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PHILOSOPHICAL FOUNDATION OF EDUCATION IN PAKISTAN CONTEXT

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ABSTRACT

One of the aims of education accepted by all the nations is to transfer knowledge from one generation to the other with the help of educational institutions. Educational systems are based on many foundations. Philosophical foundation is one of the core foundations. No education system can be said complete without philosophical foundation. Philosophy provides the vision to think beyond imagination. Education is important and compulsory according to Islamic philosophy of education. In this research, critical review through documentary analysis was done about the progress of education system in Pakistan since 1947. The aim of the study was to investigate the role of philosophical foundation in Pakistan education system. It was analyzed that how education system can be improved in the light of the nation's philosophy and ideology. This was analyzed that how the nation can make progress keeping in mind its creation. The nations of the world are in a great competition in the field of different type of advancement and inventions. Guidelines were proposed about how the nation can keep balance to move forward without affecting the philosophy and ideology of the nation. These guidelines will be helpful for the policy makers to make policy according to the philosophy of the nation.

KEY WORDS

Philosophy, Ideology, Imagination, Advancement, Inventions

1. INTRODUCTION

Philosophy of education is a branch which deal with the philosophical application to solve the education related problems. In this regard it examine the vision of the researcher and the people who are the policy maker which mostly address the debates and innovation in the process of learning [1]. Considering an academic field, study contain the philosophical study of education and its problems. Centrally the subject is education. The methods related to philosophy[2]. The philosophy mean either the philosophy of the process or of the discipline of education. It mean that either it deal the aims and goals of the process. Or its may be the dealing with the aims, goals of the discipline[3][4]. Philosophy of education may be further either with the imagination comes under meta physics or the knowledge comes in epistemology. This may cover the values in axiology. The philosophy also address the logic that how to reason. Philosophy try to answer to the question that what will be the pedagogy. What will be the curriculum. Philosophy also try to answer the question regarding educational policy as these all contribute to learning process [5] It answer that what values and norms to be developed it also study the limitation of the discipline of education. This also address the relation between the theory and practices[6]. It does not mean the philosophy which is thought in the schools of philosophy. It deals with the philosophy of education thought in educational schools[7][8][9] same like the philosophy of law is one of the subject in the schools of laws.

Philosophy is a very diverse and broad even in education. This deal with the teaching learning process as a whole just like there may be some schools who follow idealism and the some will follow realism.

1. PHILOSOPHY OF EDUCATION

2.1. Idealism

Idealism is the philosophy which believes that ideas are the only realities and with the help of the ideas generated in the mind of human truth can be searched, as they consider mind is the origin of all ideas. They believes that without ideas nothing is possible. They believe that the world is exist independent of human mind. The origin of the word idea is from Greek which means “to see” this comes into English in seventh century[10] Plato is the founder who says that the world is exist independent of human mind [11][12].

2.2 Realism

Another famous philosophy is that of realism. Aristotle is consider to be the founder of realism. They believe that physical things are the only realities. They focus on body. Just like his teacher the lecture method and question and answering technique is used to teach to the students. They believe that curriculum will be subject centered. This school of thought believe that there will be a balance between theory and practice [13].

2.3 Pragmatism

This is another famous school of thought. They believe that nothing is not permanent. Dewey is one of the famous believer. Dewey believe that education is life and life is education. They believe on activity based/problem solving teaching for the learners. Dewy believe that life without education is nothing[14][15]. Some other philosophies are

- Progressivism
- Existentialism
- Reconstructionism

2.4 Islamic Philosophy of Education

In Islam Philosophy is a famous word mean Philosophy this refers to the logic as well as math’s and Physics[16]. And Kalam. Islamic philosophy based on a systematic inquiry of the problems related to any filed

Life
Universe
Society
and so on.

The famous philosopher in the recent history of Islam are Ibne Khaldoon, Imam Ghazali, Allama Iqbal etc.

Islamic philosophy search the solution of problems in the light of the well-known and famous sources

- Quran (The last Holy Book)
- Hades (By the last prophet (S.A.W))
- Qyas (Told by The last prophet (S.A.W))
- Ijmaa (Told by the last Prophet (S.A.W)).

3. RESULTS AND DISCUSSION

3.1 Basis of the Creation of Pakistan

Pakistan came into being on 14th august 1947 on the basis of two nation theory. Two nation theory clearly state that Muslims is a separate nation having their own rich philosophy of education and culture. According to a universally accepted definition one of the main aim of education is to transfer the culture and values from one generation to the next. First educational conference held in Karachi from 27th to first Dec 1947[17] decided that the basis of education in Pakistan will be based on pure Islamic philosophy.

3.2 Ideology of Pakistan

By definition ideology are the common understanding and beliefs accepted by a group to live their lives.

The ideology of Pakistan is discussed to promote the Islamic values and culture as accepted by the majority of the people in this region. To make at the part of constitution a resolution was passed in 1948 that Allah will be the supreme power and all the rules will be created according to Quran and Sunnah.

About teaching the lord first teach

وَعَلَّمَ آدَمَ الْأَسْمَاءَ كُلَّهَا ثُمَّ عَرَضَهُمْ عَلَى الْمَلَائِكَةِ فَقَالَ أَنْبِئُونِي بِأَسْمَاءِ هَٰؤُلَاءِ إِنْ كُنْتُمْ صَادِقِينَ
(Albaqara)

Now the demand of the human is given in first surah Fateha that is

''بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ
الْحَمْدُ لِلَّهِ رَبِّ الْعَالَمِينَ
الرَّحْمَٰنِ الرَّحِيمِ
مَالِكِ يَوْمِ الدِّينِ
إِيَّاكَ نَعْبُدُ وَإِيَّاكَ نَسْتَعِينُ
اهْدِنَا الصِّرَاطَ الْمُسْتَقِيمَ
صِرَاطَ الَّذِينَ أَنْعَمْتَ عَلَيْهِمْ غَيْرِ الْمَغْضُوبِ عَلَيْهِمْ وَلَا الضَّالِّينَ''
(Alfateha)

Now the human want guidance from the lord a guidance which truly lead them to success

In reply the very first verses of albaqara The lord replied

"الم
ذَٰلِكَ الْكِتَابُ لَا رَيْبَ ۛ فِيهِ ۛ هُدًى لِلْمُتَّقِينَ
الَّذِينَ يُؤْمِنُونَ بِالْغَيْبِ وَيُقِيمُونَ الصَّلَاةَ وَمِمَّا رَزَقْنَاهُمْ يُنْفِقُونَ
وَالَّذِينَ يُؤْمِنُونَ بِمَا أُنْزِلَ إِلَيْكَ وَمَا أُنْزِلَ مِنْ قَبْلِكَ وَبِالْآخِرَةِ هُمْ يُوقِنُونَ"

(Albaqara 1-4)

That the Lord give a 4th Holy book not only but also a messenger (Muhammad S.A.W) to show them that how to practice it in their lives. This also guide the economy and will lead to the economical foundations of education.

About the creation and recreation The Lord says

كَيْفَ تَكْفُرُونَ بِاللَّهِ وَكُنْتُمْ أَمْوَاتًا فَأَحْيَاكُمْ ۖ ثُمَّ يُمِيتُكُمْ ثُمَّ يُحْيِيكُمْ ثُمَّ إِلَيْهِ تُرْجَعُونَ

(Albaqara)

It means that Quran guide us about the Medical sciences. Further The life is not end with death but a new life start after death. The efforts and work which has been done not finish with death. Allah says that the human will be recreated and they will be rewarded. About the creation of the universe Allah Guide them and told them that all created at according to a systematic method.

"هُوَ الَّذِي خَلَقَ لَكُمْ مَا فِي الْأَرْضِ جَمِيعًا ثُمَّ اسْتَوَىٰ إِلَى السَّمَاءِ فَسَوَّاهُنَّ سَبْعَ سَمَاوَاتٍ ۚ وَهُوَ بِكُلِّ شَيْءٍ عَلِيمٌ"

(Albaqara)

Now Islamic philosophy have its own vision and give hope in all type of situation.

Islamic Philosophy is the best suitable philosophy of education for Pakistan

"اقْرَأْ بِاسْمِ رَبِّكَ الَّذِي خَلَقَ
خَلَقَ الْإِنْسَانَ مِنْ عَلَقٍ
اقْرَأْ وَرَبُّكَ الْأَكْرَمُ
الَّذِي عَلَّمَ بِالْقَلَمِ
عَلَّمَ الْإِنْسَانَ مَا لَمْ يَعْلَمْ"

(Alquran)

These are the initial verse which are about Knowledge and another place Allah swear with Qalam

To show the importance of it to the creation

"ن ۚ وَالْقَلَمِ وَمَا يَسْطُرُونَ
مَا أَنْتَ بِنِعْمَةِ رَبِّكَ بِمَجْنُونٍ
وَإِنَّ لَكَ لَأَجْرًا غَيْرَ مَمْنُونٍ
وَإِنَّكَ لَعَلَىٰ خُلُقٍ عَظِيمٍ
فَسَتُبْصِرُ وَيُبْصِرُونَ"

(Alquran)

Allah told about the creation of the universe about death and life. Futher about the beauty of creation. And also Allah says to the humanity to think and search in this universe.

"تَبَارَكَ الَّذِي بِيَدِهِ الْمُلْكُ وَهُوَ عَلَىٰ كُلِّ شَيْءٍ قَدِيرٌ ◌
الَّذِي خَلَقَ الْمَوْتَ وَالْحَيَاةَ لِيَبْلُوَكُمْ أَيُّكُمْ أَحْسَنُ عَمَلًا ۚ وَهُوَ الْعَزِيزُ الْغَفُورُ
الَّذِي خَلَقَ سَبْعَ سَمَاوَاتٍ طِبَاقًا ۖ مَا تَرَىٰ فِي خَلْقِ الرَّحْمَٰنِ مِنْ تَفَاوُتٍ ۖ فَارْجِعِ الْبَصَرَ هَلْ تَرَىٰ مِنْ فُطُورٍ
ثُمَّ ارْجِعِ الْبَصَرَ كَرَّتَيْنِ يَنْقَلِبْ إِلَيْكَ الْبَصَرُ خَاسِئًا وَهُوَ حَسِيرٌ
وَلَقَدْ زَيَّنَّا السَّمَاءَ الدُّنْيَا بِمَصَابِيحَ وَجَعَلْنَاهَا رُجُومًا لِلشَّيَاطِينِ ۖ"

(Alquran)

The creation of the universe and stars give the scientific foundation of education. That Quran encourage science and research.

Allah create the human for work. Regarding this Allah says to The prophet(S.A.W)

"يَا أَيُّهَا الْمُزَّمِّلُ
قُمِ اللَّيْلَ إِلَّا قَلِيلًا
نِصْفَهُ أَوِ انْقُصْ مِنْهُ قَلِيلًا
أَوْ زِدْ عَلَيْهِ وَرَتِّلِ الْقُرْآنَ تَرْتِيلًا
إِنَّا سَنُلْقِي عَلَيْكَ قَوْلًا ثَقِيلًا
إِنَّ نَاشِئَةَ اللَّيْلِ هِيَ أَشَدُّ وَطْئًا وَأَقْوَمُ قِيلًا
إِنَّ لَكَ فِي النَّهَارِ سَبْحًا طَوِيلًا
وَاذْكُرِ اسْمَ رَبِّكَ وَتَبَتَّلْ إِلَيْهِ تَبْتِيلًا
رَبُّ الْمَشْرِقِ وَالْمَغْرِبِ لَا إِلَٰهَ إِلَّا هُوَ فَاتَّخِذْهُ وَكِيلًا
وَاصْبِرْ عَلَىٰ مَا يَقُولُونَ وَاهْجُرْهُمْ هَجْرًا جَمِيلًا
وَذَرْنِي وَالْمُكَذِّبِينَ أُولِي النَّعْمَةِ وَمَهِّلْهُمْ قَلِيلًا
إِنَّ لَدَيْنَا أَنْكَالًا وَجَحِيمًا
وَطَعَامًا ذَا غُصَّةٍ وَعَذَابًا أَلِيمًا
يَوْمَ تَرْجُفُ الْأَرْضُ وَالْجِبَالُ وَكَانَتِ الْجِبَالُ كَثِيبًا مَهِيلًا
إِنَّا أَرْسَلْنَا إِلَيْكُمْ رَسُولًا شَاهِدًا عَلَيْكُمْ كَمَا أَرْسَلْنَا إِلَىٰ فِرْعَوْنَ رَسُولًا"

(Alquran)

In the above verses Allah says that wake and pray to your lord. An read the holy book day and night.

Now what will be the effect of reading. So it is clear from the previous verses that with the help of this its will be possible to know the lord and also to study the universe. Allah says that you are the messenger just like the messenger send towards Firown. And another verse

"أَمْ تُرِيدُونَ أَنْ تَسْأَلُوا رَسُولَكُمْ كَمَا سُئِلَ مُوسَىٰ مِنْ قَبْلُ ۗ

"(Alquran)

In short it can infer that Quran is the basic source of knowledge for Islamic philosophy. The second is one is

Sunnah which explain the Holy book.

After this Islam give permission to make analogy (Qyass)

And the fourth major source is Ijmaa it is clear that Islamic Philosophy is the best for Pakistan context. It covers all aspect of life.

4. COMMENTS AND CONCLUSION

From the above discussion it is clear that Islamic philosophy is the only suitable philosophy for Pakistan education system. The only thing is that to make a proper research to make the right direction to the education system of Pakistan. If such type of roles are incorporated in Pakistani educational philosophy so definitely this will make Pakistan a prosperous country and the aims and goals of its founder will be achieved in true sense, which will be fruitful not only for Pakistan but specially for Islamic world and for the rest of the world. In future the study can be extended to extract in detail about the philosophical roots of education more deeply, Like how to make the curriculum and teaching methods make more efficient.

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REFERENCES

1. Aamna Irshad and Irshad Ullah (2018). Today education system and Quaid vision of the first international Conference held in 1947. Presented in 16th International Conference on Statistical Sciences at Islamia College University, Peshawar.
2. Ullah, I. and Irshad, A. "*Doctor of Philosophy in Education*". Harvard Graduate School of Education. Retrieved 2017-04-29.
3. Hassan Hassan (2013). *Don't Blame It on al-Ghazali*. qantara.de. Retrieved 5 June 2017.
4. Frankena, W.K.; Raybeck, N. and Burbules, N. (2002). Philosophy of Education. In Guthrie, James W. *Encyclopedia of Education*, 2nd edition. New York, NY: Macmillan Reference. ISBN 0-02-865594-X
5. "*idealism*, n. : Oxford English Dictionary".
6. McNair, J.D. *Plato's Idealism*. Students' Notes. Miami-Dade Community College. Retrieved 7 August 2011.
7. Mason, M. (2008). Complexity Theory and the Philosophy of Education. *Educational Philosophy and Theory*, 40(1), 4-18. doi:10.1111/j.1469-5812.2007.00412.x. ISSN 1469-5812.
8. Neil, J. (2005). *John Dewey, the Modern Father of Experiential Education*. Wilderdom.com. Retrieved 6/12/07.
9. Noddings 1995, pp. 1-6
10. *Philosophy and Education*. Teachers College - Columbia University. Retrieved 2017-04-29.
11. *Philosophy of Education* - Courses - NYU Steinhardt. steinhardt.nyu.edu. Retrieved 2017-04-29.
12. Philosophy of education Retrieved https://en.wikipedia.org/wiki/Philosophy_of_education 2017.
13. UNESCO (1998). *Prospects: the Quarterly Review of Comparative Education* (Paris, UNESCO: International Bureau of Education), 23(1/2), 1993, p. 39-51. © UNESCO: International Bureau of Education.
14. *Stanford Encyclopedia of Philosophy*. https://plato.stanford.edu/archives/fall2010/entries/consciousness-temporal/

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STATISTICAL TECHNIQUES TO DETERMINE INTEGRITY OF A BURIED CORRODING PIPELINE

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ABSTRACT

The use of trans country buried steel pipelines plays an extremely important role in the economy of a country. Since the start of their use in the early nineteenth century the companies owning such pipelines have always been concerned about their integrity over an extended period for the purpose for which they were used. The improvement in their integrity has been occurring gradually throughout the history based on the statistical information gathered at each stge. Since corrosion was the main cause of damage to the pipeline integrity, the focus of the pipeline engineers has been to develop effective corrosion mitigation techniques. The corrosion data have been analyzed by whatever statistical methods were known to the engineers. Although recently computer based advanced statistical techniques have been successfully used in many fields, only limited work has not been done in the field of buried pipeline corrosion. Large amount of pipeline corrosion data has accumulated over the last eighty years, but it has escaped the attention of professional statisticians. Recently few papers are published where the pipeline corrosion data is analyzed to develop statistical models. This paper highlights the importance of developing such models to inspire the professional statisticians to take interest in this crucially important field. Summaries of the two recently published case studies are also included.

1. INTRODUCTION

A buried pipeline's integrity means its uninterrupted good performance for a specific time for the purpose it is being used. Management of a pipeline's integrity is extremely crucial considering its operators' safety, environmental damage and the pipeline's replacement cost. Thus, maintaining a high level of the integrity of the industrial pipelines must be given a top priority.

1.1 Crucial Importance of Pipelines

In USA, the Department of Transportation's agency **Pipeline and Hazardous Materials Safety Administration** (**PHMSA**) is responsible for developing and enforcing regulations for the safe, reliable, and environmentally sound operation of the 2.6 million miles pipelines[1] crossing the country, transporting natural gas and hazardous liquids from sources such as wells, refineries, and ports to customers. In Pakistan's Natural Sui gas supply infrastructure more than 11 thousand kilometers[2] underground pipelines have been used as shown in Figure-1 and Table-1.

Obviously, the maintenance of an uninterrupted energy supply to the public requires the operation of these pipelines in such a manner that corrosion does not result in an unscheduled interruption to the flow of these energy supplying materials to the nation. With this objective the pipeline operating companies try to maintain high level of pipeline integrity by following standards, codes, and practices set out by variety of regulatory agencies, and standards developing organizations. The Pipeline Standards Developing Organizations Coordination Council (PSDOCC) coordinates the activities of these groups and the Department of Transportations, Office of Pipeline Safety (OPS) acts as the regulatory agency with final responsibility over this system of codes and practices[3].

Corrosion[4] is the main cause to damage the integrity of buried pipelines, so they must be protected from corrosion which may be internal, external or both. Generally internal corrosion is controlled by chemical inhibitors and external corrosion is controlled by the application of coatings and cathodic current. Currently the law requires that all trans-country industrial pipelines must be coated with an appropriate coating system and be protected by cathodic current.

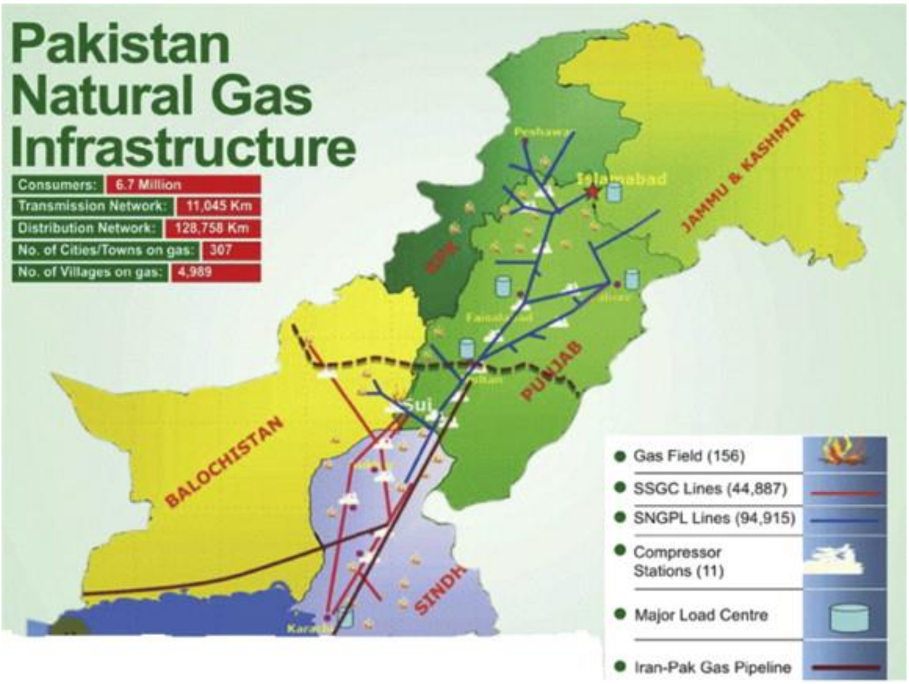

Fig. 1: Pakistan's Natural Gas Transmission Infrastructure.

SNGPL, SSGC SYSTEM ASSETS

Asset	SNGPL — Totals	SSGC — Totals
Transmission pipelines	6.121 Km	2.942 Km
Distribution pipelines	42.192	25.764 Km
Gas supply source	28	13
Total installed compression (ISO)	185.800 hp	--
Total installed compression (site rated)	152.800 hp	62.600 hp
Compressor stations	11	6
Distribution regions	8	6

Source Sui Northern Gas Pipeline Ltd (SNGPL), Sui Southern Gas Co. Ltd (SSGC)

To improve public safety and stimulate improvements in pipeline technologies, regulations, and standards; the U.S. Congress passed the Pipeline Safety Improvement Act of 2002 (PSIA)[1]. This act resulted in the formation of the PSIA Coordination Council, which communicates and coordinates pipeline relevant research in four government agencies: The Department of Energy, The Department of Transportation, The Department of Interior, and The Department of Commerce. This project is a result of this collaboration. The objectives of this project were to (1) reexamine the original NBS underground bare pipe corrosion studies to determine if the results from this study could be used to develop better empirical models for prediction of bare pipe corrosion rates and (2) to seek new in-sights that could lead to the development of pipeline external corrosion prediction models, or soil corrosivity indexes, that could be used in the future for computer-aided pipeline integrity management.

The U.S. pipeline infrastructure is steadily increasing. Pits and holes development in the pipeline wall because of corrosion of the external surface resulted in a significant portion of pipeline failures. However, the pipeline industry has been able to reduce or hold failure rates constant over recent years. This has been due to the accumulation of pipeline operation experience, and improvements in technologies including inspection, repair, coating, and information technologies. The policy of openly sharing their experience has enabled this industry to make improvements and repairs before failures occur. The trend of further improvements is expected to continue.

1.2 Meaning of Statistics during 18th Century

As per Wikipedia the **History of Statistics** started around year 1749 although, over time, there have been changes to the interpretation of the word *statistics*. In early times, the meaning was restricted to information about states. This was later extended to include all collections of information of all types, and later still it was extended to include the analysis and interpretation of such data. In modern terms, "statistics" means both sets of collected information, as in national accounts and temperature records, and analytical work which requires statistical inferences[5].

In the 18th century, the term "statistics" designated the systematic collection of demographic and economic data by states. These data were mainly tabulations of human and material resources that might be taxed or put to military use. In the early 19th century, collection intensified, and the meaning of "statistics" broadened to include the discipline concerned with the collection, summary, and analysis of data. Today, data is collected, and statistics are computed and widely distributed in government, business, most of the sciences and sports, and even for many pastimes. Advanced computers have expedited more elaborate statistical computation and have facilitated the collection and aggregation of data.

The term "mathematical statistics" designates the mathematical theories of probability and statistical inference, which are used in statistical practice. Applied statistics can be regarded as not a field of mathematics but an autonomous mathematical science, like computer science and operations research. Statistics was applied early in demography and economics but today large areas of micro- and macroeconomics are also "statistics" with an emphasis on time-series analyses. Learning from data and making best predictions is also part of statistics. It is applied in the areas of academic research including psychological testing and medicine. The ideas of statistical testing have considerable overlap with decision science. With its concerns on searching and effectively presenting data, statistics has overlap with information science and computer science.

1.3 History of Corrosion Recognition

Corrosion is the natural tendency of metallic structures to rust and deteriorate. It is the main cause to damage the integrity of buried pipelines. The history of corrosion recognition also seems to start during 18th century. The collection of the information of various incidents of corrosion occurring on buried steel pipelines was a statistical procedure according to the meaning of statistics during that period as stated above. From the collective information of randomly occurring incidents of corrosion, certain inferences and conclusions were drawn by the scientists and engineers. The phenomenon of metallic corrosion was recognized during the 1800s during which several papers were published that suggested to the fact that **corrosion** was electrochemical in nature. In 1902 the Electrochemical Society was founded, and the first convincing proof that corrosion is an electrochemical process was published by Whitney[6] in 1903.

1.4 History of Cathodic Protection

The electrochemical concept of corrosion introduced by Whitney led to the emergence of cathodic protection technology which in nutshell is reversing the corrosion reaction by supplying an external cathodic current using an auxiliary anode. The anode could be a galvanic (selected from the galvanic series or an inert anode and a Rectifier-Transformer). In US cathodic protection was applied to steel gas pipelines beginning in the 1928 and by the 1930s *it* had come into widespread use to control external corrosion on *underground pipelines*[7].

Currently all hazardous product pipelines are routinely protected applying a coating system supplemented with cathodic protection. An impressed current cathodic protection system (ICCP) for a pipeline consists of an AC powered transformer rectifier and an anode, or array of anodes buried in the ground (the anode ground-bed).

1.5 Early Methods of Integrity Management

Integrity management has been of great concern throughout the history of underground pipelines. Recognizing the damage due to corrosion two methods were put into practice to improve the pipeline integrity:

1. Increasing the thickness of pipeline sheet. This increase in thickness is known as ‘Corrosion Allowance’.
2. Using some sort of a barrier coating on the pipeline. In earlier stages coal tar or coal tar-soaked wrappers were used.

2. CORROSION ALLOWANCE

By knowing the expected general corrosion rate and the anticipated plant or service life of a part, the designer can calculate the extra thickness required for corrosion resistance of the process equipment being designed. After determining a wall thickness that meets mechanical requirements.

A *corrosion allowance* of 0.063" (1/16") is typical[8].

2.1 History of Coating Application on Buried Steel Pipelines

The use of buried steel pipelines in the ground started in the late 1800s, and the owners of these pipelines soon realized that corrosion quickly caused leaks on bare pipelines. So, to avoid direct contact of the pipes with the ground they started applying different coatings on them with or without using wrappers over them[9].

2.2 The Early Coatings

The very first coating used around 1940 was coal tar or coal tar enamel[9]. The coal tar coatings have been used up to 1970. However, from 1950 to 1960 asphalt coatings have also been used. Later other coatings were developed and used as shown in the Figure 2.

2.3 Development of Coating Systems from 1930 to 1950[9]

From 1930 to 1950, with the industrialization in the Midwest and Northeast of USA, the demand for energy greatly increased[9]. This demand was fulfilled largely by the Texas oil fields and refineries that used buried steel pipelines for transportation. This required great reliability of the integrity of the pipelines which depended on the effectiveness of corrosion control measures. As the pipeline industry matured, so did the technology of protective coatings, and one of the early innovations was the use of a built-up system where the hot products (asphalt or coal tar) were reinforced by application of a tar-saturated felt mat that was worked into the hot matrix.

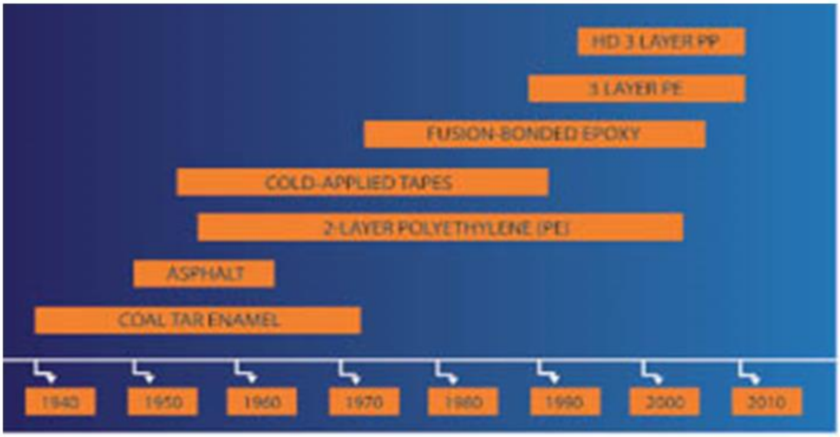

Fig. 2: Application of various coatings on buried pipelines during 1943-2010

The system was completed with a spiral wrap of Kraft paper. Coating application typically took place over the trench. For many years, the built-up coal tar and asphalt coatings were the predominant systems used on buried pipelines. Another popular system was based on using petroleum-based wax coating reinforced with fiber mesh, and these two systems accounted for nearly all the protective coatings applied to buried pipes from 1930 through 1950.

The first large-scale pipeline project in the U.S. was the “Big Inch” (24-inch) crude oil and the “Little Big Inch” (20-inch) petroleum products pipelines project, constructed from August 1942 through August 1944. The federal government funded the project through a public company called “War Emergency Pipelines, Inc.” At the time, the “Big Inch” was the longest and most expensive pipeline in the world. It was machine-wrapped over-the-ditch with a three-part coating system consisting of a hot-applied coal tar primer, fiber-reinforced coal tar tape and a reinforced fiber outer wrap.

2.4 Developments of Coating Systems from 1950 to 1970

By the mid-1950s, new hydrocarbon polymers were introduced into pipeline coating products to improve performance in the underground environment. Epoxy resins were formulated using coal tar-based pigments to make a liquid-applied pipeline coating. Coal tar epoxy exhibited superior resistance to penetration by water. Over the same period, application methods also changed. Systems like the multiple-layered coal tar tape, applied hot and over-the-trench, gave way to cold-applied, prefabricated tapes which used adhesives to bond to the pipe surface. These tapes used advanced polymers like vinyl and polyethylene, with butyl rubber adhesives. They were flexible, tough and highly resistant to water penetration. The 1960s also saw the introduction of fusion-bonded epoxy (FBE), which would eventually replace many of the earlier pipeline coating systems.

ars have gone by since the first rudimentary coating systems, utilizing coal tar and asphalt, were applied to underground pipelines to control corrosion. As time passed, the pipeline-coating industry has matured to the point where reliable coating materials are now available that can be applied at high-production rates in a coating mill with an equally high level of quality control. Depending upon the service requirements, coating materials like epoxy, polyethylene and polypropylene will be the building blocks for most of the pipeline coating demands brought about by the hydraulic fracturing process.

Case Study-1

CASE Study-1 Analysis of Pipeline Steel Corrosion Data from (NIST) Studies Conducted between 1922-1940 and Relevance to Pipeline Management

A study was conducted by the National Bureau of Standards (NBS) between 1922 and 1940 into the corrosion of bare steel and wrought iron pipes buried underground at 47 different sites representing different soil types across the Unites States. At the start of this study, very little was known about the corrosion of ferrous alloys underground. The objectives of this study were to determine:

1. Whether a coating would be required to prevent corrosion.
2. Could soil properties be used to predict corrosion and determine when coatings would be required.

This study determined very quickly that coatings would be required for some soils. However, the results were so divergent that even generalities based on this data must be drawn with care. It was further concluded that so many diverse factors influence corrosion rates underground that planning of proper tests and interpretation of the results were matters of considerable difficulty and that quantitative interpretations or extrapolations could be done “only in approximate fashion”.

Linear regression and curve fitting of the corrosion damage measurements against the measured soil composition and properties found some weak trends. These trends improved with multiple regression, and empirical equations representing the performance of the samples in the tests were developed with uncertainty estimates. The uncertainties in these empirical models for the corrosion data were large, and extrapolation beyond the parameter space or exposure times of these experiments will create additional uncertainties. It is concluded that equations for the estimation of corrosion damage distributions and rates can be developed from these data, but these models will always have relatively large uncertainties that will limit their utility. These uncertainties result from the scatter in the measurements due to annual, seasonal, and sample position dependent variations at the burial sites. The data indicate that more complete datasets with soil property measurements reflecting the properties of the soil and ground water directly in contact with the sample from statistically designed experiments would greatly reduce this scatter and enable more representative predictions.

Following the passage of the Pipeline Safety Improvement Act in 2002 the U.S. Department of Transportation approached the National Institute of Standards and Technology (NBS became NIST in 1988) and requested that the data from this study be reexamined to determine if the information handling and analysis capabilities of modern computers and software could enable the extraction of more meaningful information from these data. The data from the original NBS studies were analyzed using a variety of commercially available software packages for statistical analysis. The emphasis was on identifying trends in the data that could be later exploited in the development of an empirical model for predicting the range of expected corrosion behavior for any given set of soil chemistry and conditions.

Case Study-2 (Oil & Gas Journal**,** July 2009)
Statistical Analysis of Pitting Corrosion Field Data for Development of a Model for Realistic Reliability Estimations for Non-Piggable Buried Pipeline Systems

Alma Valor, Francisco Caleyo, Lester Alfonso, Julio Vidal, José M. Hallen

Published Online: April 3, 2014.

A research study was conducted at the National Polytechnic Institute (IPN) of Mexico on corrosion pitting of underground oil and gas pipelines using various deterministic and stochastic predictive models. Three different segments were selected in three types of soils. Pit depths were measured in situ to determine the maximum pit depth. The times simulated were year 5 to year 40 for each soil class.

It was found that:

1. The larger the exposure time, the higher the uncertainty in the estimates as determined by the variance of the maximum pit depth distribution.
2. The difference in the growth behavior of the deepest pits with respect to the remaining pit population increases with the corrosivity of the soil.
3. The study helped determine realistic values for the number of corrosion defects per kilometer (defect density) and obtain a better description of the corrosion defect size distributions in this system.
4. The reliability and risk of non-piggable, corroding oil and gas pipelines can be estimated from historical failure data and through reliability models based on the assumed or measured number of corrosion defects and defect size distribution. The field-gathered corrosion data could be used as input to a reliability framework for the estimation of the failure index of non-piggable pipelines and pipeline systems when different amounts of corrosion data are available.

