ٱلَّذِی خَلَقَ ٱلۡمَوۡتَ وَٱلۡحَیَوٰةَ

It is (He) who created death and life. (Al-Mulk-67:2)

This verse shows that death and life are the realities created by Allah (swt). However, the sequence shows that the death was created before the life.

Now look at the following verse:

كَیۡفَ تَكۡفُرُونَ بِٱللَّهِ وَكُنتُمۡ أَمۡوَ ٰتًا فَأَحۡیَـٰكُمۡ ثُمَّ یُمِیتُكُمۡ ثُمَّ یُحۡیِیكُمۡ ثُمَّ إِلَیۡهِ تُرۡجَعُونَ (٢٨)

How can you disbelieve in Allah when you were lifeless and He brought you to life; then He will cause you to die, then He will bring you [back] to life, and then to Him you will be returned. (Al-Baqra-2:28)

This verse strongly suggests that the death (ٱلْمَوْتَ) realm was created before the life (ٱلْحَيَوٰةَ) realm. The human beings remained in the death realm before they are granted life per the will of Allah (swt). The creatures to be created at this stage are in the 'intention realm' of Allah (swt). Whenever Allah (swt) wants to make them live He says the word 'Be' ('كُن' in Arabic) the creature moves to the 'Realm of Amr'. And the word 'فَيَكُونُ' indicates that the order is responded by entering the 'Realm of Khalq' and the creation process has begun.

The creation of numbers and numerals also went through the same sequence.

إِنَّمَآ أَمْرُهُۥٓ إِذَآ أَرَادَ شَيْـًٔا أَن يَقُولَ لَهُۥ كُن فَيَكُونُ (٨٢)

Verily, His Command, when He intends a thing, is only that He says to it, "Be!" and it is! (YaSeen-36:82)

Further Explanation of Absolute Primordial Numbers One (احد) and zero (لا).

As explained above the number 'One (1) and the number Zero (0) are absolutely primordial, eternal and are not derived from any other position. In this sense it would not be wrong to say that these two numbers are of Divine nature. Their primordial position also suggests that the reality is only in the number 'One (1) and in comparison the number Zero (0) is of very little importance.

It is not clear that what kind of numerals were being used at the time of Quran's revelation. However the numbers that are currently being used worldwide are basically Arabic numbers[6]. They were actually developed by Arabic Muslim scientists who revised the Indian version of numbers that contained only nine numbers. That took place during the 8th century (771 A.D) when an Indian Gastronomist came to the Almansour royal palace with a book – famous at that time – about astronomy and mathematics alled "Sod hanta" written by Brahma Gupta around 626 A.D. Almansour ordered to translate the book into Arabic and to explore further.

These early counting systems only saw the zero as a placeholder—not a number with its own unique value or properties. A full grasp of zero's importance would not arrive until the seventh century A.D. in India. There, the mathematician Brahmagupta and others used small dots under numbers to show a zero placeholder, but they also viewed the zero as having a null value, called "sunya." Brahmagupta was also the first to show that subtracting a number from itself results in zero. From India, the zero made its way to China and back to the Middle East, where it was taken up by the mathematician Mohammed Ibn-Musa al-Khowarizmi around 773. It was al-Khowarizmi who first synthesized Indian arithmetic and showed how the zero could function in algebraic equations, and by the ninth century the zero had entered the Arabic numeral system in a form resembling the oval shape we use today.

The zero continued to migrate for another few centuries before finally reaching Europe sometime around the 1100s. Thinkers like the Italian mathematician Fibonacci helped introduce Zero (0) to the mainstream, and it later figured prominently in the work of Rene Descartes along with Sir Isaac Newton and Gottfried Leibniz's invention of calculus. Since then, the concept of "nothing" has continued to play a role in the development of everything from physics and economics to engineering and computing.

Since Arab merchants used to trade with Indian merchants, the primordial numbers one (1) and Zero (0) came to be known by Indian intellectuals. They independently or along with Arabic intellectuals worked to further explain the concept of Zero (0).

Prophet Muhammad (saw) was born on 570 in Mecca, and demised on June 8, 632 in Medina. The earliest evidence of the use of Zero (0) appeared around 7th century. This period happens to be about the same that followed the life of Prophet Muhammad (saw). As a matter of fact long before the Islamic era, mathematicians of several countries have been trying to understand the true concept of Zero (0) However, after the Quranic revelation the Muslim mathematicians and traders accelerated this process and exchanged their views with the mathematicians of the neighboring countries like India, Malaysia and Indonesia. It seems that Indian mathematicians of that period like Brahmagupta also increased their efforts to understand the nature of Zero (0) and other numerals. Thus it would be doubtful to claim about Indian mathematicians to be the pioneers in developing the concept of Zero (0) as a number and making king advancements to develop the numerals. It was the combined effort of Muslim and Indian mathematicians with the difference that the Indian mathematicians documented their ideas in writing and Muslim mathematicians either did not document or their documents were destroyed somehow.

Another interesting point to be noted is that the first letter of both the words اللهand اَحَدresemble the written shape of number One (1) in all the numbering systems.

The word Allah itself is the most primordial among all the names of Gods. The Arabs were quite familiar with the name Allah prior to the revelation of the Quran. It is very unique since there is no plural for Allah nor is there any female gender for Allah. In English language we have plural of god as gods and female gender of god as goddess but there is no such thing for Allah (swt) in Arabic language.

Significance of the Numbers One (1) and Zero (0)

In the software languages all the numbers (including the decimal numbers too) and alphabetic letters of any spoken language have been represented with various combinations of Zeros (0) and Ones (1). The languages based on just two numbers Zero and One are called binary languages. The various systems employed in such languages are called the binary systems. In electricity the ‘light off’ is the representation of number Zero (0) and the ‘light on’ represents the number one. In electronics ‘connection off’ means number Zero (0) and ‘connection on’ means number One (1).

The Primordial Concept of Infinity

The primordial infinity may be defined as anything the dimensions of which are beyond the scope of human imagination. The Quranic narrative expressed in the following verse shows how in finite the space is occupied by Allah’s throne.

وَسِعَ كُرۡسِيُّهُ ٱلسَّمَٰوَٰتِ وَٱلۡأَرۡضَ

His (Allah’s) throne is as (infinitely) vast as the heavens and the earth.

(Al-Baqra-2:255)

Quranic Endorsement of Primordial Randomness

The absolute primordial randomness of the scientific verses in the Quran provides certain endorsement to the views of the physicist John A. Wheeler who found in his study the randomness of certain phenomena of quantum physics to be persistent and named them as of 'Irreducible or Primordial' randomness with certain objective.

Thus a primordial randomness must be persistent or eternal and serving a certain purpose.

Primordial Timeline Sequence

Persistent timeline sequence independent of statistical probability may also be found in certain Quranic narratives. An example of such a primordial timeline is the sequential development of ears, eyes and heart of human fetus. The narratives on this timeline sequence have been identified in my earlier paper[7].

Florence Nightingale born (May 1820) in Italy OM, RRC was an English social reformer and statistician, and the founder of modern nursing. Regarding the study of Statistics she wrote the following[8]:

> "To understand God's thoughts one must study Statistics, the measure of His purpose. The true foundation of theology is to ascertain the character of God. It is by the aid of Statistics that law in the social sphere can be ascertained and codified, and certain aspects of the character of God thereby revealed. The study of statistics is thus a religious service".

CONCLUSIONS

1. The earliest appearance of the primordial numbers One (1) and Zero (0) in the historical documents is found in the period subsequent to the revelation of the Quran.
2. The early Muslim scholars may have derived these primordial numbers from certain Quranic narratives.
3. Starting with these two primordial numbers, the other numbers and numerals could have been developed subsequently.
4. Recently the random distribution of certain phenomena of Quantum physics is termed primordial because of their persistent and irreducible nature with certain objective. This is in line with the random distribution of scientific narratives placed in the Quran with certain purpose, are eternal and non-changeable.
5. Further search work is needed in this field.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 21-30

DESCRIPTIVE ANALYSIS OF CHILD LABOR IN MINGORA DISTRICT SWAT

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ABSTRACT

Objective: The aim of this study is to check the frequency of child labor with respect to their ages, family income, father occupation, and father income at Mingora city district Swat.

Methods: The data were collected from 100 children working in 73 different shops in different areas of Mingora city by using cross sectional survey. The selections of shops were done through a systematic sampling procedure. At the starting location, a shop was randomly selected by the researcher, followed by systematic selection of every third shop. The 100 children were interviewed through questionnaire by arranging a scheduled interview. The data was then analyzed by SPSS 16 for frequency distribution. Charts were made through Microsoft excel 2013.

Results: Among 100 interviewed children, 22% were in the age of 8 12 years and 78% were found in the age group of (13 17). Out of 100 working children only 23% used to go to school. The family income 16% child laborers were less than 10,000. 61% child laborers' family incomes were 10,000 to 20,000. The 12% child laborers were found having 20,000 to 30,000 family incomes. The family income of 9% was 30,000 40,000. The 2% child laborers' family income was above 50,000.

Conclusion: Results indicate that poverty is the main cause of child labor in the area. It is also observed that most of the child laborers tend to father's occupation which means that father's occupation is responsible for child labor. The parents of child labors are mostly illiterate, which can cause child labor in the society.

1. INTRODUCTION

A child is considering as human wealth in every family. Any saving made for educating and train a child is mostly taken as the first top choice because the long runs taking on this investment are high. But this investment does have a longer growth period. The other situation is one in which the families cannot provide to set a limit for the development era and may not be capable to accept the cost either. They want the child to provide maintain the family and so the child is determined to work. This leads to gone opportunity not just for the child and family but moreover for the people. This is the reason that education and basic health is the reliability of the state. Several states fail to convey these services and therefore suffer from poor human resource. There is a body of

literature that focus on the financial determinant of child labor, but very little studies focus on point out the public and traditional aspect of the problem (Khan & Ejaz 2001).

Every child is born with right and slowly grows and develops with public responsibilities. The public and civilizing tradition and ethics reveal that today's children are tomorrow's good nation in political logic and good parents in public sense. The fact is if these traditions and ethics care children properly, educate them then they can develop their selves qualified and in reasonable ways. But literatures on child enlargement and others confirm that child labor is a power full and necessary public problem in all upward and less than developing countries as well as undersize areas of the modern countries (Bibi, S. 1980).

In the examination of determinants of child labor, the deliver elevation of children can be explain in term of lack of earning of the related household (Basu, 1999). Poverty is the major cause of provider of child labor (Hyder, 1998).

The definition of the tricks that represent labor and the meaning of a child itself are contentious issues. Analysis of Pakistan's national plan on child labor and recent lawmaking events on child labor reveal that Pakistan follow international principles by defining a man of age 14 and below as a child labor. While the meaning of a child appears clear, the description of work does not. Pakistani law states that no child under 14 should be busy in any plant or mine or other harmful employment or in any job or employment which prejudice his or her physical condition education or interferes with his or her physical, mental or good development. Given the elasticity of this definition, it is not blow that size of the level of child labor in Pakistan span an ample range of estimate.

Difficulty with measuring the occurrence of child labor is exacerbating by data limits. The sample and the Labor Force Survey (LFS) in Pakistan do not together labor information persons below the age of 10. A variety of international agencies and limited sources have attempt to fill this record gap, provide estimate of between 6 and 10 million child employees in the country(a labor force involvement rate of 15 20%). A survey shown that in Pakistan 3.3 million out of 40 million 5 14 year old children were cheaply active (Ali, 2000).

All children below 15 years of age who are reasonably active excluding (i) those who are below 5 years old and (ii) those among 12 14 years old who use less than 14 hours a week on their jobs, unless their behavior or occupations are harmful by nature or circumstances (Bhalotra, S. 2007).

2. LITERATURE REVIEW

In this chapter we deal with literature review the topic, issues relating child labors, economic activities, less working age and these entire concepts are used and child labor definition will be discussed. In the literature review we have to identify the others work which has been already done in the child labor area.

According to Weiner, M. (1991), without making education obligatory in a right way is impossible to remove child labor in a country. According to O`Donnell et al. (2003), most of the children working conditions which are very harmful to their health which can damaged permanently by working in mine fields, contraction site in manufactures etc

which is very harmful to their health. If such policies are bought forward which can ban child labor, it will defiantly reduce or even finish the child labor market but are the families prepare for that? The reason the families would not be prepared for this is because the families come from a poor background and they need their children to slave away just to bring that extra bit of cash to support their families.

According to Bibi (1980), most of child labors are influenced by their parents because they live a very poor and harsh lifestyle and they need their children to earn that little bit of money. Child labor ranges from the ages less than 15 years of age in which there are more female workers than the male workers. These children work up to 70 90 hours a week and they do not get paid enough to even support their families.

According to Jillian (2000), the number of child labor is very high in rural areas then in urban areas and most of these are less than 8 years of age. UNICEF stated that even though they work in very hard and hazardous conditions, that even for them to be around pollution or unhealthy environments affect their health and these children spent countless hours in these environments. It was stated that by the human rights commission that the majority of these children are forced by their parents to work so that they can earn money so the families can benefit from it. In such cases the parents actually beat their children so that they do not quit their jobs just so that the families can pay for the basic needs in life that is food, cloths etc.

Sadaqi and Patrinos (2002), conducted a study that child labor is a common problem worldwide, mainly in developing countries which cover 90% of totals child labors like in Africa and Asia together. Child labor is dominant mostly in rural areas where schooling and age limitation for work is deficient. Poverty and schooling contributing a child to work being poor child work for their family to help in income in lack of quality education or access to it motivate their parents to work instead for going to school which increase child labor. Child labors are often miss use in many terms like long hours of working in pay less. Work in severe condition which cause health problem. To avoid such thing state has to implement laws strictly and make it possible for child to attend the schools.

According to Osment (2014), poverty is the main cause of child labor this problem is been facing by the worlds now a days. Child labor is very harmful for child although child cannot help their family in such a way. They cannot pay for their work nor do they help in their family income properly. Most of the family started engaging their child in work to contribute financial support but their health problems often refused. The author took example of two countries India and Nigeria that how they took steps to reduced child employment. That cooperation among different community help to reduced child labor.

According to Sheeba, P. (2008), concluded that the state of child labor is a multi dimensional composite problem and spread widely in a socio economic framework of the society. So selecting single approach to child labor is impossible. There are many components which are creditworthy to this phenomenon, so a broad sense approach is required to handle the problem of child labor. This can be done by start the awareness programmers the negative effect of child providing quality education and support all the concerned that are working to stop this problem with a complete honest and effort and strong commitment.

According to study of Christenson & Juarez, F. (1987), that most a child labor are engaged in working area from 9 to 12 hour a day. That nearly 70% of child labor put to work by their parents to work in earn for them.

Rahatullah (2001) describe socio economic causes of child labor in swat. He took 100 respondents as samples and disturbed it randomly into four categories workshop brick kilns, hotels and self employments. He mostly found that respondents families was often large size and their parents income levels was not that much to admit their child to school so they start their children to wok and earn for them. His study recommended that a campaign should start to aware people of negative effect of child labor and state should take a keen action against child labor to implant law forcefully to stop child labor and made possible for them quality education incentives to their parents. Family planning should be follow. State should provide job opportunities to overcome the causes of child labor.

2.1 Aims and Objectives

The main objective of this research study is to check the frequency of child labor with respect to their ages, family income, and father occupation at Mingora city district Swat.

2.2 Methodology

This study was conducted in Mingora city, district Swat. The data was collected through a questionnaire from shops in the city arranging scheduled interview from different children working at different shops of Mingora city.

The selections of shops were done through a systematic sampling procedure. At the starting location, a shop was randomly selected by the researcher, followed by systematic selection of every third shop. 73 shops were surveyed and included in the research study. From 73 shops 100 children were interviewed and were include in the study. The researcher ask different question about their age, nature of work, educational background, family income and father occupation.

Descriptive statistics was used in the analysis. The researcher tabulates the data using frequency distribution and construct charts through Microsoft excel.

3. RESULTS

Table 3.1
Frequency Distribution of Child Labor at Mingora City District Swat

Child information		Frequency	Percent
Age	08 12	22	22%
	13 17	78	78%
Family Income	<10,000	16	16%
	10,000 20,000	61	61%
	20,000 30,000	12	12%
	30,000 40,000	9	9%
	<50,000	2	2%

School Going	Yes	23	23%
	No	77	77%
Father Occupation	Daily wages	26	26%
	Driver	17	17%
	Former	7	7%
	Government servant	1	1%
	Hotel	2	2%
	Mason	7	7%
	No work	23	23%
	Shop	12	12%
	Tailor	2	2%
	Workshop	3	3%
Father Income	No income	27	27%
	<10,000	48	48%
	10,000 20,000	22	22%
	20,000 30,000	1	1%
	30,000 40,000	1	1%
	>50,000	1	1%

Table 3.1 shows age distribution of the child laborers. The ages of child laborers are divided into two groups 8 12 and 13 17 respectively. The family and the father incomes are also given in the table. The table also shows the occupation of child laborers' fathers. The percentages of all the information are given in the table.

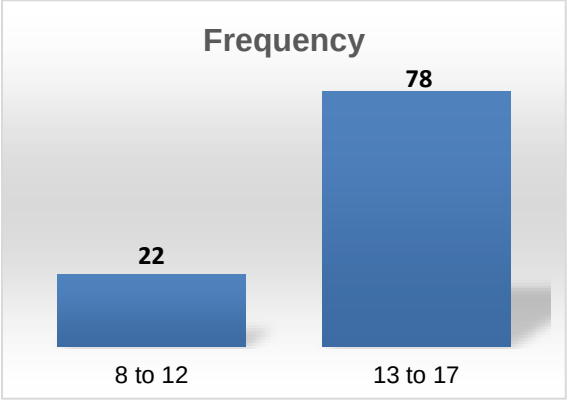

Figure 3.1 Age Distribution of Children

The graph figure 3.1 has two categories that are 8 12 and 13 17 on x axis. The first value which is 13 17 show that 78% of children are in the age ranges between 13 17 years do a regular work on daily wages, and the second group show that 22% of children doing work at the age among 8 12 years.

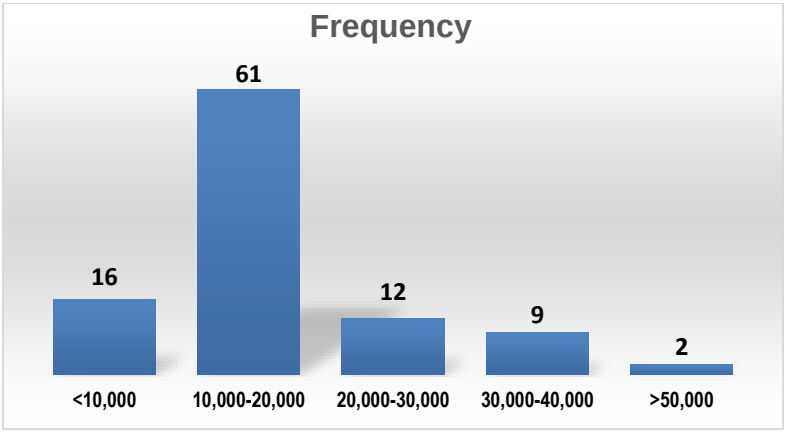

Figure 3.2: Family Income of Children Working in Different Shops of Mingora City

The graph figure 3.2 shows us variation among the family income. the 61% of the family income in the graph is 10 20 thousand ,which is a biggest value in the graph, 16 % of the family income ranges from less than 10 thousands , 12% of family income ranges from 20 30 thousands, 9% of family income ranges from 30 40 and the rest of 2% family income is more than 40 thousand.

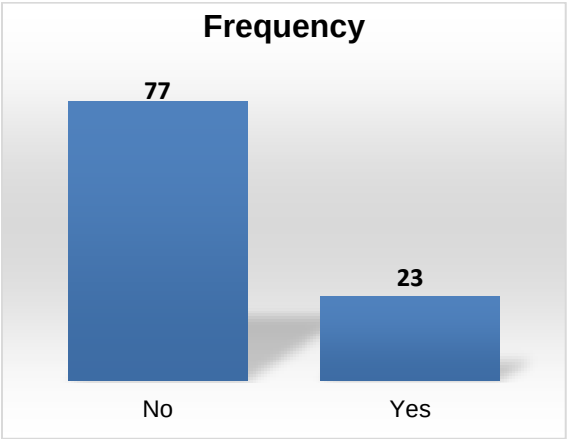

Figure 3.3: Frequency of Children Going to School Working in Various Shops of Mingora City

The graph figure 3.3 shows us that how much of children go to school and how many of them do not. In the graph 77% of children does not go to school and the rest of 23 % used to go school.

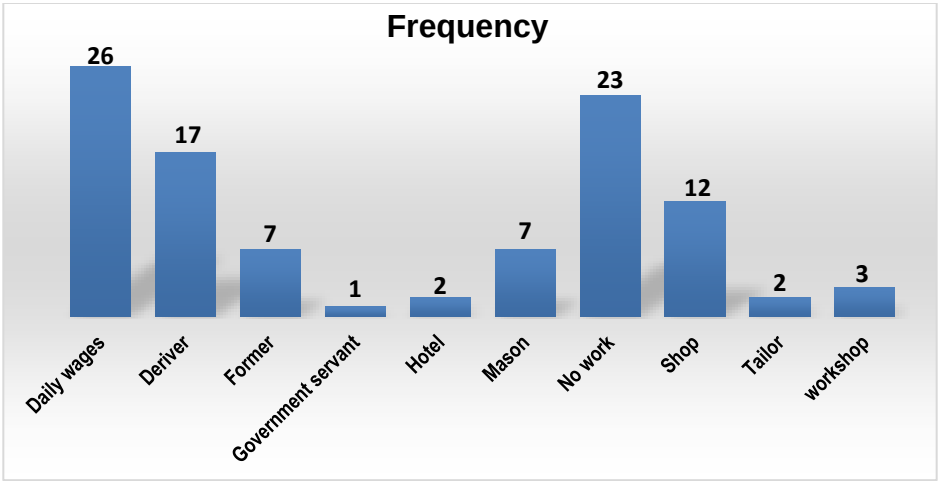

Figure 3.4: Father Occupation of Children Working in Different Shops at Mingora City

The graph figure 3.4 shows us father occupation. In the graph 26% of them work on daily wages, 23 % of them have no work, 17% of them work as a driver, 7% of them are farmer, 12% of them work in shop, 7% of them are masons, 2% of them work in a hotel, 3% of them work in workshop, 2% of them are tailors and 1% of them is a government servant.

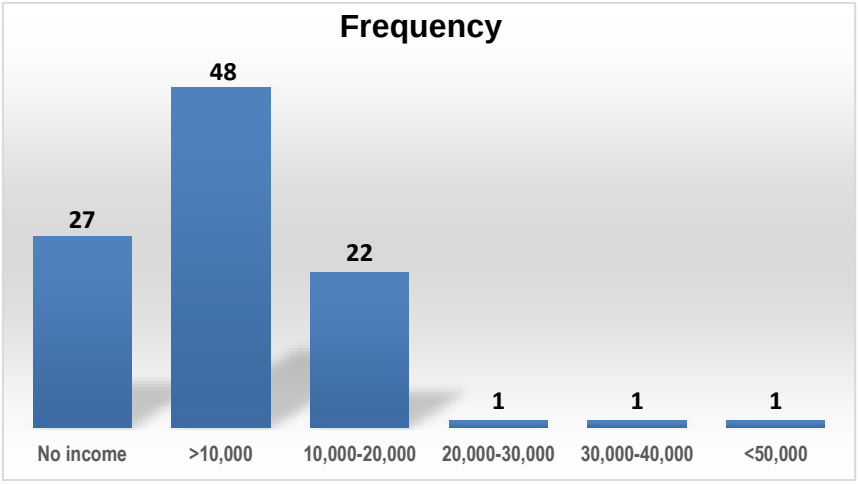

Figure 3.5: Father Income of Children Working in Different Shops of Mingora City

The graph figure 3.5 shows us father income of a child labor .27% of them have no income, 48% they have less than 10 thousand of income, 22% of them have 10 20 thousand of income, 1 % of them have 20 30 thousand income, another 1% of them have 30 40 thousand of income and the rest of 1% percent have more than 50 thousand of income.

4. DISCUSSION

In this study the data used is primary data which is collected through a proper questionnaire (see Appendix for Questionnaire). The data were collected from children working in different shops in different areas of Mingora city. At first, the shops were randomly selected. Then 73 shops were selected systematically i.e. every third shop. A total of 100 children were investigated working in these 73 shops. Among these 100 children, 22 (22%) children were in the age of 8 12 years and 78 (78%) children were found in the age of 13 17 years. The 77 children (77%) were limited only to work while 23 (23%) child laborers were going to school. The family incomes of 16 (16%) child laborers were less than 10,000. 61 (61%) child laborers' family income was 10,000 to 20,000. The 12 (12%) child laborers were found having 20,000 to 30,000 family incomes. The family income of 9 (9%) were 30,000 to 40,000. The 2 (2%) child laborers' family income was above 50,000. The 27 (27%)of child laborers revealed that their fathers earn no money. The father's income of 48 (48%) child laborers was less than 10,000. There were 22 (22%) of child laborers whose father's income was in the range of 10,000 to 20,000. There was 1 (1%) each having father income of 20,000 to 30,000, 30,000 to 40,000 and below 50,000 respectively.

This study supports many of the findings of previous studies in which child labor is responsible for low income of family, low literacy level and huge family size.

4.1 Major findings

Poverty is the root cause of child labor. But besides that there are several reasons which are responsible for the child labor.

i. Children are working in every field of life as child laborers.
ii. 78 percent of child laborers are in the age of 13 17 years.
iii. 61 percent of child labors family income is in between 10 to 20 thousand.
iv. 77 percent of child labors are not going to school.
v. 26 percent of child labors family income is based on daily wages.
vi. 27 percent of child laborers have no father income.
vii. 48 percent child labors father income is less than 10,000.
viii. Majority of child labor lives in nuclear family.
ix. Most of the children`s works in workshops.

4.2 Conclusion

Results indicate that poverty is among the main cause of child labor in the area. It is also observed that most of the child laborers tend to father's occupation which means that father's occupation is responsible for child labor. The parents of child laborers are mostly illiterate, which is a cause of child labor.

4.3 Limitation of the Study

The study was limited to only one city. Limited information is available to interpret the issue correctly. This study used small amount of data for the analysis.

4.4 Future Work

We have taken an initiative from Mingora city district Swat and hope to extend the study to the whole district Swat to investigate the main causes of child labor and to obtain maximum information on this issue and to explore more information about child labor.

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APPENDIX

FACTORS CONTRIBUTING CHILD LABOR
A Cross Sectional Survey in Mingora City District Swat.

Please answer each question to the best of your ability. Thank you in advance for your participation

PART-1
(Personal Information)

Name _____ **Father Name** _____
Religion _____ **Date of birth** _____
Gender: _____ **Area** _____
Your Number in Family _____ **Family Income** _____

What is the nature of your work? _____
Are you going to school? Yes ( ) No ( )
Duration of your work? _____
Nature of your family? _____
Your family spent most of the money on? _____

PART-2
Below is a list of common factors that can cause child labor .please read each question carefully & write your best answer.

What is the occupation of your father? _____
What is your father income? _____
Is there a trend of child marriages in your family? _____
How many brothers & sisters do you have ? _____
What is education of your parents ? _____
What is the behavior of your parents with you ? _____
Are you engaged? _____
Are your parent battering you? _____
Are any members of family beating you? If yes tell their relation with you? _____
What is your mother doing at home? _____
Do your other brother or sister also involve in child labor? _____
What is the basic reason behind your labor? _____
Have ever your parents told you to quit this labor & start going to school? _____
Are your parents happily sending you for this work? _____
Are you happy with your work? _____
What you want to be in future? _____

Thank You!

Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 31-34

STUDY OF SUNSPOTS CYCLES USING SINUSOIDAL MODELING

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ABSTRACT

The study of sunspots and its cycles has a uniquely important in the study of space weather and their impact on the earth's climate. The research in this paper includes the sunspots time series monthly data from 1960 to 2002. The sunspot cycles together used in the study and sinusoidal model is fitted to analyze the sunspot behavior in all cycles. The oscillation behavior of sunspots in consecutive cycles is examined in this research paper. This study is not useful to understand the peak of the cycle. The correlation coefficient (0.867) for sinusoidal model found significant in the data used. This study can be beneficial to understand the long term sunspots behavior.

1. INTRODUCTION

Sun is the main source of energy for earth and other different stars. Therefore the effects of different activities of the sun have been focused on the view of the earth climate variability in different eras. The effects of sunspot on earth's climate is earlier verified by Hassan *et al*., 2014b. The average sunspot cycle length is 11.1 years. Sunspot cycles include 9 years minimum and 14 years maximum duration (Hassan *et al*., 2014b). The oscillation behavior of sunspots have been observed by its dynamics. The space weather depends on the solar activities. Sunspots is the main solar activity on which other activities associated. Significant modeling of sunspot cyclic time series data is important to understand the expected space weather variation in future (Hassan *et al*., 2014a; Mariska and Oster, 1972).

2. MATERIAL AND METHOD

The research includes the sunspots time series monthly data from 1960 to 2002. The sinusoidal model is fitted to the time series data.

2.1 Sinusoidal Modeling

Sinusoidal models are important and useful for modeling any periodic phenomena that are repeating events, like the motion of waves of ocean, seasonal fluctuations in sales, or sunspot time series data (Anderson *et al*., 1990).

The representation of the sinusoidal model in algebraic form is described by eq. (1)

$$y = a + b \times \cos(cx + d) \quad (1)$$

The parameters are effectual for the model in a different manner (Stahn, 2011). The parameter ***a*** is known as the amplitude. The larger value of ***a*** is representing the larger difference between the troughs and peaks in the graph. The parameter, ***b*** affecting the frequency in graph. The larger value of ***b*** is showing the faster graph goes up and down. The parameter ***c*** of model is associated with the phase of motion. The changing value of ***c*** moves the graph right and left. Finally, the ***d*** parameter is called the bias. The average of ***y***-value is represented by ***d*** in the data. The entire graph will shift upwards by increasing ***d*** (Stahn, 2011; Anderson *et al*., 1990).

The model sinusoidal differs from the quadratic model and linear model in the sense that the parameters are not themselves linear. This makes it so that it is much harder to find the values of the parameters that minimize the sum squared error (Anderson *et al*., 1990).

3. RESULT AND DISCUSSION

The research work done in this paper regarding sunspots time series data show that the sinusoidal is best fitted. The correlation coefficient 0.867 for the fitted model represents that the significant condition. The oscillation of sunspots time series data is basically examined through sinusoidal modeling. The result indicates the sinusoidal fit defines sunspot data more than 86 % that is strongest one. Through the obtained sinusoidal model, long term behavior of sunspots time series data can be easily observed. The parameter of estimated for sunspots time series is mentioned in Table 1.

Table 1

A	B	C	D
70.68807	60.9927	0.594713	3.78149

Fig. 1 indicates the fitting of the sinusoidal model on sunspots time series data.

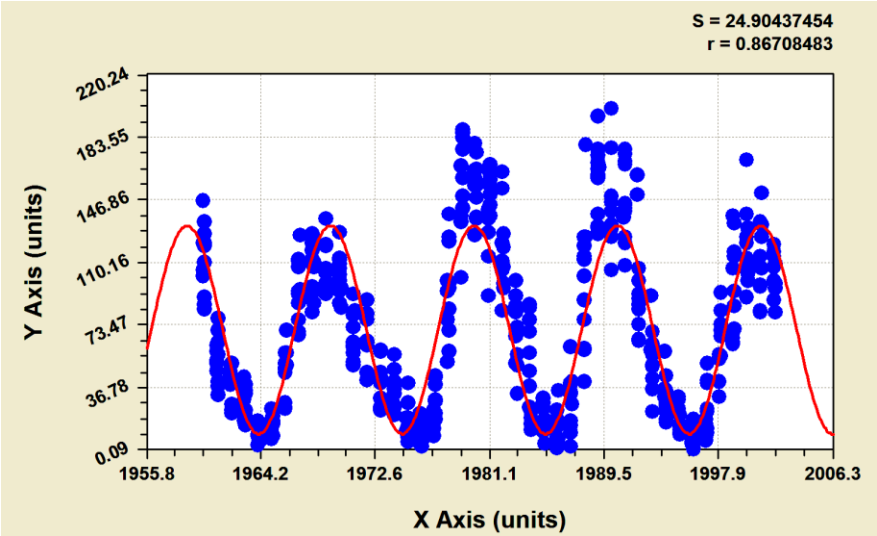

Fig. 1: (Sunspots Time Series Data along with Sinusoidal Fitting)

Fig. 1 represents that the sunspots behavior is almost same as the sinusoidal model. Fig. 2 shows the model generated data along with the actual sunspots data.

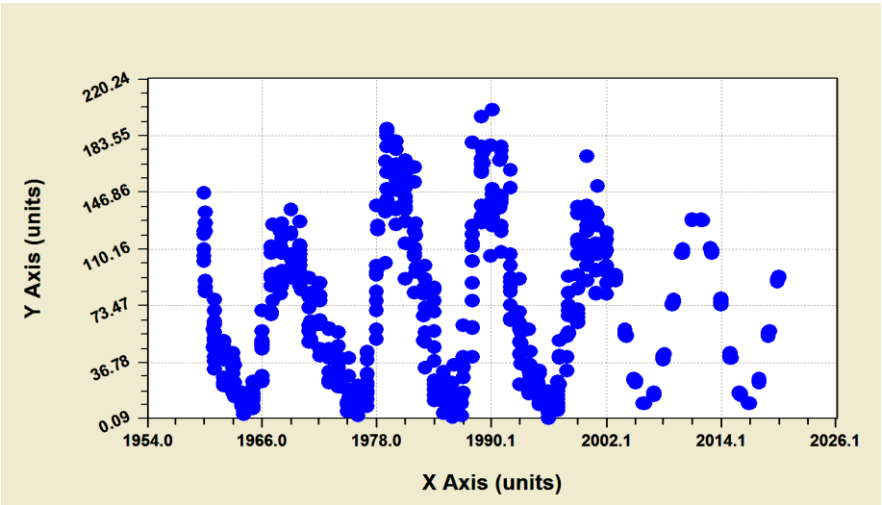

Fig. 2: (Model Generated and Actual Sunspots Data)

The residuals plot is also depicts in Fig. 3. The above and down fluctuation of residuals from the mean line shows the adequacy of the fitted model.

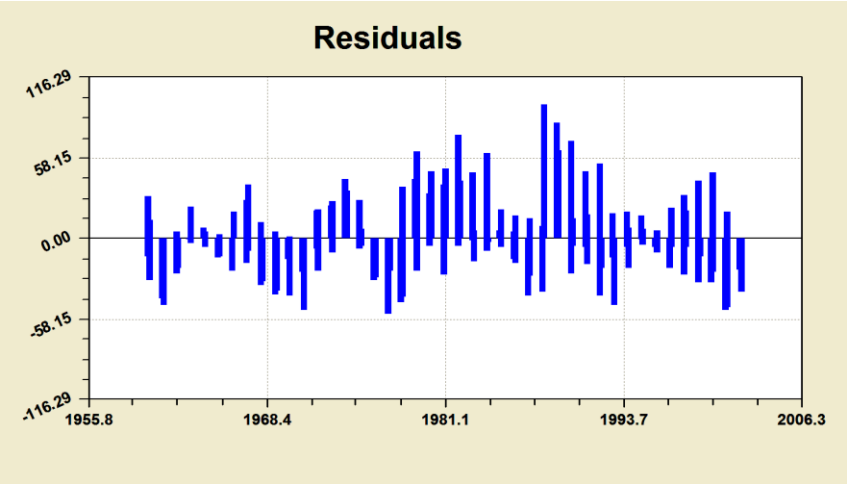

Fig. 3: (Residuals Analysis of Sunspots Time Series Data)

Although the sinusoidal model is not suitable to understand the peak of sunspot cycles, but for total time series data of sunspots its shows almost sinusoidal behavior.

4. COMMENTS AND CONCLUSION

The result obtained indicates the sinusoidal model is best fit for the sunspots time series data. The model can be useful to understand the oscillation behavior of sunspot data.

5. ACKNOWLEDGEMENT

The authors are thankful to the World Data Center for providing the Sunspots Monthly Data and ISOSS Conference Organizing Committee.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 35-42

DESIGNING SIGNIFICANT FACTORS FOR SUCCESS IN SPORTS AND GAMES

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ABSTRACT

Experimental design is powerful technique in understanding the process, response variable and factors affecting the response. It is proved that experimental design is very effective in improving process, performance of the process as well as in reducing variability. This research article presents Trash Can experiment that can easily be performed by any individual in class room. The results of the experiment proved to be true in real life and these results can easily be applied in sports and games like Basket Ball, Snooker, Cricket, Soccer and American Football.

KEYWORDS

Experimental Design, Response Variable, Variability, Trash Can, Sports, and Game.

1. INTRODUCTION

Many classroom activities have been performed by different statisticians in regression analysis, Logistic Regression, Sampling Theory and Time series analysis. (Hogg, 1991), proposed the involvement of students in entire statistical process. According to Robert Hogg, "projects give students experience in asking questions…formulating hypothesis… and analyzing data.

(Cobb, 1993), summarized many classroom projects that were founded by the National Science Foundation and from these projects he built a lot of information related to statistics. (Caroline Brophy, Lukas Hahn, 2014) engaged students in large lecture of Sudoku and with the help of this experiment they taught a lot of descriptive statistics like Bar diagram, Box Plots, Mean, Median, Mode and survival analysis to students. (Andrew Gelman, Mark E.Glickman, 2000) also emphasized on classroom activity and they performed Cards Experiment which describes Descriptive and Inferential Statistics in detailed. (Christopher H.Morrel and Richard E. Auer, 2007) from Loyal College in Maryland performed Thrashball activity describing Logistic Regression.

This article is based on the article of Christopher H.Morrell and Richard E.Auer, which describes Design of Experiment. Design of Experiment (Fractional factorial Design) is being used in this experiments to analyze the effects of input variables. In this paper plots like Main Effects, and Interaction Plots are being discussed. ANOVA seems to be helpful to discuss the significant input variables.

There are two replicates (Experiment will be performed in same conditions for two times). During analysis for low level of any input is denoted by -1 and while high level is denoted by +1.

For The Designing of Experiment (Montgomery)'s book Design and Analysis of Experiments, Eight Edition and (George E.P Box , J Staurt Hunter and William G. Hunter)'s book Statistics for Experimenters: Design, Innovation, and Discovery, 2nd were useful.

2. METHODOLOGY OF EXPERIMENT

In this experiment, a ball is thrown from two distances (8ft and 12ft) for two time two intervals (30 seconds and 50 seconds). Position of trash can, way of throwing and weight of the ball are three other input variables. In this experiment our responsible variable (Dependent or Output Variable) is proportion of success (How many times ball hit into trash can in total number of throws).

Every input variable is checked on two levels (high and low level). The level of input variables can be summarized from the following table.

Table 1

Input Variable	Input Variable	Low Level	High Level
Time	A	30 seconds Interval	50 seconds Interval
Distance	B	8 feet	12 Feet
Position of Trash Can	C	Narrow	Wide
Way of Throwing	D	Under Arm	Above Arm
Weight of the Ball	E=ABCD	Low Weight	High Weight

Fig 1: Wide Position of Trashcan

3. RULES OF THE EXPERIMENT

The thrower is allowed to bow 10 degree (Maximum), if he does not want to bow it is up to him. The experiment is performed on certain day and certain temperature. Thrower should not waste time when time is started.

Once ball is hit into can and comes out will be counted as a success.

Balls at each level will be five or more because bringing balls back from trash can, can waste time.

4. DESIGN OF EXPERIMENT

There are many designing techniques; these techniques include The Randomized Complete Block Design (RCBD), The Latin Square Design, Two-Level Factorial Designs and Two-level Fractional Factorial Designs. In our experiment Two Level Factorial Design or Two-Level Fractional Factorial Design can be used, because experiment contains input variables at two levels. But Fractional factorial will be used for the analysis of input variables.

As the number of input variables in 2^k Factorial Design increases, the number of runs also increases rapidly. In 2^5 design, there are 32 runs. In this design 5 of the 31 degree of freedom are responsible for the main effect, only 10 factors with 26 degree of freedom are responsible for two-factor interaction. Remaining factors with 16 degree of freedom are responsible for three or higher interaction effect. In our experiment we are not interested in higher interactions because higher order interactions are negligibly interacted. The factorial design which only depends upon main effect and low-order interactions is called Fractional Factorial design.

The fraction factorial design 2^{5-1} has sixteen run of each factor and E is Defining Relation which is always equal to set of all columns. The table is given below:

Table 2
Design Matrix

Run	Basic Design				E= ABCD	Treatment Combination	Response	
	A	B	C	D			Proportion of Success	
							Replicate 1	Replicate 2
1	+	-	-	-	-	a	0.48	0.45
2	+	+	+	+	+	abcd	0.70	0.80
3	-	-	+	-	-	c	1.00	0.90
4	-	-	+	+	+	cde	0.88	0.80
5	+	+	+	-	-	abc	0.64	0.66
6	-	+	-	+	+	bde	0.60	0.64
7	-	+	+	-	+	bc	0.90	0.95
8	-	+	+	+	-	bcd	0.58	0.54
9	-	-	-	-	+	e	0.70	0.65
10	+	-	-	+	+	ade	0.45	0.50
11	+	-	+	-	+	acd	0.62	0.64
12	+	-	+	+	-	abd	0.60	0.56
13	+	+	-	-	+	abe	0.58	0.60
14	+	+	-	+	-	abd	0.56	0.64
15	-	+	-	-	-	b	0.54	0.54
16	-	-	-	+	-	d	0.64	0.70

5. MAIN EFFECT

Main effect is the difference between the high level and low level response variable. For example the main effect of time interval (A) can be calculated as:

Main effect of A = Average at high level - Average at low level

$$\textbf{Main effect of } A = \frac{\frac{0.48+0.7+0.64+0.45+0.62+0.60+0.58+0.56}{8} + \frac{0.45+0.80+0.66+0.5+0.64+0.56+0.60+0.64}{8}}{2} - \frac{\frac{1+0.88+0.60+0.90+0.58+0.70+0.54+0.64}{8} + \frac{0.90+0.80+0.64+0.95+0.54+0.65+0.54+0.70}{8}}{2}$$

$$\textbf{Main effect of } A = \frac{\frac{4.63}{8} + \frac{4.85}{8}}{2} - \frac{\frac{5.84}{8} + \frac{5.72}{8}}{2}$$

$$\textbf{Main effect of } A = \frac{0.57875 + 0.60625}{2} - \frac{0.73 + 0.715}{2}$$

$$\textbf{Main effect of } A = 0.5925 - 0.7225 = -0.13$$

For Time interval we use A in design matrix. As with every job, the more you do the more bored you get. For low interval of time the proportion of success was larger but for high interval time, the proportion of success is smaller. Whereas for distance (B), the

main effect is negligible. This means that for given distances at two levels, the proportion of success is almost same.

The Position of trash can in design matrix is denoted by C. When trash can is at narrow position, the proportion of success is very small and when trash can is at wide position, then the proportion of success was higher.

Way of throwing is denoted by (D) in design matrix. From Main effects plot it is clear that under arm throwing is responsible for higher proportion of success. High weight of the ball (E) is also responsible for the higher proportion of success. The remaining main effects and two way interaction effects are given below in the form Minitab Output.

6. ESTIMATED EFFECTS AND COEFFICIENTS OF RESPONSE

Table 3
Shows Significant Factors

Term	Effect	Coefficient	SE Coefficient	T	P- Value
		0.6575	0.007043	93.35	0.00*
A	-0.13000	-0.06500	0.007043	-9.23	0.00*
B	-0.00625	-0.00312	0.007043	-0.44	0.663
C	0.156250	0.07812	0.007043	11.09	0.00*
D	-0.04125	-0.02062	0.007043	-2.93	0.010*
E	0.061250	0.03062	0.007043	4.35	0.00*
A*B	0.116250	0.05812	0.007043	8.25	0.00*
A*C	-0.03625	-0.01813	0.007043	-2.57	0.020
A*D	-0.05875	0.02938	0.007043	4.17	0.001*
A*E	-0.02375	-0.01187	0.007043	-1.69	0.111
B*C	-0.02250	-0.01125	0.007043	-1.60	0.130
B*D	-0.00250	-0.00125	0.007043	-0.18	0.816
B*E	0.072500	0.03625	0.007043	5.15	0.00*
C*D	-0.06500	-0.03250	0.007043	-4.61	0.00*
C*E	0.040000	0.02000	0.007043	2.84	0.012
D*E	0.007500	0.00375	0.007043	0.53	0.602

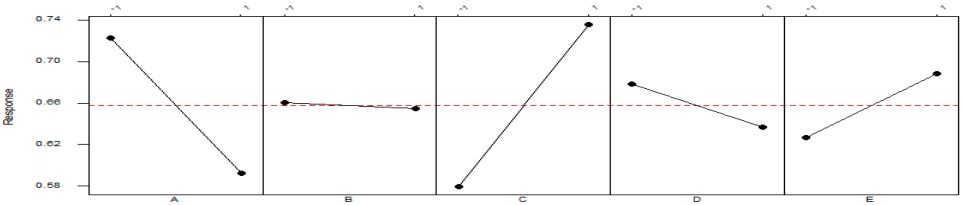

Figure 2: Main Plots and its Effect

7. ANALYSIS OF VARIANCE FOR RESPONSE (ANOVA)

Tabl: 5

Source	DF	SS	MS	F	P-Value
Main Effects	5	0.37445	0.074890	47.17	0.000
2-way Interactions	10	0.24395	0.024395	15.37	0.000
Residual Error	16	0.02540	0.001589		
Total	3	0.64380			

ANOVA table shows Main effects and 2-way interactions effects are Significant, but more than 2-way interactions effects are not significant. Which main effects and two-way interactions effects are significant? For this we draw Pareto chart and Normal Probability Plot. These two plots will identify the significant main-effects and significant 2-way interaction effects.

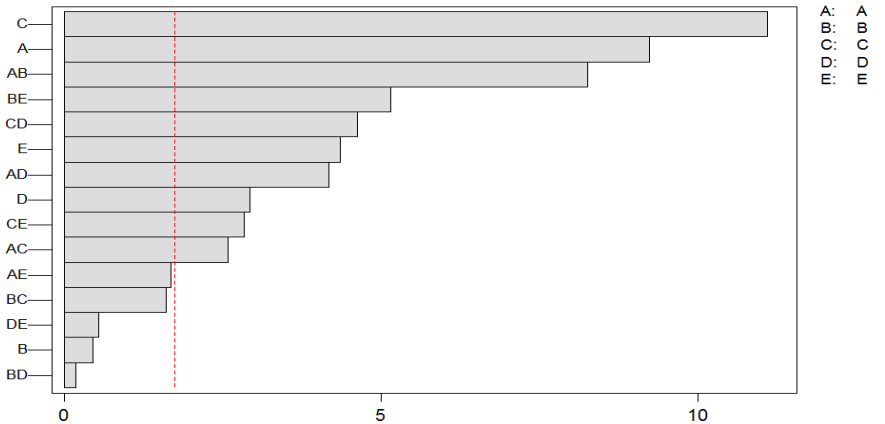

Figure 3: Pareto Chart for the Standardized Effect

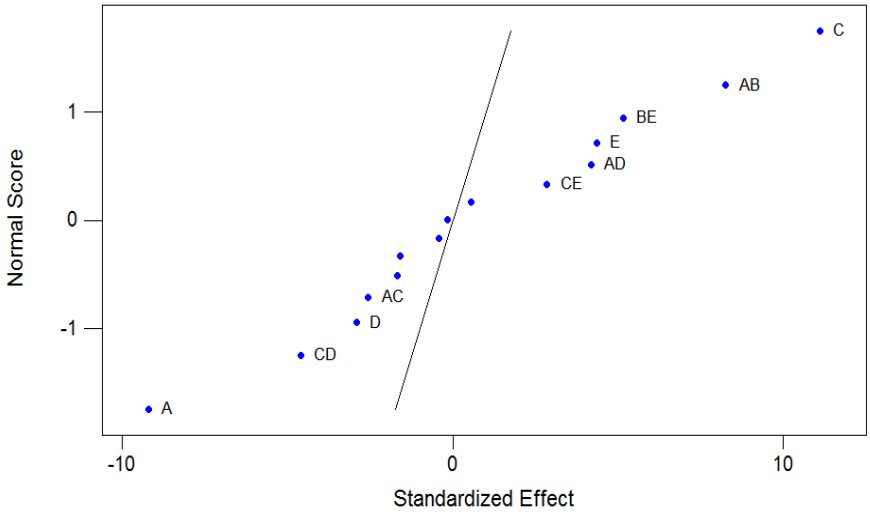

Figure 4: Normality Plot of Standardized Effects

Pareto Chart and Normal Probability Plot specify the significant factors very clearly. The difference between these two plot is very is very easy to note. Pareto Chart only identify the significant factors, while Normal Probability Plot not only identify the significant factors but also tell that which factors are responsible for higher proportion success at low level and high level. The factors at right side to the straight line in Normal Probability Plot are responsible for higher Proportion of Success when they are treated at high level, while factors at the left side to the straight line in Normal Probability Plot are responsible for higher Proportion of Success when they are treated at low level. The information is provided by Normal Probability Plot, same information is provided by Table 3 by calculation.