From the study it was further concluded that:

1. No significant difference could be determined between different pipe segments due to the scatter in the results from the environmental factors. Linear regression and curve fitting of the corrosion damage measurements against the measured soil composition and properties found some weak trends.
2. These trends improved with multiple regression, and empirical equations representing the performance of the samples in the tests were developed with uncertainty estimates. The uncertainties in these empirical models for the corrosion data were large, and extrapolation beyond the parameter space or exposure times of these experiments will create additional uncertainties.
3. The equations for the estimation of corrosion damage distributions and rates can be developed from these data, but these models will always have relatively large uncertainties that will limit their utility. These uncertainties result from the scatter in the measurements due to annual, seasonal, and sample position dependent variations at the burial sites.

3. CONCLUSIONS

1. Transmission pipelines integrity management is extremely important because of their role in supplying the energy to industrial and domestic sectors.
2. Corrosion creates pits and hole in the pipelines and is the main cause of damage to the pipelines integrity. Thus, corrosion control is mandatory as per standards and codes set by regulatory authorities of the country.
3. It is not easy to develop statistical predictive models for pipelines integrity due to complex nature of corrosion that is influenced by many variables. The larger the exposure time is, the higher the uncertainty in the estimates as determined by the variance of the maximum pit depth distribution.
4. With the advancement of computer technology, it seems possible to develop pipeline integrity predictive models. Some attempts[12,13] have already been made recently to develop such models. Professional statisticians should take interest in this ignored but highly important area.

REFERENCES

1. Wikipedia: *Department of Transportation, the Pipeline and Hazardous Materials Safety Administration* (PHMSA), May 2014.
2. Google and Google Images: *Pakistan Natural Gas Infrastructure*.
3. Google: *History of NACE International 1943-2018*.
4. Eckert, R. (2015). Corrosion: An Integrity Threat to the Entire Oil and Gas Asset Value Chain. *Inspectioneering Journal*, November/December 2015 issue.
5. Wikipedia: *History of Statistics*.
6. Willis R. Whitney (2002). A Brief History of Corrosion Science and its Place in the Electrochemical Society, Reported by Robert P Frankenthal. Published in 2002 by *Electrochemical Society*. Inc., 65 South Main Street, Pennington, NJ 08534, USA.
7. Wikipedia: *Cathodic Protection*.
8. *NACE Standard* MR0175.
9. Bud Senkowski, Pipeline Coatings, Posted on KTA University site, "*Meeting Demands of Gas Exploration: The Evolution of Pipeline Coatings*". March 1, 2016.
10. Ricker. R.E. (2010). Materials Science and Engineering Laboratory, National Institute of Standards and Technology, Gaithersburg, MD 20899; *J. Res. Natl. Inst. Stand. Technol*. 115, 373-392.
11. Valor, A., Caleyo, F., Alfonso, L., Vidal, J. and Hallen, J.M. (2014). Statistical analysis of pitting corrosion field data and their use for realistic reliability estimations in non-piggable pipeline systems. *Corrosion*, 70(11), 1090-1100.
12. Tan, H.Y. (2017). *Statistical methods for the analysis of corrosion data for integrity assessments* (Doctoral dissertation, Brunel University London).
13. Hwei-Yang, T., Keming, Y. and Bharadwaj, U. (2015). Predictive model for corrosion rate in process piping using proportional hazard density regression. *SIAM Journal of Applied Mathematics*.

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AN IMPACT OF WESTERNIZATION ON ISLAMIC CULTURAL OF UNIVERSITY STUDENTS IN PAKISTAN

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ABSTRACT

Western culture dominate Islamic culture through globalization, especially in urban areas of Pakistan. There are much more difference in both cultural values beliefs and other fields of human life. This influence change Pakistani youth in serious way. The study in hand investigate the current situation of the universities about cultural change, positive and negative impacts of western culture on students and recommend suitable suggestions for situation. Researcher collect data from university teachers on their observations and experiences in university. Through stratified random sampling 100 teachers were selected from four universities of the Punjab. An instrument were developed for assessing impact level and intensity, positivity and negativity of western culture on students of the universities. Findings of the study shows a serious situation, negative impacts destroy the student's life and our students receiving a very less positive benefits of western culture due to society background, none guidance, and double mindedness. At the end of the study researcher suggest recommendation for students that how they avail all benefits of Islamic culture in this modern era and compete the world.

1. INTRODUCTION

A culture is the collection of beliefs, symbols, values, norms, behaviors and artefacts shared by a group of people. The term 'culture' is both used to designate such collections or systems as they have developed historically in particular regions (sometimes spanning multiple countries; e.g., African culture, Polynesian culture, Asian culture), amongst particular ethnic groups (Native American culture, Jewish culture, Tuareg culture), or amongst certain subgroups in society (non-ethnic 'subcultures', such as hacker culture, hippie culture, Internet culture).

As we mentioned before that culture can be regarded as a people's complete way of life, including beliefs, customs, language and traditions, there are numerous cultures and subcultures present in the world, of primary concern to us in this discussion are the two main opposing cultures in present-day society. One is the culture of Islam — the "Complete way of life" set forth in the Qur'an and Sunnah.

On the other opposing end is the Western culture. Both cultures present a “complete way of life.” However, both ways of life differ drastically. The difference in the two cultures stems primarily from the source of the cultures. Islamic culture — the way of life of a Muslim — is defined by the Qur’an and Sunnah. It is the culture of Rasulullah (Sallallahu Alaihi Wasallam). It is that way of life upon which he established the Sahaaba after having turned them away from the culture of jahiliyyah (ignorance).

Before this time many Europeans from the north, especially Scandinavians, remained polytheistic, though southern Europe was predominately Christian from the 5th century onwards. Pakistan is a Muslim country and Pakistani culture is the unique pattern of belief, ideas, values highly influenced by religion of Islam. Islam in Pakistan sets the code of ethics for the cultural life for the people of Pakistan. But People gained western and Indian culture due to acculturations modernization and westernization.

Pakistani society is adopting Western influences and reshaping it according to local needs and concerns. For instance, wearing jeans with short eastern style shirt and scarf, English mixed Urdu, Western Eastern mixture of music, love marriages then arranged by families etc.

The societal conditions show that the hybridization of culture is ‘Westernizing’ Pakistani society. Westernization is the process which weaker societies adopt while attempting Modernization. The elite (rich and privileged) class of Pakistani society has become westernized to a large extent, which is inspiring middle and lower middle classes to imitate them. The young generations’ priority is to get a job either in a Western country or a multinational firm operating in Pakistan.

Pakistani universities also influenced by this modernization or western culture. This behavior shows in their students look, dressings, openness in relationships, bold conversations and get together parties.

2. LITERATURE REVIEW

2.1 Culture

The word culture comes from the Latin, cultura or cultus. Cultus means to *cultivate*. The original meaning of cultus was closely linked to thecultivation of soil, *agricultura*. The understanding of the word culture has changed from its root meaning as an activity to a condition, a state of being cultivated. Though the meaning of culture was originally tied to activity, the question of the activity of what and to what end is inevitable.

The Romans linked culture to *humanitas*; man vs. animal, *urbanitas*; city vs. rural and *civilitas* as in civil and good manners opposed to barbaric ways. Many, however, felt this to be a superficial concept, and interpreted culture as being *Bildung*, the cultivation of a complex inner life. Kroeber and Kluckhohn in Culture a Critical Review of Concepts and Definitions state that: “The most generic sense of the word “culture” – in Latin and in all the languages that have borrowed the Latin root – retains the primary notion of cultivation or becoming cultured. This was also the older meaning of “civilization”. A second concept to emerge was that of German *Kultur*, roughly the distinctive “‘higher’ values or enlightment of society.” (Kroeber & Kluckhohn 1952, 35) At the time that Kroeber and Kluckhohn wrote their book, the word culture was still under construction.

Different academic disciplines could not agree upon a common meaning: "few explicit definitions were given. Usage was rather consistently vague, and denotation varied from very narrow to very broad." (Kroeber & Kluckhohn 1952, 36.) Though anthropology, sociology and the other social sciences have come a long way since then, a "fully systematic scientific theory of man, society and culture has yet to be created" from the perspective of the process of westernization.

2.2 Effect on Pakistani Society

Social change in context of Pakistani society has different meaning and contents. Society in Pakistan has undergone significant transformations since the times of its creation, and this process is still going on. This time the agent of change is globalization. The society in Pakistan started experiencing major transformations since the early 1990s, a period when after end of Cold War a free and open world emerged, macroeconomic policies in this period were aimed at the liberalization and privatization of the domestic market and trade, banking sector was liberalized by permitting private banks to operate and compete with nationalized commercial banks, competition was promoted by privatization of national assets, scientific developments in communication and information technology took a boom, and reach to foreign television channels became possible through dish antenna.

The Western ways of life is inspiring less developed societies, because of the quality of life which is achieved by them through scientific/ technological advancements and economic development. This inspiration is causing the weakening of national identity especially in African and Asian societies.

Pakistanis are adopting Western styles in their daily lives, for instance frequent use of English as a common language of people, preference of English as a medium of instruction in all levels of education, Western styled dresses, Western fast food restaurants, increasing trends of single family and love marriages, making independent relationships between man and woman like friendship, and rising aspirations of migration to Western countries. Westernization is the process which weaker societies adopt while attempting Modernization. The elite (rich and privileged) class of Pakistani society has become westernized to a large extent, which is inspiring middle and lower middle classes to imitate them. The young generations' priority is to get a job either in a Western country or a multinational firm operating in Pakistan.

A. Cvetkovich and D. Kellener (2000) identify the situation of identity crisis emerging in the world societies:

[Today, under the pressure of the dialectics of the global and the local, identity has global, national, regional and local components, as well as the specificities of gender, race, class and sexuality…This situation is highly contradictory with reassertions of traditional modes of identity in response to globalization and a contradictory mélange of hybrid identities-and no doubt significant identity crisis-all over the world. (P.135)]

2.3 Reasons of Western Culture Prevailing

All societies of world are not obviously equal because of differences in economic and scientific/ technological capabilities. The impact of hybridization is especially different on societies of developed states mainly due to their economic conditions. Economic factor is largely responsible for the change in social lives of people, and it determines the socio-political conditions and power of a society and state.

Some societies possess more resources than others, and are to great extent able to influence others, some because of lack of resources are not in condition to influence or even resist the powerful influence of other societies.

Most importantly, the process of globalization has been made mandatory through powerful institutional arrangements. For instance, the influence of powerful international monetary institution like World Bank and International Monetary Fund is affecting many nation-states. The agreement of World Trade Organization (WTO) has been implemented regardless the domestic conditions of the members of community of nation-states. In open competition, everybody is competing everybody; underdeveloped and developing states have to compete highly developed states. Rules are same for weak and strong competitors and even participating in the game does not depend on anyone's will, it is necessary for the survival of all. This situation is generating highly asymmetrical and conflicting impact especially for weak states.

Less developed states bear grave disparities that directly affect their citizens. It is becoming a difficult task for governments of the less developed states to satisfy their citizens, in the highly competitive world. This economic inequality is making the world a divided place.

3. POSITIVE IMPACTS

3.1 Widening the Horizons of Knowledge in Pakistani Society

Pakistan like other countries is enjoying the benefits of globalization has provided in form of technology for example, easy access to modern technological innovations like personal computer, internet, mobile phone, fax, and cable television.

Communication has become very easy and cheap. Internet is making people aware of world, new innovations and transformations. Advanced communication means are providing enormous exposure to globe, which is resulting in overall awareness of people in all matters of life especially regarding their rights. Issues of human, women and children rights are being emphasized more and brought in the media frequently.

People are more aware about their choices of system. Because of increased exposure to media, even an illiterate member of society can discuss the socio-political condition of country, and can be clear about his/her priorities and dislikes.

3.2 Literacy

Westernization has impacted the field of education in Pakistan. Education is emphasized more by all the sections of society in Pakistan regardless of their economic status. People are aware that if they have to change and improve their life they have to get education. Research has become quite easy because latest knowledge and analysis of different issues**,** new books, scholarly online research papers, and guidelines of conducting research is available on internet. Admission in foreign educational institutions can be obtained very easily on internet. Foreign scholarships are available on the basis of open competition. Students are forming educational groups on internet and exchanging knowledge. Job opportunities are increased because doors of developed world are open for a skilled and educated individual in shape of immigration and work permit schemes for highly skilled individuals. Job vacancies are advertised on internet; open for all over the globe.

Learning of English is emphasized; increased knowledge of English has made easy access to modern knowledge and information. Competition in education and introduction of foreign degree system/ Cambridge system is making students to become able to compete on global level. Foreign NGOs and donor agencies are working in Pakistan country for betterment of education in Pakistan, and being funded by global financial institutions.

3.3 Condition of Women

The awareness globalization is bringing to the society is aiding the condition of women in Pakistani society. The status of women has been reformed and opportunities have been enlarged for them. Frequent law are being introduced to safeguard women rights in the society. The conscious amongst people have increased and they are educating their daughters equally as their sons. Job opportunities for women are available more than ever, since gender discrimination is lessening and emphasis is given on ability. Women rights are being protected and legislations have been done in this regard.

3.4 Improvement in Quality of Life

The improvement of quality of life can be seen visibly in society. Financial globalization has paved the way in Pakistan for loaning of various types like mortgage loan, business financing, credit cards, and installments plans for different luxury items like electronics, and vehicles. People can now get the luxuries which they could not afford in past.

3.5 Responsibilities of State are shared

In westernization, world's common problems have been identified. For instance, poverty eradication is not any more a state's problem rather it has become a global problem and efforts are being taken by financial institutions to solve it. Pakistan is getting aid, loan, development program strategies, expert financial opinions, and other ways of improving the economic situation of country. Betterment of women and children in Pakistan is on the agenda of international donor agencies.

3.6 Openness is Improving the Life of People

The people belong to middle class of Pakistani society are benefited more by the westernization openness. They are availing opportunities based on merit in the world of competition and successfully changing their quality of life in presence of prevailing negative trends of corruption and personal influence in Pakistan.

3.7 Media has Become Very Active

Media in Pakistan has become very active; it is exposing the weaknesses of the society and representing its all sections. The debates and analysis on socio-political and economic issues are making society aware of societal conditions. It is also affecting the thought of members of society. Education, cooking, and religious channels are providing the information which was never available by easy means.

4. NEGATIVE IMPACTS

4.1 Increasing the Power of English in Pakistani Society

In Pakistan like other developing countries, learning English opens up more jobs for those who know it. These jobs are controlled by multinationals, which are dominated by the developed states. Emphasize on English is making national and regional languages weak and unimportant, and their literature is also vanishing. In Pakistani society, mixture of English with national and regional languages is popularizing, which sounds alien and destroying the original languages and affecting the vocabulary.

Subjects regarding knowledge of Pakistan like Pakistan Studies, religion, and regional languages are considered inferior, and below average students are supposed to take admission in these subjects. Study of national and regional languages like Urdu, Sindhi, Pushto, Punjabi or Balochi has been given no importance; ratio of students is very low in these departments. English literature is widely taken as a major subject on graduate and postgraduate levels. Tuition centers for teaching English have become a sort of profitable business and joining them a craze in the society.

4.2 Projections of the Western Societies as Model Societies

Media in Pakistan projects conditions of western societies as best in terms of human rights and individual freedom conditions, economic stability, free and cohesive society, and best political system. It is causing the devaluation of individual cultures and providing citizens of weak societies with a sense of inferiority. It is producing a negative image especially for the new generation in less developed countries, that their national language, culture and history is inferior. In Pakistan, people give preference to foreign goods, education, language, foods, dresses/ fashion, celebrations, way of making relations, family system. They reject native language, traditions, values, and socio-political systems on the bases of adopting modernization and keeping pace with global development.

4.3 Spreading Materialism amongst People

The pursuance of West is making people try to get same luxuries. Materialism has become dominant social trend in Pakistani society in order to reach the level of modernity equaling to developed countries. It is increasing the trends towards individualism (pursuit of personal happiness rather than collective interest), emphasis on own selfish motives, and the fragmentation of joint family system.

The indirect result of rising materialism is competition, which compels members of society to compete in obtaining maximum luxuries. Pakistani People spend more than on foreign luxury items like luxury cars, imported make up and perfumes, electronic items, clothes**.** Huge money is being spent in luxuries, for buying imported items. As a result people are becoming prey of superiority and inferiority complexes within the society, the people who can afford these luxuries feel that they are superior, and the people who cannot afford it have feeling of inferiority, and try to seek alternative ways like bribe, corruption or even crime to fulfill the desires, and become a prestigious member of the society.

4.4 Creating a Cultural Gap ‘within’ the Society

The rich or elite class in Pakistan has become more westernized because they can afford the luxurious life style of West; they adopt western fashion and also behave accordingly. The people who are not able to afford the Western life style look different in appearance and also have different attitudes.

It is creating a cultural gap ‘within’ the society, where origin of people is same but many are trying to look different or more Western. This situation is weakening the ties amongst people and leading to disharmony in the society because of emerging cultural differences (language, dress, food, fashion) among the people of same origin living in a same territory.

4.5 Clash of Values

Every society has its own values, grown and nurtured in long times having influences of religion, civilization, history and other factors. For example changes in education sector are very welcomed and adopted in Pakistani society, on the other hand changes in the marriage system like love marriages or marriages in different cast and creed are not accepted at all. The traditional values of Pakistani society are in clash with Western values that are increasingly being adopted by different sections of society but are not accepted socially. For instance, the trend of love marriage on the basis of right of a woman is rising and creating a situation of great clash in the society. People who prefer these kinds of marriage are not acceptable for society, and they and their families experience continuous hatred and social pressure.

4.6 The Increased Exposure of Developed World becomes a Source of Deep inspiration for Weak Societies

The development of other societies creates the feeling of dissatisfaction from prevailing system which leads to blind pursuance of other societies and weakens the ties of people and state. Consequently, this dissatisfaction causes the shift in national loyalties and fragmentation of societies sometimes on ethnical, religious or sectarian lines in order to obtain a better alternative; which is happening in Pakistan. State is not providing support to the people which they need to cope the challenges of modern world. So in turn people are redirecting their support and loyalty and rejecting political and social systems, the situation of Frontier and Balochistan provinces is evident of the logic presented here. Process of westernization is creating powerful non-state actors, before that nation-state was the only institution on which people were dependent in terms of their economic, psychological and social wellbeing.

4.7 When a Society absorbs too much Foreign Influence People Feel Lost or Dislocated from their own Place

The adoption of foreign influence generates feelings of confusion and insecurity amongst people. In case of Pakistani society, there is feeling of confusion in terms of their identity, major portion of society prefers foreign cultural attributes, one section prioritize religion as a best solution, and somewhere ethnicity is dominating the lives of people. Kinvall (2001) gives reason that “Principles at a time when modern society is making increasing demands on individuals as religion-like nationalism supplies existential answers to individuals’ quest for security providing order from the chaos and uncertainty in the world.”(P.89)

The trends of rising fragmentation on ethnic, religious and sectarian bases are quite visible in Pakistani society.

4.8 Westernization divide the Society and Fragmenting It into Classes

The upper class is becoming is highly Westernized, and other classes are trying to imitate them. People pertain to high class are completely different from others. Arif Hasan (2003) analyzes the problem of class division in Pakistani society and writes:

[“Pakistan’s elite and upper middle classes are ‘westernized’. They could not relate to the changes that were taking place around them, especially in the educational institutions where their children studied. Consequently elite families stopped sending their children to public sector universities and colleges. As a result, both rural and urban culture suffered and there was a serious decline in standards of education and in the maintenance and growth of public sector real estate and recreational facilities.” (P.423)]

Pakistani society has been divided into classes on economic bases. The two rich upper classes have become westernized; their social lives are entirely different from the life of common people of society. The upper rich classes follow Western life style in, fashion, dress, food education and making relationships. They have created their separate world, which has cut them from local conditions and places. The lack of participation of upper classes in public life is resulting in gap in society and affecting educational standard, and growth of public sector.

5. STATEMENT OF THE PROBLEM

In following piece of work, the impact of western culture over national identity and culture of Pakistan is discussed by observing the prominent changes in lives of Pakistani youth people. The impact is categorized in negative and positive aspects in order to assess the impact more clearly and to analyze that to what extent and in which way it is affecting the national identity of Pakistani society. Pakistani society and especially youth is adopting western influences and reshaping it according to local needs and concerns. For instance, wearing jeans with short eastern style shirt and scarf, English mixed Urdu, Western Eastern mixture of music and love marriages then arranged by families. Same position also show in universities of Pakistan. The study define the impact of western culture on Pakistani Islamic culture in universities.

6. OBJECTIVES OF THE STUDY

- To find out the change among university students of Pakistan caused by westernization.
- To explore the negative effects of western culture on university students of Pakistan.
- To discover the positive effects of western culture on university students of Pakistan.
- To assess an effective culture for Pakistani students.

6.1 Methodology

Survey research technique was used to document the responses of respondents. The population of the study consisted of teachers of four universities, University of Punjab, LUMS, University of Lahore and Minhaj-ul-Quran University. Stratified random sampling technique was used to select the sample. 100 teachers, 25 from each university were selected for seeking the responses to assesses positive and negative effects of western culture on students.

6.2 Instrument of the study

- A Binary Likert scale was developed for assessing impact level and intensity, positivity and negativity of western culture on students of the universities.
- Tool for assessing the three major objectives of the study had three parts.
- First part of tool was used to assess the level of the influence of the western culture on university students.
- The 2nd portion of tool was constructed for assessing positive factors of western culture on university students.
- The 3rd portion of tool was constructed for assessing negative factors of western culture on university students.

6.3 Analysis of data

Tool for assessing the three major objectives of the study has three parts.

First part of tool was used to assess the level of the influence of the western culture on university students.

PERCENTAGE OF THE STUDENTS

Statement	20%	40%	60%	80%
How many students are influence by western culture in university?	10	20	65	5

Out of hundred respondents 10 says that 20% students are affected by western culture in university, while 20 respondents assess 40% impact of western culture on students, 65 assess 60% and 5 assess 80%.

When all results concluded, it is assessed that are 65 % students of Pakistani universities are westernized.

The other portion of tool was constructed for assessing positive factors of western culture on university students.

POSITIVE FACTORS OF WESTERN CULTURE

Statement	Agree	Disagree
Westernization widening the horizons of knowledge in Pakistani students	55%	45%
Westernization improve the literacy level of society	50%	50%
Westernization improve the quality of life of students	27%	73%
Westernization improve the conditions of Women in our society	35%	65%
Westernization open the boundaries of information which increase quality of education in university	33%	67%
Westernization increase responsibility of the university administration	36%	64%
Westernization increase performance of students	20%	80%

Above findings of the study shows that according to the teacher's observations positive impacts of westernization on students of Pakistani universities are very less, only 2 out of seven factors show positivity in this portion of tool.

There are seven positive fields that discussed as western culture positivity on students but our students shows no positivity as constructive element of a culture. Only 55 percent agree that westernization widening the horizons of knowledge in Pakistani students. Same as half respondents assess that literacy improve in western culture adopted students.

On the other hand 5 factors shows that73% disagree that westernization improve the quality of life of students, 65% disagree that westernization improve the conditions of Women in our society, 67% disagree that Westernization open the boundaries of information which increase quality of education in university, 64% assess Westernization decrease responsibility of the university administration and a major part of respondents 80% observe that Westernization decrease performance of students.

NEGATIVE FACTORS OF WESTERN CULTURE

Statements	Agree	Disagree
Westernization dislocate the students from their roots.	88%	12%
Westernization is increasing the power of English in Pakistani students	83%	13%
Westernization projects the westernize students as model students	76%	24%
Westernization is creating a cultural gap 'within' the students	81%	19%
Westernization spreading materialism amongst students	84%	16%
Westernization is cause of clash of values in students	83%	17%
Westernization destroying the character of the students	77%	23%

Above findings of the study shows that according to the teacher’s observations negative impacts of westernization on students of Pakistani universities are very high, all seven factors shows negativity in this portion of tool.

There are seven negative fields that discussed as western culture negativity on students and results shows that 88% agree that Westernization dislocate the students from their roots, 83% agree that Westernization is increasing the power of English in Pakistani students, 76% agree that Westernization projects the westernize students as model students, 81% agree that Westernization is creating a cultural gap ‘within’ the students, 83% agree that Westernization is cause of clash of values in students, 84% agree that Westernization spreading materialism amongst students and 77% agree that Westernization destroying the character of the students.

All results shows that western culture destroy the students in every field of their life.

FINDINGS

- Out of hundred respondents 10 said that 20% students are affected by western culture in university, while 20 respondents assessed 40% impact of western culture on students, 65 assessed 60% and 5 assessed 80%.When all results concluded, it was generalized that 65 % students of Pakistani universities are westernized.
- According to the teacher’s observations positive impacts of westernization on students of Pakistani universities are very less, only 2 out of seven factors showed positivity in this portion of tool.
- There are seven fields that were discussed as positivity of western culture on students but our students showed no positivity as constructive element of a culture. Only 55 percent agreed that westernization widen horizons of knowledge among Pakistani students, and 45% assessed that literacy has improved in the case where students adopted western culture.
- On the other hand 5 factors showed that73% disagreed that westernization improve the quality of life of students, 65% disagreed that westernization improve the conditions of Women in our society, 67% disagreed that westernization opens the boundaries of information which increase quality of education in universities, 64% assessed that westernization decrease responsibility of the university administration and a major part of respondents 80% observed that Westernization decrease performance of students.
- According to the teacher’s observations negative impacts of westernization on students of Pakistani universities are very high, all seven factors showed negativity in this portion of tool.
- There are seven negative fields related to western culture negativity on students and results showed that 88% agreed that westernization dislocate the students from their roots, 83% agree that westernization is increasing the power of English among Pakistani students, 76% agreed that westernization projects the westernized students as model students, 81% agreed that westernization is creating a cultural gap ‘within’ the students, 83% agreed that westernization is cause of clash of values students, 84% agreed that westernization is spreading materialism amongst students and 77% agreed that westernization is destroying the character of the students.

CONCLUSION

A large number of Pakistani University students are effected badly by western culture. No doubt every culture has positive effect but Pakistani University students get nothing form positivity of this culture. Like widening knowledge literacy improvement, quality of life and education and performance of students. On the other hand students are badly effected from negativity of western culture like dislocation from their roots, cultural gap in society, materialism among relations and destroying the character and values of the students.

All results of the study showed that western culture destroys the students in every field of their life.

RECOMMENDATION

The social and political systems of a state are always in accordance with each other; the situation is opposite in Pakistan and society is badly divided on this matter. The point of view of religious groups is different, and the liberals have their own liberal pursuits. State in its constitution and law making has to keep Islamic principles as organizing principles of the state but practically social domain situation is opposite to this, the inclusion of Western values is redirecting the society around its own principles.

The state of Pakistan is a result of aspirations of Muslim society in India; it is supposed to be an Islamic state. Islam was the only force, which tied the people belonging to different ethnicities as one nation, and it is the only and ultimate identity of Pakistani society. Pakistani society is essentially in need of modernization, but the path towards this is not the pursuance of western concepts, rather it needs to implement Islamic modernity and achievements in economic and scientific fields.

An ideal Islamic society advocates and provides the concept of an open, modernized and universal society that does not consider differences of ethnicities, religions, races, or cultures.

Islam provides a whole system based on the respect for human dignity, the development of his personality, the defense of personal freedom, the advancement of his material life and the care of the physical environment, the provision of basic necessities, the eradication of poverty, and the prevention of crime and suicide. It is very concerned with the development of education and equally focuses on the betterment of family and the overall society. The cohesiveness of family, and promotion of good values is greatly stressed and societal harmony, morality and collective interest is preferred.

The idea of Islamic progress emphasize upon social justice, harmony, and overall prosperity. It allows private property and honest market competition. Islam is open to change and accommodates positive changes.

REFERENCES

1. Arif Hasan, (2003). *The Political Alienation of Pakistan's Elite*. Paper published in Akbar Zaidi's Continuity and Change, Karachi: City Press.
2. Bernard, H.R. (1989). *Research Methods in Cultural Anthropology*. California: Sage Publications Inc.
3. Black, C.E. (1966). *The dynamics of modernization: A study in comparative history*, New York: Harper and Row.
4. Carrithers, M. (1992). Why humans have cultures: *Explaining anthropology and social diversity*. Oxford University Press.
5. Featherstone, M. (1990). Global Culture: Nationalism, Globalization and Modernity, SAGE Publications.
6. Gardezi, H. N. and Mumtaz, S. (2004). Globalisation and Pakistan's dilemma of development. *The Pakistan Development Review*, 423-440.
7. Inglehart, R. and Baker, W.E. (2000). Modernization, Cultural Change, and the Persistence of Traditional Values. *American Sociological Review*, 65(1), 19-51.
8. Kinvall, K. (2002). Nationalism, religion and the search for chosen traumas: comparing Sikh and Hindu identity constructions. *Ethnicities*, 2(1), 79-107.
9. MacBride Commission (1980). Cultural domination and the threat to cultural identity.
10. Mead, R. (1998). *International Management: Cross-cultural Dimensions*, 2nd ed. (Oxford: Blackwell, 1998).
11. Mole, J. (1996). *Mind Your Manners: Managing Business Cultures in Europe London*: Nicholas Brealey,.
12. Rosenau, J.N. (1980). *The study of global interdependence: essays on the transnationalization of world affairs*. Nichols Publishing Company.
13. Schneider, S.C. and Barsoux, J.L. (2003). *Managing across cultures*. Pearson Education. 2nd ed. London: Prentice Hall.
14. Shenkar, O. (2001). Cultural distance revisited: Towards a more rigorous conceptualization and measurement of cultural differences. *Journal of International Business Studies*, 32(3), 519-535.
15. Sirmon, D.G. and Lane, P.J. (2004). A model of cultural differences and international alliance performance. *Journal of International Business Studies*, 35(4), 306-319.
16. Sweeney, B. (2002). Hofstede's model of national cultural differences and their consequences: A triumph of faith-a failure of analysis. *Human Relations*, 55(1), 89-118.
17. Triandis, H.C. (2004). The many dimensions of culture. *The Academy of Management Executive*, 18(1), 88-93.

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ROLE OF ACADEMIC MOTIVATIONAL TECHNIQUES IN TEACHING AT SECONDARY LEVEL

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ABSTRACT

The study highlighted the role of academic motivational techniques in teaching at secondary level. The major objectives of the study were; (1) To explore the academic motivational techniques used in teaching (2) To make comparison of academic motivational techniques used by urban and rural secondary school teachers. The teachers who are teaching to 10th class were the population of the study. The study was conducted in three districts: Okara, Pakpattan and Sahiwal. The researchers adopted the questionnaires with a little modification, one for Secondary school teachers and other for Secondary School students. Statistical tool of frequency, mean score and standard deviation used for analysis. The study concluded that students feel enlivening when teachers acknowledge their achievements while urban secondary school teachers value the achievement of secondary school students often. Urban secondary school teachers stimulate their students by taking interest in solving students' problems. The study concluded that rural secondary school teachers make more used of academic motivational techniques in their teaching as compared to urban secondary school teachers.

INTRODUCTION

Motivation is seen as a pre-requisite of and necessary element for student engagement in learning. Students' motivation in learning is not only an end in itself but it is also a means to the end of students' performance (Russell, Ainley & Frydenberg, 2005; Ryan & Deci, 2009). This is important because students' motivation may lead to higher academic achievement throughout students' career (Zyngier, 2008). If teachers want to know and resolve the students' problems and to make schools learning place (Meyer, 2010; Smyth and McInerney, 2007 &. (Mitra, & Serriere, 2012; O'Brien, & Lai, 2011; Potter & Briggs, 2003; Zyngier, 2011). Ryan and Deci (2000) said that motivation means to be moved to do something. A student who feels no impetus to act is thus considered as unmotivated, whereas someone who is energized or activated toward an end is considered motivated.

It is very teachers for educators to understand the different types of extrinsic motivation and how they may work as they cannot always rely on intrinsic motivation to promote learning. There many of the class room activities that a teacher wants students to improve learning skills are not necessary for them or enjoyable, therefore, using more actively extrinsic motivation such as electronic media sources are effective strategies for successful teaching (Ryan & Deci, 2000). They proposed that some types of extrinsic motivations are weak, whereas, some are active. They also described various forms of extrinsic motivation; i.e. interjected regulation, identification and integrated regulation. Integrated motivations share with intrinsic motivation, but are still considered as an extrinsic from of motivation. Intrinsic motivation is at the end of this continuum. Extrinsic motivation show how much a student is self-determined during a learning process and also shows the quality of effort or putting into a task (Reeve, Deci & Ryan, 2009).

Cole (2000), “Motivation is a term that is used to describe those processes, both initiative and rational, by which people seek to satisfy the basic drives, perceived needs and personal goals, which trigger off human behavior”. The concept of motivation is situational and varies from student to student at different situations. Motivating the students to learn is a topic of great concern for teachers of modern era.

Motivation directs the behaviour of the individual towards certain goals. Motivation has been defined as: the psychological process that gives behavior purpose and direction (Kreitner, 1995). Teachers motivate students through different techniques based on understanding of the students’ growth and development patterns, individual abilities, and internal and external factors that may compel and sustain the need to learn more (Luthans, 1998). School heads need to provide the conducive environment to ensure that their teachers can see that by working towards institutional goals they are also achieving some of their objectives. These achievements could be such as financial rewards or personal rewards such as the respect of their colleagues or job satisfaction (Peters, 1992).

The teacher is the one that transfers educational philosophy into knowledge and skills to students in the classroom. Classroom environment is important in student motivation. Teachers consider that classroom is safe, healthy, happy place with supportive resources for teaching and learning, they tend to participate more than expected in the process of teaching and overall improvement of the institutions. The teachers transmit knowledge and the physical conditions of the classroom through discipline and control. They diagnose students’ feelings and motivate them to positively response in the classroom environment (Griffin, 1994). To summarize, there have been demonstrated that motivation is one of the most important factors in learning and achievement. This means that if students are more motivated, their achievement is likely to increase. Moreover, the above discussion showed that teachers need to focus on students' preferences when planning to embark on teaching.

REVIEW OF LITERATURE

Mahmood (2013), compared teachers’ motivation as pedagogical technique between Public and Private Sector secondary schools in Pakistan. He noted that qualification and salary packages of government teachers were far better than that of private teachers, but the output was reverse. It was found that teachers of both the sectors did not create intrinsic motivation among students. They also showed anger and punished students during teaching.