8. CONCLUSION

From analysis we have concluded that our factors should be adjusted in such a way that give a high proportion of success of response variable and this was our goal before starting experiment of trash can. For high proportion of success we should have: Small time duration for an experiment, any distance high or small, but be careful that the difference should not be more than 4 units, the position of trash can should be wide, ball should be thrown under arm and high weight balls should be used.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 43-48

THE ROLE OF FOREIGN DIRECT INVESTMENT AND IMPORTS IN ECONOMIC GROWTH OF PAKISTAN

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ABSTRACT

The aim of this study is to understand the impact of foreign direct investment (FDI) and imports on Gross domestic product (GDP) of Pakistan. FDI and Imports contributes in GDP growth of the country by bringing knowledge and technology. Import of capital goods is the lifeblood for the survival of local industries in less developed countries. Factors we used in this study are; foreign direct investment, imports and gross domestic product. Data is taken on yearly basis from 1980-2015, using multiple regression analysis to check the relationship among variables. Results are explaining the positive effects of foreign direct investment and imports on GDP of Pakistan.

KEYWORDS

GDP, FDI, Imports and Regression analysis.

1. INTRODUCTION

Economy of Pakistan is heavily dependent on foreign trade and investment. Pakistan will get benefit from multiple dimensions as the economic ties gets stronger with neighboring countries[1]. Local industry will bloom with the increase in the investment, expansion of capacity, technological and knowledge advancements, foreign inflows strengthen the weak economies of less developed countries, therefore, governments take different initiatives to attract foreign investors, like reduction in tax rate, increasing subsidies and infrastructure building. The belief behind offering these special incentives is that foreign investment brought spillovers and the transfer of technology.

Geographical location of Central and South East Asia of Pakistan gives it an edge of strategic nature accompanied with some other advantage like, availability of cheap labor and large population makes it attractive consumer goods market, backed by supportive

[1] http://www.finance.gov.pk/survey/chapters_15/Highlights.pdf

purchasing power[2]. Modern Firms invest in foreign markets as part of strategic plan; they can attain cheaper factors of production, gain access to new marketing channels, financing facilities and technology. Less developed countries can grab this as an opportunity and offer a conducive business environment, with higher growth potentials. Economic growth and exports are linked closely with Imports. Production possibilities expands due to imports in developing economies because of heavy reliance on importing intermediate and capital goods for local industrial consumption. Export sector gets bolstered by the import of new technology and skills, hence, export-led growth could be achieved. Economy face downward-trend in output if it suffers from reduction in factors of production imports from foreign markets ((Hentschel, 1992); (Lee, 2010))

FDI positive impact is not always the same in every country, there has to a favorable government policy for financial sustainability. This study can be beneficial to understand the structure of the complex economic environment and to better exploit the future usefulness of FDI, imports and economic development in today's rapidly changing South Asian region. This paper divided into five parts; introduction, literature review, methodology, results and conclusion.

2. LITERATURE REVIEW

Several past studies have analyzed the relation among output growth, FDI and imports. The model of growth and trade discussed by (İşcan, 1998), indicates that economic growth is stimulated by trade, as a result of uplift in input variety, market expansion and knowledge enhancement. Asafu-Adjaye & Chakraborty (1999), supported the positive association between export and import because exports are considered as a source of funding for imports, in other words, earning of foreign exchange is a major motivation for export especially in case of LDCs, to get the capital imports for the country. Hence, imports play a vital role in productivity growth. Therefore, keeping both trade variables in a same model of regression may cause the problem of multicollinearity.

FDI plays a complementary role for the imports in the country. Several studies have been published explaining the bond among Gross Domestic Product, Foreign direct investment and imports. Lee, (2005) discussed the need of barriers reduction on global trade due to the potential of huge benefits for trading countries, also emphasized the importance of structural changes and social policies implementation for achieving success in trade liberalization. Various Studies like ((Ozawa, 1992); (Zarotiadis; Mylonidis, (2005); appears to suggested the complementary role of investment and global trade, because of the vertical integration in production by cross border operations due to the price difference in factor of productions, argues that substitute role of FDI for trade, stimulates the imports at level of gross production, both at industry and product level. Some seems to support the role of substitution, like (Brainard, 1993), foreign investment flow starts towards cross-border horizontal increase in production activity due to heavy transportation and transaction cost due to barriers on trade. Azeem, (2013), found the complementary effect of FDI on imports, in the perspective of bilateral trade between American and Pakistan. According to (Sachs, 2000) the low income and less technologically advanced countries reduces the income gap with developed countries

[2] http://www.indexmundi.com/pakistan/gdp_(purchasing_power_parity).html

through a process of capital flows and technological transfer from the rich countries to the less developed regions. Theory suggest that earning of foreign exchange is essential for the import of the technology in developing economy where technologies are upgraded by foreign inflows irrespective of their industrial policy, can be achieved through export-promotion policy. Studies show the higher immersion of technologies from abroad is evident in less developed countries with large foreign direct investment and efficient export-promotion policies.

Existing studies on FDI show that spillover and transfer of technologies are two major channels for enhancing production efficiency that benefit domestic companies through inflows from outside the country positive effect theoretically justified. Improvement in reliability and quality of input known by healthy competition among local companies with less efficiency and attributes of public goods, knowledge and Labor training are some ways of occurrence of spillovers. Parent firm having foreign origin transfers technology through R&D, innovations in product and process and addition of more capital stock of production in the host country's' firms (Balasubramanyam, Salisu, & Sapsford, 1996). The "idea gap" between poor and rich countries argued by (Romer, 1993). According to him the potential effects of spillover are effective for poor countries for knowledge and technology transfer. (Rappaport, 2000) argues the limitation of these positive effects to inflows from abroad.

Doubts raised by (Carkovic & Levine, 2002) on the true essence of positive impact of spillovers to domestic firms from foreign inflows. He argued that the impact of FDI on GDP of some countries has negative impact, and FDI does not contribute to economic boost. Study conducted by (A. E. Harrison & Aitken, 1999) on Venezuela between 1979 and 1989; found no proof of favorable spillover of technology from abroad based companies to domestic companies. (A. Harrison, 1996) found the negative but statistically insignificant impact on growth of local firms of Morocco of spillover from the foreign inflows, study was conducted over the period of 1985 to 1989. The positive impact of foreign direct investment on the economic growth is observed by several studies ((Borensztein, De Gregorio, & Lee, 1998); (Sun & Parikh, 2001)).

3. METHODOLOGY

3.1 Data

Analyze the relationship among foreign direct investment, imports and economic growth of Pakistan. Annual data of thirty five years is taken from time period of 1980 to 2015 in Pakistani rupees.

3.2 Date Source

Data of variables, gross domestic product, foreign direct invest and imports of Pakistan is taken from handbook of statistic, 2015, published by State Bank of Pakistan.

3.3 Statistical Tool/Technique

Multiple regression analysis has employed by using statistical software package SPSS version 20 to analyze the impact of the independent variables; foreign direct invest and imports on dependent variable; gross domestic product.

3.4 Model Specification

Following is the suggested regression model:

$$GDP=\beta_0+\beta_1(FDI)+\beta_2(Imp)+\varepsilon$$

where:

GDP = Gross Domestic Product
FDI = Foreign Direct Investment
Imp = Imports
ε = Random error term

4. RESULTS

Regression analysis conducted to predict impact of FDI and imports on GDP of Pakistan. We performed a preliminary analysis to assure that assumption of linearity and normality has not violated. Results indicate a significant regression equation ($F = 114.164$, $p = 0.000$) and adjusted $R^2 = 0.866$.

Table 1
Model Summary[b]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.935a	.874	.866	944321.05487

a) Predictors: (Constant), Imports (Rs. m), FDI (Rs. m)a
b) Dependent Variable: GDP (Rs. m)b

R^2 = .866 shows the aggregated effect of independent variables on the dependent variables, heavier percentage shows the stronger prediction ability of independent variables. Its value ranges between 0 to 100 percent, values greater than 0.7 considered strong. Stronger R-squared values show the model fitness. Results in table 1, shows the predictors FDI and Imports (IVs) account for 86% of variance in GDP (DV).

Table 2
ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	203609466547711.120	2	101804733273855.560	114.164	.000[b]
	Residual	29427494404348.766	33	891742254677.235		
	Total	233036960952059.880	35			

a. Dependent Variable: GDP (Rs. m)
b. Predictors: (Constant), Imports (Rs. m), FDI (Rs. m)
c. ANOVA table shows the model significance, should be < 0.05,
in this case, F =114.164 and p = .000, indicates that our model is significant.

Table 3
Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	3650966.211	204219.345		17.878	.000
	FDI (Rs. m)	5.888	2.287	.217	2.574	.015
	Imports (Rs. m)	1.285	.140	.774	9.173	.000

a. Dependent Variable: GDP (Rs. m)

Coefficient Table

1. FDI have positive impact on GDP, significant (p=.015)
2. Imports have positive impact on GDP, significant (p=.000)

5. ANALYSIS

A multiple linear regression was conducted to predict association among selected macroeconomic variables from economy of Pakistan, (DV) Gross Domestic Product, (IV1) foreign Direct Investment and (IV2) Imports. A significant regression model was found at $F = 114.164$, $p = .000$, $R^2 = .866$.

Gross Domestic Product (GDP) = 3650966.211 + 5.888 (FDI) + 1.285 (Imports).

6. COMMENTS AND CONCLUSION

This study was conducted to analyze the impact of FDI, Imports on GDP of Pakistan by employing regression technique. Results show that FDI and Imports positively affect the economic growth of Pakistan. Economic growth is made up of contribution of different factors including FDI and imports, so it suggests that these factors can play an important role in the changing scenario of the regional trade. Collaboration among government, social and private sector along with sincerity can enhance the growth prospects of poverty-stricken Southeast Asian countries. Despite of the bright prospects of capital imports, Pakistan's industrialists have some future survival concerns due to rapid technological changes are expected from Chinese market at cheaper rates, for example, textile sector has concerns with rapid invasion of new technologies[3] reason may be the industrialists' nature of technology-aversion. Industrial cooperation will be the crucial factor for mitigating future concerns of local business community. Policy maker should take unbiased steps to start Industrial Corporation and growth oriented programs to attract more foreign investment and encourage imports of capital goods, may achieved through supporting new entrepreneur.

7. ACKNOWLEDGEMENT

I am thankful to ALLAH ALMIGHTY, the most compassionate and benevolent, who blessed me with the capabilities and power required for the process of completion for this project. I am grateful to my supervisors, for their guidance for making my research an

[3] http://www.cpecinfo.com/cpec-news-detail?id=MTU3Mg==

excellent learning opportunity. I also acknowledge my parents, colleagues and friends for their support and prayers for my success.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 49-56

THE IMPACT OF ADVERTISEMENT AND PROMOTION ON SUPPLY CHAINS: MANUFACTURER AND RETAILER PERSPECTIVE

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ABSTRACT

Marketers analyze and tap ways to positively divert buying behavior of consumers through well designed advertisement and promotion of their products. Advertisement carries a psychological impact on the mind of users. Well-designed advertisements motivate consumers to purchase more. Companies spend extensively on advertisement in an effort to multiply consumer base, increase sales, improve brand image and enhance profitability. It means the more the companies advertise the more demand they get for their goods which increase sales and result in more profits. However, all goods are not useful for consumers especially children and youngsters. Socially responsible organizations must exercise extreme care by limiting the publicity of "bad" and "unhealthy" products. Junk food, mostly considered as "worse food" has become essential inventory item of convenience and grocery stores and fast-food outlets across the country. Advertisement and promotion of such items in print, electronic and social media give an appealing touch to the youngsters to consume more. It is believed that such foods are the main cause of obesity in children. This study explores whether there is any impact on utility of junk food by limiting advertisement and promotion of "bad" or "unhealthy" products by the manufacturers. Are producers ready to sacrifice their sales base by reducing advertisement of junk food items for the health sake of our young generation? Karachi has been taken as sample of the study however result is applicable on whole Pakistan. The sample size is 100 respondents from Nestle Pakistan Ltd (main producer of kids' junk food), grocery stores of the city, cafeterias/canteens of universities as retailers and colleges/university students of class 10th to graduate level as consumers. The survey recommends way forward for a win-win strategy for both the manufacturer and retailer.

KEYWORDS

Junk food, obesity, tweens, teens, Nestle, canteen, fast-food, fiber.

1. INTRODUCTION

Role of advertisement and promotion in affecting consumer demand needs no emphasis. It is critical to raise the profitability graph of an organization in the

marketplace. Companies of all stature allocate huge amounts out of scarce resource for expenditure on advertisement and promotion of the brand inventory. Globalization of the corporate sector has further brought the managers under tremendous pressure to explore new promotion avenues to get superior edge over competitors. Simultaneously, business entities are also required to remain focused on the ethical manners of advertisement and promotion and refrain from publicity of products hazardous for the public in general and for children and teens in specific. To this effect a mutually agreed win-win strategy is to be adopted by the two layer supply chain members i.e. manufacturers and the retailers.

Tweens, teens and school going children exert considerable influence over family purchases, as marketers are keenly aware, particularly in the area of food. As a promotion strategy, companies attract a wider range of target customers (in this case teens & children) by highlighting the different lucrative characteristics of their product. Due to this excessive publicity, children, as young as 2 years old, can begin to make choices about what they want to eat, play with, and wear. Breakfast cereals, snack foods, and beverages of all kinds are designed to attract the attention of children, who in turn try to persuade their families to purchase them. With childhood obesity on the rise, we must feel concerned about the types of beverages available to children in schools, colleges and universities. In advance societies, on the pressure of civil societies and law makers, beverage marketers Cadbury Schweppes, Coca-Cola Company, Kraft Foods, General Mills and PepsiCo voluntarily agreed to remove high-calorie soft drinks and snacks from academic institutions, substituting lower-calorie choices in smaller serving sizes. These organizations are making efforts to market more healthful alternatives for foods, children traditionally like. Retailers are equally responsible for selling hazardous products to children by displaying these items in specially fabricated displaying stands in front portion of their stores. Retailers are therefore required to be socially responsive in this aspect.

This study explores whether there is any impact on utility of junk food by limiting advertisement and promotion of "bad" or "unhealthy" products by the manufacturers and retailers. Are producers ready to sacrifice their sales base by reducing advertisement of junk food items for the health sake of our young generation? This study differs in several aspects from the prevailing surveys undertaken on the subject. Few of the distinctive aspects are that it uses actual data collected from the manufacturers, retailers and consumers at Karachi. Work on the subject matter has been done in technological advanced countries; however, Pakistan's chapter is lacking the related research on the subject. Therefore, this survey is unique in the sense that it is Pakistan specific. We have examined the effect of less or no advertisement and promotion of food products which are harmful for the healthy growth of children and youngsters and its final impact on supply chains.

2. LITERATURE REVIEW

Supply chain management is the methodical and the tactical harmonization administration for providing products, goods and services according to the needs and wishes of users. Through SCM efficiency and effectiveness can be brought in the vital activities and operations of the chain which will ultimately result in company's reliability and users' satisfaction. Advertisement and promotion reside within the marketing

premises. Despite the digitalization era and online communication systems, advertising is vital source of promotion of productions of any organization, whether small, medium or a big company.

According to Charlton, Kähkönen, Sacks, & Cameron (2015) supermarket marketing activities have a major influence on consumer food purchases. Unhealthy foods were heavily promoted in circulars from leading supermarket chains in all regions of the world. Liu, Sun, Lei, Leong, & Deng (2016) disclose that many studies examine information sharing in an uncertain demand environment in a supply chain. However there is little literature on cost information sharing in a dual-channel structure consisting of a retail channel and a direct sales channel. Assuming that the retail sale cost and direct sale cost are random variables with a general distribution. In both single- and dual-channel structures, the retailer has little motivation to share its private cost information which is verified to be valuable for the manufacturer.

Coordinating a dual-channel supply chain could not only achieve the integrated profit of the supply chain but also alleviate the channel conflict. Although some researches addressed this area, there is scant literature to discuss the coordination issue in the situations of disruption. To fill this void, we utilize a contract with a wholesale price, a direct channel's price and a lump sum fee to coordinate a dual-channel supply chain under the cases of demand disruptions and production cost disruptions (P. Zhang, Xiong, & Xiong, 2015).

P. Zhang et al., (2015) claims that in recent years, there has been considerable research on cooperative (Co) advertising in a single-retailer-single-manufacturer supply chain. Due to the scarcity of research on game theoretical models with multiple retailers, they discuss the case on how the manufacturer coordinates the channel in which two identical retailers compete on local advertising investment.

Advertising plays an important role in affecting consumer demand. Socially responsible firms are expected to use advertising judiciously, limiting advertising of "bad" products (W. Zhang, Li, Zhang, & Hou, 2014). According to the writers, an example is the advertising initiative adopted by several major food manufacturers to limit the advertising of unhealthy food categories to children. Such initiatives are based on the belief that less advertising will lead to less consumption of these unhealthy food categories. However, food manufacturers usually distribute products to consumers through retailers whose advertising is not restricted by those initiative programs.

Borrell Associates Report (2015) claims that cooperative advertising programs represent a significant investment for distribution channels. According to the statement, recent reports estimate that about $36 billion are being paid by manufacturers to retailers in cooperative advertising funds, which represents about 12% of total advertising costs.

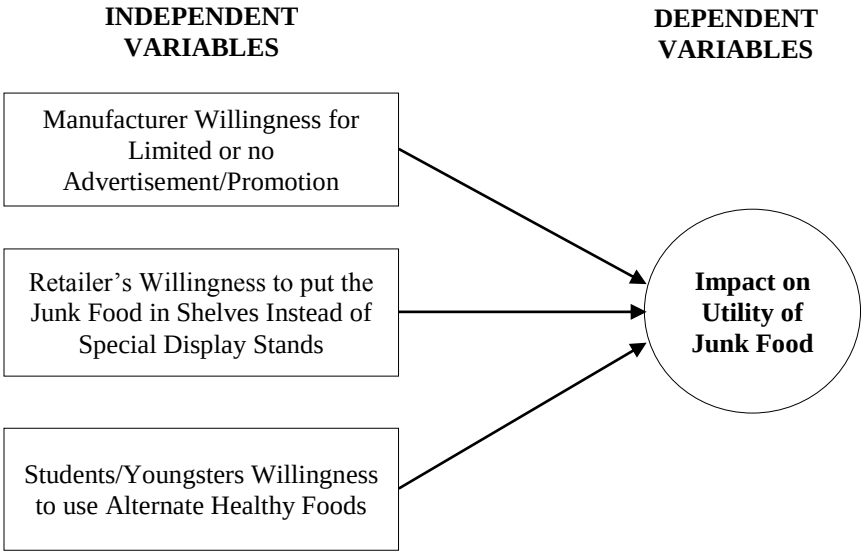
Laric, Michael V., Peter M. and Pikman, (2009) disclose that approaches to marketing were based on the flow of goods, e.g.., how wheat moved from the fields to the grocery store. Eventually the study began to stress the managerial approach, and was formed around the marketing concept. The concept was based on finding out the wants and needs of the target market, and then satisfying these wants and needs. This definition was

massaged to take account of the social and ethical impact on the satisfaction of consumer wants and needs.

Cameron, Charlton, Ngan, & Sacks, (2016) disclose that the supermarket is increasingly recognized as a key environment to promote healthy eating. According to them most high-quality studies targeting the supermarket food environment reported improvements in the healthiness of consumer purchases in response to the intervention. To add further Bogomolova, Szabo, & Kennedy (2015) say that consumer price promotions account for more than half of many manufacturers' marketing budgets, and require a significant time investment to manage. Amidst the considerable research on price promotions, little academic attention has been paid to how manufacturers and retailers make price-promotion decisions. They found that intuition and untested assumptions are the main inputs into these decisions, practitioners lack solid empirical evidence to guide their actions and their beliefs are often in stark contrast with academic knowledge about the effectiveness of price promotions and price promotions are typically not evaluated against the objectives according to which they were justified.

It has been shown that while promoting a retailing innovation, the stronger a retailer's operations capability, the more successful the retailer's promotion in its early stage (Shi., 2016). According to the writer, the results showed that the interaction between efficiency in advertising function and that in operations function did not appear to play any significant role towards success in the USA while promoting a retailing innovation.

3. THEORETICAL FRAMEWORK



4. OBJECTIVES

The objective of undertaking this study is to analyze whether less or no advertisement and promotion has any impact on the supply chains of junk foods i.e. less utility by children and youngsters at Karachi - Pakistan. The topic has been studied by the researchers around the globe however, Pakistani context lacks it. All the researchers have deduced some conclusions from their studies. Children obesity problem, due to excessive consumption of junk food, is on the rise in Pakistan. This study has examined the mindset of manufacturers, retailers and consumers to finds ways for less utility of bad food by children and youngster.

5. RESEARCH DESIGN

Sampling Procedure

Keeping in view the data, a number of organizations were identified, short listed and randomized into a representative unit of manufacturers and producers of junk food and beverages producers. Retailers and consumers were also included in the respondents. Keeping in view the requirement for the data a comprehensive study device was designed with due reference to prevailing trend of junk food consumption and its unethical publicity at Karachi - Pakistan. This research is specific to Karachi. However, results are application to all regions of Pakistan.

Date Source

Ten organizations, retailers, schools, colleges and universities were shortlisted from Karachi. The shortlisted industries and organizations comprised of manufacturers, producers, retailers and consumers.

After identifying the potential respondents, questionnaires were sent directly to them. After a gap of 3-4 days, a trained data collector personally visited the respondents to clarify the queries, if any, and collect the filled survey forms. The factors of flexibility and adaptability were catered for in the methodology.

Statistical Tool Used

Multiple regression analysis was used through the well-known software package SPSS version 20 to assess the impact of the IVs on the DV.

6. RESULTS

We estimated the impact of advertisement and promotion on less utility of junk foods through three independent variables. Only two out of the three IVs have negative impact on the DV. The below tables show the results;

Table 1
Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.318[a]	.101	.073	27.92721

a. Predictors: (constant), students / youngsters willingness to use alternate healthy foods, retailer's willingness to put the junk food in shelves instead of special display stands, manufacturer willingness for limited or no advertisement / promotion
b. Dependent Variable: Impact on utility of junk foods

Model Summary Table

R^2 =.101 taken as a set, the predictors (IVs 1-3) account for 10% of variance in DV. It does not show how much individual IV accounts for. R^2 is significant if greater than zero (0).

Table 2
ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	8451.820	3	2817.273	3.612	.016[b]
	Residual	74873.180	96	779.929		
	Total	83325.000	99			

a. **Dependent Variable: IMPACT ON UTILITY OF JUNK FOOD**
b. predictors: (constant), students / youngsters willingness to use alternate healthy foods, retailer's willingness to put the junk food in shelves instead of special display stands, manufacturer willingness for limited or no advertisement / promotion

ANOVA Table (test using alpha = .05)
The overall regression model is significant
$F(3,96) = 3.612$, $p=.016$, $R^2 =.101$

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	T	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	46.219	37.161		1.244	.217		
	Manufacturer willingness for limited or no advertisement / promotion	-3.406	4.474	-.075	-.761	.448	.975	1.026
	Retailer's willingness to put the junk food in shelves instead of special display stands	-4.767	6.503	-.072	-.733	.465	.983	1.017
	Students / youngsters willingness to use alternate healthy foods	10.232	3.434	.291	2.979	.004	.982	1.018

a. Dependent Variable: IMPACT ON UTILITY OF JUNK FOOD

Coefficient Table
(Test each predictor at alpha = .05)

1. Manufacturer willingness for limited or no Advertisement / Promotion - not sig. (p = .448)
2. Retailer's Willingness to put the junk food in shelves instead of special display stands - not sig. (p=.465)
3. Students' / Youngsters' willingness to use alternate healthy foods - sig. (p=.004)

7. OVERALL ANALYSIS

A multiple liner regression was calculated to predict "Impact on utility of Junk Food" (DV) based on Manufacturer willingness for limited or no Advertisement / Promotion (IV1), Retailer's Willingness to put the junk food in shelves instead of special display stands (IV2), Students' / Youngsters' willingness to use alternate healthy foods (IV3). A significant regression equation was found at $F(3,96) = 3.612$, $p=.016$ and $R^2 = .101$.

Participants' predicted (DV) is equal to 46.294 – 3.406(IV1) – 4.767 (IV2)+10.232 (IV3) whereas all (IVs) are coded or measured as 1 = Strongly Disagreed, 2 = Disagreed, 3 = Neutral, 4 = Agreed and 5 = Strongly Agreed.

8. COMMENTS AND CONCLUSION

Aim of the investigation was to assess the impact on utility of junk food by students / youngsters. The findings of the regression model enable us to conclude; that the advertisement and promotion of junk foods through various means of publicity motivates the consumers to use more. The students, teenagers and teenagers demand the junk foods by brand name. Brand name has created psychological effects in the users' mind due to excessive advertisement and promotion of these worse products. The manufacturer and retailers are ready to adopt policy of limited or no advertisement of the declared junk foods for the health sake of new generation, though they may face drastical decrease in their sale base. On the other hand the students may shift to alternate healthy foods in absence and scarcity of junk foods in the school canteens, grocery stores, super markets etc. Keeping the junk foods in shelves instead of placing them in the promotion stands can benefit school going children, aged 3-10 years, by not seeing the products directly. It is an accepted truth that obesity in our young generation is on the rise which is mainly because of excessive use of junk and fast foods.

Based on the findings of this study, it is considered mandatory on part of the policy makers that manufacturers and retailers may be compelled to cease or limit the advertisement and promotion of bad (junk/ fast) foods. The supply chain managers will thus promote and supply healthier items. This is will discourage the users to insist for junk foods rather will shift to alternate healthy foods.

9. ACKNOWLEDGEMENT

All thanks to ALLAH ALMIGHTY, the benevolent and compassionate, who blessed me with the power and capabilities and remained contented on all intricacies found during the successful completion of my task. I extend my special thanks to my supervisors for their encouraging response and guidance to make my Research Project a real learning experience. I also acknowledge my parents and kids for their prayers, my Wife for her throughout support in fulfilling social life obligations and teachers for their guidance because I think all these are indispensable for success in every stage of life.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 57-72

NUMERICAL RESULTS OF VISCOELASTIC FLUID FLOW IN A PIPE FILLED WITH AND WITHOUT POROUS MEDIA VIA DARCY-BRINKMAN MODEL ASSOCIATED WITH OLDROYD–B CONSTITUTIVE MODEL

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ABSTRACT

This paper presents numerical methods to solve differential equations arising from mathematical model in flow of viscoelastic fluid through a circular pipe filled with porous medium through by using Darcy–Brinkman model comprises of the conservation of mass and conservation of momentum transport coupled with the Oldroyd–B constitutive model. The research is communicated by means of numerical results of the problems of given differential equations associated by means of initial conditions and boundary conditions, happening in the knowledge of viscoelastic flow in pipes through porous media and without porous media. Most important work of this paper is represented in three jointly partial differential equations. The results are comparison to the known finite element method. Numerical calculations of the governing differential equations are accomplished accepting solver ND–Solve purpose in Mathematica.

KEY WORDS

Viscoelastic fluid flow with Porous media in circular pipe, Darcy-Brinkman model, Oldroyd–B constitutive model, Numerical Solutions.

1. INTRODUCTION

Fluid mechanics is the learning of fluids and the forces on them. Fluid mechanics can be divided into fluid statics and fluid dynamics. It is a branch of continuum mechanics, a subject which models matter without using the information that it is completed out of atoms. Fluid dynamics is an active field of research with several unsolved or partially solved problems. Fluid dynamics can be mathematically complex and it can best be solved by numerical methods at times, typically by the use of computers. A modern discipline, described computational fluid dynamics (CFD), is devoted to this approach to solving problems of fluid mechanics. Also obtaining advantage of the highly graphic representation nature of fluid flow is particle image velocimetry, an investigational method for visualizing and analyzing fluid flow. In the fluid dynamics, the solutions of the partial differential equations leading the flows of compressible or incompressible, Newtonian and non-Newtonian fluids are expressively worried large to interest in the literature. The results of Newtonian and Non–Newtonian fluid flow in problems classically depend on calculation of different properties of the flow of fluid. Flows of

Newtonian and non-Newtonian fluids linked with some important analysis are organized by way of Al-Fariss, T. and Pinder, K.L. (1987), Abel-Malek et al. (2002), Vafai, K. (2002), Kakac, S., Kilkis, B., Kulacki, F. and Arine, F (1991), Rajagopal, Na and Gupta (1984, 1985) and Wafo S.C. (2005) [1, 2, 11, 27, 28, 29] and others.

Material of viscoelastic can be prospected as the transitional states among the viscous fluids and elastic solids. The substance exhibits behavior of elastic, such as the effects of memory, as well as the properties of fluid. Viscoelasticity can be modeled by connecting Newton's rule for fluids of viscous by means of Hook's law used for elastic solids, since presented with the original Maxwell model and extended with the Convicted Maxwell models employed for the viscoelastic nonlinear fluids. Greatly difficult rheological and hydrodynamic behavior of compound fluids can be considered similar to effects of the inner elastic possessions. The differential equations relating to the viscoelastic fluids flow enclose the fundamental rules of continuum mechanics and constitutive equation or the condition of rheological equation, stating a picky fluid and explain the viscoelastic stress in the direction of deformation. The investigation is to derive a model that is as easy as likely, relating the minimum number of parameters and variables, and until now containing the facility to determine the viscoelastic activities in compound fluid flows [12, 14, 15.]. It is felled out that the enormously expensive normal stress ratios and differences described as extensional to shear viscosity associated by means of polymeric fluids will produce increasing values of clear viscosity as flow in the porous media, flow pipes are of quickly changing cross section. Viscoelastic fluids have been investigated by R.G. Larson (1999) and Taha Sochi (2009, 2010) [13, 24, 25] due to their huge purposes. Oldroyd-B model is the nonlinear viscoelastic model and is a second simplest model and it appears that the most admired in fluid flow viscoelastic modeling. Here viscoelastic behaviour will be modelled by the Oldroyd-B (Oldroyd [1958]) and Phan-thien / Tanner (PTT) [1977] differential constitutive models and simulation developed by R.G.M. van Os; T.N. Phillips (2004) [19, 26] and others.

Numerical methods for solving differential equations are the main research area in the computational mathematics. It becomes more and more important with the computer techniques. The objective of this paper is to present investigative numerical solutions for unsteady incompressible laminar viscoelastic fluid flows in pipes through porous medium adopting the constant viscosity. The unsteadiness in the fluid flow is due to a suddenly inflicted pressure gradient which drives the flow. The viscoelastic fluid flow through porous media, supposed to be isotropic and homogeneous, can be modelled by using Darcy–Brinkman model in the absence of body forces, comprises of the conservation of mass and conservation of momentum transport attached with the model of Oldroyd–B constitutive. Oldroyd-B constitutive model is the nonlinear viscoelastic model and is a second simplest model and it appears that most admired in fluid flow viscoelastic modeling and imitation.

This research is concerned with how to find numerical calculations of a system are determined by the use of computers adopting Mathematica solver ND-Solve. Section 2 concerns with the mathematical formulations. Section 3 associated with first case or solutions of viscoelastic flow in pipes without porous space; section 3.1 connected with study State Solution of first case, as 3.2 connected with numerical solution of viscoelastic flow in pipes without porous space.

Section 4 associated with second case of solutions of viscoelastic flow in pipes filled with porous space. Section 4.1 related with study state solution for second case. As section 4.2 deals with numerical solution of viscoelastic flow in pipes filled with porous space. Finally the conclusions of this paper are identified in section 5.

2. FORMULATION IN MATHEMATICS

Consider a tubular porous layer apprehended in a pipe drenched with the incompressible laminar viscoelastic fluid flow in radial route. A polar coordinate system is applied with radius-axis vertically upward. The system of flow whose main equations consists of the mass preservation and momentum preservation transport joined with the Oldroyd–B constitutive model. The viscoelastic fluid flow through porous medium is considered to have homogeneous and isotropic.

Velocity field for unidirectional flow is known as $\bar{u} = \left(u(r,t), 0, 0 \right);$ wherever the above velocity mechanically satisfies the incompressibility state. The generalized Darcy–Brinkman model has been utilized for the momentum equation and in the absence of body force; continuity equation, general equation of momentum during porous media and the Oldroyd–B equation identifies the stresses of viscoelastic in the fluid flow in vectorial form can be agreed as under

$$\nabla . \bar{u} = 0, \tag{1}$$

$$\frac{\rho}{\varepsilon}\frac{\partial \bar{u}}{\partial t} = \frac{1}{r}\nabla\left(\left[\frac{\mu_2}{\varepsilon} r \underline{\underline{d}}\right] + \tau\right) - \nabla p - \rho \bar{u} . \nabla \bar{u} - \frac{\mu}{K}\bar{u} \tag{2}$$

where as t symbolizes time so that $\frac{\partial}{\partial t}$ is a temporal derivative, $\bar{u}$ is applied for the field of velocity vector, ρ is the fluid density and is total viscosity of viscoelastic fluid respectively, ∇ represents a spatial operator for differential, μ_2 is denoted for the Newtonian solvent viscosity, $\underline{\underline{d}}$ is the rate–of–strain tensor, τ is the extra stress tensor, p is the isotropic fluid pressure, ε is porosity of porous media and K is related with intrinsic permeability within the porous media.

The equation of Oldroyd–B constitutive model describes the stresses of viscoelastic in the fluid flow may be given in the form as below,

$$\lambda\frac{\partial \tau}{\partial t} = \left[2\mu_1 \underline{\underline{d}}\right] - \tau - \lambda\left\{ \bar{u} . \nabla\tau - \nabla\bar{u} . \tau - \left(\nabla\bar{u}\right)^T . \tau \right\} \tag{3}$$

Hence the rest time for the viscoelastic fluid is specified by λ and μ_1 is used for viscoelastic solute viscosity. As μ is used total viscosity which is taken constant i.e. $\mu = \mu_1 + \mu_2 = 1.$

On the basis of constitutive equation contained by an Oldroyd-B flow fluid, the equations are achieved by leading the unidirectional viscoelastic fluid flow in porous

pipes. The derivation of these equations by using the transport equation of momentum and constitutive equations of Oldroyd–B assume that pressure gradient is constant and body force is absent, the following equations in the governing problem are considered for mathematical modeling.

$$\left.\begin{aligned} \frac{\rho}{\varepsilon}\frac{\partial u}{\partial t} &= \frac{\mu_2}{\varepsilon}\frac{\partial^2 u}{\partial r^2} + \frac{\mu_2}{\varepsilon r}\frac{\partial u}{\partial r} + \frac{1}{\varepsilon}\frac{\partial \tau_{12}}{\partial r} + \frac{\tau_{12}}{\varepsilon r} - \frac{\partial p}{\partial z} - \frac{\mu}{K}u \\ \lambda\frac{\partial \tau_{11}}{\partial t} &= 2\lambda\tau_{12}\frac{\partial u}{\partial r} - \tau_{11} \\ \lambda\frac{\partial \tau_{12}}{\partial t} &= \mu_1\frac{\partial u}{\partial r} - \tau_{12} \end{aligned}\right\} \tag{4}$$

where the component of velocity is indicated by u(r, t) in the axial direction and $\tau_{11}(r,t)$, $\tau_{12}(r,t)$ and $\tau_{22}(r,t)$ are used for components of the stress tensor in axial, shear and radial direction, where second normal stress $\tau_{22}=0$ and r is the radial direction.

The above equations system (4) can be completed dimensionless by introducing the following dimensionless variables and parameters

$$Vc\,u^* = u,\ \frac{uVc}{R}\tau^* = \tau,\ Rr^* = r,\ \frac{R}{Vc}t^* = t,\ K^* = K,\ \frac{R}{Vc}\lambda^* = \lambda,\ \mu\mu_1{}^* = \mu_1$$

and $\mu\mu_2{}^* = \mu_2$.

Hence u^* and τ^* are dimensionless velocity and dimensionless stress tensor, r^* is radial coordinates, t^* is the time using for non-dimensional and K^* is the adapted permeability concern with the porous medium for non-dimensional. As R is a radius of the pipe and Vc is applied for the feature velocity assumed since reference redial velocity $Vc = \dfrac{\varepsilon R^2\left(-\dfrac{\partial p}{\partial z}\right)}{\mu}$.

After dropping stars, we can write equations (4) with the non-dimensional variables and parameters as under,

$$\left.\begin{aligned} \mathrm{Re}\,\frac{\partial u}{\partial t} &= \mu_2\frac{\partial^2 u}{\partial r^2} + \frac{\mu_2}{r}\frac{\partial u}{\partial r} + \frac{\partial \tau_{12}}{\partial r} + \frac{\tau_{12}}{r} - \frac{1}{Da}u + 1 \\ We\,\frac{\partial \tau_{11}}{\partial t} &= 2\,We\,\tau_{12}\frac{\partial u}{\partial r} - \tau_{11} \\ We\,\frac{\partial \tau_{12}}{\partial t} &= \mu_1\frac{\partial u}{\partial r} - \tau_{12} \end{aligned}\right\} \tag{5}$$

For completing well posed problem, initial and boundary conditions are written as

$$u(t,1)=0, \quad \text{and} \quad \frac{\partial u}{\partial t}(t,0)=0 \qquad \text{When t > 0} \tag{6}$$

And initial conditions are taken as

$$u(0,r)=\tau_{11}(0,r)=\tau_{12}(0,r)=0 \qquad \text{When } 0<r<1 \tag{7}$$

where the dimensionless Reynolds number (R*e*), dimensionless Weissenberg number (*We*) and non-dimensional Darcy's number (*Da*) are described as

$$\mathrm{Re}=\frac{R\rho Vc}{\mu},\ We=\frac{\lambda Vc}{R},\ Da=\frac{K}{\varepsilon R^2} \text{ and } \mu_1^*+\mu_2^*=1.$$

3. FIRST CASE: NUMERICAL SOLUTIONS OF VISCOELASTIC FLUID FLOW IN PIPES WITHOUT POROUS MEDIA

If $Da\to\infty$ (i.e., the last Darcy's term vanishes), then the system (5) is written as

$$\left.\begin{aligned}
\mathrm{Re}\ \frac{\partial u}{\partial t}&=\mu_2\frac{\partial^2 u}{\partial r^2}+\frac{\mu_2}{r}\frac{\partial u}{\partial r}+\frac{\partial \tau_{12}}{\partial r}+\frac{\tau_{12}}{r}+1\\
We\ \frac{\partial \tau_{11}}{\partial t}&=2\ We\ \tau_{12}\frac{\partial u}{\partial r}-\tau_{11}\\
We\frac{\partial \tau_{12}}{\partial t}&=\mu_1\ \frac{\partial u}{\partial r}-\ \tau_{12}
\end{aligned}\right\} \tag{8}$$

Subject to initial and boundary conditions are

$$u(t,1)=0 \text{ and } \quad \frac{\partial u}{\partial r}(t,0)=0, \qquad \text{when t > 0} \tag{9}$$

and initial conditions are given as

$$u(0,r)=\tau_{11}(0,r)=\tau_{12}(0,r)=0, \qquad \text{when } 0<r<1 \tag{10}$$

3.1 Study State Solution of First Case

At $\frac{\partial}{\partial t}=0,$ gives the systems of equations which is given as

$$\left.\begin{aligned}
&\mu_2\frac{\partial^2 u}{\partial r^2}+\frac{\mu_2}{r}\frac{\partial u}{\partial r}+\frac{\partial \tau_{12}}{\partial r}+\frac{\tau_{12}}{r}+1=0\\
&2We\,\tau_{12}\frac{\partial u}{\partial r}-\tau_{11}=0\\
&\mu_1\frac{\partial u}{\partial r}-\tau_{12}\ =0
\end{aligned}\right\} \tag{11}$$

Subject to boundary conditions $u(1)=0$ and $\frac{\partial u}{\partial r}(0)=0$, (12)

Steady state solution which is found numerically and the steady state solutions of (11) subject to boundary conditions (12) is resolved adopting Mathematica' solver NDSolve and are plotted in figures 1–3 for some parameters and $u(r)$ at the value of range $0\le r\le 1$ and $\tau_{11}(r)$, $\tau_{12}(r)$ at different values of μ_1.

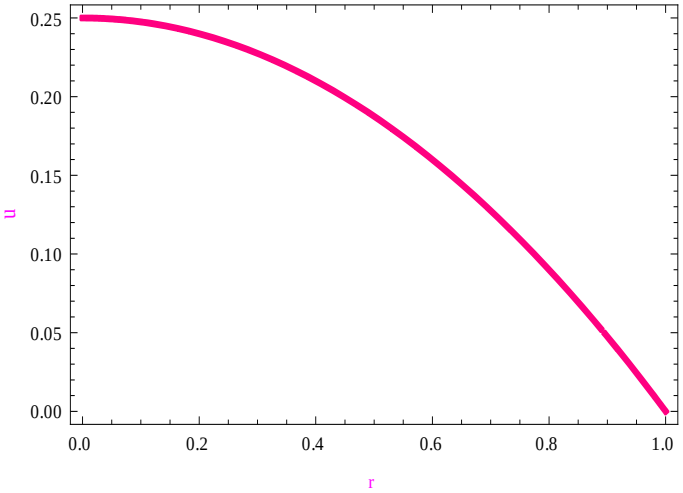


Figure 1: Steady–State Solution of Velocity *u*(r) of the Relation (11 and 12)

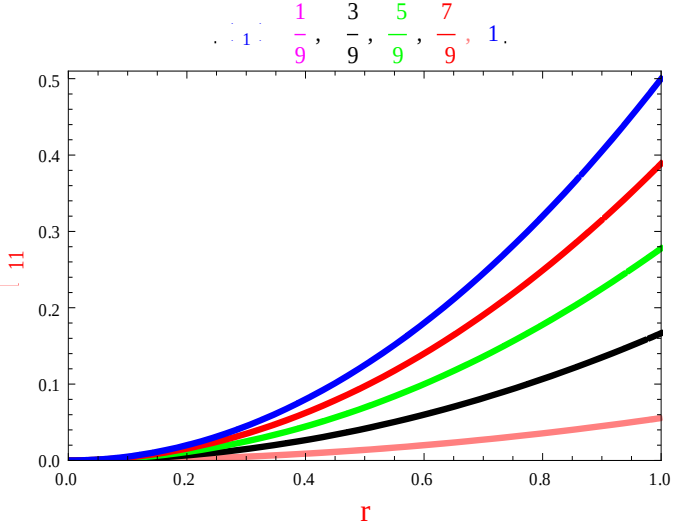


Figure 2: Steady–State Solution of the Normal Stress Component τ_{11} (11 and 12) with $We=1$ and at Different Values of μ_1

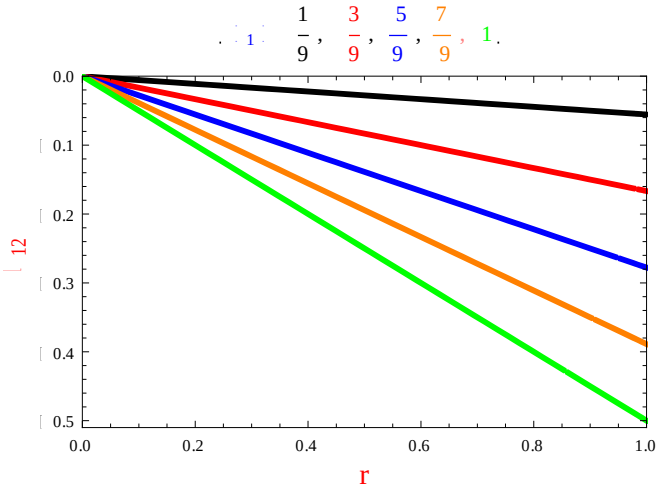


Figure 3: Steady–State Solution of the Shear Stress Component τ_{12} (11 and 12) with $We = 1$ and at Different Values of μ_1

The result of velocity *u*(r) is given in figure–1 at the range $0 \le r \le 1$. The figure–1 displays that velocity profile in the steady state has a fixed curve at the range $0 \le r \le 1$ and no further change in velocity. Steady–state solutions τ_{11} and τ_{12} are conspired in figures 2–3 at different values of μ_1 with $We = 1$. As the normal stress component τ_{11} is showed in figure–2 which explains that component of normal stress τ_{11} has small when small values of Viscoelastic solute μ_1 has taken i.e. τ_{11} decreases and as component τ_{11} increases with big values of Viscoelastic solute μ_1 and has big values and at highest values of Viscoelastic solute $\mu_1 = 1$, then maximum value of normal stress component τ_{11} is equal to 0.5. Whilst, the component of shear stress τ_{12} is showed in figure–3 and this figure displays that during the steady–state, if with large values of Viscoelastic solute μ_1, then the component τ_{12} has small values and while Viscoelastic solute μ_1 decreases, then shear stress component τ_{12} increases and at highest value of Viscoelastic solute $\mu_1 = 1$, then the shear stress component τ_{12} has its minimum value and minimum value of τ_{12} is equal to - 0.5 and finally there is no stress component when Viscoelastic solute μ_1 come up to vanishing value.

3.2 Numerical Solution of Viscoelastic Flow in Pipes without Porous Space

For the governed system of partial differential equations (8)) associated with viscoelastic flow in pipes without porous media subject to boundary and initial conditions (9 and 10), numerical solution is determined adopting Mathematica' solver NDSolve and

numerical solutions for little variations of time are conspired in the graphs 4 to 6, with growing time.

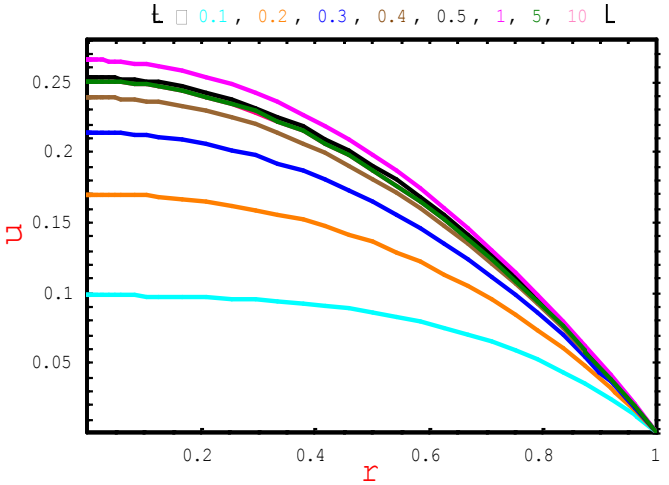


Figure 4: Numerical Solution of the Velocity *u* (t, r) of the System (8) Subject to Initial and Boundary Conditions (9 and 10) with $We = 1$, $\mu_1 = 1/9$, $\mu_2 = 8/9$, and at Different Time t.