In present era, motivation is very important for quality education. There are very few chances to produce encouraging results until unless they are not motivated through five key ingredients of motivation impacting students' learning are student, teacher, content, method/process, and environment and they must have access, ability, interest, and value education. Furthermore, teachers must be well trained, focus on monitoring of the educational process, be dedicated to the students, and be inspirational. The content must be accurate, timely, stimulating, and fulfill the student's current and future needs. The method must be inventive, encouraging, interesting, supportive, and provide tools that can be applied to the student's real life. The learning environment needs to be approachable, safe, positive and empowering. Motivation can be applied when students are exposed to a large number of these motivating factors on a regular basis (Palmer, 2007; Debnath, 2005; D'Souza and Maheshwari, 2010).

The focus of this article is to provide the teachers with suggestions that can be used to motivate students. As such, suggestions are provided for each of the five key ingredient areas impacting student motivation: student, teacher, content, method/process, and environment.

Students are the raw materials for education and the primary products of educational transformations; and most important...students are key members of the labor force involved in creating education" (Lengnick-Hall and Sanders, 1997, p. 1335). Motivation also, the increase diversity of individual differences among students can be seen in time management, learning styles, maturity, cultural orientation, and interests. Senge et al. (1994, suggest that teachers should be "producers of environments that allow students to learn as much as possible".

The role of teachers seems to be shifting from preprogrammed knowledge mentors of students' learning. Teachers must be empowered to exercise professional motivation in the classroom to attain clearly expressed goals. Academic motivation should be given latitude to test individual approaches based on students' performance. Teachers should be provided motivational training to support them in enhancing students' learning and performance.

Content must be accurate and timely. It should be should be relevant and useful to the students' learning. Olson (1997) described that student motivation depends on the extent to which the teacher is able to satisfy the student's needs. Content also must be included to satisfy students' needs.

The method/process is the way in which content is presented. It is an approach used for instruction. There are two basic approaches of motivation in the classroom are creating a classroom structure and institutional method and helping the students to develop tools that will enable them to be self-regulated (Alderman, 1999).

School environment must be conducive and be available and accessible to both the teachers and students. It must be quality oriented caliber that contributes to the motivation of the students. On the other hand, environment of freedom and freedom to learn from mistakes can foster motivation to learn (Rumsey, 1998).

It is necessary for the teachers to have teaching and learning process effective, they must use maximum all the above motivational ingredients in the class room properly.

Motivation in the classroom plays function of five components: student, teacher, content, method/process, and environment. Aspects of any of these five components could contribute to hinder motivation. Teacher should start just by choosing and trying new possibilities for enriching student motivation. More importantly, teachers could watch themselves and their own behavior to become self-aware of new understandings about motivation (Robinson in Friedman, 1999, p. 2).

Research Objectives

- To explore the motivational techniques used in teaching
- To compare motivational techniques used by urban and rural secondary school teachers.

Research Methodology

- Researchers used quantitative research method for this study.
- Secondary school teachers of three districts i.e. Okara, Pakpattan and Sahiwal were population of the study.
- Simple random sampling technique was used and 100 secondary school teachers were selected from each district (100x3 = 300) who were teaching to the 10th class
- Questionnaire was the tool of data collection which Comprised of 15 items related to the academic motivational techniques.
- Statistical techniques of mean score, standard deviation and t-value were used for the data analysis and computation of results

ANALYSIS AND INTERPRETATION OF DATA

Table 1
Appreciate Students' Work

Respondent	N	Mean	St. Deviation	t-value
Urban Teachers	300	4.84	.365	0.95
Rural Teachers		4.91	.286	

Table 1 highlighted the values of mean score, standard deviation and t-value regarding teachers' appreciation to students' work. The mean score and standard deviation goes from (4.84 to 4.90) and (.365 to .286) respectively. The t-value (0.95) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of rural teachers appreciate students' work.

Table 2
Acknowledge Students' Achievement

Respondent	N	Mean	St. Deviation	t-value
Urban Teachers	300	3.24	1.49	1.93
Rural Teachers		3.99	0.67	

Table 2 highlighted the values of mean score, standard deviation and t-value regarding teacher acknowledges students' achievement. The mean score and standard deviation goes from (3.24 to 3.99) and (0.67 to 1.49) respectively. The t-value (1.93) is greater than at p-value 0.05, so, there is a significant difference exist. Therefore, difference was found between the responses of urban and rural teachers. It is reflected that majority of the rural teachers acknowledge students' achievement.

Table 3
Teacher Gives Appropriate Homework

Respondent	Mean	St. Deviation	t-value
Urban Teachers	3.07	1.07	2.94
Rural Teachers	3.65	1.14	

Table 3 highlighted the values of mean score, standard deviation and t-value regarding appropriate homework. The mean score and standard deviation goes from (3.07 to 3.65) and (1.07 to 1.14) respectively. The t-value (2.94) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of rural teachers give students appropriate homework.

Table 4
Competition among Students

Respondent	Mean	St. Deviation	t-value
Urban Teachers	4.00	0.90	1.87
Rural Teachers	3.49	1.29	

Table 4 highlighted the values of mean score, standard deviation and t-value regarding competition among students. The mean score and standard deviation goes from (3.49 to 4.00) and (0.90 to 1.29) respectively. The t-value (1.87) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of urban teachers make competition among students.

Table 5
Role Model for Students

Respondent	Mean	St. Deviation	t-value
Urban Teachers	4.28	1.01	1.01
Rural Teachers	3.84	1.07	

Table 5 highlighted the values of mean score, standard deviation and t-value regarding role model for students. The mean score and standard deviation goes from (3.84 to 4.28) and (1.01 to 1.07) respectively. The t-value (1.01) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of urban teachers act as a role model for students.

Table 6
Habit of Self-Study

Respondent	Mean	St. Deviation	t-value
Urban Teachers	4.52	0.75	1.23
Rural Teachers	3.84	0.94	

Table 6 highlighted the values of mean score, standard deviation and t-value regarding habit of self-study. The mean score and standard deviation goes from (4.52 to 3.84) and (0.75 to 0.94) respectively. The t-value (1.23) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of urban teachers develop habit of self-study among students.

Table 7
Proper Feedback

Respondent	Mean	St. Deviation	t-value
Urban Teachers	4.05	0.94	4.23
Rural Teachers	4.29	0.91	

Table 7 highlighted the values of mean score, standard deviation and t-value regarding proper feedback. The mean score and standard deviation goes from (4.05 to 4.29) and (0.91 to 0.94) respectively. The t-value (4.23) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of rural teachers provide proper feedback.

Table 8
Awareness about success

Respondent	Mean	St. Deviation	t-value
Urban Teachers	3.45	1.24	1.30
Rural Teachers	4.07	1.05	

Table 8 highlighted the values of mean score, standard deviation and t-value regarding awareness about success. The mean score and standard deviation goes from (3.45 to 4.07) and (1.05 to 1.24) respectively. The t-value (1.30) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of rural teachers create awareness about success among students.

Table 9
Encourage Freedom of Expression

Respondent	Mean	St. Deviation	t-value
Urban Teachers	3.51	1.37	1.17
Rural Teachers	4.24	0.78	

Table 9 highlighted the values of mean score, standard deviation and t-value regarding freedom of expression. The mean score and standard deviation goes from (3.51 to 4.24) and (0.78 to 1.37) respectively. The t-value (1.17) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of rural teachers encourage freedom of expression.

Table 10
Use of Variation in Instructional Strategies

Respondent	Mean	St. Deviation	t-value
Urban Teachers	3.89	1.24	1.68
Rural Teachers	4.22	0.72	

Table 10 highlighted the values of mean score, standard deviation and t-value regarding instructional strategies. The mean score and standard deviation goes from (3.89 to 4.22) and (0.72 to 1.24) respectively. The t-value (1.68) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of rural teachers use variation in instructional strategies.

Table 11
Active Involvement of Students

Respondent	Mean	St. Deviation	t-value
Urban Teachers	3.97	0.71	2.93
Rural Teachers	3.23	1.53	

Table 11 highlighted the values of mean score, standard deviation and t-value regarding active involvement of students. The mean score and standard deviation goes from (3.23 to 3.97) and (0.71 to 1.53) respectively. The t-value (2.93) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of urban teachers encourage active involvement of students.

Table 12
Develop Self-Respect

Respondent	Mean	St. Deviation	t-value
Urban Teachers	3.01	1.13	2.92
Rural Teachers	3.93	1.05	

Table 12 highlighted the values of mean score, standard deviation and t-value regarding develop self-interest. The mean score and standard deviation goes from (3.01 to 3.93) and (1.05 to 1.13) respectively. The t-value (2.92) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of rural teachers develop self-respect.

Table 13
Check Homework Regularly

Respondent	Mean	St. Deviation	t-value
Urban Teachers	3.69	1.22	2.18
Rural Teachers	4.23	0.90	

Table 13 highlighted the values of mean score, standard deviation and t-value regarding check homework regularly. The mean score and standard deviation goes from (3.69 to 4.23) and (0.90 to 1.22) respectively. The t-value (2.18) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of rural teachers check homework regularly.

Table 14
Interest in Solving Problems

Respondent	Mean	St. Deviation	t-value
Urban Teachers	4.52	1.01	1.02
Rural Teachers	3.71	1.23	

Table 14 highlighted the values of mean score, standard deviation and t-value regarding interest in solving problems. The mean score and standard deviation goes from (3.71 to 4.52) and (1.01 to 1.23) respectively. The t-value (1.02) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of urban teachers take interest in solving students' problems.

Table 15
Arrange Instructional Material Properly

Respondent	Mean	St. Deviation	t-value
Urban Teachers	4.48	0.62	3.12
Rural Teachers	3.78	0.98	

Table 15 highlighted the values of mean score, standard deviation and t-value regarding instructional material. The mean score and standard deviation goes from (3.78 to 4.48) and (0.62 to 0.98) respectively. The t-value (3.12) is greater than at p-value 0.05, so, there exist significant difference between urban and rural teachers. It is reflected that majority of urban teachers arrange instructional material properly.

FINDINGS AND CONCLUSION

- The study found that majority of the rural teachers appreciate students' work and acknowledge students' achievement in their academia
- Rural teachers motivate the students through their teaching and give appropriate homework to the students
- The study revealed that urban teachers provide motivation by creating environment of competition among students
- The study reflected that urban teachers acts as role model for students as a result students develop self-study habits were improves learning skills whereas majority of rural teachers provide proper feedback to the students
- The study revealed that majority of rural teachers create awareness about success among students and encourage freedom of expression
- The study reflected that urban teachers act as role model for students as a result students develop self-study habits were improves learning skills. Whereas majority of rural teachers provide proper feedback to the students
- The study revealed that majority of rural teachers create awareness about success among students and encourage freedom of expression

CONCLUSION

It was concluded that rural secondary school teachers make more use of motivational techniques in their teaching as compared to urban secondary school teachers.

REFERENCES

1. Alderman, M.K. (1999). *Motivation for Achievement: Possibilities for Teaching and Learning.* Mahwah, NJ: Lawrence Erlbaum Associates, Inc. Publishers.
2. Cole, G.A. (2000). *Management Theory and Practice*. Lens Educational Aldine Place: London.
3. D'Souza, K.A. and Maheshwari, S.K. (2010). Factors Influencing Student Performance in the Introductory Management Science Course. *Academy of Educational Leadership Journal,* 14(3), 99-120
4. Debnath, S.C. (2005). College Student Motivation: An Interdisciplinary Approach to an Integrated Learning Systems Model. *Journal of Behavioral and Applied Management,* 6(3), 168-189.
5. Friedman, S. (1999). The Importance of Maintaining Your Motivation. *Financial Services Advisor,* 142(4), 4-7.
6. Kreitner, R. (1995). *Management 6th ed.* Houghton Mifflin Company. New York, USA.
7. Lengnick-Hall, C. and Sanders, M. (1997). Designing Effective Learning Systems for Management Education. *Academy of Management Journal*, 40(6), 1334-1368.
8. Luthans, F. (1998). *Organizational Behavior 7th ed*. New York: McGraw-Hill, Inc. USA. 141-165.
9. Mahmood, T., Ahmed, M., Shoaib, H. and Ghuman, M.A. (2013). Motivation as Pedagogical Technique for Teachers: A Cross Comparison between Public and Private Sector. *Mediterranean Journal of Social Sciences*, 4(2), 563-70.
10. Masaud, Z. Ali, M.Q., Hassan, M.U. and Nazli, S. (2015). Comparison of motivational techniques used by elementary school teachers in Pakistan. *Journal of Policy Research,* 1(1), 1-8.
11. Meyer, E.J. (2010). *Transforming school cultures. Gender and Sexual Diversity in Schools*, X, 121-139. http://dx.doi.org/10.1007/978-90-481-8559-7_7
12. Mitra, D.L. and Serriere, S.C. (2012). Student Voice in Elementary School Reform Examining Youth Development in Fifth Graders. *American Educational Research Journal,* 49(4), 743-774.
13. Montalvo, G.P. (1998). Pleasing the Teacher. Dissertation Abstracts International Section A. *Humanities & Social Sciences,* 58(8), February, 3002.
14. Olson, G. (1997). *Motivation, Motivation, Motivation - Secondary School Educators*. Retrieved from sysiwyg://934/http://7-12educators.about...12educators/library/ weekly/aa071897.htm.
15. Palmer, D. (2007). What Is the Best Way to Motivate Students in Science? Teaching Science. *The Journal of the Australian Science Teachers Association,* 53(1), 38-42.
16. Peters, T. (1992). *Liberation Management*. Pan Book Publisher Ltd. London: UK
17. Potter, G. and Briggs, F. (2003). Children talk about their early experiences at school. *Australian Journal of Early Childhood*, 28(3), 44-49.

18. Rumsey, D. (1998). Business Statistics: Decision Making with Data. *The American Statistician*, 52(1), 85-86.
19. Russell, V.J., Ainley, M. and Frydenberg, E. (2005). *Student motivation and engagement. Schooling Issues Digest*. Australian Government, Department of Education, Science and Training.
20. Ryan, A.M. (2000). Peer groups as a context for the socialization of adolescents' motivation, engagement, and achievement in school. *Educational Psychologist*, 35(2), 101-111.
21. Senge, P., Kleiner, A., Roberts, C., Ross, R. and Smith, B. (1994). *The Fifth Discipline Fieldbook: Strategies and Tools for Building a Learning Organization*. NY: Doubleday Currency.
22. Smyth, J. and McInerney, P. (2007). *Teachers in the middle: Reclaiming the wasteland of the Adoloscent years of schooling.* New York, USA: Peter Lang.
23. Wentzel, K.R. and Miele, D.B. (2009). Promoting self-determined school engagement: Motivation, learning, and well-being. *In Handbook of motivation at school* (185-210). Routledge.
24. Zyngier, D. (2008). Conceptualizing student engagement: Doing education not doing time. *Teaching and Teacher Education*, 24, 1765-1776.
25. Zyngier, D. (2011). Conceptualising risk: left numb and unengaged and lost in a no-man's-land or what (seems to) work for at-risk students. *International Journal of Inclusive Education*, 15(2), 211-231.
26. Zyngier, D. (2012). Pygmalions, Galateans, and Golems: Why pre-service teacher beliefs about children from diverse backgrounds (still) matters. *International Journal on School Disaffection*, 9(1), 23-38.

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COMPARATIVE STUDY OF STUDENTS' SOCIAL DEVELOPMENT IN PUBLIC AND PRIVATE SECTOR COLLEGES

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ABSTRACT

Social development is defined as arranging human needs in the growth and development of society. Social development also governs the norms and conventions that govern human interaction. The focus is on improving the lives of regular citizens, especially the poor, to make society a better place for everyone. The researcher explored major aspect of personality development such as social development of students. The aims of the study were (1) To compare the perception of male and female students about social development in public and private sector colleges. (2) To explore which sector is more efficient in doing better social development at intermediate level in public and private sector colleges. The study was descriptive in its nature with survey design. All public and private colleges of Punjab Province were population of the study. Out of 640 students, 626 students participated in the research, 160 teachers and 30 administrators out of 32 participated in the research. Three questionnaires were used for data collection. Pilot testing of the instruments was also conducted. Reliability of instruments of students, (α=.82), teachers, (α=.76) and administrators (α=.80) was accordingly. Each questionnaire was on five point Likert scale. To check the assumption for parametric statistics and summary statistics, boxplot and normality tests were applied. Non-parametric test used for comparison of public and private sectors. The results of the study revealed that there is no significant difference in perception of male students and female students regarding social development in public sector colleges as well as private sector colleges. It was also concluded from the study that social development of students is comparatively better in private sector as compared to public sector.

KEY WORDS

Social Development, Personality, Public Sector, Private Sector

INTRODUCTION

Education, commonly, is a sort of learning wherein information, talents and habits of both people or organization of humans are transferred to the approaching generations truly thru teaching and training or research. Training usually nurtures below the umbrella of others however, in a few cases, it is able to also show it autodidactic under positive situations. Usually, reviews are considered academic in the event that they have full-size impact on ones thinking, feeling and acting. Training sustains evolutionary stages i.e. pre-college, number one faculty, secondary faculty and then university and college.

The college years are a time of extensive increase and trade for college kids as they address current thoughts and studies that enhance their previously amassed knowledge and ideals. School contributors who apprehend those changes may additionally increase such courses and sports that meet students' wishes and guide their sluggish development (Bhatia, 2005). If the purpose of college is to develop the personality of the students, it should first set norms. Norms are the equally accepted by the society. When we hear someone says that "Marge has a lot of personality", this tells us that Marge is a colorful and interesting person. But the term personality is incorrectly used, because one's personality includes all of one's behavioral characteristics. Used correctly, a person does not have more personality than others.

"Personality" is a derivation from the Latin term "Persona". "Persona means a mask that theatre actors wear while playing a role in the play; (2) it is also the true self, that is a combination of one's integral motivations, emotions, conducts, and ideas (Robbins, 2001). Personality includes the ways in which one shows interaction and reaction to certain things".

Personality initiates very important role in the decision of self-employment. Personality is a combination of distinguish characteristics of a person from other is called personality. A person has the charm, positive attitude or smiling face, it does not mean he has personality; rather it describes the growth and development of a person's whole psychological system (Kinicki & Kreitner, 2003).

Maqbool (2011) states that development means a continual process, which begins with conception and ends with the death of an individual. Although one changes physically and these changes are the most notable, one also changes personally socially intellectually, cognitively and educationally. The subject of human development has been the basic primary concern of educational psychologists.

Statement by Mangle (2002) reveals that the process of engaging individuals into the social world is called socialization. In common sense, socialization is an eternal training of an individual in adjustment of oneself in society.

Socialization includes multiple processes i.e. learning norms, rules, techniques and other cultural patterns. There are two stages of learning. Social development takes place in the consequence of learning values, knowledge and skills. This process relates to others and people by this play their positive role in family, school and community. This learning process is transferred directly to the children.

Mangle (2002) stated that the process of inducting the individual into the social world is called socialization. In common sense, socialization is a lifelong training for the adjustment of one's life in society. This process of socialization is the process of learning norms, rules, techniques and other cultural patterns. In two ways, the process of learning takes place.

Social development is a kind of learning which is transferred to the children via friends, teachers and family members they are part of. This social relation making process helps children enhance their awareness level of social values and expectations. By this, they come to know themselves and also get acknowledge about their role in society. When children develop from social point of view, they become active to respond influences around them and play a significant role in formation of social relationships (Sadler, 2004).

Socially accepted values and activities are subject matter of social development. Social activities go hand in hand with cultural heritage and social values. For example, giving response to their elders is fine stance here. Abilities are demonstrated by individuals to meet their social needs, in fact is known as social development. Organized set of activities for awakening social relationship refers to social development. Home is lap of social life. Home is a model of society for a child. Soon after it, the child moves to his nearby street or school. It is also an inevitable fact that one who develops himself socially, leads a significant life. It is responsibility of school to infuse and develop social abilities among children because man is sociable creature in his nature. (Maqbool, 2011).

LITERATURE REVIEW

Humans are vital social beings. In modern life man has come to feel the necessity of human relationships more than ever earlier. Almost, nobody can to any extent further live in a country of entire or even partial isolation. It is by way of living in society that man comes to expand human traits of which the social section is the fundamental first-class which he should broaden for better adjustment in a civilized society.

In the beginning, the human child is blind to the social section, though he's born with in a social institution, is surrounded with the aid of the ones, whose case social development has already taken place or is under the way. From simply a biological organism, the human baby develops in his total make up in the context of social environment. As the kid grows up, he does not only develop in physical, intellectual, emotional and attitudinal conduct, but additionally in his social behavior. It is the suited social behavior that makes him an acceptable social being, Hameed & Aziz (2015).

Social Development, not apart from other phases of Development

Social boom and development cannot be taken into consideration aside from different levels of development inclusive of physical, mental and emotional. The linkage among these factors is so near that, whatever may be the character of a man or woman’s growing social self, his degree of mental alertness and the volume of his emotional maturity as those affect and are laid low with different humans, either develop or hinder the achievement of applicable social attitudes and behavior at any level of development. Consequently, social improvement places emphasis upon the truth that the growth can be modern (Jack, 2012).

Two Important Aspects of Social Development

Zanting (2003) claimed that Social growth and improvement cannot be taken into consideration other than one-of-a-kind levels of improvement including bodily, mental and emotional. The linkage amongst those elements is so close to that, a few element may be the individual of a man or woman's developing social self, his degree of mental alertness and the volume of his emotional adulthood as the ones have an effect on and are tormented by extraordinary people, either develop or avoid the success of relevant social attitudes and conduct at any level of development. Consequently, social development places emphasis upon the truth that the growth can be progressive.

Social Development

The maturing and growing child develops now not merely in bodily intellectual and emotional conduct however also corresponding in social development. The child's variety of social activities is interwoven with other capabilities of his growth, i.e., bodily, mental and emotional. Language is the end result of interrelation among mental and social behavior. Jealousy, shyness, affection and sympathy are the result of interrelation among social and emotional forms of behavior.

Influence of Social Group

Each child, like each adult, relies upon other humans for his existence. The depended is whole at delivery and during the early year of baby hood as an infant comes to be older; he becomes less based upon the social organization, however, records desires then group and contact with others. All through every succeeding year his relations with others turn out to be more complicated and should keep in touch with greater human beings as well as with human beings of various types.

Now not first-class is the child structured upon the social organization, however of even extra importance, the social institution upon which he depends determines to a huge make bigger what type of individual he may be. Maqbool (2011) concluded that an infant is bendy, both bodily and intellectual, his development can be motivated and formed into a sample determined with the aid of members of organization with whom he is usually associated at no age is he loose from the impact exerted by using his associates. This influence is specifically pronounced at some point of the early years of life, due to the fact this is the time of finest flexibility. At this time the child's family is a most influential socialist agency in his existence. When he is going to high school his instructors and his friends begin to exert and influence over his persona and a process of socialization. Peer influence is more effective than that of trainer.

one of the most essential approaches in which the child peer's organization affects him is by supporting him to achieve independence from his discern and turn out to be a man or woman in his very own style. Over his institutions together with his peers, he learns to suppose independently to make his very own designs, points of views and values now not shared together with his family and to examine kinds of behavior accredited with the aid of the group which he belongs to.

Pattern of Social Development

In line with Chartier (2003) Social development follows a patterning an orderly collection, now not most effective the form of social conduct presentations at each age but also in the type of companions selected which means typically each infant need to bypass thru certain stages of becoming soiled at approximately the identical age as youngsters pass via the identical phases. Consistent with vessel proper kids are expanded in social improvement, even as dull youngsters are began in their development in the direction of social maturity.

knowing what a pattern of social improvement is, you'll radially predict that at a certain age the kid may be timid in the presence of strangers, at another age he'll crave the companionship of people of his own age and intercourse, even as at nevertheless every other age, his interest can be centered on contributors of contrary sex. Siddiqui (2008) concluded that studies of organizations of children have discovered that there are age ranges in social improvement. Even as two years old it solitary in his play he is however influenced with the aid of older youngsters to the quantity that he imitates their behavior each in play and conduct. 2.5 years old refuses to share toys with others and grasp toys from them, ignores requests and refuses to comply. The 3years old suggests the fundamentals of team play and the 4 year old suggests the beginning of group affect by way of being copious of the opinions of the others and seeks to benefit and interest through "displaying of", first, young youngsters lack organization feeling. Then segment of partial adjustment takes place all through which the child starts off evolved installed his function and to play in extremely coordinates fission with others. Within the 0.33 section of social development organization family members are hooked up and baby enjoys institution lifestyles. At this time, the institution invents and organizes its personal organization names and becomes an independent unit, free from adults' supervision and interface. For the duration of the later factors of formative years there's an increase in social contacts. The dimensions of the organization and frequency of participation likewise increase.

OBJECTIVES OF THE STUDY

Objectives of the study were;

1 To compare the preparation of male and female students about social development in public and private sector colleges.
2 To explore which sector is more efficient in doing social development of students at intermediate level.

Significance of the Study

- The study may provide a guideline for policy makers to improve the general set up in public sector.
- It might also set a guideline for higher management in private sector.
- The findings may be helpful for the administrators of the institutions to design their institutional objectives and suitable guideline for teachers might also be provided to build more balanced personality of the students.
- The results may prove to be beneficial for the growth of healthy society and might open new horizons for policy makers as well as curriculum developers.

Research Methodology

The study was descriptive in which and survey design was used for data collection.

Population of the study

All Public and Private Colleges in Punjab Province were the population of the study but it was delimited to Lahore division.

Sample of the study

Sixteen public and sixteen private colleges of Lahore division were selected randomly from where data was to be collected. From each college, an administrator, five teachers and twenty students were selected as sample. All the teachers and students were selected randomly.

Data Collection

Out of 640 students, 626 students responded in the research, 160 teachers and 30 administrators provided responses out of 32 colleges selected for research.

Instrument of the study

Data were collected through questionnaire. Three questionnaires were developed for the students, teachers and administrators respectively on five-point (Likert) scale. Pilot testing of the instruments was also conducted. Coefficient of reliability of instruments for students was, (α =.82) that of teachers was, (α=.76) and administrators (α= .80). To check the assumption for parametric statistics, summary statistics, boxplot and normality tests were applied. Mann-Whitney U test for comparison of public and private sector colleges of intermediate level was also affected.

Table 1.1
Comparison between Male and Female of Private and Public Colleges Regarding social Development

Scales	Male		Female		Mann-Whitney U	
	M	***SD***	***M***	***SD***	**Z**	**P**
Private	4.17	.47	4.11	.49	-.92	.354
Public	3.68	.27	3.90	.31	-1.55	.121

Table 1.1 shows that the results of Mann-Whitney U test indicate that there is no significant difference (p>.05) between the perception of male students (M=4.17, SD=.47) and female students (M=4.11, SD=.49) of private sector and there is also no significant difference (p>.05) between the perception of male students (M=3.68, SD=.27) and female students (M=3.90, SD=.31) of public sector colleges regarding social development. The results of the study reveal that there is no significant difference in perception of male students and female students regarding social development in public sector colleges as well as private sector colleges.

Table 2
Overall Comparison between Perception of Male and Female Students Regarding social Development

Scales	Male		Female		Mann-Whitney U	
	M	***SD***	***M***	***SD***	Z	P
Social Development	4.01	.41	4.01	.43	-.27	.781

Table 1.2 shows that the results of Mann-Whitney U test indicate that there is no statistically significant difference (p>.05) between male students (M=4.01, SD=.41) and female students (M=4.01, SD=.43) regarding social development.

Table 3
Comparison between Public and Private Sector Colleges Regarding social Development

Scales	Public		Private		Mann-Whitney U	
	M	***SD***	***M***	***SD***	**Z**	**P**
Students	3.88	.29	4.14	.48	-8.3	<.001
Teachers	4.14	.38	4.36	.33	-3.7	<.001
Administrators	3.87	.35	4.31	.34	-3.2	.000

Table 1.3 reflects that social development among students of public sector (M=3.88, SD=.29) is less than private sector (M=4.14, SD=.48). According to the perception of teachers, Social development among teachers of public sector (M=4.14, SD=.38) is less than private sector (M=4.36, SD=.33).According to the perception of administrators the social development among students of public sector (M=3.87, SD=.35) is less than private sector (M=4.31, SD=.34).Mann-Whitney U test was conducted to compare perception of students, teachers and administrator of Public and Private Sector colleges on social development of students at intermediate level. There is statistically significant difference (p<.05) between students of public and private sector colleges regarding social development of students at intermediate level. Results of the study show that social development of students at intermediate level in private sector colleges is better than public sector colleges.

DISCUSSION

- Allport (2003) reported that youth in colleges should be provided with equal opportunities that may lead to healthy development of the students. Present study also reveals that type of colleges affected the male students as well as female students.
- Statement by Golash (2010) reveals that the process of engaging individuals into the social world is called socialization. In common sense, socialization is an eternal training of an individual in adjustment of oneself in society.
- Aim of present study is examine about social development of intermediate students in public and private sector colleges and to show the difference between in-practice norms of public and private sector colleges. The findings of the present study support the literature reviewed, with some exception. Mean score of social development shows private colleges play a major role in social development of intermediate students. The college years are fertile ground for the rapid expansion of social development process in adults; results of several studies have shown this. According to Zwart (2007) Social development follows a patterning an orderly sequence, not only the type of social behavior displays at each age but also in the type of companions selected.

CONCLUSION

- There was no significant difference in preparation of male and female students regarding social development in public sector colleges as well as private sector colleges.
- Social development among students at intermediate level from private sector colleges was found better than public sector colleges.

REFERENCES

1. Ahmad Maqbool (2011). Development and Cognition: Ilmi Kitab Khana. Urdu bazar, Lahore.
2. Allport G.W. (2003) A Study of values, Boston, Houghton, Mittilin Company.
3. Bhatia, (2005). *The philosophy of childhood*. Cambridge, Ansari Road Darya Ganj New Delhi.
4. Chartier, M.J., Walker, J.R. and Stein, M.B. (2003). Considering comorbidity in social phobia. *Social Psychiatry and Psychiatric Epidemiology*, 38, 728-734.
5. Golash, D. (2010). *Freedom of expression in a diverse world*. Dordrecht: Springer.
6. Hameed & Aziz. (2015). Sociology and Social Problems. Abdul Hafeez Taga & Sons Publishers, Al-Fazal Market, Urdu Bazar, Lahore.
7. Kluckhohn (2004). *Values and Value Orientation in the theory of Action*. Free Press, New York.
8. Kreitner, R., Kinicki, A. and Cole, N. (2003). *Fundamentals of organizational behavior: Key concept, skills, & best practices*. McGraw-Hill.
9. Mangle, S.K. (2002). Statistics in Psychology and Education, New Delhi, Prentice Hall of India (2nd Revised Edition).
10. Mangle, S.K. (2002). Advanced Educational Psychology, New Delhi, Prentice Hall of India (2nd revised Edition).
11. Robbins, S.P. (2001). *Organizational behavior* (9th ed.). New Jersey: Prentice-Hall.
12. Sadler, M. (2004). Gender equity in the class room, In M. Kimmel (Ed.), The Gender Society Reader, 2nd edition. York: Oxford University Press.
13. Siddiqui, M. (2008). Educational Psychology. APH Publishing Corporation, 4435-36/7, Ansari Road Darya Ganj New Delhi.
14. Zanting, A., Verloop, N. and Vermunt, J.D. (2003). Using interviews and concept maps to access mentor teacher practical knowledge. *Higher Education*, 46(2), 195-214.
15. Zwart, R.C., Wubbels, T., Bergen, T.C.M. and Bolhuis, S. (2007). Experienced teacher learning within the context of reciprocal peer coaching. Teachers and Teaching: *Theory and Practice*, 13(2), 165-187.

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STUDY WAVELET MANIFESTATION OF INTERACTION OF RADIO WAVE WITH THE IONOSPHERE AT PAKISTAN AIR SPACE

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ABSTRACT

In this study we are in a position to recognize the property of ionized upper atmospheric region ranging up to 600 Km, the ionosphere the part of upper atmosphere interferes high frequency radio signals. The fluctuation in electron-ion concentration is function of temperature, solar radiation and altitude in the ionosphere region The data sets obtained from local and global sources that contain the periods mentioned as Karachi Ionosphere station (KIS) 1996, 2000, 2006, Islamabad Ionosphere station (IIS) 2005, 2006 and Wakanai (WK) 545-2005.

This work also counts the parametric variability of the ionosphere. The local data has been acquired from the space agency of Pakistan (SUPARCO) using Digisonde-256. The implementations of exploratory data analysis provided insight into the ionosphere. We have performed wavelet analysis. This approach has been utilized both in space and frequency components of ionospheric variables. The principal so far it is known as 1-Dimentional Haar wavelet transform one-to-five level.

1. INTRODUCTION

This work suggests important ionosphere mechanisms used in high frequency wave propagation research and also provides insight of dynamics of the ionosphere process. The ionosphere is an ionized part of the Earth's upper atmosphere (60-600 Km) above the earth. It is 0.1% of the total mass of our Earth's atmosphere in which the ultra-violet rays cause the formation of ions and free electrons. In 1883 Balfour Steward discussed presence of ionization as an electronically conducting medium furthermore, the existence of ionosphere was confirmed by O. Heaviside in England and A.E. Kennelly in the USA in 1902. Heaviside summarized that the formation of ionosphere is due to photo-ionization of atoms of gases of the air present in upper atmosphere. His theory was practically demonstrated after 20 years by Dr. Edward Appleton, who investigated the electrical properties of the atmosphere and discovered two layers. Lower one is called the Heaviside layer and the upper is known as Appleton layer shown in figure 1.