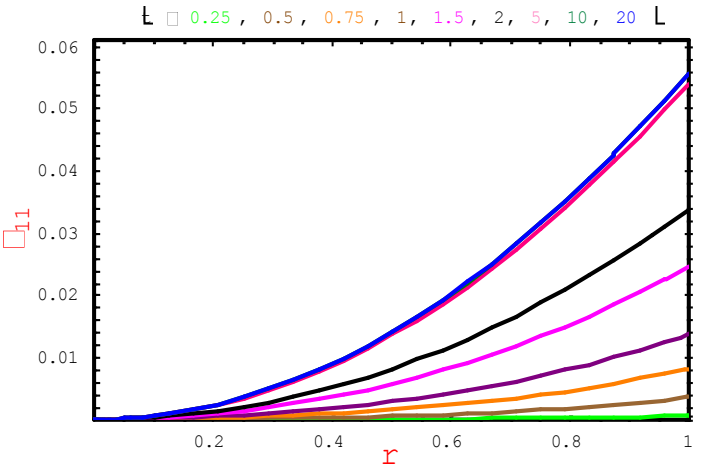


Figure 5: Numerical Solution of the Normal Stress Component τ_{11} of the System (8) Subject to Conditions (9 and 10) with $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ and at Different Time t.

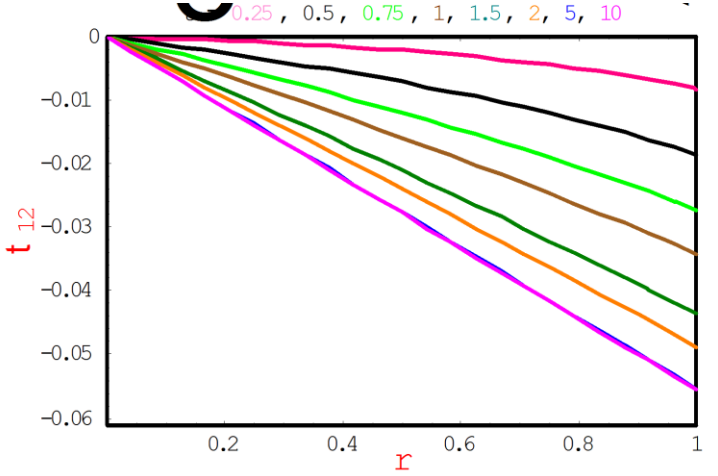


Figure 6: Numerical Solution of the Shear Stress Component τ_{12} of the System (8) Subject to Conditions (9 and 10) with $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ and at Different Time t

The result of numerical solutions such as velocity, components of normal stress and shear stress as are presented in figure–4 to 6 respectively. Figure–4 shows if time continues from rest, then pipe velocity profile enlarges and attained at maximum value of u = 0.265 and then some level decreases from the value of u = 0.265 and reached at steady–state whose value of u = 0.25 as time comes up to further than five units ($t > 5$). Also, in figure–5 the component of normal stress τ_{11} show extends with respect to time t and reached on maximum value of $\tau_{11} = 0.056$ from transition to steady–state with time level beyond seven units ($t > 7$) with comparable non–linear style. While, in the figure–6 the actions of component of shear stress τ_{12} is shown. Hence τ_{12} obviously point out the linear style and increases in negative direction and achieved at steady–state of minimum value of $\tau_{12} = -0.056$ as time approaches beyond five units ($t > 5$).

4. SECOND CASE: SOLUTION OF VISCOELASTIC FLOW IN PIPES THROUGH POROUS SPACE

The partial differential equations system (5) communicates to the viscoelastic flow in pipes filled with porous space accepting constitutive model of Oldroyd–B revise as under:

$$\left.\begin{aligned} \mathrm{Re}\, u_t &= 1 + \mu_2 u_{rr} + \tfrac{\mu_2}{r} u_r + \tau_{12r} + \tfrac{\tau_{12}}{r} - \tfrac{1}{Da} u \\ We\, \tau_{11t} &= 2We\, \tau_{12} u_r - \tau_{11} \\ We\, \tau_{12t} &= \mu_1 u_r - \tau_{12} \end{aligned}\right\} \qquad (13)$$

where $u_t = \frac{\partial u}{\partial t}$, $u_r = \frac{\partial u}{\partial r}$, $u_{rr} = \frac{\partial^2 u}{\partial r^2}$, $\tau_{11\,t} = \frac{\partial \tau_{11}}{\partial t}$, $\tau_{12\,t} = \frac{\partial \tau_{12}}{\partial t}$, $\tau_{12\,r} = \frac{\partial \tau_{12}}{\partial r}$, etc.

Subject to boundary conditions

$$u(t,1) = 0 \text{ and } u_r(t,0) = 0, \text{ when } t > 0 \tag{14}$$

and initial conditions are $u(0,r) = \tau_{11}(0,r) = \tau_{12}(0,r) = 0$, when $0 < r < 1$ (15)

4.1 Study State Solution of viscoelastic flow in pipes through porous media.

At $\frac{\partial}{\partial t} = 0$, the systems of equations (13) takes the form as under

$$\left.\begin{aligned} &\mu_2 u_{rr} + {}^{\mu_2}\!/_{r}\, u_r + \tau_{12\,r} + {}^{\tau_{12}}\!/_{r} - {}^{1}\!/_{Da}\, u + 1 = 0 \\ &2We\ \tau_{12} u_r - \tau_{11} = 0 \\ &\mu_1 u_r - \tau_{12} = 0 \end{aligned}\right\} \tag{16}$$

Depending on boundary conditions $u(1) = 0$ and $u_r(0) = 0$, (17)

The numerically results of steady state solutions (16) subject matter to boundary conditions (17) is resolved adopting Mathematica' solver NDSolve are plotted in figures 7–9 at different values of Darcy's number *Da*.

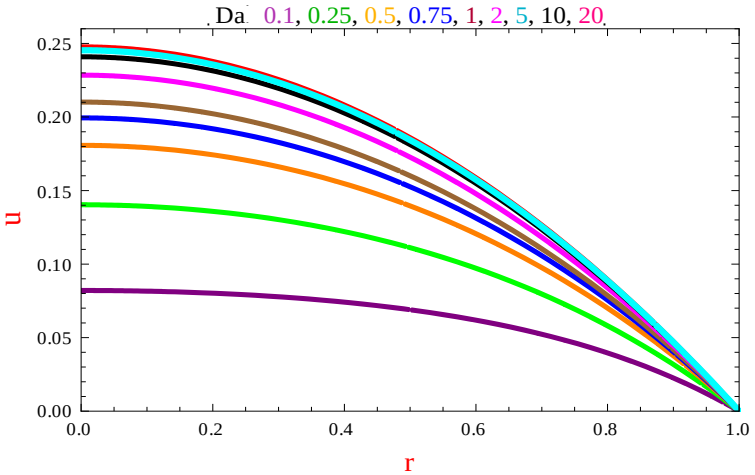
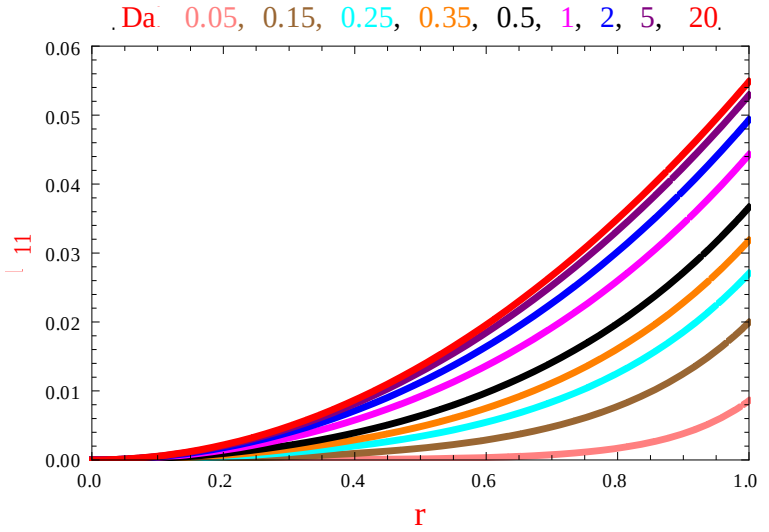


Figure 7: Steady State Solution of the Velocity *u* (19) at different values of *Da*.



Figure–8: Steady State Solution of the Normal Stress Component τ_{11} (19) with $\mu_1 = 1/9$, $We = 1$ and at Different Values of *Da*.

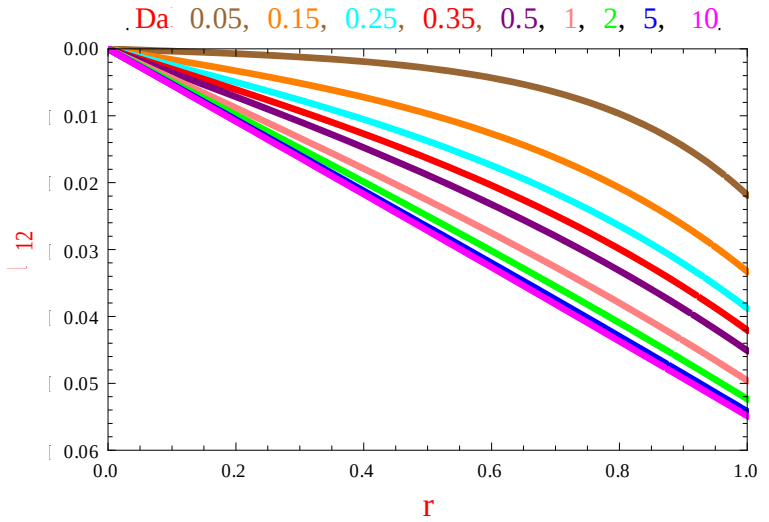


Figure 9: Steady State Solution of the Shear Stress Component τ_{12} (19) with $\mu_1 = 1/9$, and at Different Values of *Da*.

The results of steady state like velocity, components of normal stress and shear stress show in figures 7, 8 and 9 respectively. As the figures 7 and 8 illustrate in order to the velocity profile *u* and steady normal stress component τ_{11} at high Darcy's number (*Da*) whilst, as Darcy's number porosity of porous media (*Da*) decreases and flow resistant of

the fluid enlarges and here velocity u and component τ_{11} decreases within the steady condition and as figure–9 shows to build possible in the steady state, when pipe flow containing small values of *Da*, then component of shear stress τ_{12} contains large values so that if permeability decreases, then component τ_{12} increases in the steady state and finally there is no flow when Darcy's number come close to vanishing value.

4.2 Numerical solutions of viscoelastic fluid flow in pipes through porous media

For numerical solutions, the system of partial differential equations (14) subject to initial and boundary conditions (15 and 16) is solved through Mathematica solver NDSolve and solutions are designated in the figures–10 to 12 with increasing value of time.

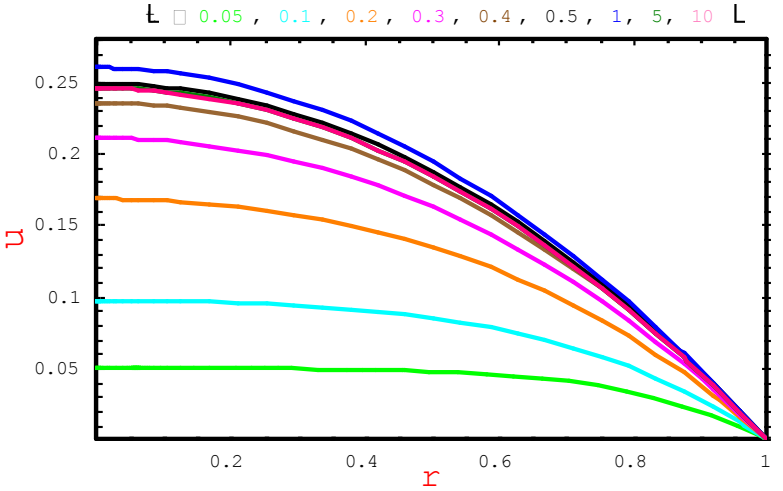


Figure 10: Numerical Solution of the Velocity *u* of System of PDE's (14) Subject to Conditions (15–16) with $Da = 10$, $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ **and at Different Values of Time t**

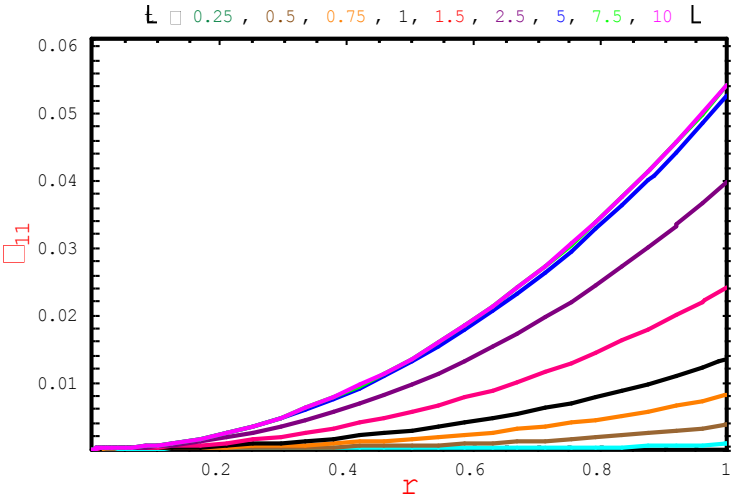
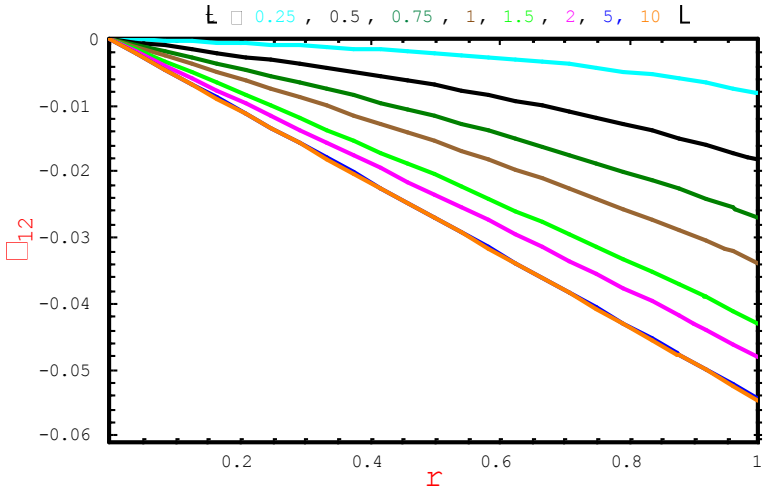


Figure 11: Numerical Solution of the Normal Stress Component τ_{11} of the System of PDE's (14) Subject to Conditions (15–16) with $Da = 10$, $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ and at Different Time t



Figure–12: Numerical Solution of the Shear Stress Component τ_{12} of the System of PDE's (14) Subject to Conditions (15–16) with $Da = 10$, $\mu_1 = 1/9$, $\mu_2 = 8/9$, $We = 1$ and at Different Time t

In Figure 10 to 12, the numerical solution of partial differential equation (14 to 16) for fixed parameters and at different values of time t is displayed. The results show in Figure 10, the velocity profile of pipe flow through porous media expands as time increases and arrives at the value of 0.26 as time come up to one unit and then eventually

flow of velocity profile simply decreases and achieves steady state whose maximum value is 0.245. Also, in the shape–11, the component of normal stress τ_{11} shows enlarges with respect to time *t* and reaches at maximum value of 0.0545 with similar non–linear fashion. While, within figure–12, the component of shear stress τ_{12} demonstrates linear propensity of increment in opposite direction and reach at the minimum value of –0.055.

5. CONCLUSIONS

The purpose of this paper was to make mathematical models and to find the numerical solutions of the problem arising in the study of viscoelastic fluid flow in a pipe filled with and without porous medium. Most vital work of this paper is represented in three jointly partial differential equations. The complexities and difficulties in the computational fluid dynamics problems, to obtain numerical solutions of conduct partial differential equations have been motivated to the research in this field. In our investigation, we concerned the basic equations, methods and literature survey with numerical solutions of viscoelastic fluid flow in a pipe by using Darcy–Brinkman model coupled with Oldroyd–B constitutive model. The numerical solutions of governing PDE's were solved by means of the important solver NDSOLVE in Mathematica. In the paper, firstly steady state solutions have been found in each case of the problem related with pipes subject to suitable boundary conditions. Also, the numerical calculations of the problem of PDE's systems subject to initial and boundary conditions are accomplished taking on computer by ND–Solver in Mathematica and solutions are designated in the figures with increasing value of time in each case with and without porous media in pipe.

Graph of all solutions of each case were given and discussed. Numerical calculations can provide some useful insights into the structure of solutions and sometimes may help to arrive at particular solutions in many cases. We hope that the results may be useful for other workers in the field.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 73-82

IMPROVED ROBUSTNESS OF RGB IMAGE CONTENT WATERMARKING USING RDWT-SVD DOMAIN

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ABSTRACT

Digital Image Watermarking (DIW) is a technique that is designed to protect the images from illegal manipulations. It is considered as one of the secured technique for copyright protection and ownership of the image. In this paper, invisible robust digital watermarking technique was proposed using Redundant Discrete Wavelet Transform (RDWT) and Singular Value Decomposition (SVD) in RGB color space. It protected the contents against different geometrical attacks (cropping, noise addition, median filtering etc) and provided better watermark concealment. The performance of this technique resulted in the improvement of the MSE (Mean Square Error), PSNR (Peak Signal-to-noise Ratio) and NC (normalized correlation) values.

KEYWORDS

Redundant Discrete Wavelet Transform, Singular Value Decomposition, Non-Blind Technique.

1. INTRODUCTION

With the rapid growth of the internet and the development of digital multimedia technologies, exchange and copy of digital multimedia have become quite convenient [1]. Hence, the copyright protection of digital multimedia has become an important issue. A technique to solve this problem is digital watermarking, which embeds directly some digitized information into digital media by making small modification to the media, where the watermark information remains detectable after attack. Thus the digital watermarking can be used to identify the rightful owner [2]. Furthermore, it is an important issue to develop a robust watermarking scheme with a better trade-off between robustness and imperceptibility. Recently many literatures have reported the watermarking schemes based on DWT [3]. Among these schemes, the one which require the original information and secret keys for the watermarking extraction are called private watermark schemes. Schemes which require the watermark information and secret keys are called semiprivate or semi-blind schemes. Schemes which need secret keys rather than the original information are called public or blind watermark schemes.

In general, the robustness of private watermark scheme is good to endure signal processing attacks. However, they are not feasible in real application, such as DVD copy

protection where the original information may not be available for watermark detection. On the other hand, semiblind and blind watermark schemes are more feasible in that situation. However, they have lower robustness than the private watermark schemes.

A new approach is proposed in this paper, where the watermarking method is based on the use of filter parameterized RDWT for hiding the information. In general, RDWT produce an over complete, over sampled expansion system which is used to embed watermark to the transformed coefficients. The filters used in the transformation process are designed using Pollen's filter parameterizing technique. The perceptual model is applied with stochastic approach for watermark embedding. This is based on computation of a noise visibility function (NVF) which has local image properties where the strength of watermarking is controlled. The result is that watermarks at texture and edge areas are stronger than flat areas

2. PREVIOUS WORK

Watermarking technology has a key role in the prevention of copyright infringement because it allows the user to hardly notice the watermark or sometimes totally perceptible. The technique is usually implemented on the major sub-parts of the host image, i.e. modification in low frequency band or subbands will results in the effective and robust image. The information is encoded in the low subbands. This helps in the progress of watermarking schemes that are to be embedded in the frequency domain. Most of the image transforms were measured like DCT (Discrete Cosine Transform), DHT (Discrete Hartley Transform). With the help of the homogeny process of JPEG2000, Moving from DCT to DWT domain, it becomes more mesmerizing to perform watermarking and make the image more robust rather than DCT. A translucent and robust watermark should be such that the watermark is perceptible in the original image.

It is fascinating approach that helps for tracing copyright infringements and for authentication. Currently, digital watermarking has wide area of research and has different applications such as broadcast supervision, owner identification (proofs of own image), data validation, help to protect from copying, and file renovation. Watermark images can be embedded in both spatial domain and frequency domain, but it is describable that spatial domain is less stout to attacks rather than in frequency domain. Different techniques works in frequency domain in which the image can be transformed using DCT, DWT, DFT and the frequency coefficients embeds the watermarks image onto the transformed image. Addition of the watermark to different subbands of the host image, the quality should be enhanced and by adding the qualitative value of noise in those bands outranged by the decomposition of the image, the watermarked image quality will remain preserved in very efficient manner. It is highly-adaptive way, which will increase the robustness of the image even after applying some kinds of distortions i.e. image solidity, geometric attacks, and noise accumulation. The discrete wavelet transform (DWT) converts a signal into low (L) and high (H) frequency sub- bands. The image can be partitioned by performing a DWT in both vertical and horizontal directions, that will result into one low frequency sub-band (LL) and three high frequency sub-bands (LH, HL, and HH).

Similarly, a few wavelet-based algorithm were intended for the embedding of the watermark signal into the lower level sub-bands. An important way to protect rational properties of digital media is digital watermarking. Digital watermarking figure-out the problem of how to protect copyright. And it is the key for the shelter of permissible rights of digital content owner and customer.

2.1 Discrete Wavelet Transform:

Wavelet Transform is a contemporary technique normally used in digital image processing, solidity attacks, watermarking etc. Transforms on small waves, called wavelet are basically of changeable frequency and limited duration. A wavelet series is a depiction of a four-sided figure integral function by a certain ortho-normal sequence generated by a wavelet.

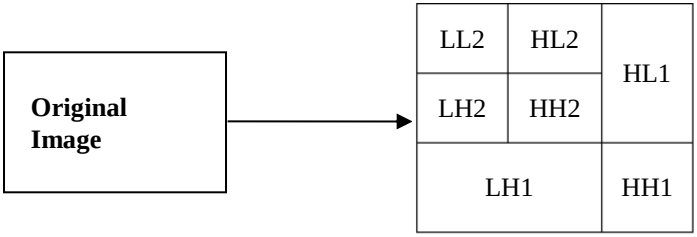

Fig. 1: Co-Efficient in Sub Bands

2.2 Redundant Discrete Wavelet Transform

The shift variant characteristic of DWT implies the lack of samples during reconstruction. Unlike DWT, RDWT gives an over complete representation of the input sequence and functions as a better approximation to the continuous wavelet transform. The RDWT is shift invariant, and its redundancy introduces an over complete frame expansion. It is known that frame expansion increases the robustness to additive noise, that is, addition of noise to transform coefficient results in less signal distortion for frame expansions than for orthogonal expansion [6]. RDWT has been proposed for signal detection and enhancement, since the RDWT maintains uniform sampling rate in time domain and in some respects, is a discrete approximation to the continuous wavelet transform. RDWT removes the down-sampling operation from the traditional critically sampled DWT

2.3 Wavelet-Based Watermarking Schemes

Basically the overview of wavelet based watermarking schemes , the popularly accommodated algorithms is taken into a general gibbet by dissecting the algorithms into common practical modes and hence calculating a decisive embedding form that is as follows:

$$I'_{m,n} = I_{m,n} + \Delta_{m,n,} \text{ --------} \quad [1]$$

where

$I'_{m,n}$ is the modified coefficient at (m,n) position, $I_{m,n}$ is the coefficient to be modified and $\Delta_{m,n}$ is the modification due to watermark embedding. These modification algorithms were broadly categorized into two groups.

- Direct modification
- Quantization based modification.

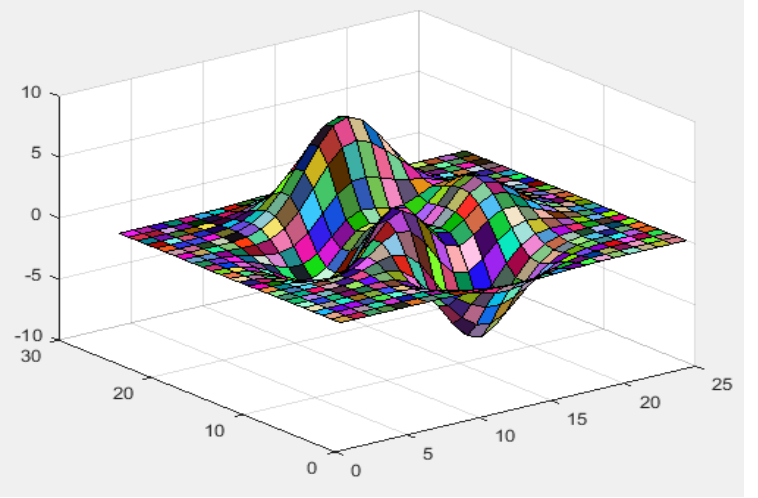

Figure 2: Mesh Chart

Watermarking algorithms are also divided into two categories.

- Blind watermarking
- Non-Blind Watermarking

Because of the nature of embedding algorithms, most of the direct modification algorithms are laid in non-blind category whereas most of the quantization based schemes are referred as blind.

3. EXPERIMENTAL RESULTS

The experimentations describes that the watermarked image are liable towards this proposed scheme against common image processing attacks. 512x512 RGB and 256 x256 RGB images are directly used that are used as cover image and watermark image correspondingly. Different images are shown as follows:

3.1 Watermark Embedding Process

Watermark image is firstly embedded into the host or cover image. By using alpha blending technique, image is embedded into host and makes it invisible for the users to detect the presence of the watermark. Given image dimension is 512x512 RGB for both host image and image to be watermarked.

Input Color Image

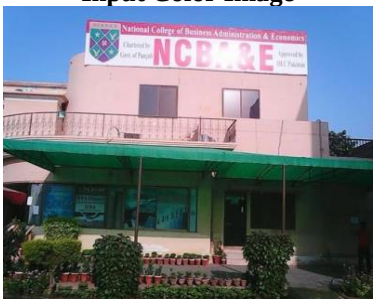

Fig 3: a)
Original Color Input Image

Watermark Color Image

Fig 3: b)
Watermark Image

Watermarked Image. JPG

Fig 4: Embedded Watermarked Image

Input Image

Watermarked Image

Fig 5: Comparison of Input Color Image and Watermarked Image

Table 1
Watermarking Embedding Process

	DWT	DCT	SVD	DCT-DWT	DWT-SVD	PROPOSED
PSNR_RED	24.0712	24.727	25.615	28.969	28.6238	35.1480
PSNR_GREEN	22.1091	25.838	26.176	29.284	27.5755	35.6223
PSNR_BLUE	27.9651	27.688	28.222	31.376	27.2327	35.1988
NC_RED	0.9724	0.7107	0.9115	0.9596	0.9984	0.9989
NC_GREEN	0.9539	0.9384	0.9924	0.9785	0.9980	0.9988
NC_BLUE	0.9738	0.8861	0.9994	0.9341	0.9985	0.9992

Watermark Extraction Process:

Watermark image can be extracted using original host or cover image by excluding the alpha blending values (scale factors).

Input Image

Fig 6: a)
Original Color Input Image

Watermarked Image

Fig 6: b)
Watermarked Image

Extracted Watermark Image

Fig 7: Extracted Watermark Image

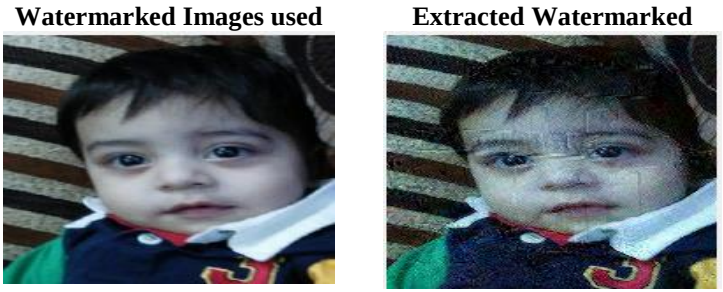

Fig. 8: Comparison of Watermark Color Image and Extracted Watermark Image

Table 2
Watermarking Extraction Process

	DWT	DCT	SVD	DCT-DWT	DWT-SVD	PROPOSED
PSNR_RED	21.9473	22.919	22.944	23.324	24.2229	22.7668
PSNR_GREEN	21.3326	23.344	23.388	29.011	31.3539	24.9857
PSNR_BLUE	24.7136	23.331	23.424	23.616	29.0073	24.7378
NC_RED	0.9522	0.7207	0.9955	0.8942	0.9970	0.9653
NC_GREEN	0.9946	0.7870	0.9980	0.9287	0.9975	0.9764
NC_BLUE	0.9758	0.7742	0.9978	0.9219	0.9957	0.9752

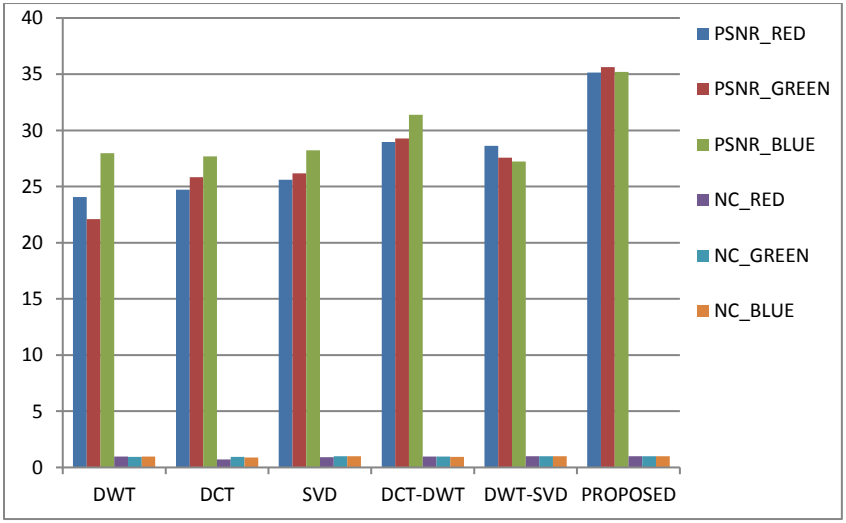

Fig 9: Comparison of the Proposed Scheme to Earlier Used Techniques during Embedding Process

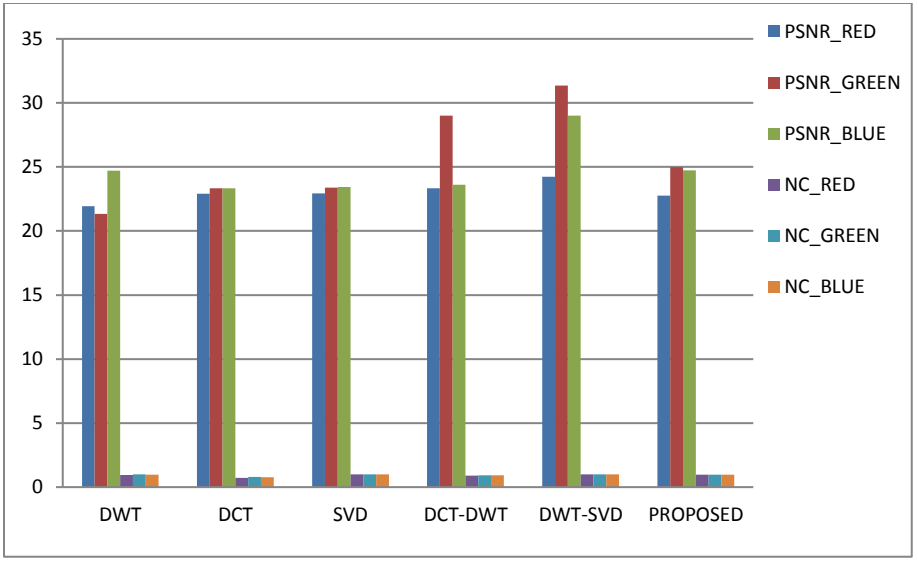

Fig 10: Comparison of the Proposed Scheme to Earlier Used Techniques during Extraction Process

4. COMMENTS AND CONCLUSION

A robust watermarking using RDWT and is presented in this paper. The results obtained are compared with existing Daubechies filter to evaluate the performance of the proposed scheme. From the results, it is inferred that imperceptibility and robustness of the proposed scheme is improved in terms of high PSNR values when compared to the conventional existing scheme. The proposed algorithm is adaptive by implementing NVF, in which the watermark in texture and edges are stronger in flat areas.

5. ACKNOWLEDGEMENT

The completion of this undertaking would not be possible without participation and inspiration of so many people whose names all may not be enumerated. Their contributions are greatly appreciated and sincerely acknowledged. However I would like to express my deep appreciation to the Directorate of Research Centre of NATIONAL COLLEGE OF BUSINESS AND ECONOMICS Multan, for their endless support i.e technical and financial support.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 83-90

ROBUSTNESS OF IMAGE CONTENT USING LWT-SVD BASED WATERMARKING TECHNIQUE

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ABSTRACT

The perceptual quality and robustness of RGB image was evaluated using Lifting Wavelet Transform- Singular Value Decomposition (LWT-SVD) technique. Watermark was separated into RGB channels and scaled. Low band (LL) is decomposed from original image by using LWT. SVD was operated on each channel of RBG image and scaled watermark was embedded in it. Real time application is the basic advantage of LWT as it uses the properties of spatial domain while SVD uses non-invertible property for watermarking so that fake watermarked image be stopped. Outcomes of Peak Signal to Noise Ratio (PSNR) and Normalized Correlation (NC) seems satisfactory against geometric attacks.

KEYWORDS

LWT, SVD, PSNR, NC.

1. INTRODUCTION

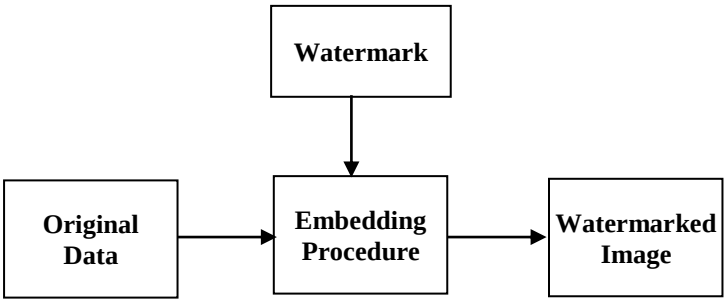
Over the past two decades, there has been a tremendous growth in computer networks and more specifically, the World Wide Web. This along with the exceptional increase in Computer performance has facilitated extensive distribution of multimedia data such as images over the internet quite easily. Publishers, artists, and photographers, however, may be unwilling to distribute pictures over the Internet due to a lack of security, images can be easily duplicated and distributed without the owner's consent. Digital watermarks have been proposed as a way to tackle this tough issue. In a traditional watermarking the watermark is intended to be visible due to identifying the manufacturing attacks.

The digital watermarking is intended to be imperceptible in the noise tolerant signal of image, audio and video file in such way that the host data does not distort and also provides a possible solution to the problem of easy editing and duplication of images, since it makes possible to identify the author of an image by embedding secret information in it [2]. Digital watermarking need to be within an acceptable limit and robust when different types of processing is applied in digital content namely adding noise, cropping, compression, resizing [1].

Ideal properties of a digital watermark:

1. A digital watermark should be imperceptible, meaning that it should be perceptually invisible to prevent obstruction of the original image.
2. Watermarks should be robust to filtering, additive noise, compression, cropping and other forms of image manipulations.

BLOCK DIAGRAM FOR WATERMARKING



Watermarking Scheme

Watermarks can be categorized into blind, semi-blind and non-blind schemes[15].

There are mainly two general methods for watermarking which are commonly known as spatial domain technique and frequency domain technique. Insert the watermark in spatial domain done by direct change the brightness values of pixels through methods like LSB. Watermark in frequency domain inserted in coefficients of transformation domain [4][20]. The Sample Transformations of this domain can be noted to discrete wavelet transformation (DWT), discrete cosine transformation (DCT), Discrete Fourier transformation (DFT), integer wavelet domain (IWT) [14]. The obtained frequency component is then modified to hide the watermark. The RGB, YIQ and YUV color space are used for embedding watermark in an host image. LWT which is the second generation fast wavelet transform is a substitute method for DWT to transform images into the transform domain for real time applications. In lifting wavelet transformation, up sampling and down sampling are replaced simply by split and merge in each of the levels. Split and merge process in LWT reduces computational complexity to 50%. Information loss is less as compared to DWT algorithm, because in LWT based algorithm up sampling and down sampling have not been used. The odd poly-phase and even poly-phase components of the signal are filtered in a

Specific parallel process by using the corresponding wavelet filter coefficients, producing the better result compared to up sampling and down sampling which is required in the traditional DWT approach. In comparison with general wavelets, reconstruction of images by lifting

Wavelet is a good idea because it increases smoothness and reduces aliasing effects [13]. Employing LWT reduces information loss, increases intactness of embedded watermark in the image and helps to increase the robustness of watermark. Lifting

wavelet transform also provides several advantages [14, 15] such as less memory requirements, reduced distortion and aliasing effects, good reconstruction, less computation and

Computational complexities. The lifting Wavelet Transformation (LWT) function is combined with the Singular Value Decomposition (SVD) for embedding the watermark in the YUV color space.

In the proposed method, the R planes of the watermark is converted using thresholds. Then combined with SVD and LWT to embed data in frequency domain of cover image.

2. RELATED WORK

Some researchers have used DCT, DWT and SVD technique on the R, G and B components of a host and watermark image. The watermarking technique used with SVD overcomes the weakness that was found in other methods [4]. The technique of converting the RGB color components first into the YIQ color component and then embedding the watermark image into Y and Q color space is shown by Sun and Yu[5].

The R, G and B planes of host image and watermark image are separated first. Then the R plane is used for embedding. A threshold technique is applied to R planes of watermark image and then 2D-LWT technique to R planes of cover image to decompose into four band of frequency namely LL, LH, HL and HH. The LL3 (Approximation Coefficient) of fourth level decomposition increases the PNSR by reducing the effect of noise on the cover image. SVD technique is applied on LL3 band of original image and R planes of watermark image. The equation for embedding is:

$$Swmi = Sori + Swm \quad \ldots\ldots (1)$$

Apply inverse SVD and inverse LWT to obtain the customized band R. Then construct watermarked image. The embedding procedure is shown through the Figure 1.

The watermark and the cover picture are extracted applying the reverse process on the watermarked image. The extraction procedure is described below:

Separate *Rwmi*, *Gwmi* and *Bwmi* planes from the watermarked image. Decompose *Rwmi* planes four times to receive *WLL3* band. Apply SVD on WLL3 band and perform the equation SEWM = SWMI - SORI to receive watermark. The SEWM is the extracted watermark. SWMI has come from watermarked image and SORI from cover image. Apply inversed SVD, inversed LWT and threshold technique to re-construct the watermark image.

Fig. a: Host image

b: Watermark Image

c: Embedded Watermark Image

d: Extracted Watermark Image

4. COMMENTS AND CONCLUSION

In our experiment we have used a watermark to embed into a cover image to construct the watermarked image. Figure-3 shows the watermarked image derived with Cover image and watermark image. The performance of the proposed algorithm is measured through the obtained values. The peak signal to noise ratio (PSNR) and normalized correlation (NC) are used as the performance criteria. The table-1 describes PSNR values between watermarked image and attacked image. It also demonstrates the NC values of original watermark and the extracted watermark from attacked image. The PNSR value shows the intensity of noise added on the watermarked image through different types of noise attacks. The lower is PSNR, the higher is the noise added in watermarked image. The higher is normalized correlation (NC) the better is the similarity between original and extracted watermark image.

EMBEDDING VALUES

PSNR and NC VALUES	DCT-DWT	DWT-SVD	Proposed LWT-SVD
PSNR_RED	28.969	28.6238	35.1493
PSNR_GREEN	29.284	27.5755	35.6209
PSNR_BLUE	31.376	27.2327	35.202
NC_RED	0.9596	0.9984	0.9989
NC_GREEN	0.9785	0.998	0.9988
NC_BLUE	0.9341	0.9985	0.9992

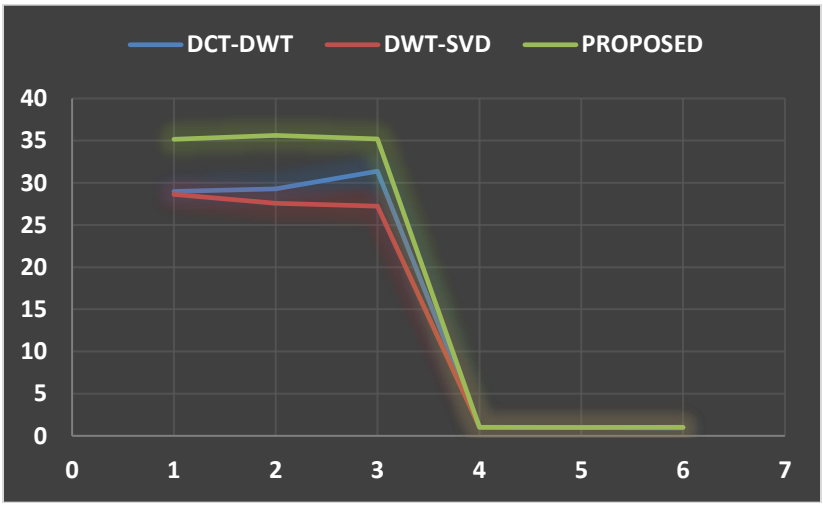


Fig. 1: Comparison of Proposed Method

EXTRACTION VALUES

PSNR and NC Values	DCT-DWT	DWT-SVD	PROPOSED
PSNR_RED	23.324	24.2229	24.7614
PSNR_GREEN	29.011	31.3539	29.9858
PSNR_BLUE	23.616	29.0073	29.7133
NC_RED	0.8942	0.997	0.9653
NC_GREEN	0.9287	0.9975	0.9764
NC_BLUE	0.9219	0.9957	0.9752

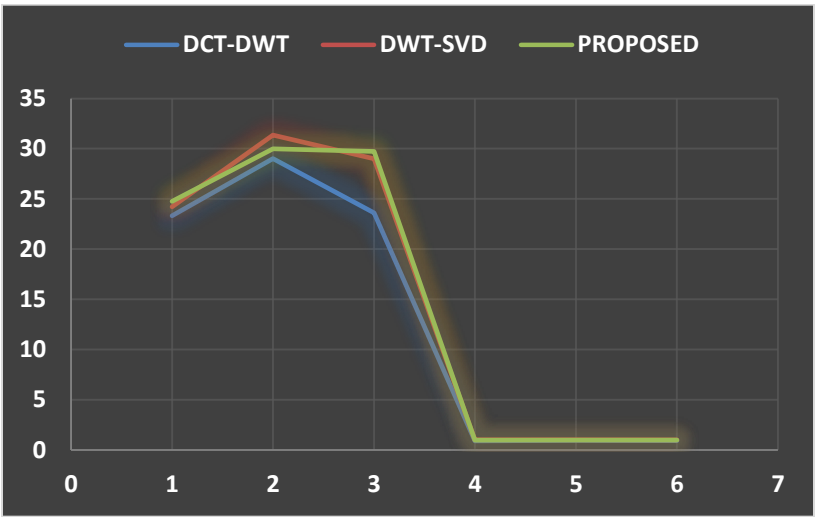


FIG 2: Comparison of Proposed Method

5. ACKNOWLEDGEMENT

The completion of this undertaking would not be possible without participation inpiration of so many people whose names all may not be enumerated. Their contribution are greatly appreciated and sincerely acknowledge. However I would like to express my deep appreciation to the Directorate of Research Centre of National College of Business and Economics Multan, for their endless support, i.e technical and financial support.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 91-98

INVESTIGATE THE OZONE LAYER DEPLETION FROM THE PERSPECTIVE OF FRACTAL DIMENSION

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ABSTRACT

We intend to analyze the nonlinear stratospheric ozone layer dynamics that is an important issue for predicting and forecasting the concentration to verify the alteration in the concentration of ozone and its influence on the biosphere (marine life, agriculture, and human being) for our region.

In this paper, we used monthly time series data of stratospheric ozone over Pakistan region from 1970 to 2013 and applied fractal dimension associated with Hurst exponent process. Fractal dimension (FB) is space filing ability of the curve and indexing of measuring rigidness of the Hurst exponent in the measure of classifying the time even. Thus, we can find the behavior of the ozone in this region by calculating fractal dimension using Hurst exponent and prove rescaled range analysis.

Results obtained from this research have shown that the Hurst exponents value of monthly time series data of stratospheric ozone are anti-persistent i.e. deceasing trend in time series and negatively correlated.

This study will be useful to the researchers working in the same field to verify the complex nature of stratospheric ozone.

KEY WORDS

Nonlinear dynamic, Fractal dimension, Hurst exponent, rescaled range analysis,

1. INTRODUCTION

Since, Ozone (O_3) is a minor constituent in the atmosphere that has considerable importance to human being due to its role as a shield over the biosphere against toxic incoming ultraviolet radiation from the sun and outer space [1] . Furthermore, ozone may have essential links to weather and climate. Events of previous decades have made a prevalent consciousness of the importance of ozone from overall new perception specially the possibility that anthropogenic actions may pollutant the stratosphere and there by extensively perturb the ozone layer [2]. The stratospheric ozone layerplaysa crucial role in terrestrial life mostly in climatic variation and being very important element for sustaining life in the vicinity of the earth environment by absorbing havoc UV radiation within the stratosphere [1,3]. Ozone is existed initially due to chemical process that is dissociation of O_2 by solar UV radiation[4]. The dynamics of stratospheric

O_3 is a universal issue of the day because of its nature to shield ecological life from a range of UV radiation (Jan et al., 2014, Whitten and Prasad, 1985) [1, 2] and the dynamic of ozone over Pakistan region has been studied by several researchers [5, 6, 7].

Fractal analysis plays a distinctive role on a broad range of natural events that is it displays' self-similarity' which emphasize that the broad-spectrum of the object is usually repetitive at randomly smaller and smaller levels[8,9].The FB present show much intricacy is being continual at each level. A fractal nature with a high intricacy is much complex or 'rough' as compared to a lesser dimension and fills much space [10]. The Fractal geometry applies in different areas i.e. Space Weather (Sunspot or flares), Climate Change, Biological Science, Quality Control System, Population Dynamics, Astronomy, Rbeology, Economics, Industry and Engineering [11].

Mandelbrot and Van (1968) used Hurst exponent approach for FB and this methodology has practical application in time series data. Kassem [12] applied it to investigate persistency in time series data of hourly and monthly surface ozone concentrations in Egypt on perspective of Rescaled range analysis. Sachithananthem [9] predicted behavior of ground O_3 level concentration as random series in south Chennai based on fractal and Hurst exponent. Suresh Kannan et al. [13] studied the concept of fractal analysis, Hurst exponent on time series data of Ozone. He observed that the total column of O_3 over tropical region shown persistence behavior.

In this paper, an attempt has been made to examine complex nature of time series data of stratospheric O_3 in terms of nonlinear dynamics over Pakistan atmospheric region by applying fractal dimension. This study distributed in two sections where first section described time series analysis with basic statistics and second section fractal analysis.

2. DATA AND METHOD OF ANALYSIS

The monthly time series data of O_3 from 1970 to 2013 for Pakistan region were taken from http://woudc.org/home.php World Ozone and Ultraviolet Radiation Data Centre (WOUDC) and Pakistan Meteorological Department (PMD) Quetta Centre.

2.1 Estimation of Hurst Exponent and Fractal Dimension

In 1951, Hurst studied first time the well-known technique Hurst exponent concept by means of rescaled range analysis (R/S). This estimates the cumulative series of time series data that is the deviations of data from its mean [14]. Many researchers [8, 12, 13] used to compute Hurst exponent of time series data by the Rescaled range analysis procedure.

2.1.1 Resealed Range Analysis

Consider the time series $Z = \{z_1.z_2, z_3, ..., z_N\}$ with complete size N such that N classifies further small samples (n) so that the series become accordingly as $N, \frac{N}{2}, \frac{N}{4}, \frac{N}{8}, ...$ and so on. The cumulative series $\left(y(n)\right)$ (deviation of the time series Z_i from its mean $\bar{Z}$) can be proposed for each of value of n. The procedure can be calculated as follow,

Step: 1

Computed mean ($\bar{Z}$) and adjusted cumulative series $y(n)$ for $i = 1,2,3,\ldots,n$,

$y(n) = \sum_{i=1}^{N}[z_i - \bar{z}]$, where Z_i= original time series data, $\bar{Z}$ = mean and N is discrete time.

Step: 2

Find range (R)

The range (R) depends on N and generally increased by rising N i.e.