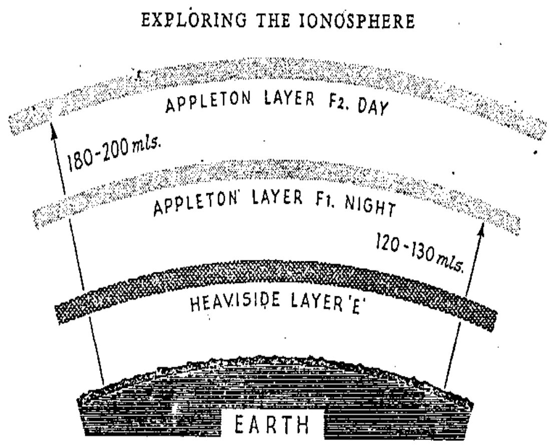

Fig. 1: The Distribution of Layers Proposed by Heaviside and Appleton

Figure 2 depicts variations of electron density with height. Figure 3. Illustrates the thermal structure of the atmosphere. The Figure 4. manifests the effects of sunspot dynamics on the ionospheric layers at solar and terrestrial environment. Figures 5 and 6 indicate the inosode stations at Pakistan and Japan from we obtained data sets of these variables [1-8].

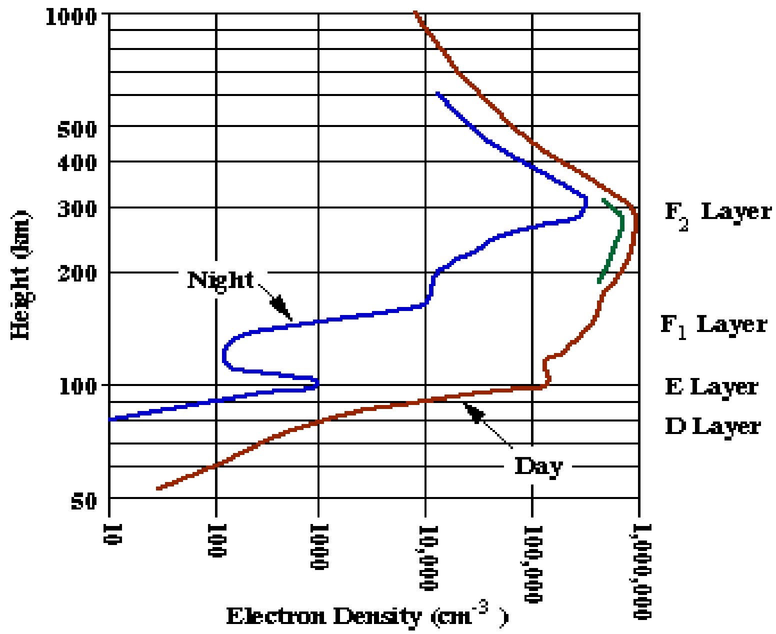

Fig. 2: The Electron Density Profile Variation with Altitude

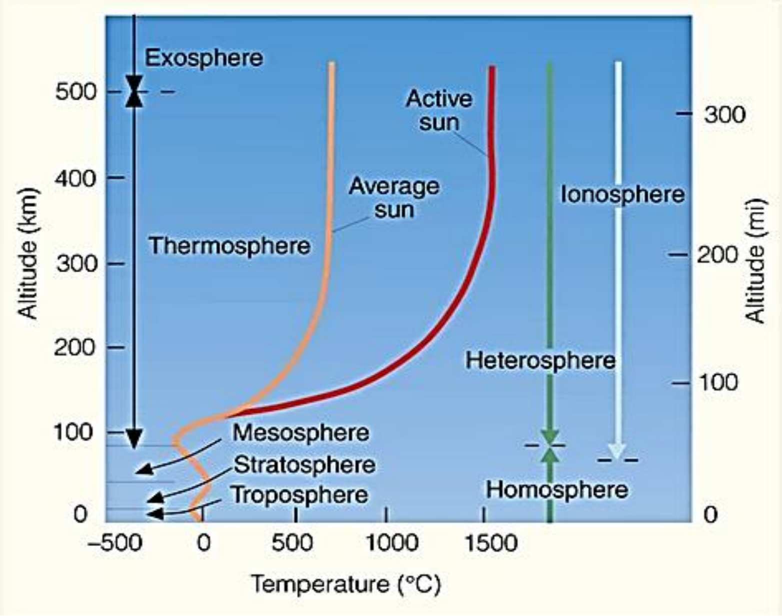

Fig. 3: Thermal Structure of Atmosphere

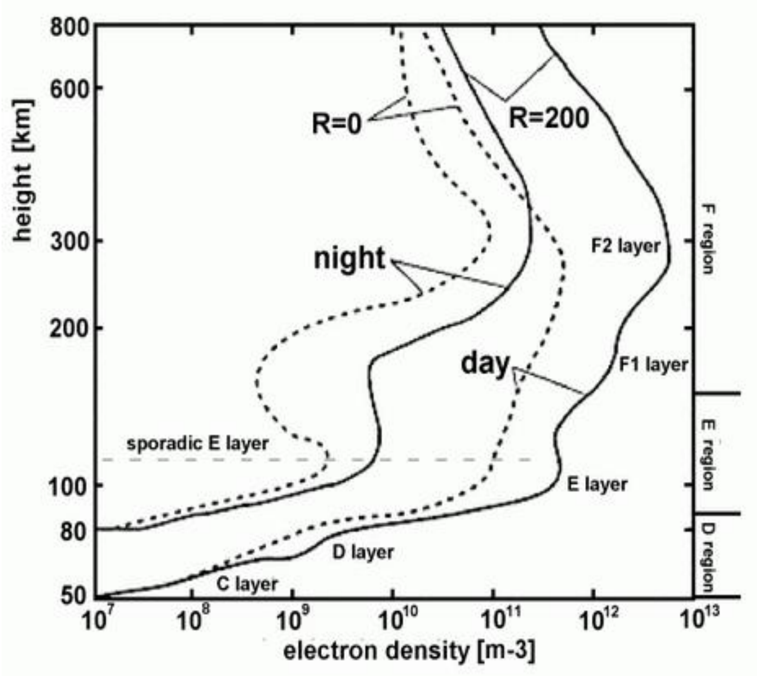

Fig. 4: Effect of Sunspot Dynamics on Ionospheric Layers

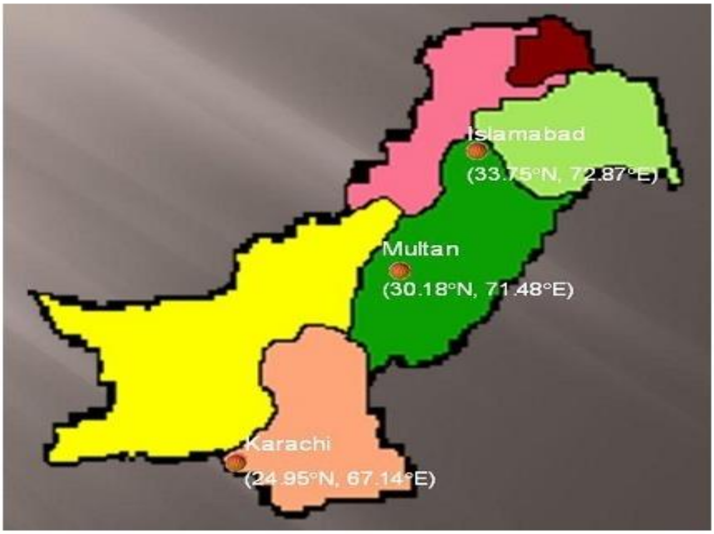

Fig. 5: Exhibition of Location of Ionospheric Stations in Pakistan

Fig. 6: Ionosphere Stations in Japan

Figure 7 shows the vertical and oblique paths used for recording and monitoring ionospheric layers within respects to, their concentration, degree of ionization, and heights from the sea level.

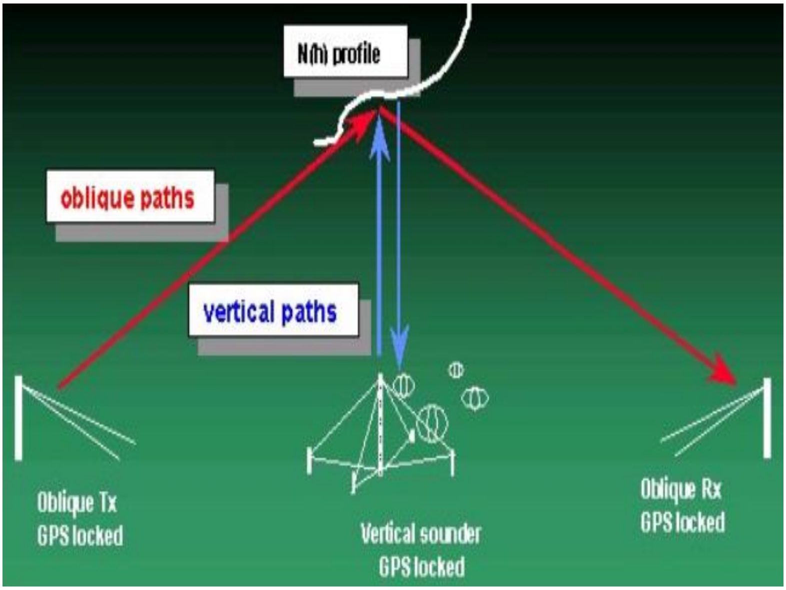

Fig. 7: Oblique and Vertical Incidence Paths

3. DESCRIPTION OF IONOSPHERIC PLASMA

The term plasma is used for an ionized gas; the ionosphere form space plasma defined as the quasi neutral gas of charged particles that exhibits collective behavior and also follow the criteria:

1) Debye length , $ʎ_D$ <<< L
2) number of particles in space N_D >>>1
3) $\acute{\omega}\tau > 1$

where $\acute{\omega}$ is angular frequency and *t* is relaxation time. The L represents dimension of the system. In this thesis our calculation based on space plasma which is strongly ionized [3-8]. In order to explore the dynamics of ionosphere and its interaction with radio wave signals we have to examine the strength of the variations (energy) as function of frequency to analyze that at which frequency variation of ionospheric signals strong and weak [9-10]. The estimation of the spectral density is computed displaying periodogram using Fourier transform. The periodogram illustrated the periodicity in the ionospheric signals as shown in Figures 8 and 9. For Pakistan and Japan in frequency domain. Furthermore, ionospheric signals have been characterized using Wavelet transform level 1-5 which defined the discontinuities in ionosphere variables. After de-noising we have identified the exact. The spectral analysis is used to estimate the energy of periodic components for all possible frequencies. This analysis suggests sines and cosines series including their frequencies and amplitudes. We have pursued the use of ionospheric data by considering the set of possible frequencies wj for j= 1,2,3 ..; these are called harmonics frequencies. These data sets are represented in the figures 10, and 11 for Pakistan and Japan for comparison. This information provided help in evaluating the dynamics and behavior of EM waves with ionosphere [11-13].

4. SPECTRAL DOMAIN OF IONOSPHERIC DATA

We know that the periodogram of a time series is given according to the data sets below:

$$X_t = \sum_{j=1}^{\frac{n}{2}} \left[A\left(\frac{j}{n}\right) \cos(2\pi\omega_j t) + B\left(\frac{j}{n}\right) \sin(2\pi\omega_j t) \right]$$

The estimates of coefficients, A_i ,B_i ,are computed such that square of the sum of these amplitudes are significantly greater than zero. The amplitude value if equal to zero it corresponds to non-existence of coefficients in the periodogram expressions as depicted in Figures 12 and 13 for time domain functions obtained for Pakistan and Japan.

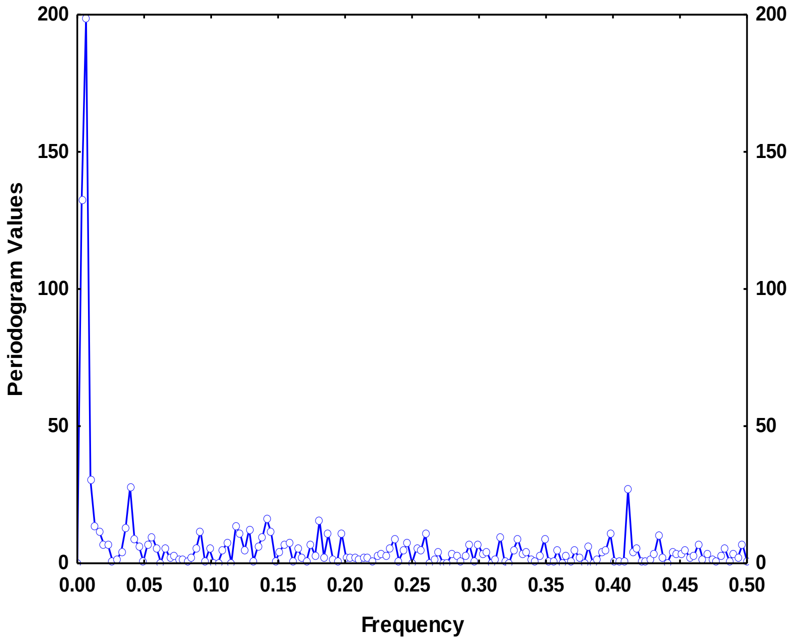

Fig. 8: Period-gram for the Series (KIS-2000) of ICF in Frequency Domain.

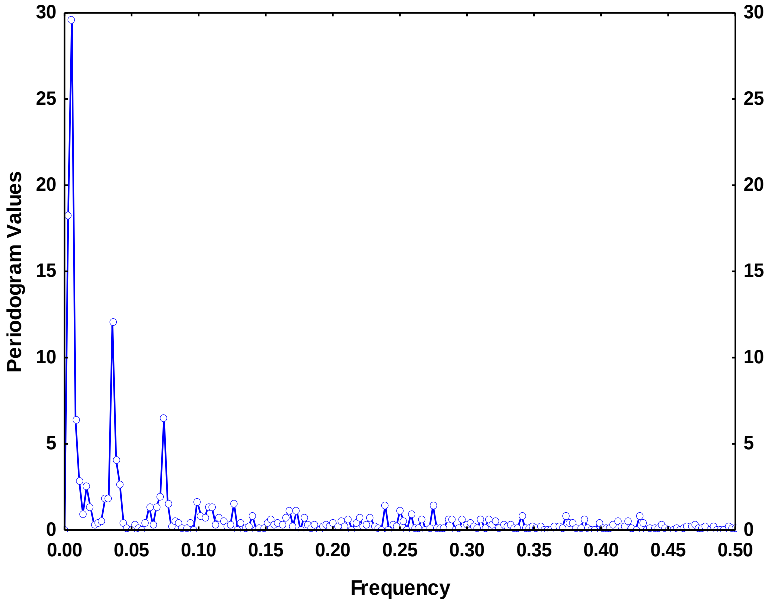

Fig. 9: Period-gram for the Series (WK-2005) of ICF in Frequency Domain

Fig. 10 and 11 illustrate the Spectral density for the series in Pakistan and Japan for the year 2000 of ICF in frequency domain

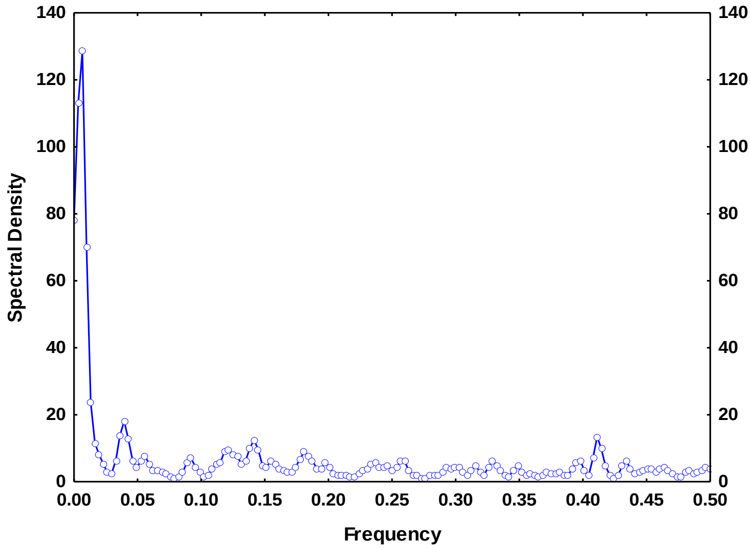

Fig. 10: Spectral density for the Series (KIS-2000) of ICF in Frequency Domain

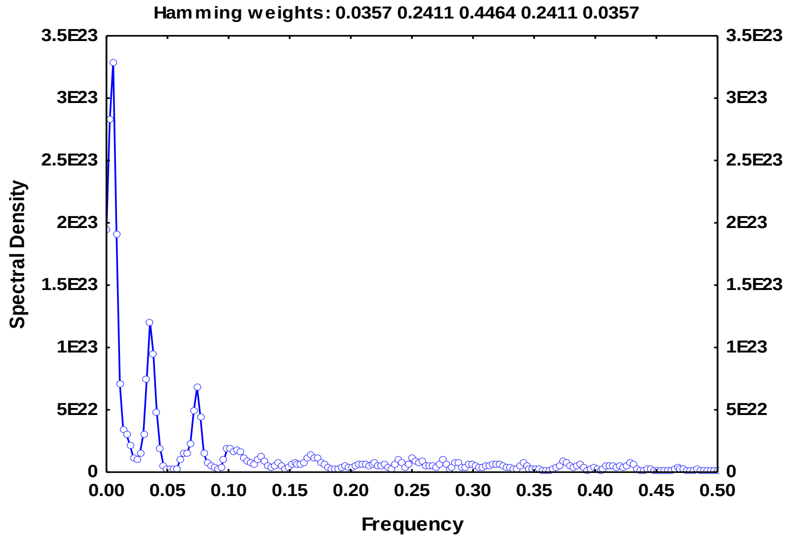

Fig. 11: Spectral density for the Series (WK-2005) of IEC in Frequency Domain

Figures 12 and 13 manifest the Periodogram for the time series at Pakistan and Japan (KIS-2000) (WAK-) of ICF in time domain

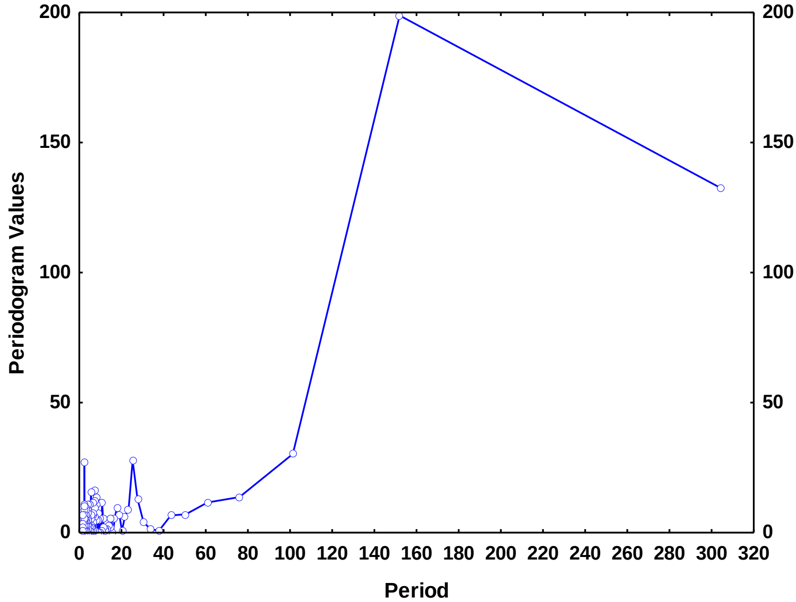

Fig. 12: Period-gram for the Series (KIS-2000) of ICF in time domain

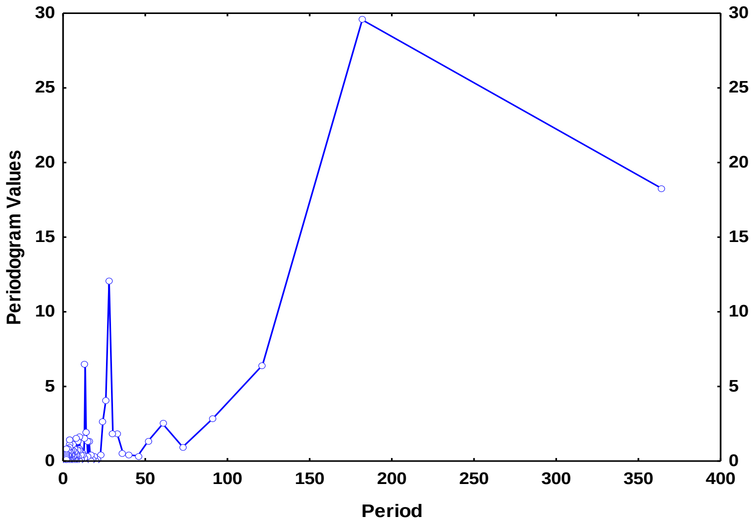

Fig. 13: Period-gram for the Series (WK-2005) of ICF in time domain

5. WAVELET ANALYSIS

Wavelet is a wave-like oscillation with an amplitude that begins at zero, increases, and then decreases back to zero. It can typically be visualized as a "brief oscillation" like one might see recorded by a seismograph or heart monitor. Generally, wavelets are purposefully crafted to have specific properties that make them useful for signal processing. Wavelets can be combined, using a "**reverse**, **shift**, **multiply** and **integrate**" technique called convolution, with portions of a known signal to extract information from the unknown signal

The word wavelet was adopted to express the idea of small waves in an intuitive which is associated with localized waves like function whose values fluctuate in short period, in order to a function be called a wavelet. Wavelet analysis is a particular time-or space-scale representation of signals that has been found in atmospheric application like here in this case for the characterization of ionospheric layer concentration. Actual wavelet analysis is a mathematical technique, which is very useful for numerical analysis and manipulation of multidimensional discrete signal sets as shown in Figures 14 and 15. Wavelet are functions that satisfy certain requirements and are used in representing and processing functions and signals as well as in compression of data and images in different of science and engineering. Wavelet transforms have shown superiority over the classical Fourier transforms In many applications, wavelet transforms converge faster than Fourier. They lead to more efficient processing of signals and data.

Fourier contributed a lot that is obviously recognized that it transforms a function *f* that does depend on time into a new function that depends on frequency. Wavelet transforms are more accurate and faster in convergence. When wavelet analysis is used to study a given signal, it is essential to choose the best wavelet representation for the signal under study. In statistical applications, the wavelet transform is linear and covariant under translation and dilatation transform [14-18].

6. TYPES AND GENERATION OF WAVELETS

We have studied the interaction of radio wave with ionosphere for layer on the basis of their time and frequency profiles and their periodic and aperiodic variability. We have investigated the periodic components from ionospheric data using Fourier transform in the previous section. The wavelet has brought new revolution in signal processing with respect to time-frequency and time-scale transforms.[18-20]

First generation wavelets transform include the Daubechies wavelet, Haar wavelet, Shannon Wavelet, Coiflets Wavelet and the Meyer wavelet. Wavelets transforms (WTs) identifies pure frequencies, de-noising signals, detecting discontinuities and breakdown points, detecting self-similarity and compressing images. Alfred Haar contribution is very evident these wavelets are simplest and easy to understand. Scale-varying basis function were discovered as created by Haar were better basis that Fourier basis functions. Unlike the HAAR basis function, that can be chopped up into different 9 intervals such as o and 1 or interval from o to ½ and ½ to 1, Fourier basis function have only one interval. Therefore, the HAAR wavelets can be much more precise in modeling a function

We have mentioned distinct features of wavelet analysis but Haar wavelet was considered suitable as compared to other wavelets. The use of Haar wavelet is dependent upon behavior of ionospheric signal received at destination location. In 1970s and 1980s, the signal processing has been introduced using wavelet analyses. The wavelet basis, constructed from the scaling function and location function of the wavelet. Figures 16 and 17 illustrate the decomposition of ionospheric critical frequency signals for Pakistan and Japan for d_5-d_1 levels of ICF (KIS-WAK -2005)

Similarly, Figures 18 and 19 show original and de-noised signal of ICF (KIS-WAK , 2000) by scaled white noise for Ionospheric Critical Frequency (ICF).

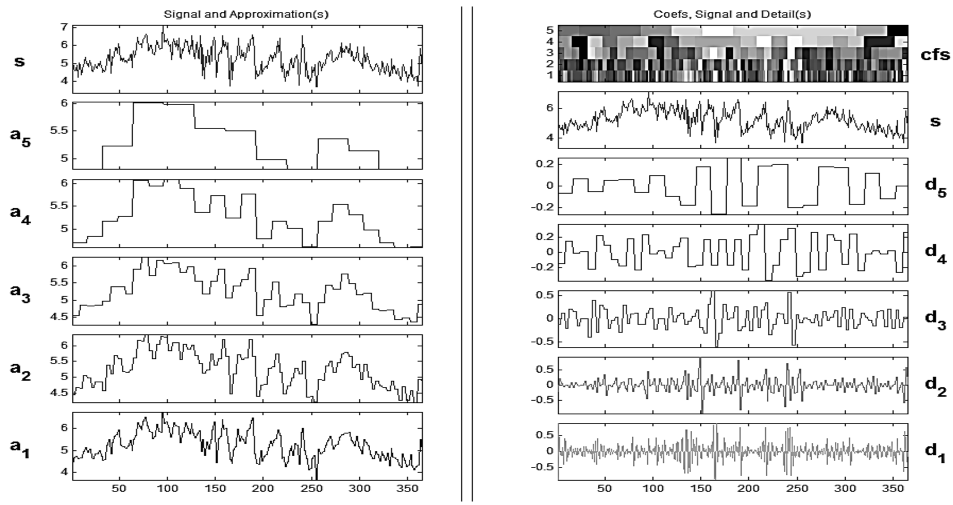

Fig. 14: Approximation and Decomposition for Pakistan and Japan -2005

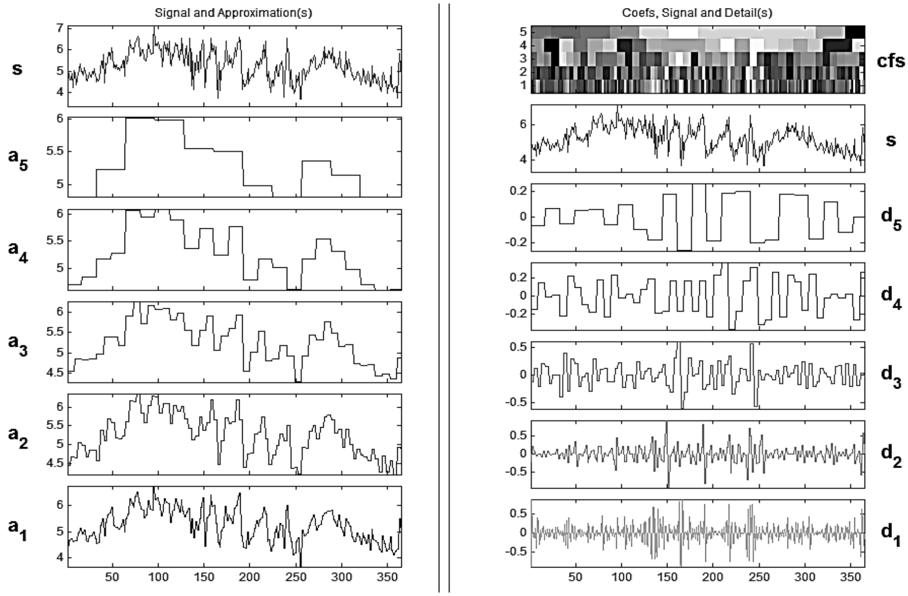

Fig. 15: Approximate and Decomposition of ICF (KIS-2005)

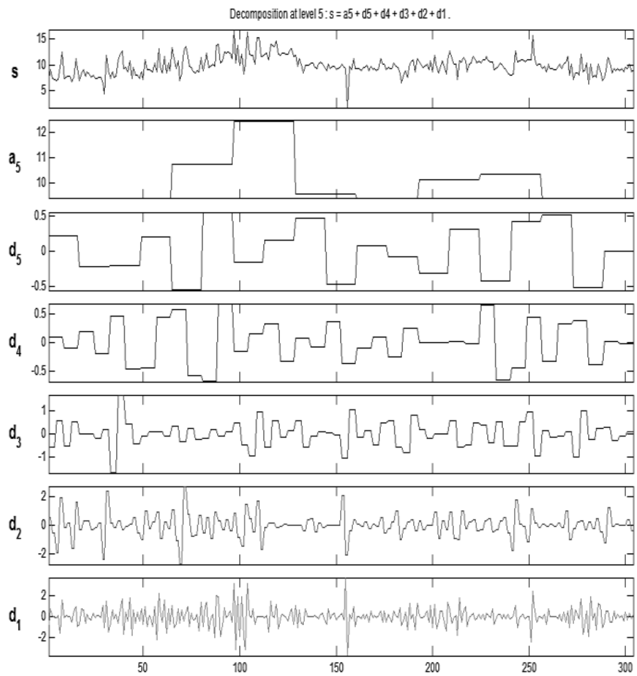

Fig. 16: Decomposition at Level 5, s = a_5+ d_5+d_4+d_3+d_2+d_1 of ICF (KIS-2005)

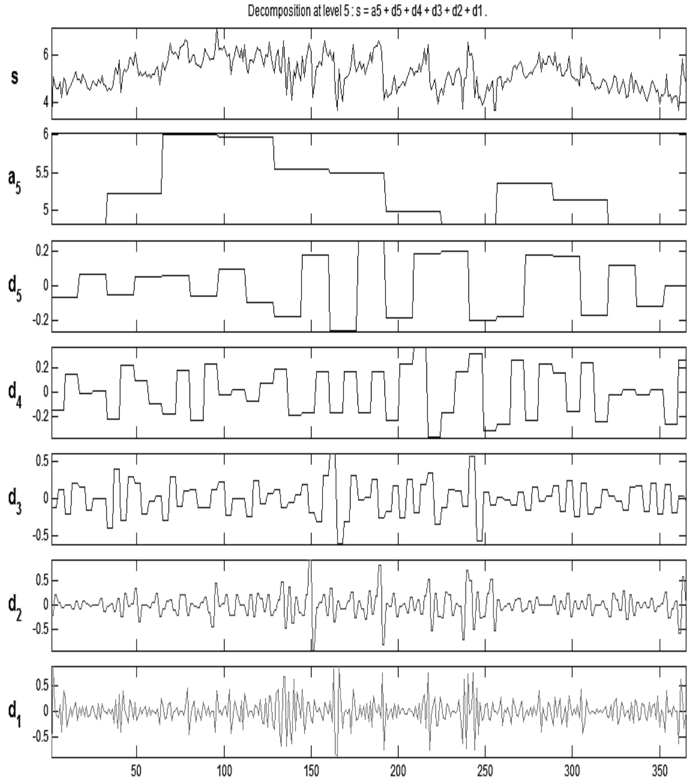

Fig. 17: Decomposition at level 5, s = a_5+ d_5+d_4+d_3+d_2+d_1 for ICF (WK-2005

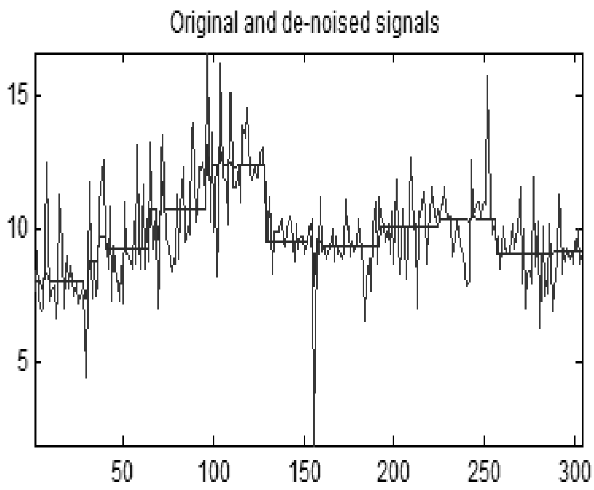

Fig. 18: Original and de-noised signal of ICF (KIS-2000) by scaled white noise

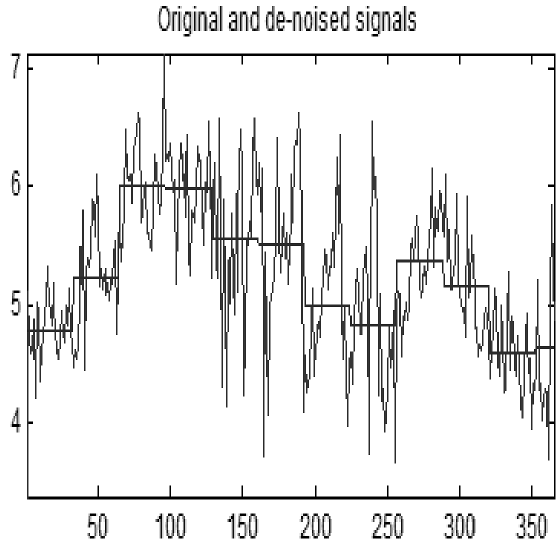

Fig. 19: Original and de-noised Signal of ICF (WK-2005) by scaled white noise

We have characterized the processed signals using wavelet approach. The components of the processing are termed as autocorrelation, and threshold coefficient plots as mentioned for these variables in figures 20, 21 and 22. We have also implemented the operation of thresholding for scale white noise which leads to regularize the signal [21-26].