$R(N) = Max.y(N) - \min.y(N)$

Step: 3

Calculated Standard Deviation (S) as follow

$$S(N) = \sqrt{\frac{\sum_{i=1}^{N}[z_i - \bar{z}]^2}{N-1}}$$

Step: 4

The Rescaled range is then computes from Step: 2 and 3

$$R/S = \frac{\max.y(N) - \min.y(N)}{\sqrt{\frac{\sum_{i=1}^{N}[z_i - \bar{z}]^2}{N-1}}}$$

Step: 5

Compute Hurst exponents (H)

Suppose that the range (R) of given time series data dependent on a sequence of arbitrary variables that has fixed standard deviation (S) and both R and S are independent, then the ratio R/S fallow power law

$$R/S \alpha (N)^H$$

Applying log on both side, we have

$$H = \frac{\log_2(R/S)}{\log_2(N)}$$

By applying least square procedure against $\log_2$ (R/S) vs. $\log_2$ (N) where the slop of the best fitted curve determined value of H [13].

The procedure is repeated for whole samples (n) over the time series data and diving each sample interval by two and determining R/S for each sample (n) as shown in Fig. 1

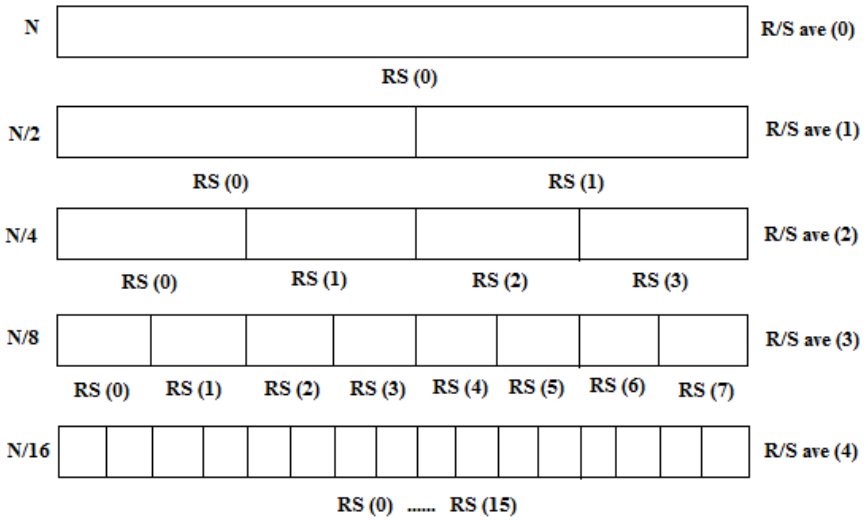

Fig. 1: Estimation of Rescaled Range Analysis Method to Computing Hurst Exponent (H)

2.1.2 Estimation Fractal Dimension (D)

Fractal dimension is a statistical measurement of roughness of an object [15]. The fractal dimension (D) associated with Hurst exponent (H) as follow

$$D = 2 - H \quad \ldots\ldots\ldots\ldots \quad (1)$$

where, D stand for fractal dimension, H represent Hurst exponent while 2 stand for Euclidean dimension. Fractal dimension and H characterized the dynamical behavior of the time series data[14].The H is restricted to the range $0 \leq H \leq 1$; this compares nature of the time series data[10]. When the value of $H > 0.5$ ($D<1.5$), we have a persistent time series (positive correlation), if the value of $H < 0.5$ ($D >1.5$), the process is considered as anti- persistent (negative correlation) time series and if $H=0.5$ ($D=1.5$) indicates that the system is "random walk" i.e. the time series is independent and uncorrelated. Furthermore, as the value of H moves towards zero, it implies that the irregularity of the system rose up [8, 13, 14].

3. RESULTS AND DISCUSSION

Since, the aim of the study focused on the nonlinear dynamics of Ozone level over stratospheric region of Pakistan, this section introduces brief description of monthly stratospheric ozone variations and its dynamics using statistical test in defined area during the period from 1970 to 2013. The monthly variation of stratospheric ozone in Pakistan region revealed in Fig. 2.

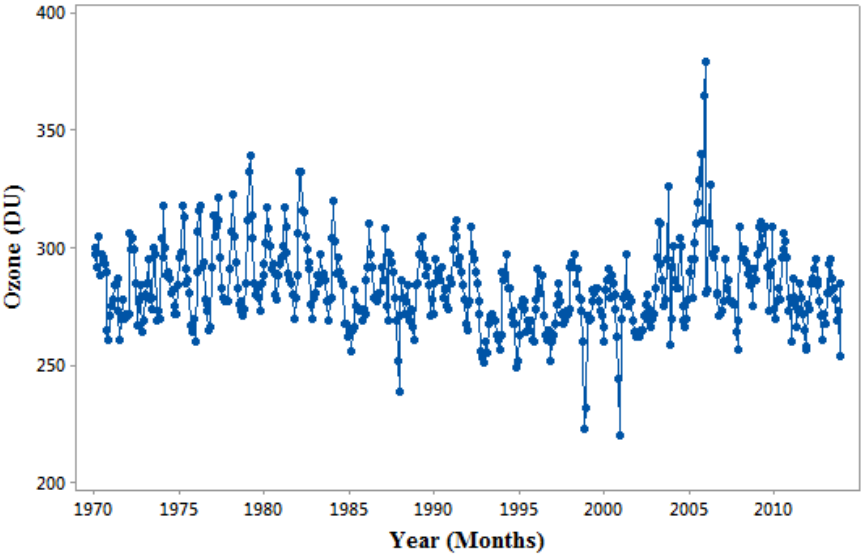

Fig. 2: Plot of the Monthly Average time Series Pattern of Stratospheric Ozone at Pakistan Region from 1970-2013

The maximum ozone concentration recorded 379 DU and minimum 220 DU as indicated in summary Table 1. Skewness is an indicator applied in distribution analysis as a sign of irregularity and deviation from normal distribution [8].The positive skewness indicates a sign of right skewed allocation of ozone concentration. This shows that mainly of values is determined on left of the mean with extreme values. Kurtosis illustrates the vertical peakedness or smoothness of a distribution compared with the normal distribution. The positive kurtosis in this case as shown in table: 1 that indicated the data a quite fat-tailed which is one of the sign of fractal brown nature [16] and representing more intermittency in ozone level over Pakistan air space. The variance is also quite irregular and large.

Table 1
Statistical Summary of Average Monthly Stratospheric Ozone (O_3)
at Pakistan Region from the year 1970-2013

Statistic	Value
Mean	283.428
Standard Error	0.751
Median	281
St. Deviation	17.250
Sample Variance	297.565
Kurtosis	3.0172
Skewness	0.731
Minimum	220
Maximum	379
Count	528

In this section intricacy of ozone concentration have been discussed over considered region using the rescaled range analysis which applied independently on the average monthly time series stratospheric ozone data over the period from 1970 to 2013. The procedure of rescaled range analysis for different samples of ozone concentrations in the study period are given in Table: 2. The slope of the best fitted line to the curve of $\log_2(R/S) vs \log_2(N)$ gives the estimate of Hurst exponent as indicated in Fig. 3 and obtained value of Hurst exponent is H=0.2031 that gives the fractal dimension value from equation (1) is D= 1.7969.

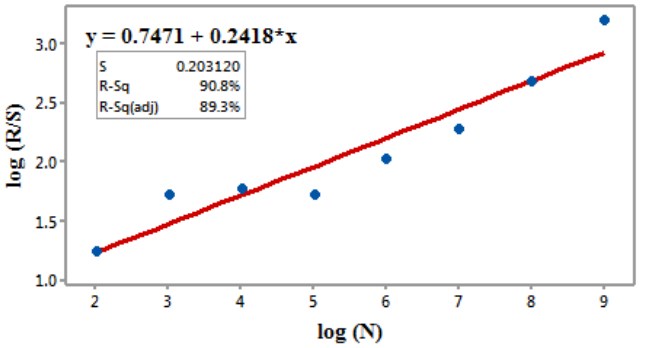

Fig. 3: The Least Square Procedure for Ozone Concentration for Pakistan Atmospheric Where the Slope of Fitted Line Estimates the Hurst Exponent for the Year 1970-2013

This revealed that as the value of H is less than 0.5 and fractal value D >1.5, the result emphasized that the ozone concentration has shown anti-persistence behavior i.e. negative correlation with time.

Table 2
The Detail Procedure of Rescaled Range Analysis for Ozone Concentration Over Pakistan Atmospheric Region to Estimate Hurst for Year 1970-2013

Level	Data size (N)	R/S	$log_2(N)$	$log_2(R/S)$
1	512	9.15	9	3.1938
2	256	6.40	8	2.6781
3	128	4.83	7	2.2720
4	64	4.07	6	2.0250
5	32	3.30	5	1.7225
6	16	3.42	4	1.7739
7	8	3.28	3	1.7137
8	4	2.38	2	1.2376

4. CONCLUSION

To analyze the nonlinear stratospheric ozone dynamics over Pakistan region, time series analysis with basic statistics, fractal analysis and Hurst exponent methodology has been performed using rescaled range analysis. Several year data of stratospheric ozone from 1970-2013 are used. The time series analysis and their basic statistics described the fluctuation of ozone thickness and intermittency over the supposed region.

Finally, considering the debate presented above, the fluctuation in ozone thickness in the stratospheric region of Pakistan have been investigated in terms of the fractal analysis. The results obtained have shown that all data set of stratospheric ozone concentration have anti-persistent behavior (negative correlation). The Hurst exponent results obtained from ozone time series is a valuable numerical estimation that can used to illustrate the time series behavior of the randomness of ozone in different atmosphere.

The basic objective of this research was to identify the fractal nature, extracted through Hurst exponent using Rescaled range analysis for time series of monthly ozone data in the stratospheric region of Pakistan. In future, this study may be extended to the investigation of the natural complexity of the stratospheric ozone by correlation analysis and multi-fractal techniques.

5. ACKNOWLEDGMENT

We are thankful to World Ozone and Ultraviolet Radiation Data Centre (WOUDC) and Pakistan Meteorological Department (PMD) Quetta Centre for providing us important ozone data.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 99-107

MONTE CARLO SIMULATIONS OF FINANCIAL TIME SERIES MODELS

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ABSTRACT

In finance it is often required to find the proposed out of sample data streaming for the risk analysis of a financial time series. Most of the time financial time series are characterized by long memory, skewed heavy tailed and mean reverting. In order to capture such anomalies mostly nonlinear and auto regressive type models are used for financial time series. We applied Monte Carlo simulation techniques to study the characteristics of Pakistan equity market.

1. INTRODUCTION

Monte Carlo simulation is a procedure involving repeated simulation of a system with random sampling from specific probability distributions of real life phenomenon.

This method is often very suitable for big data mining and complex data streaming processes to find relative solutions to real life situations. The method is now widely used in financial engineering applications, risk analysis and profit at risk. The viability of results are achieved by large number of iterations. The valuation of multidimensional options, path-dependent options, and stochastic volatility or interest rate options can be tackled by Monte Carlo simulations. Most of the time this method is used when numerical and analytical methods are not possible or they are so complex to model and analysis. This method provides simulation based simple iterative solution to the problem (Cornelia, 2004).

We have used MC simulation technique to simulate Pakistan stock market Index for one week ahead based on the large numbers of iterations generated during simulation process. The trace plots and the Auto correlation plots are also given to characterize the Pakistan stock market Index. We follow the methodology proposed by (Dagpunar, 2007).

2. LITERATURE REVIEW

(Daniel E Rush, 2015) used Monte Carlo method to simulate the information system investment in a global context. They used simulation methods to find the relationship between international financial events relevant to US firms for financial information system investment.

(Arnold & Yildiz, 2015) perform Monte Carlo simulation to measure the renewable energy project risk. They conclude that Monte Carlo simulations provide methodological advantage over classical historical measures. Further the technique allows several fuzzy or uncertain input parameters for simultaneous analysis. The probabilities of events with

in confidence intervals can be determined easily. Simulation technique gives investors to evaluate risk-return ratio more accurately, also can simulate the design and risks associated with the investment projects.

(Zhenxi & Lux, 2015) proposed a simulated methods of moments approach to estimate financial indices and prices. They empirically test model to artificially simulate foreign exchange rates, gold prices, Stock market indices like German DAX, S&P 500 and Japanese Nikkei. They tried to generate various scenario based simulated results by simulated Methods of Moments (SMM). They further conclude that various scenario basically provide similar insight due to some inherent nature of the Noise ratio in simulations, however according to asymptotic theory more data will be required than typically available for financial markets. But the versatility of simulated estimators cannot be ignored in financial application.

(Neaime, 2015) studied the mean reversion in MENA stock markets through Monte Carlo simulations. By assuming uniform distribution prevailing in all stock markets they rejected the null hypothesis in favor of mean reversion. Trading strategies based on historical information prove higher earnings in MENA rather than simulation studies. Further the information in efficiency is also a cause of mean reversion.

(Tobias, Peter, Wokfgang, & Johannes J, 2009) Proposed a GPU based Monte Carlo simulations of the Ising model was used to simulate set of linear congruential random numbers on GPU device. For the two dimensional Ising model, results on the GPU can be obtained up to 60 times faster than on a current CPU core. An implementation of a three dimensional Ising model on a GPU is able to generate results up to 35 times faster than on a current CPU core.

(C & W.B, 2002) provide a detailed analysis of the stationary state and the parameter sensitivity of a trader based market model. The Monte Carlo simulations provide various market dynamics of the trader based model, like regime transitions, stationary dynamics and the stability of convergence. They also explain the volatility feedbacks with in certain ranges of asymmetries and updating of price outlooks in market. Their simulation studies also provide insight into volume generated by traders and the allocated budget for each trader in the market.

(Castiglione, R.B, & D.Stauffer, 2001) have presented a Monte Carlo simulation model that has some element of sampling in statistical physics, i.e., the distribution for the Glauber-like dynamics. It provides reasonably the price evolution and shows a power-law distribution of the tail of the price change distribution. There is a strong potential to develop this model further to make it more realistic and, bring physics and finance closer, as one of our on-going efforts.

3. METHODOLOGY

Monte Carlo simulation is simple Brownian motion application. This process for stock market follows the following equation

$$\ln\left(\frac{S_t}{S_{t-1}}\right) \sim \phi\left[\mu - \sigma^2/2\right]T, \sigma\sqrt{T}$$

The Natural Log of today's price is divided by the yesterday's price, which is continuously compounded periodic return. This continuous compounded frequency is approximately distributed with mean of drift minus variance over time, and volatility multiplied by the square root of time or in other words the volatility is scaled with the square root of time. The continuous periodic return is approximately normally distributed. The price level are log normal to make the return normally distributed. So the Brownian motion is a log normal distribution process. The equation 1 can also be written as

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = \alpha + Z_t\sigma$$

Here the periodic log return is equal to two components. One deterministic component and other is stochastic component. The deterministic component is drift which states some positive expected return over time and fixed. Other is the random shock which is a function of volatility multiplied by the Z_t, which is a random variable. The random variable is scaled by the volatility of the stock, which allows us to model the stochastic process.

In order to calculate the stochastic process we have to account for the following variables

Transaction Cost:

These variables constitute the drift component of the Brownian motion. It has the two forms one is annual and other is daily which is calculated by the number of trading days in a year. In our case of Pakistan stock market it constitutes the two hundred and fifty two days (252).

3.1 Volatility

According to Poon, 2005 the volatility is defined as the spread of all likely outcomes of an uncertain variable specifically in finance it is related to the spread of an asset returns. It is measured as

$$\hat{\sigma} = \sqrt{1/T\sum_{t=1}^{T}(r_t - \mu)^2}$$

Where r is the return on day t and u is the average return over the T days period.

The volatility is again used in two forms first is annual and the second is daily which is measured by dividing annual volatility by 252 trading days.

3.2 The Procedural Algorithm

There are the three parts of the technique

a) Monte Carlo
b) Markov Chain
c) Metropolis Hasting

3.2.1 Monte Carlo

The term Monte Carlo refers to the generation of random numbers as $Z_t \sim N(\mu,\sigma)$ if we assume to generate a series of random numbers from normal distribution with a given mean and some arbitrary variance.

We refer the distribution to be the proposal distribution; the density plot is the histogram of the samples being generated.

If we were to generate 10000 samples our density plot looks like our proposal distribution. Here we also assume that the proposal distribution does not chanting from our iteration in the next.

3.2.2 Markov Chain

Markov chain is a sequence of samples where each sample is dependent upon the previous sample as given below.

$$Z_t \sim N(Z_{t-1},\sigma)$$

In example of generation of random samples. For every generation of numbers for each iteration outcome depends upon the iteration outcome of the previous number, which are taken from the normal proposal distribution. Here the mean equals the previous value of the data.

Now if we have to generate 10000 samples using Markov chain Monte Carlo (MCMC) method the trace plot so obtain shows the behavior of the Brownian motion (random Walk). The resulting density plot so obtained does not resembles the proposal distribution.

3.2.3 Metro Polis Hasting

The Metropolis Hasting algorithm gives us a decision rule to which proposal value of the data to accept or reject according to the rule given as

$$\psi(Z_t - Z_{t-1}) = \frac{\text{Posterior\_Probability}\_Z_t}{\text{Posterior\_Probability}\_Z_{t-1}}$$

We began to calculate the posterior probability distribution of the newly generated value of Z_t .

We also calculate the posterior probability of the previously generated value of Z_{t-1}. Now we do not have to know the functional form of the posterior distribution. We simple multiply the posterior distribution with the likelihood function.

If the posterior probability is greater for the new value of the data, the ratio of probability will be greater than 1 and we always accept the new value of data.

If the posterior probability is greater for the previous value of data we will not necessarily discard the new value of data. We can treat ratio as less than 1 as an acceptance probability.

$$\text{Acceptance\_probability} = \alpha(Z_t - Z_{t-1})$$

$$= \text{Min}[\psi(Z_t - Z_{t-1}),1]$$

Draw ψ if Uniform (0,1)

If $\psi < \alpha\ (Z_t - Z_{t-1})$ then $Z_t = Z_t$, Otherwise $Z_t = Z_{t-1}$

On the basis of the above decision rule, we can draw uniform random samples; keep the new value of the data if the random sample is less than the acceptance probability. Otherwise we discard the new value of data (Dagpunar, 2007).

4. RESULTS

We have taken Pakistan Stock market 100 index for the empirical analysis of the model. The index is converted to log returns and volatility accordingly. The descriptive statistics are given as follows

Mean Estimation			**Number of obs 29**
	Mean	**Std. Error**	95% Conf. Interval
Variable Index	49448.77	88.68733	49267.1 49630.43
Log Index	4.69	0.003956007	0.003877 0.000124034
Return	1.00002149458	0.000564914	0.001107 3.54239 E-05
Volatility	0.0061111	0.004828257	
Volume	138,384,847.27	38401314.64	7621.246 243.8329297

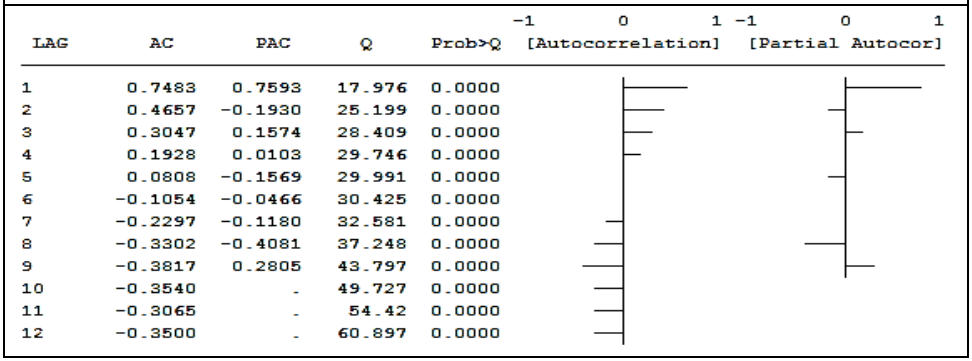
The estimated ARMA(p q) model.

The model summary is given below as

ARMA	**Regression**	**Sample**	**29-Jan-2017**	**28-Feb-2017**	**29**
Log Likelihood	**-203.3563**			**Wald Chi^2**	**17.43**
				Prob > Chi^2	0.0002
Index	Coefficient	STD. Error	z	P (z)	Confidence Interval
	49399.08	179.6451	274.98	0.0000	49046.98 49751.18
AR	0.5856222	0.2295386	2.55	0.011	0.1357348 1.03551
MA	0.382074	0.3471223	1.10	0.271	-0.2982732 1.062421
Sigma	260.0306	39.53583	6.58	0.000	182.5418 337.5194

The auto correlation and Partial Auto Correlation Graphs are given below Which shows the AR(p) and MA(q). Here according to auto correlation function and partial auto correlation function we estimated ARMA(1,1) model.