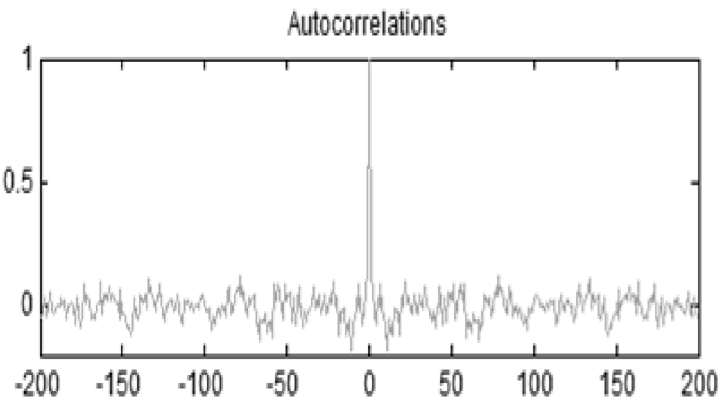

Fig. 20: Autocorrelation plot of de-noised ICF (KIS-2000) by scaled white noise

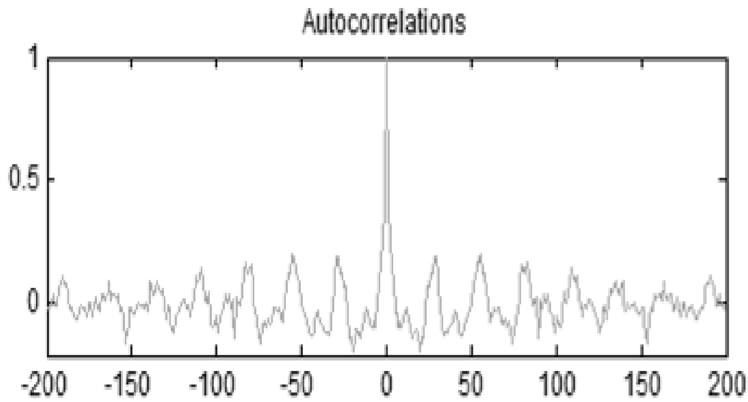

Fig. 21: Autocorrelation Plot of de-noised ICF (WK-2005) by Scaled White Noise

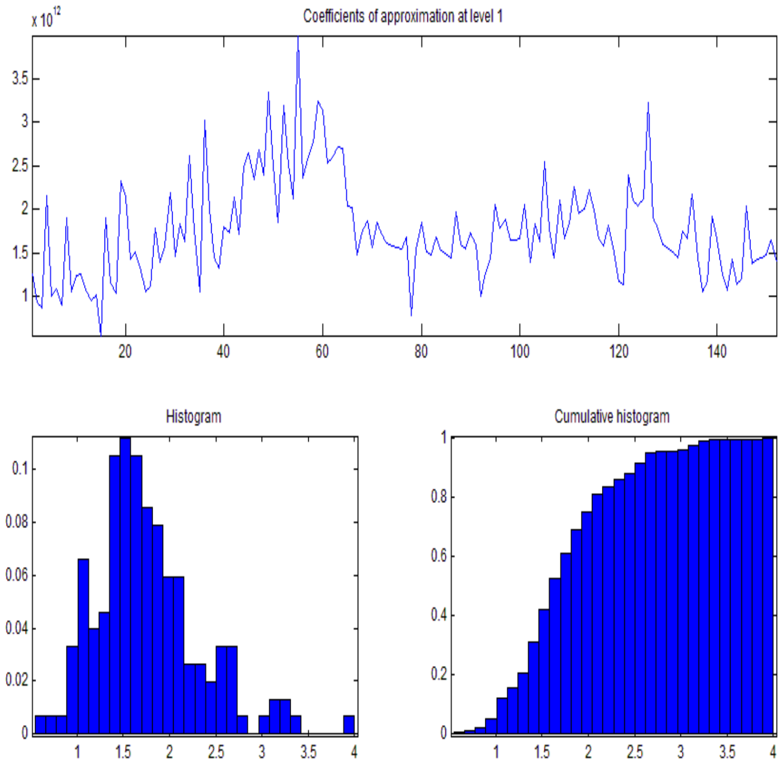

Fig. 22: Coefficient Approximation at Level 1, Histogram & Cumulative Histogram Plot of IEC KIS-2000

7. CONCLUSION

In this communication we have analyzed ionospheric predictability using currently employed Fourier analysis including spectral compartmental approach. This exhibits the ionospheric phenomena with the help of recorded data using signal processing technique. In order to disclose the hidden discontinuities in ICF, IEC and IRI signals we have used continuous wavelet analysis. We have segregated the astrophysical signals by scaled white noise using de-noising technique for multiple level of 1-D Haar wavelet transform. A comparative study of regional ionospheric communication for Karachi Pakistan and Wakkanai Japan has been carried out to establish the variation indication of the parametric values of the ionosphere. The results obtained manifest the data trend, compatible model approach, quantitative equivalence. Spectral analysis helped in focusing frequency spectrum information and results obtained from wavelet transformation strengthen the understanding of sky wave communication. Different approaches have been implemented on both Pakistan and Japan ionospheric data and observed that trends are identical with slight difference in magnitude

8. ACKNOWLEDGEMENT

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REFERENCES

1. Steward, B. (1983). *Physik*, Publisher; De Gruyter Incorporated, Walter, ISBN 3111161072, Germany.
2. Harison, J.A. (1958). *The Story of the Ionosphere, Exploring with Wireless Waves*, Hulton Educational Publications, London, United Kingdom.
3. Rishbeth, H. and Garriott, O.K. (1969). *Introduction to Ionospheric Physics*, Academic Press Inc, library of congress, 5th Avenue, New York, 10003, USA.
4. Picquenard, A. (1974). *Radio wave propagation*. John Wiley & Sons.
5. Calvert, W. and Goe, G.B. (1963). Plasma Resonances in the Upper Ionosphere. *J. Geophys, Res*., 68, 6113-6120.
6. Chandra, S. (1964). Plasma Diffusion in the Ionosphere, *J. Atmospheric Terrest. Phys*., 26, 113-122.
7. Francis, F.C. (1974). *Introduction to Plasma Physics*, Plenum Press, New York, USA.
8. Kelley, M.C. (2009). *The Earth's ionosphere: plasma physics and electrodynamics* (Vol. 96). Academic Press.
9. Hargreaves, J.K. (1979). *The Upper Atmosphere and Solar-Terrestrial relations, An introduction to the aerospace environment*, Van Nostrand Reinhold Company, 450 West 33rd Street, New York, USA.
10. Ratcliffe J.A. (1956a). The Formation of the Ionospheric Layers F-1 and F-2. *J. Atmospheric Terrest, Phys*., 8, 260-269.
11. Vanzandt, T.E. and Knecht, R. (1964). The Structure and Physics of the Upper Atmosphere, in “Space Physics” (D.P. Le Galley and A. Rosen, eds.), 166-225.

12. Yonezawa, T. (1965). Maintenance of ionization in the nighttime F2 region. *J. Radio Res. Labs Japan*, 12, 65-88.
13. Shimazaki, T. (1957). Dynamical structure of the ionospheric F2 layer. *J. Radio Res. Lab. Jpn.*, 4, 309-332.
14. James, J.F. (1995). A student's guide to Fourier transforms: with applications in physics and engineering. Cambridge; New York: Cambridge University Press.
15. Lathi, B.P. (1995). Modern digital and analog communication systems. Oxford University Press, Inc.
16. Simon Braun (2008). *Discover Signal Processing-an Interactive Guide for Engineers*, ISBN 978-0470-51970-7, John Wiley & Sons, Ltd.
17. Diggle, P.J. (1992). Time Series, A Bio-statistical Introduction, ISBN 0-19-852226-6. Published by Biddles Ltd, Guildford and King's Lynn, United Kingdom.
18. Schuster A. (1906). On the Periodicities of Sunspots. *Phil. Trans. R. Soc.*, Ser. A, 206, 69-100.
19. Yule, G.U. (1927). on a Method of Investigating Periodicities in Disturbed Series with Special Reference to Wolfer's Sunspot Numbers. *Phil. Trans. R Soc. Lond*., A226, 267-298.
20. Schuster, A. (1898). On the Investigation of Hidden Periodicities with Application to a supposed 26 Day Period of Meteorological Phenomena. *Terrestrial Magnetism,* 3(1), 11-41.
21. Bartlett, M.S. (1950). *Periodogram Analysis and Continuous Spectra*. *Biometrika*, 37, 1-16.
22. Jenkins, G.M. and Watts, D.G. (1968). *Spectral Analysis and its Applications*, Holden-Day, San Francisco, USA.
23. Fisher, R.A. (1929). Tests of Significance in Harmonic analysis, *Proceedings of the Royal Society*, A 125, 54-9.
24. Kay, S.M. (1988). *Modern Spectral Analysis: Theory and Application Englewood Cliffs*, NJ: Prentice-Hall.
25. Schunk, R.W. and Nagy, A.F. (2004). *Ionospheres Physics, Plasma Physics, and Chemistry*, Cambridge University Press, Cambridge, UK.
26. Barclay, L. (2003). *Propagation of Radio Waves*, 2nd edition, The Institution of Electrical Engineers, London, United Kingdom.

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BAYESIAN INFERENCE AND RANKING OF DETERMINANTS, INFLUENCING CORRUPTION IN PAKISTAN USING METHOD OF PAIRED COMPARISONS

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ABSTRACT

There are many problems which are destroying countries' progress and corruption is one of those. It is the most sensitive issue in the whole world. There are different factors which are also contributing in it. A variety of research works have been done to explore the different factors of corruption. The main purpose of this project is to identify the different factors of corruption and their role in its expansion. Bayesian inference has been applied to highlight the factors affecting the corruption in Pakistan. Bayes' estimators are computed which reflects the overall worth probabilities for each factors. The ranking is done and posterior analytical probabilities are computed for each of the twenty one pairs of factors affecting the Corruption in Pakistan for future single comparisons of each pair. Results for analysis are computed in C language and programs coding are designed for seven parameters' inference. According to the results, political system is at the highest level of ranking and educational system at the lowest rank which affects corruption in Pakistan. Furthermore' to ensure the appropriateness of the model, the goodness of fit criteria is used as used by Aslam (2002).

KEYWORDS

Paired comparison method; Bayesian Statistics, Posterior Means, Ranking, Posterior Predictive Probabilities Bradley-Terry Model, Non informative prior.

1. INTRODUCTION

Recently corruption has become a major issue in the world. It has spread like a fire in the whole forest. Almost all the countries and fields of life are imprisoned by it in a complex net.

Corruption is a form of dishonest and unethical means by a person to acquire personal benefits which is mostly associated with power and authority. It involves many activities like bribery, legal blunder, electoral frauds and embezzlement. According to Senior (2006), defines corruption is a covert activity or action to facilitate a good or a service to a third party so that the party could manipulate and pressure few actions which are beneficial for the corrupt. The involved parties gain their benefits and utilize almost all the sources for their personal usage. Kaufmann (2005) elaborates the same concept by including 'legal Corruption' where power is abused within the confines of the law.

Porter et al. (2004) argue that corrupt countries have workers who are less likely to know about accessible jobs and employers are less likely to know about the quantity of quality available, thus leading to higher employment.

In simple words, Corruption is the misuse of influence and authority for personal and secretive benefits. In this case it is not necessary that it is someone's private benefits it may include any party, class, tribe, group, friends, family and many others.

1.1 Corruption in Pakistan

Pakistan came into being in 1947 as the result of Pakistan movement. At that Pakistan had well disciplined bureaucracy and army from the Britisher's. The system works under those rules which are set by the British government. In different and numerous report papers of World Bank report that Pakistani bureaucracy can be observed being predominant with Corruption, incompetent and distended in size and shape with a lack of accountability and defiant to change (World Bank, 1983, 1985, 1993, 1997)

Corruption in Pakistan is very common and widespread (Ackerman, 1997) mainly in governmental organizations and lower levels of police forces. In the report of Corruption Perception Index (CPI) of Transparency International (2016), Pakistan is at the 116th place out of 176 countries where Corruption is being exercised. In the previous report, the earlier scores are of 127 out of 175 in 2013 Transparency International, 2010, 2011, 2012, 2013, 2016). The significant improvement can be seen in the statistics of 2013 when it's ranking improved by 12 indices. These previous results of 2012 are 139 out of 174, 134 out of 182 in 2011, 143 out of 178 in 2010, and 139 out of 180 in 2009. These results clearly express that Pakistan has higher level of Corruption as the time passes. The indication of corrupt departments makes the situation alarming.

1.2 Factors of Corruption

According to the above discussion, there are numerous factors which are involved in the performance of countries and role of Corruption, and they can be grouped in seven factors which are sectioned as social, economical and institutional factors.

1.2.1 Educational System

Education is the key of nation's development and necessary for its growth. Level of education is primary factor in any country's environmental performance and betterment and it also affects its level of development. Duit (2005) believes that a country with an elevated and enhanced level of educational system and social value is more able to grip environmental issues and help to initiate environmental type of cooperation and betterment programs. The poor system may give birth to poor generation and opens new doors for Corruption.

Meyer et al. (2013) develop a connection between corruption and institutional factors with respect to 117 countries. The unhelpful factors like environmental impact, is the reason of Corruption behind it. Countries with lofty levels of Corruption lean to have low levels of educational system and environmental performance (Welsch, et al., 2005).

1.2.2 Political System

Corruption in any country around the world brings political discrimination. It involves politician, political decision makers and public. The minor change at any level in any department results in grand political Corruption is at the highest level of political set up. It launches the necessary and desired involvement of the state or country and different agents and workers of state in Corruption, without any notion as to the level of authority where Corruption takes place (Heidenheimer et. al., 1993).

This situation is the outcome of poor politicians and state managers, who are part of that specific activity are themselves' corrupt. They have their own personal interests. In political Corruption, corrupt policies are introduced by decision makers to sustain money, power and status.

1.2.3 Social System

Corruption is penetrated in almost all the fields of life and more dangerous for such department's construction especially at national level, nursing as security, police, army, fire protection, etc. It is obvious these areas become the most susceptible to Corruption. Corruption not builds only wrong typecast of people's conduct but also creates the incorrect economic relations violating the fundamental laws of economic development. National economy and infrastructure is directly connected with social system and it gives most possibilities for Corruption (Rogers, 2008; Haque, et al., 2006; Haque & Knellar, 2007).

1.2.4 Unemployment

The present world is facing different problems in which unemployment is the most exigent social and economic problem. It is the strong belief of youth that their future and present unemployment is due to Corruption and its factors. This perception is originated by the jobs for pay or paid job scandal in almost all the departments by the steady reports of nepotism (GCB, 2013. Corruption at any level directly hit the unemployment and job issue. Most Pakistanis believe public officials and civil servants are corrupt and they do not support merit in job (GCB, 2013). If there was no Corruption, there would be no chance for more unemployment.

1.2.5 Inflation

Corruption is an unlawful and beneficial-oriented convention of public power which causes many macro problems in economical performance of the country. Directly there is no relationship between Corruption and inflation but economic is the core parameter of Corruption. As Karaca et al. (2012) stat that inflation is not only define as results of economical factor in Corruption but also a results of economic problem from Corruption. In this situation of inflation and economic stability growth, trade gap, the quality of legislation, the efficacy of government, political stability and responsibility variables are the outcomes of Corruption. It is resulted from empirical data that there is a strong relationship of inflation with Corruption and it has a statistically significant and also effect on Corruption is positive in these 97 countries from three different income-level groups (Karaca et al., 2012).

1.2.6 Psychological Reason

Corruption is the reason of many political, economical and socio issues but it also causes many mental health issues. It augments the sense of anxiety, insecurity, hypersensitivity, timidity and many others. It reboots the burden of mental health problems as sadness, depression, nervousness and post- traumatic stress disorder (Aarons, 2014).

Corruption causes psychological issues which grounds serious levels of disabilities regarding mental health. Fatal and non-fatal mental health emergencies are the product which is directly connected with Corruption.

1.2.7 Brain Drain

Corruption is itself a disease but it also leans to create a number of negative socio-economic and political conclusions, political insecurity and poor public sector infrastructure, administrative efficiency and governance (Lambsdorff et al. 2006). According to Hirschman's (1970) notion of "exit, voice and loyalty" among many other issues, leaving the country and migrating to a better place is the major consequence of Corruption. In this regard the core brains shift from the court and country has to bear the loss of brain drain. In other words, it may be possible to observe a "voting by feet" as a result of Corruption.

1.3 The Paired Comparison Model

An unexpected and surprising amount of model building is gained as a result of paired comparison. The representation of this experimental process is given in probabilities choice models. Let's suppose we have t objects for compared in pairs by the judge and let d_{ij} be the indicator random variable that have 0 values for ϕ_i and 1 for ϕ_j. It is implicit throughout experiments that all the comparison are statistically independent except of dij+d_{ji}=1, then the preference probability of $\phi_i \rightarrow \phi_j$ will be

$$\Pr(d_{ij}=1) = \varepsilon_{ij} \quad (1.1)$$

With Preference probability that $\phi_j \rightarrow \phi_i$

$$\Pr(d_{ij}=1) = \varepsilon_{ji} \quad (1.2)$$

More generally with replication and ordered effect

$$\varepsilon_{ij}=\varepsilon_{ijkl} \quad (1.3)$$

This means that $\phi_i$is preferred on ϕ_j and the k^{th} comparison by I^{th} judge. Here ε_{ij} and ε_{ijkl} are probabilities approaches to the following condition

$$0< \phi_{ij}\phi_{ij} < 1 \quad (1.4)$$

1.4 The Linear Model for Paired Comparisons

The model is very in the sense of pairs of comparison. In pair comparison, the object ϕ_i is preferred on ϕ_j if it fulfill the condition $w_i > w_j$ and the object ϕ_j is preferred on ϕ_i if it also follows the condition of $w_j > w_i$. Let $\zeta_j = w_j - \eta_j$ and $\zeta_j = w_j - \eta_j$ for which i=1,2…., m and j=1,2….,m that the objects have importance or worth η_j and η_j. If every pair $\left(\zeta_i - \zeta_j\right)$ has the same bi-variety distribution then $\left(\zeta_j - \zeta_i\right)$ will also have the same distribution, now

$$\Pr\left\{\left(\zeta_i - \zeta_j\right) < v\right\} = H(v) \tag{1.5}$$

It follows that

$$\omega_{ij} = \Pr\{(w_i - w_j) > 0\} \tag{1.6}$$

$$= \Pr\left\{\left(\zeta_i - \zeta_j\right) > -\left(\eta_i - \eta_j\right)\right\}$$

$$\omega_{ij} = H\left(\eta_i - \eta_j\right) \tag{1.7}$$

However the probability of preference can be expressed in that term of symmetrical cumulate density function, the term w_i fulfill to be a linear model. This model is the simplification of the Thurstone-Mosteller model (1927) in which w_j is supposed to be normal distribution $N\left(\eta_i, \sigma^2\right)$ equal-correlated with correlation co-efficient.

$$\omega_{ij} = \mathrm{H}(\eta_i - \eta_j) = \int_{-(\eta'_i - \eta'_j)}^{\infty} z(v)dv \tag{1.8}$$

where

$$\eta'_i = \frac{\eta_i}{\left[2\sigma^2(1-\rho)\right]^{1/2}} \tag{1.9}$$

and

$$z(v) = \frac{1}{\sqrt{2\pi}} e^{-1/2v^2} \tag{1.10}$$

1.5 The Bradley Terry Model for Paired Comparisons

The Bradley-Terry model for paired comparison (1952) is introduced as a basic model for paired comparison after Zermelo (1929) consideration that states, "The objects or treatments have merit η_i and η_j when judge or comparison on the basis of some characteristics on some characteristic and may be represent in the continuous random variable with the following limit.

$$w_i, (-\infty < w_i < +\infty)," \tag{1.11}$$

$$H(\eta_i - \eta_j) = \frac{1}{4} \int_{-(\ln\theta_i - \ln\theta_j)}^{\infty} \sec h^2(w/2)dw \tag{1.12}$$

where

i≠ j and i=1,2…m 1.13 $\omega_{ij} = H(\eta_i - \eta_j) = \frac{1}{4} \int_{-(\ln\theta_i - \ln\theta_j)}^{\infty} \sec h^2(w/2)dw$ (1.14)

This model indicates that the difference between two underlying variables $\left(\zeta_i - \zeta_j\right)$ has logistic density with parameter ($\ln\phi_i$ -$\ln\phi_j$) and is formulated as

$$\omega_{ij}=\phi_i/\phi_i+\phi_{ij} \tag{1.15}$$

Where ω_{ij} denotes the preference probability for object i when ϕ_i and ϕ_j are compared.

1.6 Notations of the Model

The researcher considers the following notations for the analysis of the model in this particular study.

$x_{i.ij}$ = Number of times i^{th} object is preferred on j^{th} object.

$x_{j.ij}$ = Number of times j^{th} object is preferred on i^{th} object.

$r_{ij} = x_{i.ij} + x_{j.ij}$ = Total number of comparisons between i^{th} and j^{th} objects

n_i = Total number of times i^{th} object is preferred to any other objects.

1.7 Likelihood Function of the Model

For seven factors affecting Corruption in Pakistan, the likelihood function of the model is given below which have the following form;

$$l(x;\phi_1,\phi_2,\phi_3,\phi_4,\phi_5,\phi_6,\phi_7) \propto \frac{\prod_{i=1}^{7}(\phi i)^{n_i}}{\prod_{i<j}^{7}(\phi i+\phi j)^{r_{ij}}} \tag{1.16}$$

where $n_i = \sum_{j\neq i}^{m} x_{i.ij}$ and $r_{ij} = x_{i.ij} + x_{j.ij}$ be the total number of comparisons between these objects.

1.8 Uniform Prior as a prior Distribution

Bayesian analysis is complex for paired comparison model, it is very complex. It is not simple prior distribution is assumed to conjugate prior of data from the comparison model. Subjective uniform prior is also used for this analysis. The joint uniform prior for parameters space $\underline{\theta}$ is given below;

$$P(\underline{\phi}) \propto 1 \tag{1.17}$$

$$Here \qquad \underline{\phi} = (\phi_1,\phi_2,\phi_3,\phi_4,\phi_5,\phi_6,\phi_7)$$

$$With \qquad 0<\phi_i<1$$

2. BAYESIAN INFERENCE OF THE MODEL

The target population of this study is the judges/experts/faculty of public/private sector universities in Pakistan. Sampled population of this study is faculty as experts of university of Gujrat, Gujrat. Experts' preferences are obtained from faculty members selected on a random sample of 100 using simple random sampling technique. Different faculty is selected in the sample. Programs are designed in C++ language to perform the Bayesian analysis. Following table shows data:

Table 1
Data of Preferences for Factors of Corruption

S.No	Factors	A_{ij}	A_{ji}	S.No	Factors	A_{ij}	A_{ji}
1	PS,ES	45	5	**11**	ES, BD	24	26
2	PS,SS	35	15	**12**	SS, UE	29	21
3	PS,UE	36	14	**13**	SS, In	29	21
4	PS ,In	35	15	**14**	SS, PR	33	17
5	PS,PR	39	11	**15**	SS, BD	33	17
6	PS, BD	33	17	**16**	UE, In	30	20
7	ES, SS	21	29	**17**	UE, PR	40	10
8	ES, UE	19	31	**18**	UE, BD	35	15
9	ES, In	19	31	**19**	In, PR	29	21
10	ES, PR	23	27	**20**	In, BD	34	16
				21	PR, BD	27	23

In the (Table 1), 'PS' denote the Political System, 'ES' denote the Education System, 'SS' denote the Social System, 'Un' denote the Unemployment, 'In' denotes the Inflation, 'PR' denote the Psychological Reason, and 'BD' denote the Brain Drain. In each pair of factors, Aij represent the preference of first factor and Aji represent the preference of the 2nd factors in give pairs of factors.

2.1 Posterior Distribution for Model of Factors Affecting Corruption in Pakistan

The joint posterior distribution, with the Uniform prior density takes the following form;

$$\mathrm{P}(\underset{-}{\phi}|x) = \frac{\prod_{i=1}^{7}(\phi i)^{n_i}}{M\left(\prod_{i<j}^{7}(\phi i + \phi j)^{r_{ij}}\right)} \tag{2.1}$$

Where 'M' is the normalizing constant of the distribution and the identifiability condition $\sum_{i=1}^{m}\phi_i = 1$

4.2 Posterior Means for Factors affecting Corruption in Pakistan

The general integral formula for ϕ_i is given as below.

$$E(\phi_i) = \int_{\phi_1=0}^{1} \int_{\phi_2=0}^{1-\phi_1} \int_{\phi_3=0}^{1-\phi_1-\phi_2} \int_{\phi_4=0}^{1-\phi_1-\phi_2-\phi_3} \int_{\phi_5=0}^{1-\phi_1-\phi_2-\phi_3-\phi_4} \int_{\phi_6=0}^{1-\phi_1-\phi_2-\phi_3-\phi_4-\phi_5} \frac{(\phi_i).\prod_{i=1}^{6}(\phi_i)^{n_i}}{M\left(\prod_{i<j}^{6}(\phi_i+\phi_j)^{r_{ij}}\right)} d\phi_6 d\phi_5 d\phi_4 d\phi_3 d\phi_2 d\phi_1 \tag{2.2}$$

Here $\phi_7 = 1-\phi_1-\phi_2-\phi_3-\phi_4-\phi_5-\phi_6$ is the constraint on the numerical integration.

The results of posterior means are computed and organized in Table 2 for posterior means as below;

Table 2

Factors	Political System	Education System	Social System	Unemployment	Inflation	Psychological Reason	Brain Drain
$E(\phi_i)$	0.30212	0.06558	0.15994	0.16061	0.11265	0.10911	0.09000
Ranks	1	7	3	2	4	5	6

In (Table 2) shows the contribution of different factors affecting Corruption in Pakistan and also give ranks. The probability value for each factor shows to which extent it causes amount of Corruption in Pakistan. The factor having more probability value has more contribution in Corruption. The factors with high probability values are major sources of Corruption, while factors with low probability values have minor role in promoting Corruption in Pakistan. In this specific study, political system reflects greater probability which is the interpretation that it has strong contribution in promoting Corruption in Pakistan similarly remaining all according to higher probability values.

Mostly Faculty members of University of Gujrat are preferred the political system as an affecting factor which is the highest probability of the preference that probability is 0.30212 so I give it the 1st rank order of the political system and give 2nd rank to unemployment because the preferences probability of that's factor is 0.16061 that probability is less than as compare to political system that's why I give it the 2nd rank and the similarly I give it 3rd, 4th, 5th. 6th and 7th ranks according to their probability that is Social System, Inflation, Psychology Reason, Brain Drain, Education System respectively.

2.3 Appropriateness of the Model

We use Chi-Square method to test the hypothesis about the goodness of fit of the model for factors of terrorism. We used the criteria by Aslam (2002) as follow:

H_0; This model is good fitted for the given data
H_1; This model does not fitted for the given data

Table 3:
(Observed and Expected Number of Preferences of Factor for Corruptions)

S. No	X_{ij}	$\widehat{x_{ij}}$	X_{ji}	$\widehat{x_{ji}}$	S. No	X_{ij}	$\widehat{x_{ij}}$	X_{ji}	$\widehat{x_{ji}}$
1	45	41.08240413	5	8.91759587	**11**	24	21.07597378	26	28.9240262
2	35	32.69272389	15	17.3072761	**12**	29	24.94774606	21	25.0522539
3	36	32.64538716	14	17.3546128	**13**	29	29.33709967	21	20.6629003
4	35	36.42018468	15	13.5798153	**14**	33	29.7230998	17	20.2769002
5	39	36.73370134	11	13.2662987	**15**	33	31.99567896	17	18.004321
6	33	38.52392125	17	11.4760788	**16**	30	29.38776257	20	20.6122374
7	21	14.5397304	29	35.4602696	**17**	40	29.77346878	10	20.2265312
8	19	14.4966621	31	35.5033379	**18**	35	32.0438131	15	17.9561869
9	19	18.39757617	31	31.6024238	**19**	29	25.39908009	21	24.6009199
10	23	18.77039327	27	31.2296067	**20**	34	27.7942265	16	22.2057735
					21	27	27.39942745	23	22.6005725

We can calculate the expected frequencies using the following data.

$\widehat{x_{ij}} = r_{ij}(\omega_{ij})$

$\widehat{x_{ji}} = r_{ij}(\omega_{ij})$

where For all i<j

With the significance of 5% the following of chi-square distribution is as below,

$$x^2 = \sum_{i<j=1}^{m} \left\{ \frac{(x_{ij} - \widehat{x_{ij}})^2}{\widehat{x_{ij}}} + \frac{(x_{ji} - \widehat{x_{ji}})^2}{\widehat{x_{ji}}} \right\}$$

We follow the concept by Aslam (2002) about the degree of freedom in which he considered the choice of degree of freedom using this formula d.f = m (m-2) =7(7-2)=35

Chi-Square test statistic value as

x^2_{cal}= 31.96126

The table value is $x^2_{(0.05,35)}$=49.802

Since can observe the x^2_{cal} doesn't not fall in the criteria region that is

$x^2_{cal} \ngeq x^2_{(0.05,35)}$

So on the basis of previous region we can conclude that we have not enough evidence to reject the null hypothesis. So our model is good fitted.

2.4 Posterior Predictive Probabilities for Factor effecting Corruption in Pakistan

Predictive probabilities for seven factor promoting Corruption in Pakistan are calculated for each pair of factors using a program designed for eight objects in C++. The predictive probability for **ith** and **jth** objects is given follow for, I < j as;

$$P_{(ij)} = \frac{1}{M}\int_{\phi_1=0}^{1}\int_{\phi_2=0}^{1-\phi_1}\int_{\phi_3=0}^{1-\phi_1-\phi_2}\int_{\phi_4=0}^{1-\phi_1-\phi_2-\phi_3}\int_{\phi_5=0}^{1-\phi_1-\phi_2-\phi_3-\phi_4}\int_{\phi_{6}=0}^{1-\phi_1-\phi_2-\phi_3-\phi_4-\phi_5} P(\underline{\phi}\,|\,\underline{x}).\omega_{ij}\, d\phi_6\, d\phi_5\, d\phi_4\, d\phi_3\, d\phi_2\, d\phi_1 \qquad (4.8)$$

Here $P(\underline{\phi}|x) = \dfrac{\prod_{i=1}^{7}(\phi i/\phi i+\phi j)^{n_i}}{N\left(\prod_{i<j}^{7}(\phi i/\phi i+\phi j+\phi i/\phi i+\phi j)^{r_{ij}}\right)}$ be the posterior distribution

$\omega_{ij}=\frac{\phi_i}{\phi_i+\phi_j}$ be the preference probabilities of 'I' factor over 'j' factor.

Here $\phi_7 = 1-\phi_1-\phi_2-\phi_3-\phi_4-\phi_5-\phi_6$ is the constraint on the numerical integration.

Table 4:
Posterior Predictive Probabilities for Factor effecting Corruption in Pakistan

S. No.	P(ij)	Estimate	P(ji)=1-P(ij)	Estimate	S. No.	P(ij)	Estimate	P(ji)=1-P(ij)	Estimate
1	P(12)	0.730148	P(21)	0.269852	**11**	P(27)	0.498966	P(72)	0.501034
2	P(13)	0.642163	P(31)	0.357837	**12**	P(34)	0.499093	P(43)	0.500907
3	P(14)	0.641288	P(41)	0.358712	**13**	P(35)	0.545486	P(53)	0.454514
4	P(15)	0.680348	P(51)	0.319652	**14**	P(36)	0.600542	P(63)	0.399458
5	P(16)	0.729488	P(61)	0.270512	**15**	P(37)	0.600568	P(73)	0.399432
6	P(17)	0.729502	P(71)	0.270498	**16**	P(45)	0.546337	P(54)	0.453663
7	P(23)	0.398504	P(32)	0.601496	**17**	P(46)	0.601392	P(64)	0.398608
8	P(24)	0.397661	P(42)	0.602339	**18**	P(47)	0.601416	P(74)	0.398584
9	P(25)	0.443941	P(52)	0.556059	**19**	P(56)	0.555058	P(65)	0.444942
10	P(26)	0.498939	P(62)	0.501061	**20**	P(57)	0.555004	P(75)	0.444996
					21	P(67)	0.500027	P(76)	0.499973

In Appendix A Table 4 we also see that in a single comparison the first pair of factors which is Political System and Education System, the first factor of these pair Political System has 0.730148 preference probability of this factor by faculty is 73.01 % total of 100% and second factor of this pair is Education System has 0.269852 probability of preference by faculty member of University of Gujrat for this factor is 26.99% and in this pair of factors Political System have greater preference than the Social System.

From the second pair of factors which is Political System and Social System, the first factor of these pair Political System has 0.642163 probability preferred by faculty member's of this factor is 64.21% out of 100% and second factor of this pair is Social System has 0.357837 preference probability by faculty members of University of Gujrat preference of this factor is 35.79 % and in this pair Political System have greater preference than the Social System. From the third pair which is Political System and Unemployment , the first factor of pair Political System has 0.641288 probability i.e. faculty member's preference of this factor is 64.13 % total of 100% and second factor of this pair is Unemployment which has 0.358712 preference probability, preferred by faculty members of University of Gujrat on other factor is 35.87% and in this pair of factors Political System have greater preference than the Unemployment .From the next pair of factors which is Political System and Inflation, the first factor of these pair is Political System has 0.680348 probability according to faculty member's preference of this factor is 68.03 % total of 100% and second factor of this pair is Inflation has 0.319652 probability according to faculty member's of University of Gujrat preference of this factor is 33.97% and in this pair of factors Political System have greater preference than the Inflation. From the next pair of factors which is Political System and Psychological Reason , the first factor of these pair is Political System has 0.729488 probability according to faculty member's preference of this factor is 72.95 % out of 100% and second factor of this pair is Psychological Reason has 0.270512 probability according to faculty member's of University of Gujrat preference of this factor is 27.05 % and in this pair of factors Political System have greater preference than the Psychological Reason. From the next pair of factors which is Political System and Brain Drain, the first factor of these pair is Political System has 0.729502 probability according to faculty member's preference of this factor is 72.95 % out of 100% and second factor of this pair is Brain Drain has 0.270498 probability according to faculty member's of University of Gujrat preference of this factor is 27.05 % and in this pair of factors Political System have greater preference than the Brain Drain.

CONCLUSION

A questionnaire has been used to collect data of different factors of corruption in Pakistan. The faculty of University of Gujrat has selected for data collection. According to their views and data, political system is the most preferred the factor that has the highest probability, and Unemployment is at the second level, which less probability as compare to political system. The next order like third, fourth, fifth, sixth and seventh order is Social System, Inflation, Psychological Reason, Brain Drain and Education System respectively. The most preferred and highly influenced factor of the Corruption in Pakistan is political system with the highest rank in probability value and it is affecting all the other systems. The other factors do not have much influence on the factor affecting Corruption in Pakistan. Education System is the less corrupt factor and has less probability according to the faculty of University of Gujrat are not preferred that factor and have a weak influence on affecting Corruption in Pakistan.

RECOMMENDATION

- The given methodology can be explored in detail and its other aspects can also be utilized.
- In this research study, the research has collected data only from faculty members of University of Gujrat. Other researchers can collect the same data from large group of people like management staff, senior students etc.
- Ranking of factor effecting corruption in Pakistan can be explored by different factors and by applying different technique.

REFERENCES

1. Blackburn, K., Bose, N. and Haque, M.E. (2005). The incidence and persistence of Corruption in economic development, *Journal of Economic Dynamics and Control*, 30, 2447-24467.
2. Djankov, S., La Porta, R., Lopez-de-Silanes, F. and Shleifer, A. (2010). Disclosure by politicians. *American Economic Journal: Applied Economics*, 2(2), 179-209.
3. Mauro, P. (1995). Corruption and Growth. *Quarterly Journal of Economics,* 110(3), 681-712.
4. Mauro, P. (1995). Corruption and Growth. *Quarterly Journal of Economics* 110(3), 681-712.
5. Meyer, A.L., van Kooten, G. and Wang, S. (2003). Institutional, social and economic roots of deforestation: A cross country comparison. *Int. J. Soc. Econ*, 36, 832-843.
6. Porter, M.E. (1990). *The competitive advantage of nations*. New York: The Free Press.
7. Rogers, M. (2008). Directly Unproductive Schooling: How Country Characteristics Affect The Impact of Schooling on Growth. *European Economic Review*, 52, 356-385.
8. Tanzi, V. (1995). Corruption: arm's-length relationships and markets, in Gianluca Fiorentini and Sam Peltzman (eds.) *The Economics of Organised Crime*, Cambridge: Cambridge University Press, 161-180.
9. Vinod, H.D. (1999). Statistical Analysis of Corruption Data and Using the Internet to Reduce Corruption. *Journal of Asian Economies*, 10, 591-603.
10. Welsch, H. (2004). Corruption, growth and the environment: A cross-country analysis. *Environ. Dev. Econ*., 9, 663-693.

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REQUIREMENTS PRIORITIZATION: A COMPARISON BETWEEN TRADITIONAL AND AGILE (SCRUM AND FDD)

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ABSTRACT

Development of a software intensive system is done on the premise of some required/requested capabilities, termed as ‘requirement’. The objective, for software application development, emphasizes on client's requirement, in light of set of characterized and comprehended standards. The set of requirements, when engineered legitimately, may uncover that a few requirements are of most extreme significance for user. Notwithstanding a set of requirements, might be those which are of minimum significance. Choosing the needs of requirements under certain criteria is done in prioritization phase of requirements engineering phase. This paper presents how requirements prioritization is applied in conventional software development approaches in contrast to Agile Software Development Methodologies (SCRUM and FDD). Actors, activities and exercises performed by them and artifacts created are taken as parameters of comparison.

KEYWORDS

Requirement Prioritization, Requirement Engineering, Agile, Scrum etc.

1. INTRODUCTION

The term requirement, as restricted in The IEEE Standard Glossary of Software Engineering Terminology (1990), relates with the conditions or abilities a client needs for, solving a problem or accomplishment of a goal [1]. Set requirements, for software intensive system, indicate what to implement alongside depicting the general conduct of system, a property utilized as a part of the system or a trait of the system. Numerous researchers are working in this field anyway it is difficult to make use of right technique at right time. It is meant to execute security viewpoint since the start of programming i.e. requirement engineering phase [2]

Requirements engineering then again, characterizes a set of exercises for gathering of software requirements, investigating and displaying them, through systematic approach [3].

In requirement engineering, prioritization is very important aspect and consider as a groundwork for other subsequent phases of SDLC [4]. Effective requirement prioritization techniques are very essential for planning successful arrangement of requirements in system development [5]. Prioritization of requirements incorporates positioning them in a specific order to develop them on priority. The criteria for positioning may vary from organization to organization. Some may prioritize according to time and budget constraints [6], some may characterize positioning according to stakeholder's interests [7] or some other may prioritize according to complexity of requirements.

Through the procedure of requirements prioritization, a set of most significant requirements is recognized which underpins Requirements prioritization can be done as per different aspects of a project [8]. Some ordinary perspectives are (1) Significance, (2) Penance, (3) Price, (4) Schedule and (5) Threat.

Over the procedure of requirements prioritization, a set of most valuable requirements is identified which supports [8]:

a) Selection of a subdivision of requirements and production of a setup that fulfill client's need.
b) Handling of opposing requirements through determination of differences between stakeholders.
c) Balancing the business benefits accomplished through a requirement against its cost.
d) Stakeholders to concur upon a core set of requirements for the system.
e) Planning and selection of an ideal set of requirements to be executed in successive releases.
f) Establishment of relative significance of every requirement which thus give big value at least cost.

2. LITERATURE REVIEW

Traditional Requirement Prioritization Process

In traditional software development approaches as requirements are gathered and recorded right on time in the development lifecycle, so prioritization must be done before the last draft of requirements report is agreed upon. For prioritization of requirements, in traditional approach, various gatherings with the client are directed, clashes are settled and requirements are prioritized according to stakeholder's interests. In [9] the prioritization procedure of traditional requirements engineering is presented as:

a. **The Preparation Stage**
   A team and team leader structure requirements according to the characterized standards of applied methods
b. **The Execution Stage**
   Real prioritization of requirements is done on the premise of data given from preparation stage
c. **The Presentation Stage**
   Aftereffects of second stage are processed and after that exhibited to stakeholders of project

The initial two phases guide to the set of activities performed and third stage characterizes the artifacts produced. Be that as it may, the actors playing out the activities are not distinguished while characterizing the procedure in [9].

The actors required in the requirements prioritization process are [10]:

a) **Project Manager**
- Resolves clashes among stakeholders.
- Adjust contributions from various stakeholders.

b) **Key Representative from Client**
- Supply advantages and penalty evaluations.

c) **Development Group Representative**
- Supply cost and risk appraisals.

In [11], the procedure of prioritization is characterized as a set of eight activities including, persuading and training stakeholders, classification of crude requirements, prioritization of genuine requirements, distributing priorities, estimating efforts, planning the development and looking after priorities.

The actors and activities of requirements prioritization in traditional requirements engineering methodology can be summarized as in Table-1.

Table 1
Actors and Activities in Traditional Requirements Prioritization

Actors	Activities
Project Manager	Conflict resolution, alteration of inputs from stakeholders Prepare stakeholders, publish priorities, schedule development, keeping up priorities as they change
Technical Team Lead	Supply cost and risk appraisals, categorize crude potential requirements. Prioritize actual requirements, estimate effort, plan development, maintain priorities
Development Representative	Plan development, appraise effort.
Customer Representative	Supply advantage and penalty appraisals categorize crude potential requirements, prioritize actual requirements.

Requirements Prioritization Techniques/Methods

Following is a list of various requirements prioritization techniques/methodologies available in the literature [12][13]. Despite the fact these techniques are not particularly related to traditional software development life cycle but rather these are generally applied in customary approach:

- Bubble Sort Technique
- Quality Function Deployment Method
- Analytic Hierarchy Process (AHP)
- Cost Value Approach
- WIN WIN Approach

- Qualitative WIN WIN
- Hierarchy AHP
- Multi-criteria Preference Analysis Requirements Negotiation (MPARN) Technique
- Priority Groups Approach
- Visualization Technique
- Spanning Tree Matrix
- Goals-Skills Preferences
- Binary Search Tree
- The Psychotherapy for system requirements

Agile Methods

Agile methodology is an iterative and transformative process for software development endeavoring at place of every now and again happening changes [14]. A key set in any agile methodology procedure of frameworks conveyance is the nearby, mutual and constant teamwork amongst customers and application designers [15]. Most common agile techniques utilized in industry are:

- Scrum
- Scrum ban
- Rapid Application Development (RAD)
- Feature Driven Development (FDD)
- Crystal
- Dynamic System Development Method (DSDM)
- Agle Unified Process Model
- Adaptive Software Development (ASD)
- Extreme Programming (XP)
- Lean and Kanban Software Development

We concentrate for the most part on Scrum and FDD as they are generally being fitting in software industry when contrasted with different approaches.

3. RESEARCH METHOD

We took after the principles of an organized literature survey [28], which is a very much characterized way to identify, assess and translate all pertinent studies in regards to a specific analysis query, subject range or marvel of premium. **Figure 1** demonstrates an outline of the three-staged technique we enforced in the investigation.

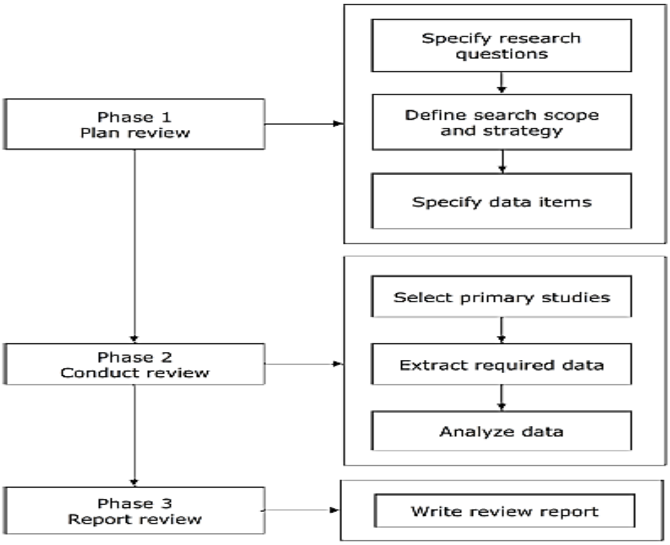

Fig. 1: Analysis of the Standardized Review Procedure (adapted from [29])

Literature study has been done by the means specified in **Figure 1**. In Phase-1, check pact was used to incorporate research inquiries, procedure and range amid information which must be gathered. The research queries convey the exploration themes of enthusiasm in the writing survey. The range of the analysis depends on the statistical collection via primary seminar, gatherings, and journals in the sector. A consolidated program was executed with non-automatic exploration technique. Automatic exploration was characterized into two-stage procedure for which two hunt strings were utilized. The main string expects to choose the reviews on classical techniques of prioritizing requirements, and the 2nd string refers to requirement prioritization in agile methodologies, with special accentuation on Scrum and FDD. For the non-automatic pursuit, incorporation and avoidance standard were characterized.

Research Questions

We detailed the generic objective review over Goal oriented-
Question-Metric (GQM) points of view (reason, problem, item and outlook) [12]:

Reason: Know and describe

Problem: Comparative Analysis between Traditional Techniques and Agile Methods (Scrum and FDD)

Item: Requirement Prioritization.

Overlook: From a specialist's and architect's perspective.
The generic research query means three solid hunting queries:

RQ1: What all general techniques of implementing requirement prioritization in Traditional software development approaches and in addition in Agile Techniques?

RQ2: How requirement prioritization in Scrum and FDD is not the same as other classical techniques/models.

RQ3: What benefits do agile methods have on prioritizing requirement when contrasted with traditional software development approaches?

With RQ1, we want to get understanding in the utilization of requirement prioritization by researchers in Traditional approaches of software development and in Agile Methodologies (Scrum and Feature Driven Development). RQ2; we aim to comprehend the distinction between prioritization in Agile and Traditional Methodologies. Solidly, we intend to assess the distinctions. At long last, RQ3 aims to differentiate between Scrum, FDD and Traditional Methods. What's more, relative analysis demonstrates what approach is better. Fundamentally, this question's reaction provided inspiration towards Agile Techniques for developers and additionally researchers, which is aim of our research.

Information Sources and Search Tactics

Multiple digital databases that we have scrutinized to yield desired results related to requirement prioritization were:

- IEEE Xplore (ieeexplore.ieee.org)
- Springer Link (www.springerlink.com)
- Science Direct– Elsevier (www.sciencedirect.com)
- Citeseer (citeseerx.ist.psu.edu)
- ACM Digital Library (portal.acm.org)

Moreover, customary measures of the International Symposiums on "Requirement Engineering" were physically sought. Apart from it, measurements of "International Requirements Engineering Conference" which is the significant event in requirements building was also sought.

Publication Selection & Screening

We additionally refined the studies related to auto search using a manual method. The target of this progression is to perceive the essential investigations that are particularly identified in relation to our examinations. Accordingly, we characterized the accompanying consideration/avoidance criteria:

✓ **Inclusion criterion 1:** We encompassed audits which used a few wordings related to requirement prioritization in classical method (requirement assembling) and Agile method (Scrum and FDD).

✓ **Inclusion criterion 2:** Papers of published journals and without a doubt comprehended gatherings have been consolidated, which bounces sufficient data on the space.

✗ **Exclusion standard 1:** Some conference papers in which much important information was not given are barred during this review.

✗ **Exclusion standard 2:** This essentially concentrate upon Scrum and FDD and avoided every one of the papers that were concentrating on the other Agile methodologies of requirement elicitation and prioritization. E.g. XP, DSDM, etc.

On the premise of the described examine measures, 72 papers left afterwards name shielding, 41 after abstract and 26 after the screening of all papers, out of aggregate 112 papers.

4. PRIORITIZATION IN SCRUM AND FDD

Scrum

A group-based process with an incremental and iterative advancement approach having center around conveyance of business esteem [13]. Scrum is more situated towards project management activities [17] [18]. Focus of scrum is brief time conveyance of project, as intermediate deliverables, called sprints. Time traverse for a sprint is from 3 to 4 weeks.

Phases of Sprint

A Sprint comprises of following stages [13].

- **Sprint Planning Meeting**
  Product pile-up is analyzed and recognized for release
- **Scrum Sprint Meeting**
  By the day's end to talk about tasks completed and task left in sprint
- **Sprint Development**
  Design, development and testing is accomplished for identified product excess
- **Sprint Review**
  Incremental functionality of the product is tested
- **Retrospective Sprint Closure**
  Sprint closure is done for all intents and purposes and the distinguished sprint build-up is added to scrum product backlog.

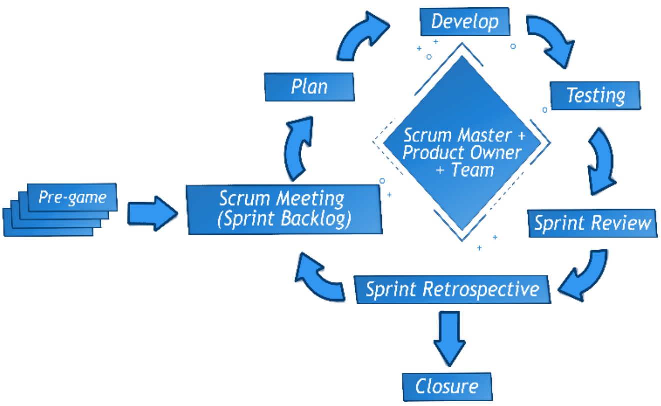

Fig. 2: Phases of Scrum [16]

Requirements Prioritization in Scrum

Product build-up is actually the rundown of requirements and product owner has the responsibility to guarantee most profitable functionality is produced and develop first. Product owner much of the time prioritize the product build-up [27]. Product owner has the privilege to scratch off or reprioritize as of now prioritized requirements. The benefit of product build-up is that it is not finished report and advances with the product. At begin, the product build-up constitutes of just high-level requirements. Everything in product build-up has an assigned priority value, which is utilized to constitute the business estimation of item in the final product. In sprint planning meeting, sprint group needs to get to the most noteworthy build-up items [19]. Methods accessible for requirements prioritization can be utilized by product owner to prioritize requirements [20].

Scrum provides the mechanism in which client can approve the pre-requisites via frequent regular scrum gatherings. These gatherings include customers to verify all the needs/requirements. [21]. The actors and activities involved in requirement prioritization [22] are introduced in Table2.

Table 2
Actors and Activities in SCRUM Requirements Prioritization

Actors	Activities
Product Owner	➢ Prioritize requirements in product backlog on the basis of business values ➢ Manages the priorities of requirements in product backlog
Self-Organizing Team	➢ Effort estimation of priority list of requirements in product backlog ➢ An executable sprint backlog is selected

The Artifact

A prioritized product build-up is created because of requirements prioritization process in scrum. Nevertheless, Agile strategies concentrate on casual documentation and more face to face communication and on location client visits. That is the reason why the product build-up continues advancing and priorities of requirements continues changing all through the development procedure.

Feature Driven Development

Feature driven development (FDD) is more situated towards components of a software intensive system. During the FDD procedure, every one of the requirements are caught and a detailed model is build. Components are produced in a couple of hours or days yet not over 2 weeks [23]. Groups in FDD are partitioned for designing and implementation of a component.

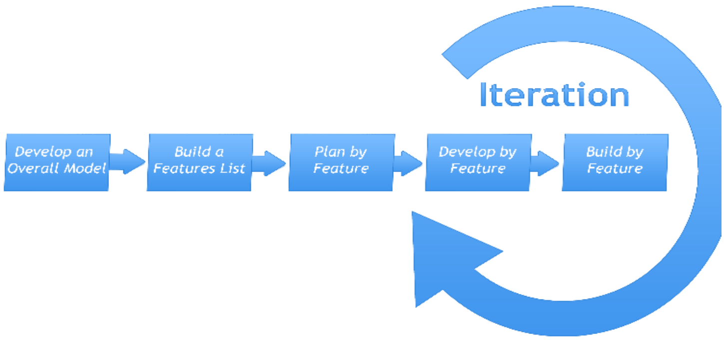

Fig. 3: Phases of FDD [23]

FDD Process Model

Process with in FDD has following five phases [24] [25].

a. Develop an Overall Object Model
b. Build a prioritize feature list
c. Planning by feature
d. Develop by feature
e. Build by feature

Requirements Prioritization in FDD

In the second phase of FDD process components are prioritized. The key activities performed in 2nd phase are:

a. Transformation of method in to features.
b. Generating feature sets

A team comprising of domain and development specialists plays out the above defined tasks. Prioritization of component is done as, A (must have), B (nice to have), C(add if we can) or D(will be added in future). Each component is considered as, as far as customer fulfillment, if included. Table 3 shows the on-screen characters and activities in FDD requirement prioritization process.

Table 3
Actors and Activities in FDD Requirements Prioritization

Actors	Activities
Developers	➢ Scheduling of development of components as per priorities ➢ Grouping of most applicable and high priority components for development
Customers (Problem Domain Experts)	➢ Identify changes amid specification of components and their priorities ➢ Selects the most high priority components

The Artifact

A prioritized list of components after interaction with clients. Like Scrum, the priorities of components are not last they may change as the development continues.

5. COMPARISON

Challenges for Traditional Software Development Approach

Difficulties distinguished in customary approach in software development are [19] [30]:

- Customer desires have grown exponentially about the items and services, which makes software development a perplexing errand.
- Due to less interaction with client, the requirements stay misty and additionally evolving requirements are overlooked.
- A criticism of the difficult technique may confuse the stakeholders.

In traditional requirements prioritization process the priorities are documented with SRS, thus, giving not even a solitary opportunity to scratch off or reprioritize requirements. In spite of the fact that there are various techniques accessible in literature yet less association of customer in prioritization procedure yields to surprising outcomes, losing confidence of customer on development team.

Agile Benefits

Agile requirements prioritization procedure responds to new data in natural way [26]. Throughout the product development change in priorities of requirements is acknowledged and appreciated. Client actively takes an interest in the procedure of prioritization of requirements which builds the level of confidence over the final product. In Agile techniques the prioritization is done by client himself; however, experts are always there to inculcate the technical needs. Documentation is casual and client is in charge of changing and dealing with the priorities in greater part of the strategies. Techniques/ methods accessible for prioritization of requirements for customary approach may likewise be applied to get the best prioritize list. Although, Agile has multiple advantages over traditional software requirements prioritization techniques; yet a lot of effort is required to formulate requirements prioritization artists, activities and artifacts. Agile methodologies are applicable and manageable for small and medium size projects however more research work is required on complex and large projects to reduce the risk factor.

REFERENCES

1. Wiegers, K.E. (2003). *Software Requirements*, 2nd Edition, Microsoft Press.
2. Batra, M. and Bhatnagar, A. (2015). Descriptive Literature Review of Requirements Engineering Models. *International Journal of Advanced Research in Computer Science and Software Engineering*, 5(2), 289-293.
3. Pandey, D., Suman, U. and Ramani, A.K. (2011). A framework for modelling software requirements. *International Journal of Computer Science Issues* (IJCSI), 8(3/1), 164-171.

4. Batra, M. and Bhatnagar, A. (2016). Requirements Prioritization: A review. *International Journal of Advanced Research in Science, Engineering and Technology*, 3(11), 2899-2904.
5. Saranya. B., Subha. R. and Palaniswami. S. (2014). A survey on prioritization methodologies to prioritize non-functional requirements. *International Journal of Computer Science and Business Informatics*, 12(1), 32-44.
6. Hassan, S., Awan, S., Jaan, F. and Akmal, H. Factors affect on Requirement Prioritization. *IOSR Journal of Engineering*, 2(12), 28-33.
7. Kukreja, N. and Boehm, B. (2013). Integrating Collaborative Requirements negotiation and prioritization processes: a match made in heaven. *In Proceedings of the International Conference on Software and System Process*, 141-145. ACM.
8. Berander, P. and Andrews, A. (2005). Requirements prioritization. *In Engineering and managing software requirements*, 69-94. Springer, Berlin, Heidelberg.
9. Karlsson, J., Wohlin, C. and Regnell, B. (1998). An evaluation of methods for prioritizing software requirements. *Information and Software Technology*, 39(14-15), 939-947.
10. Wiegers, K. (1999). First things first: prioritizing requirements. *Software Development*, 7(9), 48-53.
11. Firesmith, D. (2004). Prioritizing requirements. *Journal of Object Technology*, 3(8), 35-48.
12. Achimugu, P., Selamat, A., Ibrahim, R. and Mahrin, M.N.R. (2014). A systematic literature review of software requirements prioritization research. *Information and Software Technology*, 56(6), 568-585.
13. Nilofer, M. and Sheetal, G. (2012). Comparison of Various Elicitation Techniques and Requirement Prioritization Techniques. *International Journal of Engineering Research & Technology*, 1(3), 1-8.
14. Kaur, M.R., Choudhary, M. and Mehta, M.R. (2012). Agile Process: An Enhancement to the Process of Software Development. *International Journal of computer Science and Network Security (IJCSNS)*, 12(7), 101-105.
15. Maiden, N. and Jones, S. (2010). Agile Requirements can we have our cake and eat it too. *IEEE software*, 27(3), 87-88.
16. Selvi, K. and Majumdar, R. (2013). Scrum: An Agile Process. *International Journal of Research in Engineering and Technology*, 2(3), 337-340.
17. Batool, A., Motla, Y.H., Hamid, B., Asghar, S., Riaz, M., Mukhtar, M. and Ahmed, M. (2013). Comparative study of traditional requirement engineering and agile requirement engineering. *IEEE 15th International Conference on Advanced Communication Technology (ICACT)*, 1006-1014.
18. Bhalerao, S., Puntambekar, D. and Ingle, M. (2009). Generalizing Agile software development life cycle. *International Journal on Computer Science and Engineering*, 1(3), 222-226.
19. Düchting, M., Zimmermann, D. and Nebe, K. (2007). Incorporating user centered requirement engineering into agile software development. *In International Conference on Human-Computer Interaction*, 58-67. Springer, Berlin, Heidelberg.
20. Rosberg, J. (2005). A Requirement process for Scrum development. *Journal of Systems and Software*, 76, 251-275.

21. Qasaimeh, M., Mehrfard, H. and Hamou-Lhadj, A. (2008). Comparing Agile Software Processes Based on the Software Development Project Requirements. *IEEE International Conference on Computational Intelligence for Modelling Control & Automation*, 49-54.
22. Srinivasan, J. and Lundqvist, K. (2009). Using agile methods in software product development: A case study. *IEEE Sixth International Conference on Information Technology: New Generations, 2009. ITNG'09*. 1415-1420.
23. Pathak, K. and Saha, A. (2013). Review of agile software development methodologies. *International Journal of Advanced Research in Computer Science and Software Engineering*, 3(2), 270-276.
24. Anderson, David J. (2004). *Feature-Driven Development*. Microsoft Corporation.
25. Coad, P., Luca, J.D. and Lefebvre, E. (1999). *Java modeling color with UML: Enterprise components and process with Cdrom*. Prentice Hall PTR.
26. Racheva, Z., Daneva, M., Sikkel, K., Herrmann, A. and Wieringa, R. (2010). Do we know enough about requirements prioritization in agile projects: insights from a case study. *Requirements Engineering Conference (RE), 18th IEEE International*. 147-156.
27. Schwaber, K. (2007). *The enterprise and scrum*. Redmond, Wash, Microsoft Press.
28. Kitchenham, B.A. (2007). Guidelines for performing Systematic Literature Reviews in Software Engineering. *EBSE Tech. Report*, EBSE-2007-001.
29. Brereton, P., Kitchenham, B.A., Budgen, D., Turner, M. and Khalil, M. (2007). Lessons from applying the systematic literature review process within the software engineering domain. *Journal of Systems and Software*, 80(4), 571-583.
30. Achimugu, P., Selamat, A., Ibrahim, R. and Mahrin, M.N.R. (2014). A systematic literature review of software requirements prioritization research. *Information and Software Technology*, 56(6), 568-585.

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TIME AND SPACE COMPLEXITY OF TOMPSO ALGORITHM

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ABSTRACT

In today's world there is an increase in the demand of high data rates in communication systems, which makes capacity of channel a major problem. Multiple Input Multiple Output (MIMO) technology is gaining importance in communication world. This technology provides quality service and also increases the capacity of the channel. The technology works at physical layer. The medium through which signal passes is known as channel. As signal passes through channel, some noise is added to the signal which will deteriorate the signal. Because of this addition the receiver is unable to receive the exact message. Therefore, it becomes essential to remove this noise. Also, it is very important to analyze the characteristic of channel. This process is known as channel estimation. Semi blind, blind and training-based methods are some important techniques that are used to achieve the goal of channel estimation. Several transmitter and receiver antennas are used for improvement in communication. MIMO technology works by sending data on multiple paths using multiple antennas, therefore increasing the amount different algorithms were designed to achieve this objective. On receiver end certain algorithms are used for restoration of data. Time and Space Complexity of certain algorithms was also computed. Our main concern in this paper is to compute the complexity of newly Proposed Total Opposite Mutant Particle Swarm Optimization (TOMPSO) algorithm for MIMO system in terms of space and time.

1. INTRODUCTION

Today one of the basic issue in network communication is capacity of channel. As the demand of high data rates is growing day by day. The problem is resolved by using Multiple Input Multiple Output (MIMO) technology. Using this technology, channel's capacity is enhanced [1], [2], [3]. MIMO technique uses multiple antenna at receiver and sender end in order to improve communication. Special algorithms are used to send data on multiple paths, which will in return increases the capacity [4], [5], [6], [7], [8]. In the same way, when we talk about receiver end, there are also certain algorithms that will restore data. MIMO technology works at physical layer and is one of the important technique in wireless system. Bandwidth is optimized using this method [6].

The medium through which signal passes is known as channel. As signal passes through channel, some noise is added to the signal which will deteriorate the signal. Because of this addition the receiver is unable to receive the exact message. Therefore, it becomes essential to remove this noise. Also, it is very important to analyze the characteristic of channel. This process is known as channel estimation [6], [7].

Channel estimation includes training based, blind and semi blind methods. Certain factors like mobility f sender and receiver, changes in environment or frequency mismatch will make the process of channel estimation a time-consuming process [2], [8]. So, certain algorithms are used for this purpose. One of the important approach is Kalman Filtering. The main drawback of this process is that it is restricted to two antennas only [9]. Several algorithms are designed for channel estimation like Zero Forcing (ZF), Maximum Likelihood (ML) [10], Swarm Optimization [6], [7] etc.

Complexity of several other algorithm was already computed on other MIMO systems. In this paper, our focus is towards computing the complexity of newly proposed channel and data estimation MIMO based TOMPSO algorithm.

2. SYSTEM MODEL

There are A transmit antennas and B receiving antennas. The channel implemented is a flat fading channel. During communication process of Q symbols, the channel is expected to be stationary.

$$\begin{aligned} J_{ML}(\widetilde{D}, \widetilde{H}) &= \frac{1}{B * Q}\left[\sum_{i=1}^{Q} r^2(i) - 2\sum_{i=1}^{Q} r(i)\widetilde{H}\widetilde{d}(i) + \sum_{i=1}^{Q}\left|\left|\widetilde{H}\widetilde{d}(i)\right|\right|^2\right] \\ &= \frac{1}{B * Q}\left[\sum_{i=1}^{Q} r^2(i) - \left\{2\sum_{i=1}^{Q} r(i)\widetilde{H}\widetilde{d}(i) + \sum_{i=1}^{Q}\left|\left|\widetilde{H}\widetilde{d}(i)\right|\right|^2\right\}\right] \end{aligned} \quad (1)$$

$$C_{ML}(\widetilde{D}, \widetilde{H}) = 2\sum_{i=1}^{Q} r(i)\widetilde{H}\widetilde{d}(n) - \sum_{i=1}^{Q}\left|\left|\widetilde{H}\widetilde{d}(i)\right|\right|^2 \quad (2)$$

Put in Eq. 1

$$J_{ML}(\widetilde{D}, \widetilde{H}) = \frac{1}{B\ x\ Q}\left[\sum_{i=1}^{Q} r^2(i) - C_{ML}(\widetilde{D}, \widetilde{H})\right] \quad (3)$$

Eq. 1 can be written as:

$$J_{ML}(\widetilde{D}, \widetilde{H}) = \left[\min_{\widetilde{D}, \widetilde{H}}\left[\sum_{i=1}^{Q} r^2(i) - C_{ML}(\widetilde{D}, \widetilde{H})\right]\right] \quad (4)$$

It means,

$$J_{ML}(\widetilde{D}, \widetilde{H}) = \max_{\widetilde{D}, \widetilde{H}} C_{ML}(\widetilde{D}, \widetilde{H}) \quad (5)$$

3. COMPLEXITY OF TOMPSO ALGORITHM:

Algorithm is defined as step by step procedure to solve a problem using computer. Complexity defines how efficient an algorithm is in terms of time and space. In other words, complexity shows how fast or how slow an algorithm works. In terms of function complexity is defined as T(n) where T is time and n define the size of input. After fix number of cycles, if multiple algorithm gives same Bit error rate and Minimum mean square error than complexity is one of the factor which will determine which algorithm is best [11].

Complexity of algorithm is computed in terms of time and space using cost function given in (5). Time complexity defines how much time the algorithm will take to solve the problem [12]. And space complexity defines the memory consumed.

3.1 Proposed Algorithm

The TOMPSO algorithm is divided into 3 main phases. The steps involved in these phases are defined as:

3.1.1 Phase 1

In first phase:

3.1.1.1 Data population (P_D), Channel population (P_h), data velocity (v_d) and channel velocities (v_h) are calculated.

3.1.1.2 Population fitness is computed using eq (5) and sorted using massive merge sort.

3.1.1.3 Lower and upper bound values are obtained.

3.1.1.4 Opposite data (op_d) and opposite channel (op_h) are computed

3.1.1.5 Fitness of opposite population is computed using eq (5).

3.1.1.6 Local and Global best particle are selected from data and channel population.

3.1.2 Phase 2

In phase 2 algorithm works on channel population.

3.1.2.1 Velocities of channel population particles are updated.

3.1.2.2 Position of channel population particles are updated.

3.1.2.3 Mutated channel population particle fitness is computed

3.1.2.4 Channel population is updated

3.1.3 Phase 3

In phase 3 data symbols are estimated:

3.1.3.1 Global best particle of data population is selected and velocity is updated.

3.1.3.2 Data population particle's position is updated and mutant operator is calculated

3.1.3.3 Mutant particle fitness is computed

3.1.3.4 Data population is updated

After phase 3 new signal sample is taken and execution starts again.

3.2 Complexity in Terms of Time

3.2.1 Complexity of Phase 1

Step 1: data population and channel population initialization

```
for i → 1:a
for j → 1:k
PD (i,j) =rand ();
end
end
```

Complexity of initialization of data population:

$$I_1 = \sum_{j=1}^{k} 1 = k$$

$$I_2 = \sum_{i=1}^{a} I_1 = \sum_{i=1}^{a} k = ak$$

Complexity of initialization of channel population:

```
for i → 1:a
for j → 1:T
PC (i,j) =rand ();
end
end
```

$$I_3 = \sum_{j=1}^{T} 1 = T$$

$$I_4 = \sum_{i=1}^{a} I_3 = \sum_{i=1}^{a} T = aT$$

Complexity of initialization of data velocity and channel velocity will be same as that of initialization of channel and data population i.e.:

$$I_5 = ak \; and \; I_6 = aT$$

And the complexity of step 1 will be sum of all complexities.

$$\begin{aligned} I_1\,(n) &= I_2 + I_4 + I_5 + I_6 \\ &= ak + aT + ak + aT \\ &= 2ak + 2aT \\ &= 2a\,(k + T) \end{aligned}$$

Step 2:
In step 2 population fitness is calculated and sorted using merge sort. The complexity to calculate the fitness for channel population is d as we have d particles and that for data population is h. And complexity of merge sort is:

```
Merge-sort (ar A, int p, int r)
    if (p<q)
    then
    q← (p + r)/2
    Merge-sort (A, p, q)
    Merge-sort (A, q+1, r)
    Merge-sort (A, p, q, r)
```

Recursive Function for merge sort will be:

$$T(n) = \begin{cases} 1 \ if \ n = 1 \\ 2T\left(\frac{n}{2}\right) + n \ otherwise \ (i) \end{cases}$$

Put n= n/2 in equation no. (i);

$$T\ \left(\frac{n}{2}\right) = 2T\left(\frac{\frac{n}{2}}{2}\right) + \frac{n}{2}$$

$$T\ \left(\frac{n}{2}\right) = 2T\ \left(\frac{n}{4}\right) + n/2$$

Now, put the value of T (n/2) in equation no. (i)

$$T\ (n) = 2\ \left[2T\ \left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$$

$$= 4T\left(\frac{n}{4}\right) + 2n\ (ii)$$

$$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2n$$

Now, put n=n/4 in equation no. (i):

$$T(n) = 2T\ \left(\frac{n}{2}\right) + n$$

$$= 2T\ \left(\frac{\frac{n}{4}}{2}\right) + \frac{n}{4}$$

$$= 2T\ \left(\frac{n}{8}\right) + n/4$$

Putting the value of T(n/4) in equation no. (ii)

$$T(n) = 4\left[2T\ \left(\frac{n}{8}\right) + \frac{n}{4}\right] + 2n$$
$$= 8T\left(\frac{n}{8}\right) + 3n\ (iii)$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3n$$

Now put value of n=n/8 in equation no.(i) :

$$T\ \left(\frac{n}{8}\right) = 2\ T\left(\frac{\frac{n}{8}}{2}\right) + \frac{n}{8}$$
$$= 2T\left(\frac{n}{16}\right) + \frac{n}{8}$$

Putting the value of n/8 in equation no. (iii):

$$T(n) = 8\left[2T\ \left(\frac{n}{16}\right) + \frac{n}{8}\right] + 3n$$
$$= 16T\left(\frac{n}{16}\right) + 4n\ (iv)$$

$$T(n) = 2^4 T\left(\frac{n}{2^4}\right) + 4n$$

$$T(n) = \ 2^k T\left(\frac{n}{2^k}\right) + kn\ (v)$$

From equation no. (v);

$$2^k = n\ (vi)$$

Taking log on both sides:

$$\log 2^k = \log n$$

$$\log n = k$$

or

$$k = \log n$$

Put the value of 2^k and k in equation number (v):

$$T(n) = \ 2^k T\left(\frac{n}{2^k}\right) + kn$$
$$= \ nT\left(\frac{n}{n}\right) + logn * n$$
$$= nT(1) + nlogn$$

From recursive equation it can be seen that T(1)=1. So, above equation will become:

$$T(n) = n + nlogn$$

So, the complexity of merge sort is $nlogn$.