Auto correlation and Partial Auto correlation



LAG	AC	PAC	Q	Prob>Q
1	0.7483	0.7593	17.976	0.0000
2	0.4657	-0.1930	25.199	0.0000
3	0.3047	0.1574	28.409	0.0000
4	0.1928	0.0103	29.746	0.0000
5	0.0808	-0.1569	29.991	0.0000
6	-0.1054	-0.0466	30.425	0.0000
7	-0.2297	-0.1180	32.581	0.0000
8	-0.3302	-0.4081	37.248	0.0000
9	-0.3817	0.2805	43.797	0.0000
10	-0.3540	.	49.727	0.0000
11	-0.3065	.	54.42	0.0000
12	-0.3500	.	60.897	0.0000

Monte Carlo Simulation of the ARMA(1,1)

bayesmh var2	Likelihood (normal(10))		prior({var2:}	normal(49448.77,88))	
Burn-in ...	Simulation ...				
Model summary			Likelihood:	var2 ~ normal({var2:_cons},10)	
			Prior:	{var2:_cons} ~ normal(49448.77,88)	
Bayesian ARMA normal regression			MCMC iterations	12,500	
Random-walk Metropolis-Hastings sampling			Burn-in	2,500	
			MCMC sample size	10,000	
			Number of obs	29	
				var2 ~ normal({var2:_cons},10)	
			Prior	{var2:_cons} ~ normal(49448.77,88)	
			Acceptance rate	.4321	
Log marginal likelihood			-253999.8	Efficiency = .2148	
			Equal-tailed		
	Mean	STD.	MCSE	Median	95% Cred Interval
var2	49391.55	.5899792	.01273	49391.54	49390.38 49392.67

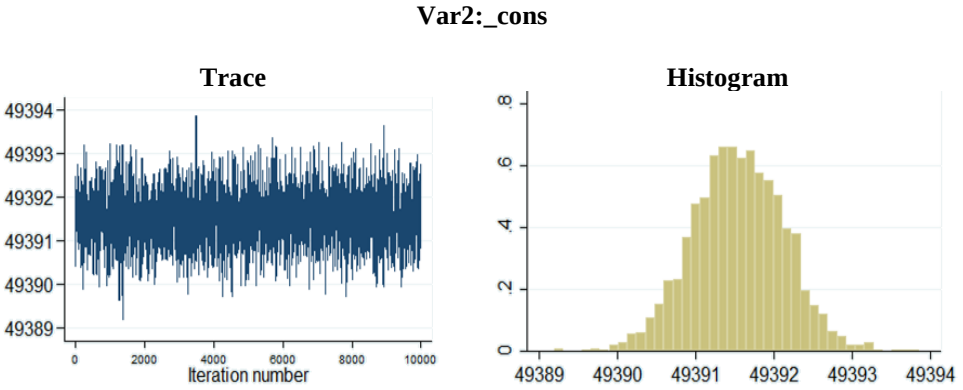


Fig. 4.1: Graphical Summaries of the MCMC ARMA Model

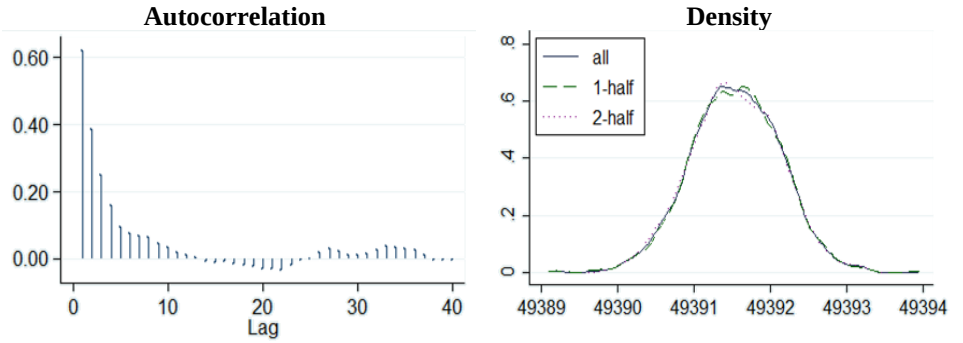


Fig. 4.2: Out of Sample Six Days MCMC Forecast

From the MCMC ARMA equation generated we have forecasted ARMA-MCMC Time series Model for next six days and the results are plotted in graphs.

$$y_{MCMC} = \beta_0 + \beta_1 y_{t-1} + \beta_2 e_{t-1} + \upsilon$$

$$y_{MCMC} = 49475.52 + 0.4016757\, y_{t-1} + 0.7798844 e_{t-1} + \upsilon$$

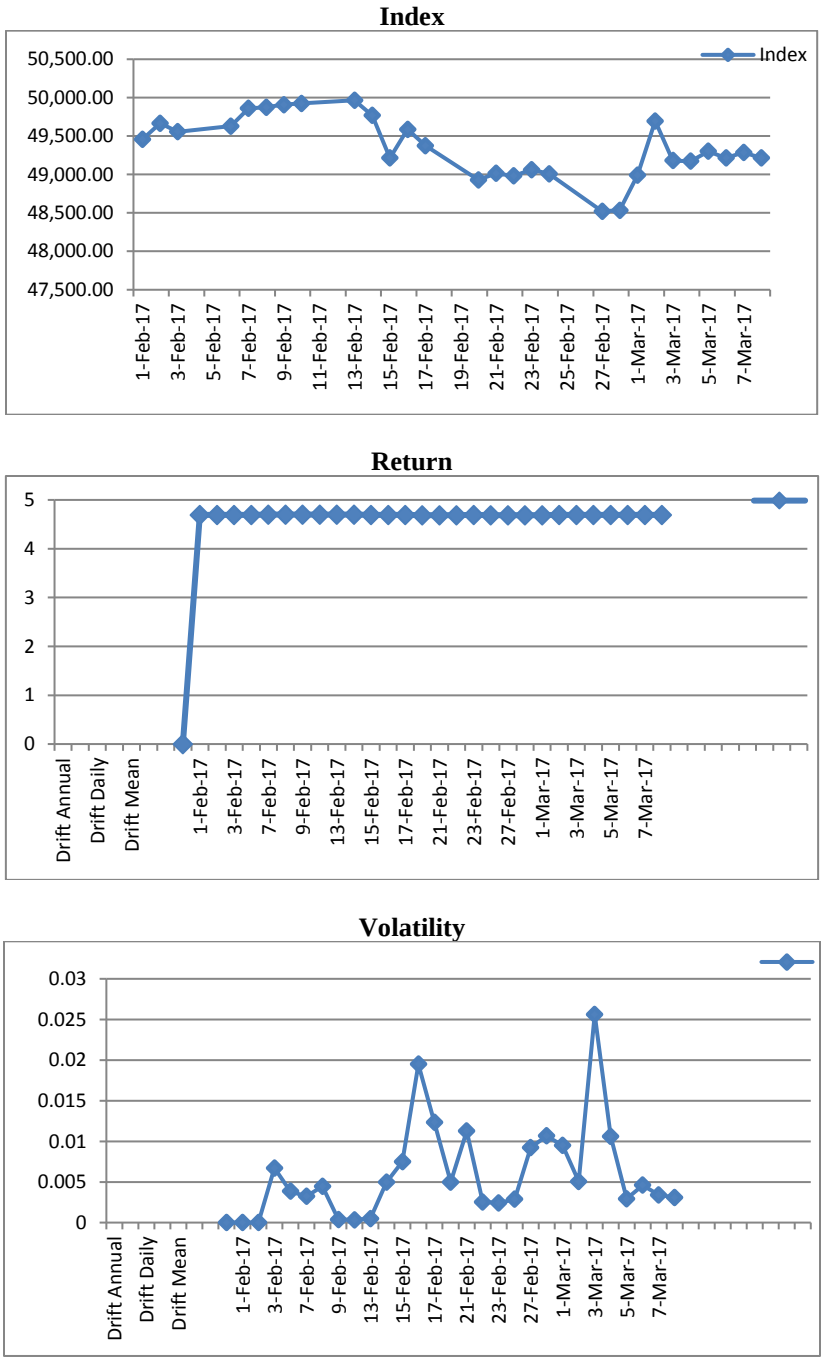


Fig. 4.3: ARMA Model Graph of PSX 100 Index, Return and Volatility

5. CONCLUSION

We have estimated and simulated Markov chain Monte Carlo ARMA(p q) model. The MCMC is a Bayesian sampling technique to generate random samples from a given prior. Our Proposal distribution for the MCMC method was normal distribution. Based on the normal prior we estimated ARMA(1,1) model on basis of Auto correlation and Partial autocorrelation functions.

The trace plot clearly gives us posterior distribution behavior of the Random walk (Brownian motion), where each subsequent iteration depends upon the preceding sample value creating a Markov chain. The trace plot further elaborates the prior and posterior belief in the Parameter space.

Our estimated ARMA model decides the forecasted value on the basis of Metropolis-Hasting Algorithm which significantly forecasted reduce mean square error.

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Proc. 15th International Conference on Statistical Sciences
Khairpur, Pakistan – March 9-11, 2017, Vol. 31, pp. 109-122

SOME CHARACTERIZATIONS OF TRANSMUTED DAGUM DISTRIBUTION

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ABSTRACT

In this paper, Transmuted Dagum distribution (Shahzad and Asghar; 2016) is characterized through hazard rate functions, reverse hazard rate functions, elasticity function, doubly truncated moments and ratio of truncated moments. The applications of characterizations of Transmuted Dagum distribution will be beneficial for scientists in different areas of science.

KEY WORDS

Characterization; Hazard rate; Reverse hazard rate; Truncated moments.

1. INTRODUCTION

1.1 Dagum Distribution

Dagum studied income and income related data using Dagum distribution. Dagum distribution has wide applications in business failure data, modeling finance, insurance data failure time modeling, reliability and acceptance sampling plans. Dagum distribution, Income, wage, and wealth distribution, inverse Burr distribution and Kappa distribution in different areas of research are also known as Burr III distribution. It belongs to generalized beta distribution. Dagum (1977) and Fattorini and Lemmi (1979) developed independently.

Dagum distribution for random variable X has following cumulative distribution function (cdf) and probability density function (pdf)

$$G(x)=\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha} \qquad \alpha>0,\ \beta>0,\gamma>0,x\geq 0, \tag{1}$$

$$g(x)=\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1} \qquad ,\alpha>0,\ \beta>0,\ x>0. \tag{2}$$

1.2 Transmuted Dagum Distribution

Transmuted Dagum distribution with parameters α, β, γ *and* λ, has the pdf

$$f(x) = \frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right] \quad x>0, \quad |\lambda| \le 1. \quad (3)$$

Transmuted Dagum distribution with parameters α, β, γ *and* λ, has the cdf.

$$F(x) = (1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha} - \lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}, \; x \ge 0, \quad |\lambda| \le 1. \quad (4)$$

.

Sub-Models

i) For $\lambda = 0$, Transmuted Dagum reduces to Dagum distribution.

ii) For $\gamma = 1$, Transmuted Dagum reduces to TBIII distribution.

iii) For $\lambda = 0$ and $\gamma = 1$, Transmuted Dagum reduces to BIII distribution.

iv) For $\lambda = 0$, $\alpha = 1$ and $\gamma = 1$, Transmuted Dagum reduces to Log-Logistic distribution.

v) For $\lambda = 0$ and $\gamma = 0$, Transmuted Dagum reduces to generalized inverse Weibull distribution.

vi) For $\lambda = 0$, $\beta = 2$ and $\gamma = 0$, Transmuted Dagum reduces to generalized inverse Rayleigh distribution.

vii) For $\lambda = 0$, $\beta = 1$ and $\gamma = 0$, Transmuted Dagum reduces to generalized inverse exponential distribution.

viii) For $\alpha = 1$, and $\gamma = 0$, Transmuted Dagum reduces to Transmuted inverse Weibull distribution.

ix) For $\alpha = 1$, $\beta = 1$, and $\gamma = 0$, Transmuted Dagum reduces to Transmuted inverse exponential distribution.

x) For $\alpha = 1$, $\beta = 2$, and $\gamma = 0$, Transmuted Dagum reduces to Transmuted inverse Rayleigh distribution.

xi) For $\alpha = 1$, $\lambda = 0$ and $\gamma = 0$, Transmuted Dagum reduces to inverse Weibull distribution.

xii) For $\alpha = 1$, $\beta = 1$, $\lambda = 0$ and $\gamma = 0$, Transmuted Dagum reduces to inverse exponential distribution.

xiii) For $\alpha = 1, \beta = 2$, $\lambda = 0$ and $\gamma = 0$, Transmuted Dagum reduces to inverse Rayleigh distribution.

xiv) For $\alpha = 1$, *and* $\beta = 1$, Transmuted Dagum reduces to Transmuted extended inverse exponential distribution.

xv) For $\alpha = 1$, *and* $\beta = 2$, Transmuted Dagum reduces to Transmuted extended inverse Rayleigh distribution.

Transmuted Dagum distribution with parameters α, β, γ *and* λ, has hazard rate function

$$h_F(x) = \frac{\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}{1-(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}+\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}}. \quad (5)$$

Transmuted Dagum distribution with parameters α, β, γ *and* λ, has reverse hazard rate function

$$r_F(x) = \frac{\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}{(1+\lambda)-\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}}. \quad (6)$$

Transmuted Dagum distribution with parameters α, β, γ *and* λ, has Mills ratio

$$m_F(x) = \frac{1-(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}+\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}}{\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}. \quad (7)$$

The elasticity of Transmuted Dagum distribution is given by

$$e(x) = \frac{d\,\ln F(x)}{d\,\ln x} = x\,r_F(x),$$

$$e_F(x)=\frac{\alpha\beta\left(\frac{x}{\gamma}\right)^{-\beta}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}{(1+\lambda)-\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}}. \quad (8)$$

Naveed and Asgher (2016) studied Transmuted Dagum distribution along with its properties. The characterization of Transmuted Dagum distribution is yet to do.

2. CHARACTERIZATION

In order to develop a stochastic function for a certain problem, it is necessary to know whether function fulfills the theory of specific underlying probability distribution, it is required to study characterizations of specific probability distribution. Different characterization techniques have developed. Glanzel (1987 and 1990), Hamedani (1993, 2002, 2011 and 2015), Ahsanullah and Hamedani (2007, 2012), and Merovci et al. (2016) have worked on characterization.

In this paper, Transmuted Dagum distribution is characterized through (i) hazard rate function; (ii) reverse hazard rate function, (iii) elasticity function; (iv) doubly truncated moments and (v) ratio of truncated moments.

2.1 Characterization Based On Hazard Function

Definition 2.1.1

Let $X:\Omega\to(0,\infty)$ be a continuous random variable having absolutely continuous cdf $F(x)$ and pdf $f(x)$ if and only if the hazard function $h_F(x)$ is twice differentiable function satisfying differential equation

$$\frac{d}{dx}\left[\ln f(x)\right]=\frac{h_F'(x)}{h_F(x)}-h_F(x). \quad (9)$$

Theorem 2.1.1

Let $X:\Omega\to(0,\infty)$ be a continuous random variable with pdf (3) if and only if the hazard function (5) satisfy the differential equation

$$h_F'(x)\frac{\left(1-(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}+\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}\right)}{\left(\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]\right)}-h_F(x)$$

$$= \left\{ \frac{-\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-1} + \frac{\beta(\alpha+1)}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1} + \frac{2\lambda\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}}{\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]} \right\}.$$

Proof:

For random variable X having Transmuted Dagum distribution with hazard rate function (5), we obtain the following equation

$$h_F'(x)\frac{\left(1-(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}+\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}\right)}{\left(\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]\right)} - h_F(x)$$

$$= \left\{ \frac{-\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-1} + \frac{\beta(\alpha+1)}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1} + \frac{2\lambda\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}}{\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]} \right\}.$$

After simplification we obtain as

$$\frac{d}{dx}h_F(x)\left[1-(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}+\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}\right]$$

$$= \frac{\alpha\beta}{\gamma}\frac{d}{dx}\left\{\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]\right\}.$$

$$\frac{f(x)}{1-F(x)} = \frac{\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}{\left[1-(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}+\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}\right]},$$

After integrating above equation, and simplifying, we obtain as

$$F(x)=(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha},\ x\geq 0,\quad |\lambda|\leq 1$$

this is cdf of Transmuted Dagum distribution.

2.2 Characterization Based on Reverse Hazard Function

Definition 2.2.1

Let $X:\Omega\to(0,\infty)$ be a continuous random variable having absolutely continuous cdf $F(x)$ and pdf $f(x)$ if and only if the reverse hazard function r_F is twice differentiable function satisfying differential equation

$$\frac{d}{dx}\left[\ln f(x)\right]=\frac{r_F'(x)}{r_F(x)}+r_F(x). \tag{10}$$

Theorem 2.2.1

Let $X:\Omega\to(0,\infty)$ be a continuous random variable with pdf (3) if and only if the reverse hazard function (6) satisfy the differential equation

$$r_F'(x)\frac{\left[(1+\lambda)-\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}\right]}{\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}+r_F(x)$$

$$=\left\{\frac{-\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-1}+\frac{\beta(\alpha+1)}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1}+\frac{2\lambda\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}}{\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}\right\}.$$

Proof

For random variable X having Transmuted Dagum distribution with reverse hazard rate function (6), we obtain the following equation

$$r_F^{/}(x)\frac{\left[(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}\right]}{\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}+r_F(x)$$

$$=\left\{\frac{-\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-1}+\frac{\beta(\alpha+1)}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1}+\frac{2\lambda\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}}{\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}\right\}.$$

After simplification we obtain as

$$\frac{d}{dx}r_F(x)\left[(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}\right]$$

$$=\frac{\alpha\beta}{\gamma}\frac{d}{dx}\left\{\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]\right\}.$$

$$\frac{f(x)}{F(x)}=\frac{\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}{(1+\lambda)-\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}},$$

After integrating above equation, and simplifying, we obtain as

$$F(x)=(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha},\ x\geq 0,\quad |\lambda|\leq 1,$$

this is cdf of Transmuted Dagum distribution.

2.3 Characterization Based On Elasticity Function

Definition 2.3.1

Let $X:\Omega \to (0,\infty)$ be a continuous random variable having absolutely continuous $F(x)$ and pdf $f(x)$ provided the elasticity function $e_F(x)$ is twice differentiable function satisfying differential equation

$$\frac{d}{dx}\left[\ln f(x)\right] = \frac{e'(x)}{e(x)} + \frac{e(x)}{x} - \frac{1}{x}. \tag{11}$$

Theorem 2.3.1

Let $X:\Omega \to (0,\infty)$ be continuous random variable .The pdf of X is (3) provided that its elasticity function, $e_F(x)$ satisfies the first order differential equation

$$e'_F(x)\frac{\left[(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha} - \lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}\right]}{\alpha\beta\left(\frac{x}{\gamma}\right)^{-\beta}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]} + e_F(x)$$

$$= \left\{\frac{-\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-1} + \frac{\beta(\alpha+1)}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1} + \frac{2\lambda\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}}{\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}\right\}. \tag{12}$$

Proof:

If X has pdf (3), then (12) holds. Now if (12) holds, then

$$\frac{d}{dx}e_F(x)\left[(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha} - \lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}\right]$$

$$= \alpha\beta\frac{d}{dx}\left\{\left(\frac{x}{\gamma}\right)^{-\beta}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]\right\},$$

or

$$e_F(x)=\frac{\alpha\beta\left(\frac{x}{\gamma}\right)^{-\beta}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}{(1+\lambda)\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2\alpha}}$$

is the elasticity function of Transmuted Dagum distribution.

2.4 Characterization Based on Doubly Truncated Moment

Theorem 2.4.1

Let $X:\Omega\to(0,\infty)$ be continuous random variable with Transmuted Dagum distribution. The pdf of X is (3) if and only if

$$E\left(\left[1+2\lambda\left(1+\left(\frac{U}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]^{-1}\middle| x<X<y\right)=\frac{\left(1+\left(\frac{y}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}}{\left[\bar{F}(x)-\bar{F}(y)\right]}$$

holds.

Proof

Necessary Part

For random variable X having Transmuted Dagum distribution with pdf (3), we proceeds as

$$E\left(\left[1+2\lambda\left(1+\left(\frac{U}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]^{-1}\middle| x<X<y\right)=\frac{\int_y^x\left[1+2\lambda\left(1+\left(\frac{u}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]^{-1}f(u)\,du}{\left[\bar{F}(x)-\bar{F}(y)\right]},$$

$$=\frac{\int_y^x\left[1+2\lambda\left(1+\left(\frac{u}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]^{-1}\frac{\alpha\beta}{\gamma}\left(\frac{u}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{u}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{u}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]du}{\left[\bar{F}(x)-\bar{F}(y)\right]},$$

$$=\frac{\int_y^x\frac{\alpha\beta}{\gamma}\left(\frac{u}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{u}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}du}{\left[F(y)-F(x)\right]},=\frac{\left(1+\left(\frac{y}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}}{\left[\bar{F}(x)-\bar{F}(y)\right]},$$

$$E\left[\left[1+2\lambda\left(1+\left(\frac{U}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]^{-1} / x<X<y\right]=\frac{\left(1+\left(\frac{y}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}}{\left[\bar{F}(x)-\bar{F}(y)\right]}.$$

Conversely taking

$$\frac{\int_x^y\left[1+2\lambda\left(1+\left(\frac{u}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]^{-1} f(u)\,du}{\left[\bar{F}(x)-\bar{F}(y)\right]}=\frac{\left(1+\left(\frac{y}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}}{\left[\bar{F}(x)-\bar{F}(y)\right]}$$

$$\int_x^y\left[1+2\lambda\left(1+\left(\frac{u}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]^{-1} f(u)\,du=\left(1+\left(\frac{y}{\gamma}\right)^{-\beta}\right)^{-\alpha}-\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}$$

Differentiating w.r.t y, we have

$$\left[1+2\lambda\left(1+\left(\frac{y}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]^{-1} f(y)=\frac{\alpha\beta}{\gamma}\left(\frac{y}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{y}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}$$

$$f(y)=\frac{\alpha\beta}{\gamma}\left(\frac{y}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{y}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{y}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right] \quad y>0,\ \ |\lambda|\le 1$$

This is pdf of Transmuted Dagum distribution.

2.5 Characterization through Ratio of First Truncated Moments

Theorem 2.5.1

Let $X:\Omega\to(0,\infty)$ be a continuous random variable with distribution function (4), let

$$h_1(x)=-\frac{\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{\alpha-1}}{\alpha\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]} \text{ and } h_2(x)=-\frac{2\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{\alpha-2}}{\alpha\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]} \quad x>0.$$

The pdf of X is(3) if and only if $p(x)$ has the form $p(x)=\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)$ (Theorem 1).

Proof:

For random variable X having Transmuted Dagum distribution with pdf (3) and cdf (4), we proceeds as

$$\left(1-F\left(x\right)\right)E\left(h_1\left(x\right)\middle| X\geq x\right)=\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1}$$

and

$$\left(1-F\left(x\right)\right)E\left(h_2\left(x\right)\middle| X\geq x\right)=\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-2},x>0.$$

$$p\left(x\right)=\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right) \text{ and } p'\left(x\right)=-\frac{\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1},$$

$$s^{/}=\frac{p^{/}h_2}{ph_2-h_1}=\frac{-2\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1} \text{ and } s=2\ln\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right),$$

Therefore in the light of theorem 1, X has pdf (3) and cdf (4).

Consequence:

Let $X:\Omega\rightarrow\left(0,\infty\right)$ be a continuous random variable and

$$h_2\left(x\right)=-\frac{2\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{\alpha-2}}{\alpha\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]}\ x>0.$$

The pdf of X is (3) provided functions $p \text{ and } h_1$ satisfy equation

$$\frac{p^{/}}{ph_2-h_1}=\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha+1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]$$

Statement:

The solution the above differential equation is

$$p=\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{2}\left[-\int\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]h_1dx\right]+D,$$

where D is constant.

Theorem 2.5.2

Let $X:\Omega\to(0,\infty)$ be a continuous random variable with distribution function (4), let

$$h_1(x)=\frac{\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{\alpha+1}}{\alpha\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]} \text{ and } h_2(x)=\frac{2\left(\frac{x}{\gamma}\right)^{-\beta}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{\alpha+1}}{\alpha\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]} \quad x>0.$$

The pdf of X is (3) with condition that $p(x)$ has the form $p(x)=\left(\frac{x}{\gamma}\right)^{\beta}$ (Theorem 1).

Proof:

For random variable X having Transmuted Dagum distribution with pdf (3) and cdf (4), we proceeds as

$$\left(1-F(x)\right)E\left(h_1(x)\middle|X\geq x\right)=\left(\frac{x}{\gamma}\right)^{-\beta} \text{ and } \left(1-F(x)\right)E\left(h_2(x)\middle|X\geq x\right)=\left(\frac{x}{\gamma}\right)^{-2\beta}$$

$$p(x)=\left(\frac{x}{\gamma}\right)^{\beta} \text{ and } p'(x)=\frac{\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{\beta-1},$$

$$s^{/}=\frac{p'h_2}{ph_2-h_1}=\frac{-2\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-1} \text{ and } s=2\ln\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)$$

Therefore in the light of theorem 1, X has pdf (3) and cdf (4).

Consequence:

Let $X:\Omega\to(0,\infty)$ be a continuous random variable and

$$h_2(x)=\frac{2\left(\frac{x}{\gamma}\right)^{-\beta}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{\alpha+1}}{\alpha\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]} \quad x>0.$$

The pdf of X is (3) provided functions p *and* h_1 satisfy differential equation

$$\frac{p'}{ph_2 - h_1} = \frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]$$

Statement:

The solution the equation

$$\frac{p'}{ph_2 - h_1} = \frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right],$$

is $p = \left(\frac{x}{\gamma}\right)^{2\beta}\int\left(-h_1\frac{\alpha\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{-\beta-1}\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha-1}\left[1+2\lambda\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\alpha}\right]\right)+D,$

where D is constant.

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APPENDIX A

Theorem 1

Suppose that probability space $(\Omega, \mathrm{F}, \mathrm{P})$ and interval $[\mathrm{d}_1, \mathrm{d}_2]$ with $d_1 < d_2$ ($d_1 = -\infty, d_2 = \infty$) are given. Let continuous random variable $X : \Omega \to [\mathrm{d}_1, \mathrm{d}_2]$ has distribution function F. Let real functions h_1 and h_2 be continuous on $[\mathrm{d}_1, \mathrm{d}_2]$ such that $\frac{E[\mathrm{h}_1(X)/\mathrm{X} \geq \mathrm{x}]}{E[\mathrm{h}_2(X)/\mathrm{X} \geq \mathrm{x}]} = p(x)$ is real function in simple form. Assume that $h_1, h_2 \varepsilon C([\mathrm{d}_1, \mathrm{d}_2])$, $p(x) \varepsilon C^2([\mathrm{d}_1, \mathrm{d}_2])$ and F is two times continuously differentiable and strictly monotone function on $[\mathrm{d}_1, \mathrm{d}_2]$: As a final point, assume that the equation $h_2 p(x) = h_1$ has no real solution in $[\mathrm{d}_1, \mathrm{d}_2]$. Then $F(x) = \int_{\ln k}^{x} K \left| \frac{p'(t)}{p(t)h_2(t) - h_1(t)} \right| \exp(-s(t))\, dt$ is obtained from the functions h_1, h_2, $p(t)$ and $s(t)$, where $s(t)$ is obtained from equation $s'(t) = \frac{p'(t)h_2(t)}{p(t)h_2(t) - h_1(t)}$ and K is a constant, picked to make $\int_{d_1}^{d_2} dF = 1$.