The complexity of step 2 for channel is:

$$I_7 = d + nlogn$$

And for data population the complexity will be:

$$I_8 = h + nlogn$$

Complete step complexity will be:

$$\begin{aligned} I_2(n) &= I_7 + I_8 \\ &= d + nlogn + h + nlogn \\ &= d + h + 2\,nlogn \end{aligned}$$

Step 3:

LB_d= P_d [1,1]

LB_h= P_h [1,1]

for i ← 1: a

for j ← 1 : k

if LB_d> P_d [i,j]

LB_d=P_d [i,j]

If LB_h < P_d [i,j]

LB_h = P_d [i,j]

end
end
end

$$I_9 = \sum_{j=1}^{k} 2 = 2k$$

$$I_{10} = \sum_{i=1}^{a} I_9 = \sum_{i=1}^{a} 2k = 2ak$$

The complexity of finding upper and lower bound from data population is 2ak. And complexity to compute upper and lower bound from channel population is:

$$I_{11} = P_h = 2aT$$

So, over all complexity of this step is:

$$\begin{aligned} I_3(n) &= I_{10} + I_{11} \\ &= 2ak + 2aT \end{aligned}$$

Step 4:
Complexity of Opposite data and channel population calculation:

for I ← 1 : a

for j ← 1 : k

$OP_{d\,(i,j)} = LB_d + UB_d - P_{d\,(i,\,j)}$

end

end

$$I_{12} = ak$$

Similarly, for opposite channel population complexity will be:

$$I_{13} = aT$$

Over all complexity of this step will be:

$$\begin{aligned} I_4(n) &= I_{12} + I_{13} \\ &= ak + aT \\ &= a\,(k + T) \end{aligned}$$

Step 5:
The complexity of computing the fitness and sorting it is already computed above. So, it will be equal to:

$$I_5(d, h, n) = d + h + 2\,nlogn$$

Step 6:
As population are sorted and best particles are selected from P_d and OP_d and P_h and OP_h. It will take some constant time. So, its complexity will be C.

$$I_6(n) = C$$

Step 7:
As total particle size is a, so the complexity to find global best particle for channel and data population is a.

$$I_7(a) = 2a$$

The complexity of phase 1 of this algorithm will be:

$$\begin{aligned} I &= I_1(n) + I_2(n) + I_3(n) + I_4(n) + I_5(n) + I_6(n) + I_7(n) \\ &= 2a\,(k + T) + d + h + 2\,nlogn + 2a(k + T) + a\,(k + T) + d + h \\ &\quad + 2\,nlogn + C + 2a \end{aligned}$$

$$T_1(k, T, d, h, n) = I = 5a\,(k + T) + 2d + 2h + 4nlogn + 2a + C$$

3.2.2 Complexity of Phase 2

In phase two the complexity of updating the velocities of channel population particles is same as initialization of channel population particles i.e.:

$$I_8(n) = ak$$

Step 10:
Complexity of this phase will be

$$I_9 = 2dk$$

Step 11:
The complexity of this step is same as that of step 3.

$$I_{10} = d + nlogn$$

Step 12:
The complexity of this step is:

$$I_{11} = ak$$

Step 13:

$$I_{12} = T_c = I_C$$

Overall complexity of phase 2:

$$I_C = I_8 + I_9 + I_{10} + I_{11} + I_{12}$$

$$= ak + 2\mathrm{dk} + \mathrm{d} + \mathrm{nlogn} + \mathrm{ak} + (ak + dk + d + nlogn + ak)* N_{cc}$$

$$T_c(a,k,n,d,N_{cc})=2ak + 2dk + d + nlogn + (2ak + dk + d + nlogn) * N_{cc}$$

Phase 3 Complexity:

The complexity of phase 3 will be same as that of phase 2. The only difference is that d is replaced with h. And ak is replaced with aT.

$$T_d(a,T,h,k,n,N_{DD}) = 2aT + 2hk + h + nlogn + (2aT + h + nlogn) * N_{DD}$$

Total Complexity of Algorithm:

$$T_c(a,k,T,n,d,h,N_{cc},N_{DD}) = T_1(k,T,d,h,n) + T_c(a,k,n,d,N_{cc}) + T_d(a,T,h,k,n,N_{DD})$$

$$T_c = 7a(k+T) + 3(d+h) + 6nlogn + 2a + c + 2k(d+h) + ((2ak + dk + d + nlogn) * N_{cc} + (2aT + h + nlogn) * N_{DD}$$

3.3 Complexity in Terms of Memory

The memory complexity of the proposed TOMPSO algorithm is following

$$M_c = 4(d+h)i + 28$$

4. SIMULATION AND RESULTS

The Time & Memory complexity of the MIMO system is instigated using BPSK signaling and data sequences length was Q=50. The transmitters are each equipped with A=3 transmit antennas, while the base station has B=3 antennas. The channel taken for simulation is a 3*k MIMO with k=10 users, data population is 100 and number of cycles for both algorithms are 5. As we have choose the population size 2 be 5*P_h, where P_h is the channel matrix size.

Fig. 1 & 2 shows the time complexity & Fig. 3 shows the memory complexity of the proposed algorithm-based solution.

Fig. 1 shows the time complexity of the proposed algorithm vs Number of Cycles (NoC) with different number of Users(K). The top most curve, middle curve & bottom curve shows the time complexity when k=10, 15 & 20 respectively. It’s clearly shows that when the number of cycle and number of users are increased then Time complexity is also increased.

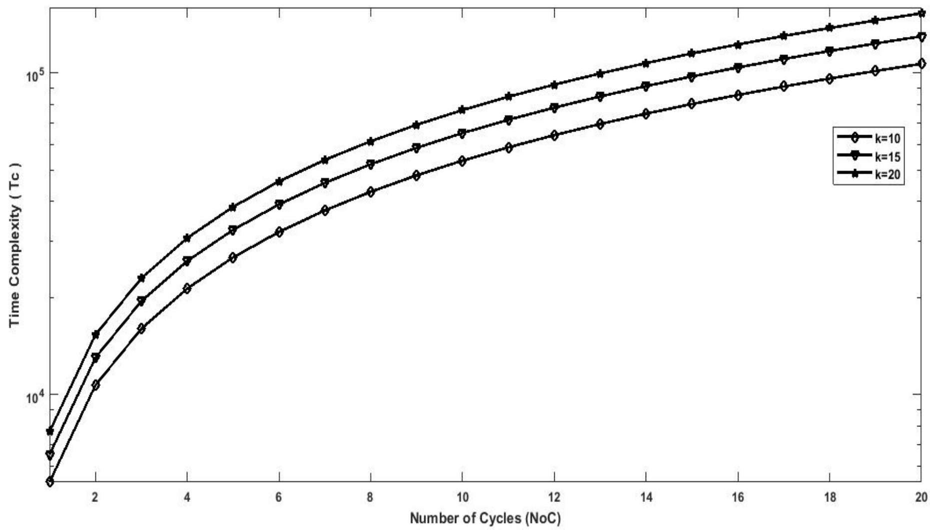

Figure 1: NoC vs Time Complexity with Different Number of Users

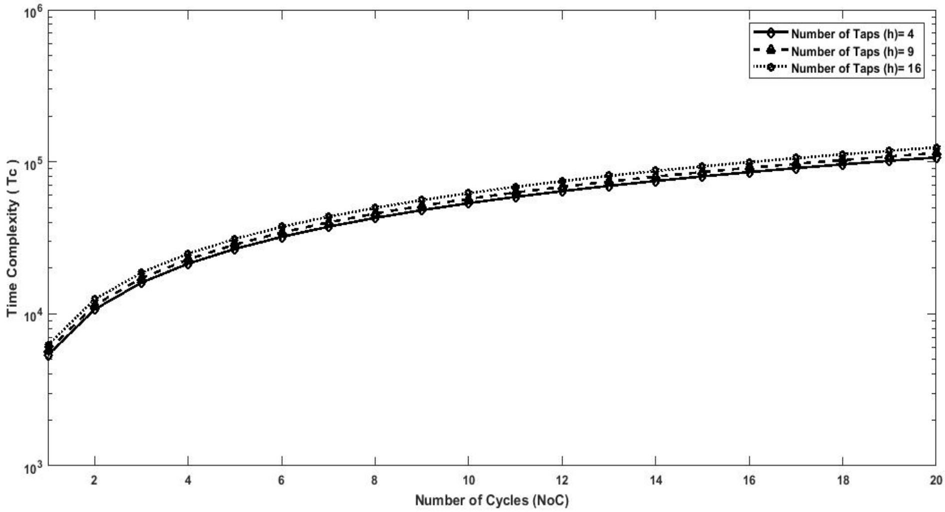

Figure 2: NoC vs Time Complexity with Different Number of Channel Taps

Fig. 2 shows the time complexity of the proposed algorithm vs Number of Cycles (NoC) with different number of Channel Taps/ paths. The top most curve, middle curve & bottom curve shows the time complexity when no of taps=4, 8 & 16 respectively. It's clearly shows that when the number of cycle and number of taps are increased then Time complexity is also increased.

Fig. 3 shows the memory complexity of the proposed algorithm vs Number of Cycles (NoC) with different number of Channel Taps/ paths. The top most curve, middle curve & bottom curve shows the time complexity when no of taps=4, 8 & 16 respectively.

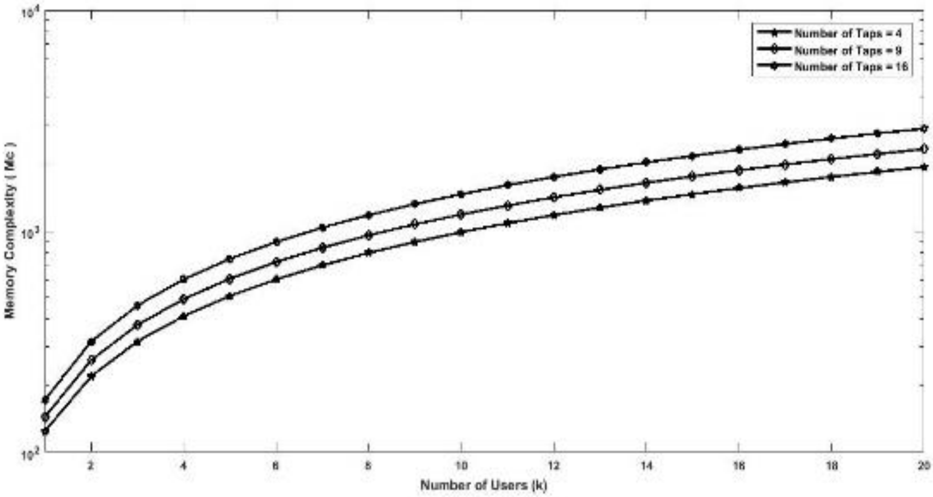

Figure 3: NoC vs Memory Complexity with Different Number of Channel Taps

It's clearly shows that when the number of cycle and number of taps are increased then Memory complexity is also increased.

5. COMMENTS AND CONCLUSION

In communication channel capacity is an important issue. MIMO technology is used to improve the capacity of channel and service quality. In this paper time and space complexity of TOMPSO is computed. When the number of users, channel paths & number of cycles are increased then Time & Memory Complexity is also increased.

REFERENCES

1. Abdelkader, T., Mokhtar, K. and Abdelaziz, O. (2014). New Space Time coding for Joint Blind Channel Estimation and Data Detection through Time Varying MIMO Channels. *International Journal of Computer Science Issues (IJCSI)*, 11(5), 33-40.
2. Abuthinien. M., Chen. S., Wolfgang. A. and Hanzo, L. (2007) Joint maximum likelihood channel estimation and data detection for MIMO systems. *In Communications, 2007. ICC'07. IEEE International Conference on*, 5354-5358.

3. Biguesh, M. and Gershman, A.B. (2006). Training-based MIMO channel estimation: A study of estimator tradeoffs and optimal training signals. *IEEE Transactions on Signal Processing*, *54*(3), 884-893.
4. Khan, M.A. (2016), Multi user detection using computational intelligence in multi-carrier communication systems, ISRA University, Islamabad campus, Pakistan.
5. Khan, M.A., Nasir, A., Umair, M. and Aamir, M. (2017). Channel and Data Estimation Using TOMPSO for MIMO Systems. Paper is accepted for publication in CMES.
6. Khan, M.A., Umair, M. and Choudhry, M.A.S. (2015). GA based adaptive receiver for MC-CDMA system. *Turkish Journal of Electrical Engineering & Computer Sciences*, 23(1), 2267-2277.
7. Khan, M.A., Umair, M. and Choudry, M.A.S. (2014). Accelerated Assistant to SubOptimum Receiver for Multi Carrier Code Division Multiple Access System. *International Conference on Future Trends in Computing and Communication Technologies* (FTCom), 26-29.
8. Khan, M.A., Umair, M., and Saleem, M.A. (2015). Island Differential Algorithm based MUD for MC-CDMA System, *6th IEEE International Conference on Information and Communication Technologies*, Electronic ISBN: 978-1-4673-8907-5, Electronic ISBN: 978-1-4673-8907-5, doi:10.1109/ICICT.2015.7469588, INSPEC Accession Number: 16004107, P. 1-6.
9. Mezghani, A. and Swindlehurst, A.L. (2016). Blind Estimation of Sparse Multi-User Massive MIMO Channels, (2), 2-6.
10. Raut, P.W. and Badjate, S.L. (2013). MIMO-future wireless communication. *International Journal of Innovative Technology and Exploring Engineering (IJITEE)*, 2(5), 102-106.
11. Seyman, M.N. and Taspinar, N. (2013). Symbol detection using the differential evolution algorithm in MIMO-OFDM systems. *Turkish Journal of Electrical Engineering & Computer Sciences*, 21(2), 373-380.
12. Umair, M., Khan, M.A. and Choudry, M.A.S. (2013January). GA backing to STBC based MC-CDMA systems. *In Intelligent Systems Modelling & Simulation (ISMS), 2013 4th International Conference on* (503-506). IEEE.
13. Umair, M., Khan, M.A. and Choudry, M.A.S. (2015). Island genetic algorithm based MUD for MC-CDMA system. *In Information and Communication Technologies (ICICT), 2015 International Conference on Information and Communication Technologies* (1-6). IEEE.
14. Zubair, M., Choudhry, M.A., Naveed, A. and Qureshi. I.M. (2008). Joint channel and data estimation using particle swarm optimization. *IEICE Transactions on Communications,* 91(9), 3033-3036.
15. http://www.belatrixsf.com/blog/introduction-to-algorithm-complexity.
16. http://www.studytonight.com/datastructures/time-complexity-of-algorithms.
17. http://algs4.cs.princeton.edu/analysis.

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MEASURING RESPONSE TIME FOR SLA BY USING MAMDANI FUZZY INFERENCE SYSTEM

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ABSTRACT

Cloud Computing is a promising technology and become a hot research area for scientist and engineer. Cloud computing provides the online services on the basis of Pay-as-you use. Cloud computing provides diverse types of services such as Software as a service (SAAS), Platform as a service (PAAS) and Infrastructure as a service (IAAS). These types of services are signed between Cloud services customer, Cloud services providers and broker agent or monitoring agent. Service Level Agreement (SLA) is the session type agreement between the user and service provider. This is pre-signed mechanism for the require fulfillment of the user. SLA is dependent on many parameters or factors. Namely response time, availability, tolerance and many more. This paper deals with the measure of Response Time (RT) for Service Level Agreement. Fuzzy is being used in much field i.e. medical, science, engineering and computing. Most used or famous fuzzy types are Mamdani and TSK. In this article Mamdani Inference System has been proposed to measure the exact value of response time of Service Level Agreement. The solution of problem is simulated on MATLAB.

1. INTRODUCTION

The cloud computing is one of the most common used service of the world of IT. It believes on the pay-as-you-use service [1]. The service is provided by the SLA. It is not necessary to have a long-term contract with the service provider. It is an agreement to provide you agreed services, maintain the level of the services and telling the violation of the services [2]. The customers don’t need to be in touch with the cloud service provider they just need to know about the SLAs and get the services of the cloud [3].

This article proposed a system for measuring SLA Response Time depends on a lot of factors. In this article in byte, out byte, packet size, available bandwidth in and available bandwidth are used to calculate Response Time. To make it more appropriate the fuzzy is being discussed to find the Response Time. The MUMDANI fuzzy logic is used to by the crew to find the response time.

2. LITERATURE REVIEW

The cloud is an IT service which is used by millions of the people these days. These services are provided as per a rule of pay and use. The concept is to provide services at any hour any day without delay [4]. Service Level Agreement (SLA) is a kind of an agreement that builds up a customer and vendor relationship. The SLA is responsible for providing all sort of services and customer has no direct connection with the cloud thing. These SLA's are those who provide and tell limits and any type pf problem with the clouds. It saves a type of documentation contract any creates a node [5]. A typical SLA will offer services like [6] a set of delivery service, each service definition and explanation, the dos of the service provider and the consumer, a set of instruction and check as the provider is offering the ordered services, the exclusion clauses, a machine to supervise the services and how the services will be charged with the passage of time [6]. For any service that is provided to us there is one thing everyone is concerned about and that is the response time [6]. How well the query is being sorted out and in how much short time the result is available for that specific task that was asked to be performed. In the language of computer and internet the response time is that time during with the given input Is processed. The SLA has criteria that measures the response time that depends upon the five characteristic as (1) In bite (2) out Bite (3) packet size (4) available bandwidth in (5) available bandwidth out. To have the minimum response time the priority of every service provider. The purpose is to manage the problem and give a quick feedback to the query or the problem that is reported. To solve this problem a lot of research is being done. Better the response time is, it is an assurance to the customer to get his loyalty and maintain a good name in the place where competition is there. The fast speed of the response time the guarantee of the efficiency of the system. All of engineers are familiar with the term name as the fuzzy logic. In the modern word a lot of the problems are discussed and are provided with the solution through the fuzzy logic. In the computing world fizzy logic is quite handy. Fuzzy has two logic MUMDANI and the TSK. These are the mathematically driven equation driven based on samples, analyzed with concern and run through a lot of experiments and proven right. The logic that is used widely in the computer technology is MUMDANI.

3. PROPOSED MFIS BASED SOLUTION

This section thoroughly explains Mamdani Fuzzy Inference System (MFIS) based on Cloud SLA Response Time (RT). The details given below explain the measuring of RT for the cloud SLA which is basis on Mamdani Fuzzy logic principles.

In this article the proposed MFIS which is capable for measuring the RT for cloud SLA algorithm is given in table-1. The five inputs and one output Mamdani Fuzzy Inference System (MFIS) is proposed to calculate RT.

In this method five inputs: In Byte (IB), Out Byte (OB), Packet Size (PS), Available Bandwidth in (ABI) and Available Bandwidth out (ABO) are taken. These inputs are used to build up lookup table given in table-2 in order to decide RT for respective algorithm for input-output relation given by MFIS.

$$\mu_{RT} = \text{MFIS}\ [\mu_{IB}, \mu_{OB}, \mu_{PS}, \mu_{ABI}, \mu_{ABO}]$$

Table I
Proposed MFIS based RT Algorithm

1.	**Inputs:** In this system 5 Fuzzy Input variables are used which are the following (In Byte, Out Byte, Packet Size, Available Bandwidth in, Available Bandwidth out)
2.	Each Fuzzy Input variable has different types of membership function.
3.	Every Fuzzy membership function is used to build fuzzy inference rules.

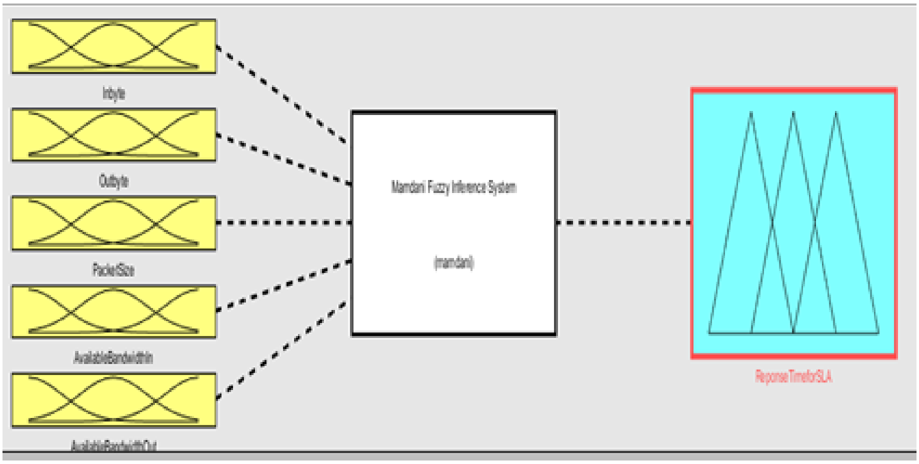


Figure 1: Input and Output Surface for MFIS

In this article, measure the Response Time (RT) using Mamdani Fuzzy Inference System (MFIS). TABLE 1 shows the proposed MFIS Based RT algorithm. The I/O surface for MFIS is given in Fig. 1.

3.1 Input Fuzzy Sets

Fuzzy input variable are statistical values that are used to calculate the Response Time (RT) in cloud computing. In this article 5 different type of fuzzy variables are used for the analysis of Response Time in cloud computing. The detail of these variables is given in TABLE II.

Table II
Input Variable Ranges

Sr #	Input Parameters	Ranges	Semantic sign
1	In Byte	0-0.4 0.2-0.7 0.5-1	Low Medium High
2	Out Byte	0-0.4 0.2-0.7 0.5-1	Low Medium High
3	Packet Size	0-0.4 0.2-0.7 0.5-1	Small Medium Large
4	Available Bandwidth in	0-0.4 0.2-0.7 0.5-1	Small Medium Large
5	Available Bandwidth out	0-0.4 0.2-0.7 0.5-1	Small Medium Large

3.2 Fuzzy Output Variable

Fuzzy output variable Response Time (RT) is used to calculate the result on the basis of the values of input variables in the world of discourse. The details of output are shown in Table-III.

Table III
Output Variable Ranges

Sr	Output of MFIS	Ranges	Semantic sign
1	Response Time for SLA	0-0.4 0.2-0.7 0.5-1	Slow Medium Quick

3.3 Membership Functions

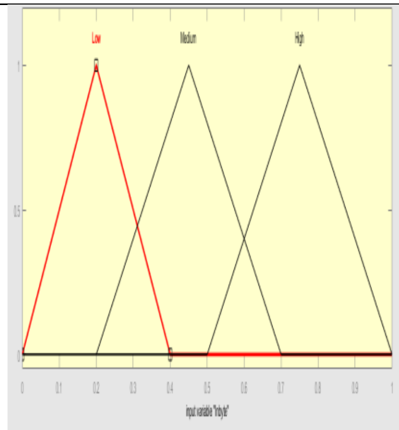
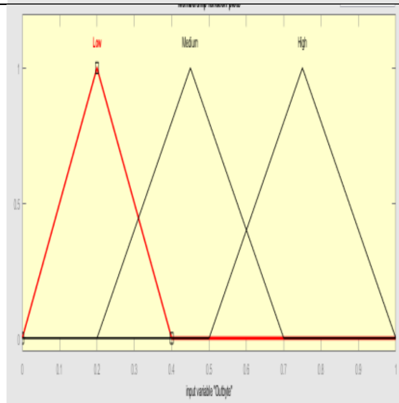
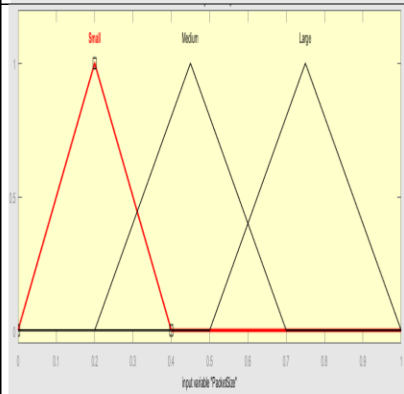
Membership function gives curve value between 0 and 1 and it provides mathematical function which provides statistical values of input and output variable. Membership functions are also available in MATLAB tool. The propose solution use the membership function which is as follow.

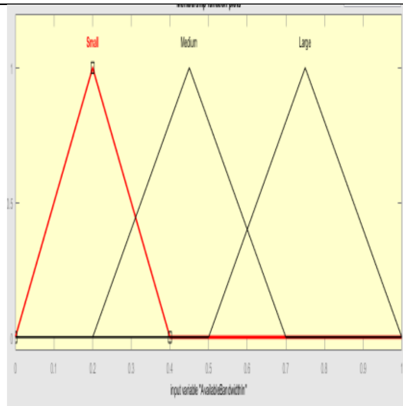
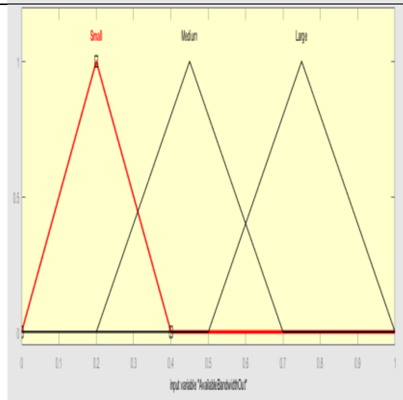
Trimf

Trimf is a triangular curve built-in MATLAB function. To calculation of this function three scalar parameters are used in the proposed solution which is Slow, Medium and Quick.

The mathematical equations and graphical representation of membership function is given in TABLE-IV.

Table IV
Mathematical & Graphical MF of MFIS Input variables

Sr. No.	i/p	Membership Function(MF)	Graphical Representation of MF
1	IB ($\mu_{IB}(x)$)	$\mu_{IB,\ Low}(x)=\begin{cases}\frac{x}{0.2} & if\ x\epsilon[0,\ 0.2]\\ \frac{0.4-x}{0.2} & if\ x\epsilon[0.2\ ,0.4]\end{cases}$ $\mu_{IB,\ Medium}(x)=\begin{cases}\frac{x-0.2}{0.25} & if\ x\epsilon[0.2\ ,0.45]\\ \frac{0.7-x}{0.25} & if\ x\epsilon[0.45,0.7]\end{cases}$ $\mu_{IB,\ High}(x)=\begin{cases}\frac{x-0.5}{0.25} & if\ x\epsilon[0.5\ ,0.75]\\ \frac{1-x}{0.25} & if\ x\epsilon[0.75,1]\end{cases}$	
2	OB(μ_{OB}(w))	$\mu_{OB,\ Low}(w)=\begin{cases}\frac{w}{0.2} & if\ w\epsilon[0,\ 0.2]\\ \frac{0.4-w}{0.2} & if\ w\epsilon[0.2\ ,0.4]\end{cases}$ $\mu_{OB,\ Medium}(w)=\begin{cases}\frac{w-0.2}{0.25} & if\ w\epsilon[0.2\ ,0.45]\\ \frac{0.7-w}{0.25} & if\ w\epsilon[0.45,0.7]\end{cases}$ $\mu_{OB,\ High}(w)=\begin{cases}\frac{w-0.5}{0.25} & if\ w\epsilon[0.5\ ,0.75]\\ \frac{1-w}{0.25} & if\ w\epsilon[0.75,1]\end{cases}$	
3	PS ($\mu_{PS}(s)$)	$\mu_{PS,\ Small}(s)=\begin{cases}\frac{s}{0.2} & if\ s\epsilon[0,\ 0.2]\\ \frac{0.4-s}{0.2} & if\ s\epsilon[0.2\ ,0.4]\end{cases}$ $\mu_{PS,\ Medium}(s)=\begin{cases}\frac{s-0.2}{0.25} & if\ s\epsilon[0.2\ ,0.45]\\ \frac{0.7-s}{0.25} & if\ s\epsilon[0.45,0.7]\end{cases}$ $\mu_{PS,\ Large}(s)=\begin{cases}\frac{s-0.5}{0.25} & if\ s\epsilon[0.5\ ,0.75]\\ \frac{1-s}{0.25} & if\ s\epsilon[0.75,1]\end{cases}$	

Sr. No.	i/p	Membership Function(MF)	Graphical Representation of MF
4	ABI($\mu_{ABI}(t)$)	$\mu_{ABI,\ Small}(t)=$ $\begin{cases} \frac{t}{0.2} & if\ t\epsilon[0\ ,\ 0.2] \\ \frac{0.4-t}{0.2} & if\ t\epsilon[0.2\ ,0.4] \end{cases}$ $\mu_{ABI,\ Medium}(t)=$ $\begin{cases} \frac{t-0.2}{0.25} & if\ t\epsilon[0.2\ ,0.45] \\ \frac{0.7-t}{0.25} & if\ t\epsilon[0.45, 0.7] \end{cases}$ $\mu_{ABI,\ Large}(t)=$ $\begin{cases} \frac{t-0.5}{0.25} & if\ t\epsilon[0.5\ ,0.75] \\ \frac{1-t}{0.25} & if\ t\epsilon[0.75, 1] \end{cases}$	
5	ABO($\mu_{ABO}(u)$)	$\mu_{ABO,\ Small}(u)=$ $\begin{cases} \frac{u}{0.2} & if\ u\epsilon[0\ ,\ 0.2] \\ \frac{0.4-u}{0.2} & if\ u\epsilon[0.2\ ,0.4] \end{cases}$ $\mu_{ABO,\ Medium}(u)=$ $\begin{cases} \frac{u-0.2}{0.25} & if\ u\epsilon[0.2\ ,0.45] \\ \frac{0.7-u}{0.25} & if\ u\epsilon[0.45, 0.7] \end{cases}$ $\mu_{ABO,\ Large}(u)=$ $\begin{cases} \frac{u-0.5}{0.25} & if\ u\epsilon[0.5\ ,0.75] \\ \frac{1-u}{0.25} & if\ u\epsilon[0.75, 1] \end{cases}$	

3.4 Rule Based

The rule base contains input output rules. The Mamdani Fuzzy Inference rules are shown in Figure 2.

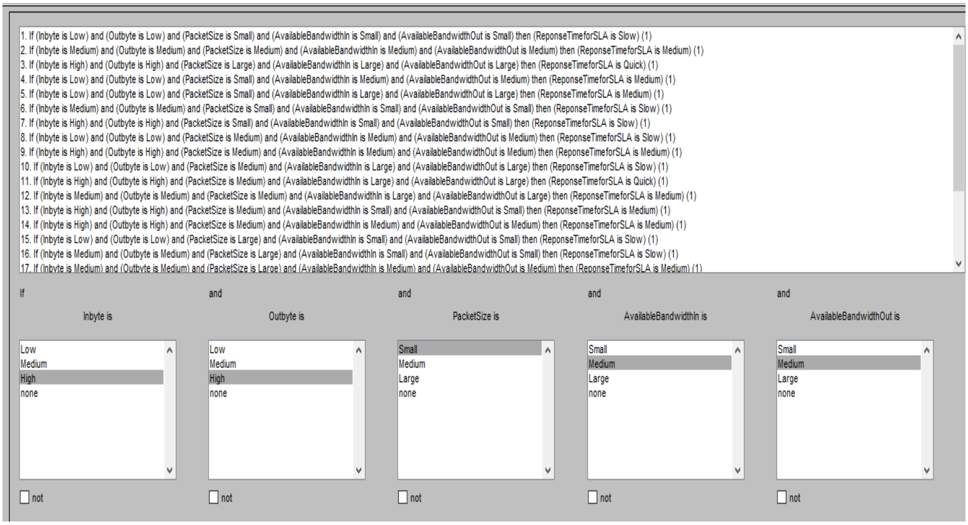


Figure 2: I/O Rules for MFIS

3.5 Inference Engine

The Mamdani Inference Engine is used in order to map 5 inputs to one output (RT).

4. SIMULATION RESULTS

MATLAB R2017a tool is used for simulation results. MATLAB is an efficient tool for programming, data analysis, visualization and computing. MATLAB is also use for modeling, simulation, algorithm development, prototyping and many other fields. For simulation results five inputs and one output RT variable is used.

In Fig. 3, it is observed that if in Byte and out Byte is Low and Packet Size is Large then RT is Slow.

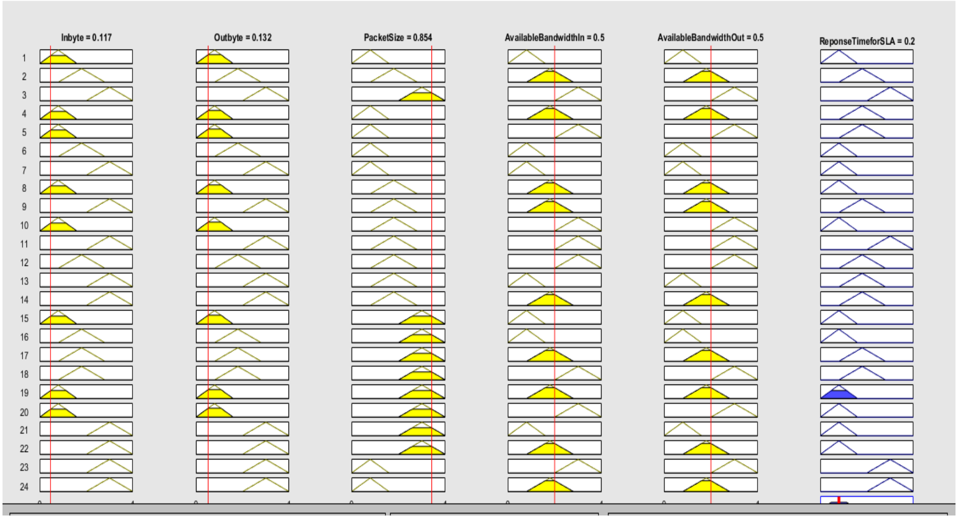


Fig. 3. Lookup Diagram for Slow RT

Fig. 4 shows the performance of RT is medium. It also observed that if in byte and out byte is High and packet size is Medium then RT is Medium.

Fig. 5 shows that the performance of RT is Quick. It clearly shows that, if in byte and out byte is 75%, Packet size is Low and all other i/p parameters are Medium then RT is High.

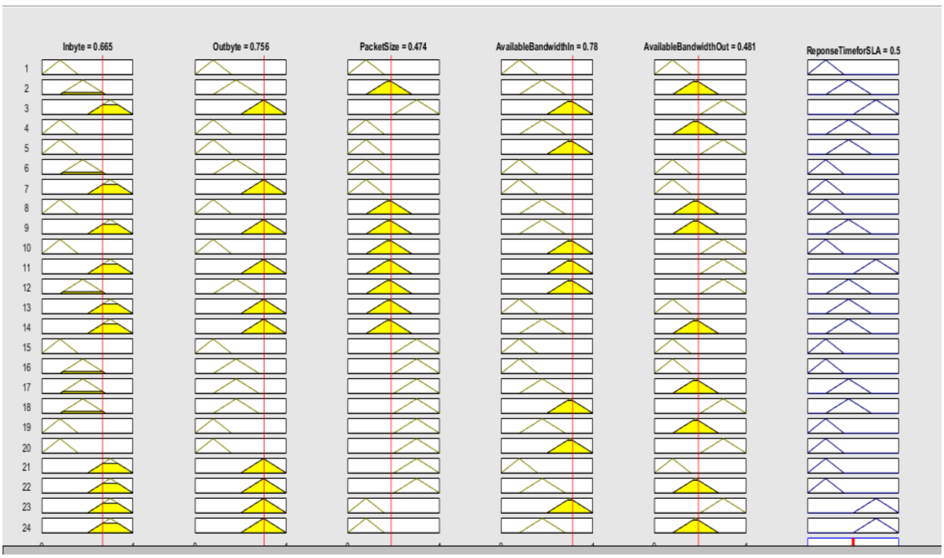


Fig. 4. Lookup Diagram for Medium RT

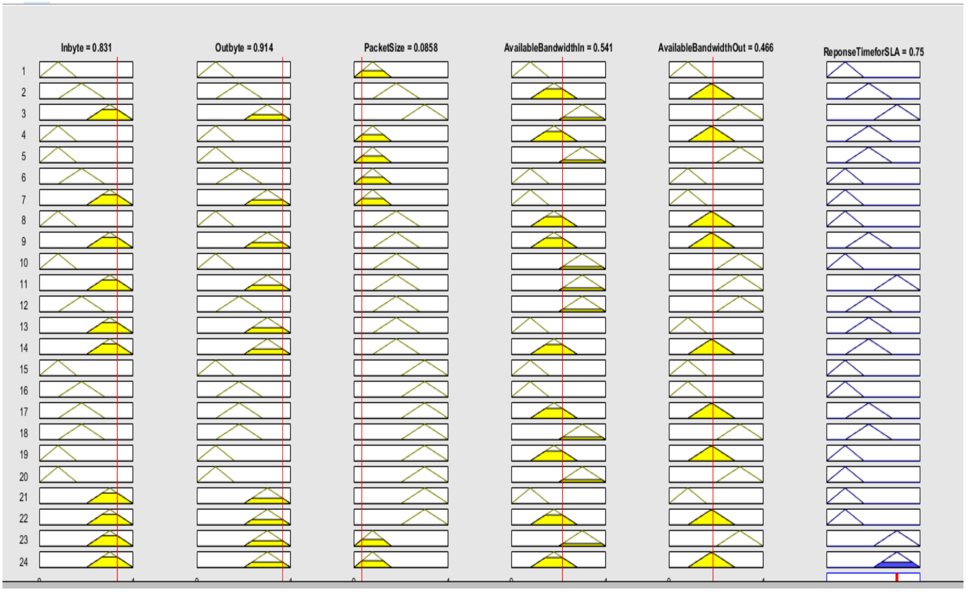


Fig. 5. Lookup Diagram for Quick RT

5. CONCLUSION AND FUTURE WORK

This RT fuzzy expert system is designed with the help of 5 input and 1 output variable. Mamdani Fuzzy Inference System (MFIS) is used to measure the Response Time (RT). The proposed system design RT been really helpful for the measure the Response Time. Through this system an anybody is able to check the any Response Time for Service Level Agreement (SLA). In future this work could be further enhanced to the different parameters of cloud like Service Level Agreement, Performance of the cloud etc. using FIS. Another expert system could also be developed which could be able to predict all-important parameters of the cloud.

REFERENCES

1. Alhamad, M., Dillon, T. and Chang, E. (2010). Conceptual SLA framework for cloud computing. *In Digital Ecosystems and Technologies (DEST), 4th IEEE International Conference on* (606-610). Curtin University of Technology, IEEE.
2. Andrieux, A. (2004). *Web Services Agreement Specification* (WSAgreement).
3. Joy, N., Chandrasekaran, K. and Binu, A. (2016). Energy aware SLA and green cloud federations. *In Distributed Computing, VLSI, Electrical Circuits and Robotics (DISCOVER), IEEE* (7-11).
4. Buck, K. and Hanf, D. (2010). *Cloud SLA Consideration for Gevernment Consumer, System Engineering at Mitre*, Tech. Rep. Case: 10-2902, Available at http://www.mitre.org /work/tech papers/2010/10 2902.

5. Mohammad. M.R. (2010). *Cloud Computing Use Cases White Paper* - Version 4.0. Available: http://opencloudmanifesto.org/Cloud Computing Use Cases Whitepaper–4 0.pdf.
6. Ghosh, N. and Ghosh, S.K. (2012). An approach to identify and monitor SLA parameters for storage-as-a-service cloud delivery model. In *Globecom Workshops (GC Wkshps)*, 2012 IEEE (pp. 724-729). IEEE.
7. Harikrishan, D. (2013). Gaussian Noise Reduction, *IJARCSSE*, 3(9), 12-18.

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DIGITAL FORENSIC, INVESTIGATION AND CRIME DETECTION

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ABSTRACT

Increasing number of computer criminalities is the foundation of computer forensics. Technology is advancing day by day and these advancing technologies are beneficial to users, as well as they also, advancement also exposed many paths for criminals. Therefore, it is necessary to secure your data from unauthorized access using some encryption methods. Digital Forensics involves knowledge of computer as well as engineering, signal processing and criminal justice. Forensics is a term interconnected with use of scientific knowledge to judicial issues. Digital Forensics is the method to obtain and inspect the material found in digital devices. The main objective of this process is to preserve the evidences in their original form while carrying out investigations. In Criminal Justice digital forensics involves recovery and investigation of hidden proofs. Before digital forensics evidences were gathered physically. But forensics involves collection and storage of digital data. Once attack on computer is proved, investigations are carried out using digital forensics techniques. Techniques does not involve only identification of attack. Proper investigations are carried out and evidences are generated and presented in courtroom. Investigator should have proper knowledge of equipment's and tools used in investigation. This knowledge is of great importance as selecting unsuitable investigation method becomes reason for imperfect or incomplete information. If any step of investigation model is missed the results cannot be concluded.

1. INTRODUCTION

Today we live in digital a world. With invention of computers, many everyday tasks become simple and easy. Communication now is not a big problem. The invention of internet has enhanced the speed of communication. Undoubtedly, advancing technology and invention of modern things makes our lives easy but they also open many ways for criminals to commit different crimes [1]. A huge number of crimes involves the use of internet and computer.

Digital forensics involves knowledge of computer as well as engineering, signal processing and criminal justice. Forensics is a term interconnected with use of scientific knowledge to judicial issues. Digital forensics is the method to obtain and inspect the material found in digital devices. The main objective of this process is to preserve the evidences in their original form while carrying out investigations. The data obtained can be used in court as an evidence. Forensics involves recovery and examination of hidden digital evidences [1,2]. These evidences include fingerprints, DNA tests obtained from bloodstains or files on hard drive. Traditionally fingerprints or other physical proofs obtained from crime scenes are saved but those methods are not successful methods for investigations. While digital forensics investigation requires digital forensics experts to collect and save evidences digitally. As data is saved in digital form, so, it is reasonable that the proof needed to sue criminal should also be in digital form. It is necessary that investigator should carry out all investigations accurately as all activities are open for inquiry to judiciary. The presence of a planned process offers a proper mechanism to be followed by forensic investigators.

Many methods are introduced to carry out the investigation process. Selection of right process is the main step to get clear and accurate results. Inappropriate selection may result to wrong outcomes. Also passing or missing any of the introduced step gives invalid conclusions [3].

2. BASIC PRINCIPLES OF DIGITAL FORENSICS

Digital forensics works on the principles that the act of collecting data should not result in any alteration of data in question. Collection, analysis and preservation of evidences must be documented. Only the authorized persons should have access to original evidences [4].

3. FORENSIC PROCESS

The main purpose of forensics investigation is to get data from computer seized as cues in criminal investigation. Proves that needs to be presented in courtroom are examined using systematic approach by experts. [4,5]. Main steps involved in forensic investigations are:

- Collection: In this step, digital evidences are searched, seized and gained.
- Examination: In this phase, techniques are applied to recognize and extract data.
- Analysis: This phase uses collected data and resources to prove the case.
- Reporting: In the last step, all obtained information is presented in court.

4. TYPES OF FORENSIC CASES

Forensic cases vary greatly. Some cases involve dealing with computer hackers who tries to steal or steals data. Some cases deal with hackers who break into websites and attack the systems. Or the hackers who tries to gain user name and password to make frauds.

Besides all these reasons, whatever the investigation reasons are the analysts goes through a series of step to prosecute the case. All the equipment’s are seized after the case is open to investigation. All-important items are gained for presenting in court, during retrieval process.

After retrieval, data is extracted and analyzed. Computer forensic investigator takes 5W's in account (Who, What, When, Where, Why) and occurring of computer crime or incident. By means of ordinary assessment measures, the examiner can recognize confidential intervals in a network environment looking for doubtful traffic and any kind of disturbances, or they can gather messages, data, pictures, and other information to be exclusively attributed to a particular user involved in a case [6].

Report writing is an important step in digital forensics. Reports are created by computer forensic examiner to discuss evidences. As, evidences collection is important for testimony, especially when cases go to trial and the examiner is called as a technical/scientific witness or expert witness.

5. A FEW COMPUTERS FORENSIC TOOLS:

Crime scene investigators uses following comprehensive software tools for collection, indexing and detailed analysis.

- Encase Forensic Edition
- X-Ways Forensic Addition
- Paraben
- Forensic Tool Kit (FTK)
- Linux DD etc.
- Wireshark is a forensic tool that is capable of interrupting traffic and logging it for analysis. It is used when network traffic is to be analyzed.
- Network Miner is another Network Forensic Analysis Tool (NFAT). It provides an alternative means to Wireshark and is used to extract or recover files.
- Snort, is another tool which tracks down hackers in real time.

6. DIGITAL FORENSIC INVESTIGATOR

A person who carries investigations on digital media is a forensic investigator. Investigator should not only have a strong knowledge about computers but they also have knowledge in many other important areas.

To lead a PC crime scene investigation examination, the individual must have a solid foundation in software engineering. The specialist should know a wide range of working frameworks work. The two most normal frameworks to explore are Windows and UNIX. [7,8,9]. Knowing these two working frameworks is an absolute necessity. It is conceivable that different sorts of frameworks will likewise must be examined other than UNIX or Windows. Next, the specialists should know an extensive variety of programming dialect, for example, C, C++, UNIX contents and others [8]. Commonly the source code is changed on the researched framework, so the examiner must realize what the progressions to code finish. Last, the agent ought to be up and coming on PC security issues. They should recognize what new vulnerabilities exist that programmers are utilizing to misuse frameworks.

7. DIGITAL EVIDENCES

Digital evidences are any piece of information or data valuable to an inquiry that is kept on, recovered or communicated by any electronic means. These evidences include audio evidences, video evidences, images, crime scene photos etc. Any person involved in investigation should not change data obtained by investigation [3,4]. All processes involved in investigation should be documented and preserved. If data is stored in any inaccessible area image cannot be obtained. So, it is important to obtain data directly for the purpose of recovery. It is important to display objectivity and integrity of evidences. In addition, it should be demonstrated in court how data was recovered. Moreover, evidence should be stored in such a way that if any other party carries out the same process, they should arrive on the same result.

8. COMPUTER FORENSIC INVESTIGATION MODELS

Numerous models are released to carry out forensic investigation. The selection of investigation model is responsibility of investigator [1,7,8,9].

Table 1
Computer Forensic Investigation Models

Model No.	Name of Model	Year
S01	Computer Forensic Investigation Process	1984
S02	DFRWS Investigation Model	2001
S03	Abstract Digital Forensic Model	2002
S04	Integrated Digital Investigation Process	2003
S05	Enhance Digital Investigation Process	2004
S06	Case Relevance Information DFM	2005
S07	Computer Forensic Field Triage Process	2006
S08	Four Step Forensic Process	2006
S09	Framework for A Digital Forensic Investigation	2006
S10	Common Process Model for Incident And DF	2007
S11	Dual Data Analysis Process	2007
S12	The Digital Forensic Investigation Framework	2008
S13	Two-Dimensional Evidence Reliability Amplification Process Model	2008
S14	Mapping Process of Digital Forensic	2008
S15	Digital Forensic Model Based on Malaysian Investigation Process (DFMMIP)	2009
S16	Network Forensic Generic DFM	2010
S17	The Systematic Digital Forensic Model	2011
S18	The Structured and Consistent DFM	2011
S19	The Proactive and Reactive DFM	2011
S20	Generic Computer Forensic Model	2011
S21	Common Phases of Computer Forensics Investigation Models	2011
S22	Comparative Digital Forensic Model	2013

9. SOME COMMON PHASES OF INVESTIGATION MODEL

The common phases of the investigation model are shown in given below table:

Table 2
Common Phases of Investigation Models

ID	Phase Name	Models with Phases
P01	Access	S12
P02	Acquisition	S01, S12, S18
P03	Accumulation	S19
P04	Admission	S01
P05	Analysis	S02, S04, S06, S09, S12, S13, S14, S16, S17, S18, S19
P06	Approach Strategy	S04
P07	Archive Storage	S15
P08	Attribution	S16
P09	Authorization	S08, S16
P10	Awareness	S08
P11	Case Specific Analysis	S10
P12	Chronology Timeline Analysis	S10
P13	Collection	S02, S04, S06, S08, S09, S14, S16, S17
P14	Communication Shielding	S17
P15	Conservation	S19
P16	Crime Scene Protocol	S13
P17	Defense	S08, S15
P18	Deployment	S05, S07
P19	Detection	S16
P20	Digital Crime Investigation	S05
P21	Dissemination of Information	S08, S14, S19
P22	Documentation	S17, S19
P23	Dynamite	S07
P24	Evaluation	S01
P25	Examination	S02, S04, S06, S08, S14, S16, S17
P26	Foundation	S19
P27	Harvesting	S13
P28	Hypothesis Creation and Presentation	S08,
P29	Identification	S01, S02, S03, S04, S06, S08, S13, S15
P30	Incident Alerts	S13
P31	Incident Closure	S09
P32	Incident Response	S09, S16
P33	Inspection	S19
P34	Individualization	S03
P35	Internet Investigation	S10
P36	Investigation	S11, S16

ID	Phase Name	Models with Phases
P37	Justification	S19
P38	Notification	S08
P39	Organization and Research	S13
P40	Physical Crime Investigation	S05
P41	Protection	S16
P42	Planning	S10, S15
P43	Post-Process	S18
P44	Pre-Process	S18
P45	Preparation	S04, S09, S11, S14, S16, S17
P46	Presentation	S02, S04, S06, S09, S11, S14, S16, S17, S18, S19
P47	Preservation	S02, S04, S06, S13, S14, S16, S17, S18
P48	Prioritize	S13
P49	Proof	S15
P50	Persuasion and Testimony	S13
P51	Readiness	S05, S07
P52	Recognition	S03, S17
P53	Reconnaissance	S15
P54	Reconstruction	S03
P55	Recovery	S13
P56	Reduction	S13
P57	Report	S12, S13, S14
P58	Result	S17
P59	Returning Evidence	S04
P60	Review	S05, S07
P61	Search and Identify	S08
P62	Secure	S17
P63	Survey	S17
P64	Traceback	S07,
P65	Transport and Storage	S08, S15
P66	Triage	S10
P67	User Usage Profile Investigation	S10

10. COMMENTS AND CONCLUSION

As digital crimes are increasing day by day. Digital forensic term came into being. In addition, basic knowledge about criminals and investigations techniques should be required. Digital investigator should have strong knowledge about computer and should use systematic approach to carry out investigation. Different investigation models are being released to carry out investigation and investigator should select appropriate model to carry out investigations as these results are to be presented in courtroom. Wrong selection may lead to incorrect results.

REFERENCES

1. Khan, M.A., Nasir, A., Ali, M.N., Farooq, U. and Malik, S.A., (2016). Crime Detection using Digital Forensic Technology, *in International Journal of Computer Science and Information Security*, 14(10), 487.
2. Pilli, E.S., Joshi, R.C. and Niyogi, R. (2010). Network forensic frameworks: Survey and research challenges. *Digital İnvestigation*, 7(1-2), 14-27.
3. Noblett, M.G., Pollitt, M.M. and Presley, L.A. (2000). Recovering and examining computer forensic evidence. *Forensic Science Communications*, 2(4),102-109.
4. Dhananjay, K. and Jain, N. (2013). Comparatıve Dıgıtal Forensıc Model, *International Journal of Innovative Research in Science, Engineering and Technology*, 2(8), 3414-3419.
5. Beebe, N.L. and Clark, J.G. (2005). A hierarchical, objectives-based framework for the digital investigations process. *Digital Investigation*, 2(2), 147-167.
6. Reith, M., Carr, C. and Gunsch, G. (2002). An examination of digital forensic models. *International Journal of Digital Evidence*, 1(3), 1-12.
7. Yusoff, Y., Ismail, R. and Hassan, Z. (2011). Common phases of computer forensics investigation models. *International Journal of Computer Science & Information Technology*, 3(3), 17-31.
8. Selamat, S.R., Yusof, R. and Sahib, S. (2008). Mapping process of digital forensic investigation framework. *International Journal of Computer Science and Network Security*, 8(10), 163-169.
9. Pollitt, M. (1995). Computer forensics: An approach to evidence in cyberspace. *In Proceedings of the National Information Systems Security Conference,* 2, 487-491.

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FACIAL EXPRESSIONS EMOTIONS RECOGNITION

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ABSTRACT

In this fast generation of technology Face Expressions recognition is very emerging field. Due to more demand of human computer interaction it plays an important role to interact with a machine more easily. We search about face recognition to detect the emotions on human face. Because emotions are the part of communication between two people and it can be in a human-to-machine.

We applied four algorithms to detect the emotions on human face. First, we applied the PCA (Principle Component Analysis). Which extract the features by creating Eigen vectors and Eigen values. Then we applied ISVM (Incremental Support Vector Machine). For more results and knowledge, we applied ICA (Independent Component Analysis), it is based on statistical methods to identify the emotions on human face. At last we applied LDA (Linear Discriminant Analysis). The simulations are performed on the Matlab.

1. INTRODUCTION

In recent ages, the demand of Human Computer interaction (HCI) has increased to a great extent. It is an emerging field of science. It provides natural ways to communicate with machines. Emotion detection is a very important and challenging field that makes human-to-computer interaction effective [1].

Facial expressions are used express the emotions of a person, they are used in biometrics, image retrieval, they provide important communication cues during social interaction, like our interest, our demand to speak. In order to solve the problems that occur in face recognition a lot of researches has been develop [2].

Although several methods have been proposed to recognize human emotions based on facial expressions or speech, Digital Image Processing includes many factors like Image enhancement, segmentation, object recognition, removal of noise and many more [3,4,5]. Facial Expression gives important information about a person. We have used seven different emotions of human face for our research which are happy, sad, angry, surprise, disgust, fear and neutral face. We took images from two different datasets (JAFFE & ECK) to test our research work properly. Emotion recognition of face is a basic and important application of image processing machine vision. We are looking for some very efficient results with these four methods.

These methods consist of two parts pre-processing part and processing parts [6]. In pre-processing part detection and extraction of face is done from the input data and pre-processing process also normalizes the data [6]. Normalization actually aligns the extracted images independent of different conditions like illumination, orientation.

We have tried to use four algorithms detect the emotions of human face. Principal component analysis (PCA) has been called one of the best valued results from applied linear algebra. PCA is used richly in all forms of analysis - from neuroscience to computer graphics - because it is a simple, non-parametric method of extracting useful information from unclear datasets. With negligible additional effort PCA provides a roadmap for how to diminish a complex dataset to an inferior dimension to reveal the hidden factors.

Support Vector Machine (SVM) was first proposed by Vapnik and Chervonenkis and it is widely used in machine learning. SVM deals with the dimensions in vector form and it is also used to classify the data [2].

Linear Discriminant Analysis (LDA) was first develop by R.A Fisher. It is also used in machine learning. It is used to classify two or more sets of data. Linear Discriminant Analysis is used in the cases where the frequencies within the class are unequal, LDA used to equalize those frequencies them and it also classify the dataset.

Independent Component Analysis (ICA) instead of the existing technologies such as fingerprint and iris recognition, the human face recognition is the most popular self-recognition technique used in biometrics. Face recognition is done without the cooperation of the user. It makes a good negotiation between performance consistency and social acceptance and it is widely used in security and privacy purpose. Other biometric systems do not hold these advantages.

For example, in fingerprint recognition biometric technique person needs to cooperate, a person should make a physical contact by touching his finger with the sensor surface.

Image edge detection is used to recognize the facial expression. In these different algorithms are used to produce the different features like lips, eyes etc. This technique consists of different steps which include normalization, feature extraction, feature selection, binary & multi-class organizations of the front face images. It also extracts lips, nose and eyes.

Our main focus is on (lips & eyes) to find out the edges of the image.

In Digital Image processing, it becomes very remarkable to recognize the human movements for all-purpose. Facial expressions exploration plays a major role for human computer interaction (HCI).

2. TECHNIQUES USED FOR EMOTION RECOGNITION

There are many types of techniques to detect the emotions of human face. In these different techniques we retrieve different results some of the results are accurate and some are not very much useful. But the techniques we used are very efficient and accurate as compare to other one.

2.1 Principal Component Analysis

Principal Components Analysis (PCA) used to identify the arrangements of the data, and it presents the data in a way that prominent their relationships and distances. The facial expression recognition using Eigen faces in which PCA is used to extract features from input image.

PCA is works on bases matrix; first input an image and find the vector matrix of an image then find Eigen vector of that vector of all points where data is located and then compare all the Eigen values with each other and then extract the on dimension where we can easily retrieve all meaningful data. Basically, PCA is used for reduced the dimension to find out useful data from huge number of datasets.

By using Eigen values, we convert our image data into Eigen vector and then work with that dimension to find the dimension.

We have used eigenvectors (the principal components) of a dataset to collect them in a projection matrix. Following are the results of PCA according to our work done.

Happy	Sad	Surprise	Anger	Disgust	Fear	Neutral
97.5490	46.4940	49.0020	57.3840	40.1100	57.6240	45.3060

Figure 1: Using PCA to Find Emotion

Above table is the example of happy face by PCA. Similarly, other expressions are also found.

2.2 Linear Discriminant Analysis

LDA based feature extraction method works on the basic principal of On and OFF, means true or false means (0, 1). The neighborhood values are threshold by the center value and the result is treated as a binary number. If the canter pixels value is greater than the neighborhood value then writes 1, if center pixel value is less than neighborhood value then writes 0. It encodes the neighborhood information very efficiently.

This method is first presented by R.A Fisher it is widely used in machine learning where we want to separate two or more classes of objects.

Linear Discriminant Analysis first calculate the mean vectors for the two-different set of data sets and then compute the matrices and within matrix, compute the eigenvectors as;

$$(ee1, ee2, ..., eed\ ee1, ee2, ...\ eed) \ldots.. \quad (1)$$

and parallel Eigen values by;

$$(\lambda\lambda 1, \lambda\lambda 2, ..., \lambda\lambda 1\lambda\lambda 1, \lambda\lambda 2, ..., \lambda\lambda d) \ldots.. \quad (2)$$

for the distributed matrices. Sort the eigenvectors by decreasing the Eigen values.

2.3 Incremental Support Vector Machine

Support Vector Machine (SVM) separates hyper-plane it works as classifier. It output the best hyper plane. For consider 2D set of points. These set of points belongs to two different classes. Then find a separate straight line;

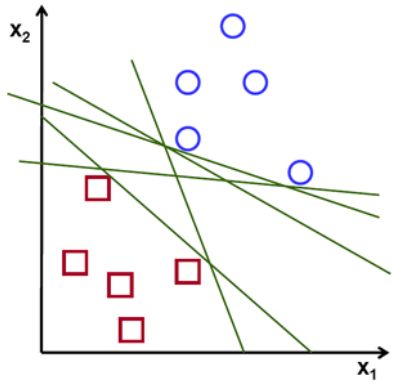

Figure 2: Hyper Plane

As you can see that there are different solutions to the problem but Support vector machine (SVM) actually find hyper plane, that gives the largest distance to other points in data sets. Furthermore, this largest distance is called margin. So, we can say that SVM maximizes the distances from the points in data sets.

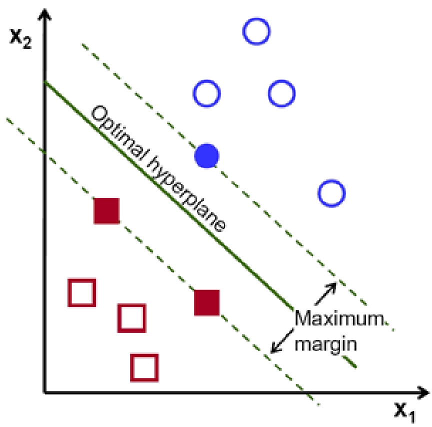

Figure 3: Finding Largest Distance

2.4 Independent Component Analysis

ICA is just like principal component analysis (PCA). But ICA is more efficient effective and then PCA. As ACA has more abilities to find hidden factors than PCA from the datasets. In this model data variables are considered as linear to some unfamiliar variables.

2.5 Edge Detection Algorithm

Edges are the adrupt changes in an image or videos. Simply we can say the sharp curves of an image are called edge detection.

Types of Edges:

Edges are of three types:

- ✓ **Horizontal Edges:** Simply it is defined as to convolve the matrix I with matrix G(x) in the x-axis.
- ✓ **Vertical Edges:** Convolving matrix I with G(y) in y-axis.
- ✓ **Diagonal Edges:** Calculating approximate value of the above G(x) and G(y) result to calculate diagonal edges.

Figure 4: Original Image

Figure 5: Edges of the Original Image

3. SYSTEM MODEL

The existing systems are based on one or maximum two methods used at a time but in this article, used basically four to five methods to test the results to find the human face emotions. We retrieve some efficient results by these methods. Existing models were very helpful for us in such a way that to make some accurate steps to find and create a new system model or can say a new method to detect the emotions on human face as shown in Figure 6.

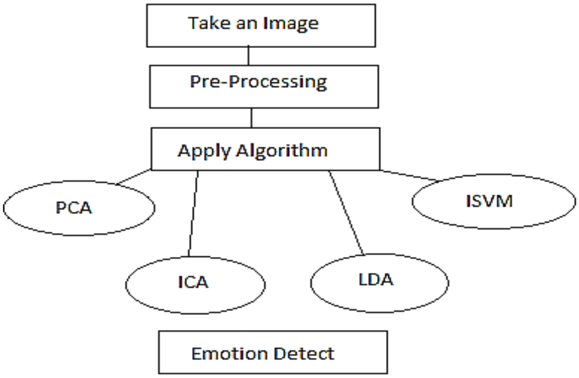

Figure 6: System Model

Following are some steps which have followed in this research to achieve our goal.

- Select an image from dataset.
- Pre-processing is a term to find out the features from face (and if the image is in 3D it converts it into 2D).
- Choose an algorithm (PCA, ICA, ISVM, LDA) by which you want to find emotion.
- Resultant Image.

4. SIMULATIONS AND RESULTS

In this article, we have used two well-known image databases like JAFFE & ECK to find the emotion recognition as shown in figure 7 & 8. And the different facial expression results are shown in figure 9-15.

Figure 7: Images from Database JAFFE

Figure *8*: Images from Database ECK

HAPPY	SAD	SURPRISE	ANGER	DISGUST	FEAR	NEUTRAL
97.5490	46.4940	49.0020	57.3840	40.1100	57.6240	45.3060

Figure 9: Happy Expression

HAPPY	SAD	SURPRISE	ANGER	DISGUST	FEAR	NEUTRAL
54.9480	95.4910	55.5660	57.2340	41.6700	52.0080	46.8660

Figure 10: Sad Expression

HAPPY	SAD	SURPRISE	ANGER	DISGUST	FEAR	NEUTRAL
54.9360	40.9980	98.8270	41.2500	43.4580	57.6240	48.6540

Figure 11: Surprise Expression

HAPPY	SAD	SURPRISE	ANGER	DISGUST	FEAR	NEUTRAL
42.7620	51.6600	47.0040	94.1050	42.6660	55.4100	47.8620

Figure 8: Angry Expression

HAPPY	SAD	SURPRISE	ANGER	DISGUST	FEAR	NEUTRAL
52.1880	42.0540	45.4620	57.6360	96.2860	57.0420	57.7680

Figure 9: Disgust Expression

HAPPY	SAD	SURPRISE	ANGER	DISGUST	FEAR	NEUTRAL
48.2220	55.1160	43.5600	53.6700	58.6860	95.6650	44.7480

Figure 10*4*: Fear Expression

HAPPY	SAD	SURPRISE	ANGER	DISGUST	FEAR	NEUTRAL
51.3720	49.5300	53.7000	53.6580	40.5300	55.4940	92.8630

Figure 11: Neutral Face Image

5. COMMENTS AND CONCLUSION

In this paper, we introduced the new design of a facial expression to recognize the emotions. It has the main characteristics and training database under the condition of different expression of eyes and lips edge by face in the ratio, thus the face recognition and then it can find the emotions. During research we studied a lot that human emotions are very important and very helpful that what they want. After doing some experiments on it we saw that if it applied on a machine everyone can use a machine more easily. We applied four algorithms and get very accurate result from PCA (Principle Component Analysis) which we have shown above in results.

REFERENCES

1. Sharma, A. (2015). Filters for image processing. *IJARCSSE*, 5(5), 12-17.
2. Hassh, P.S., Ayush, N., Amit, K.H., Aakanksha, B. and Neha, S. (2014). Average Filters, in *ICACEA*, 24-27.
3. Jebamalar, L. (2013). Salt & Peppar Noise, Reduction, *IJSRSET*, 6(05), 32-37.
4. Harikrishan, D. (2013). Gaussian Noise Reduction. *IJARCSSE*, 3(9), 12-18.
5. Shah, M.A. (2014). Reduction of speckle Noise. *JIKRESCE*, 4(15), 17-23.
6. Arfa, H., Unsa, T., Asim, I. and Muhammad, A.K. (2016). Performance Analysis of Skin Detection using GIE and Without GIE Techniques. *Journal of ISOSS*, 2(1), 85-95.

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AN EFFICIENT METHOD TO APPROXIMATE FIRST KIND OF VOLTERRA INTEGRAL EQUATION OF CONVOLUTION TYPE

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ABSTRACT

In this paper we will utilized the Laplace transform for the approximate solution of the first kind of Volterra integral equation of convolution type. The inverse Laplace transform will be calculated numerically. The concept of contour integral will also be utilized and using the parabolic and hyperbolic contour. The quadrature rule will be also used in the formation of the proposed algorithm. The method will be constructed for both linear and nonlinear Volterra integral equation of first kind of convolution type

Some example of linear and nonlinear type is solved. The absolute error between numerical and exact solution is also calculated for the each problem. The error bound using hyperbolic contour is also calculated for each problem. The examples show the accuracy of the proposed method.

KEYWORDS

Laplace transform, Volterra integral equation, Trapezoidal rule, Contour.

1. INTRODUCTION

The Volterra integral equation (VIE) have many application in mathematics as well as in engineering. The Volterra integral equation play a very important role in the study of fluid dynamics, heat conduction, population growth and in spread of different diseases.

The Volterra integral equation of first kind of convolution type has been solved by different methods. M.T Rasheed (Rasheed, An expansion method to treat integral equations, 2003) using expansion method to solve the numerically the VIE of convolution type, in (Rasheed, Lagrange interpolation to compute the numerical solutions of differential, integral and integro-differential equations, 2004) M.T Rasheed used Lagrange interpolation for the approximate solution of the VIE of convolution type.

E Babolian in (Babolian, E;Masouri, Z, 2008) used operational matrix with block-pulse function for the numerical solution of 1st kind of VIE of convolution type also the E Babolian (Babolian E; Salimi shamloo A, 2008) used the operational matrices with piece wise constant orthogonal function for the approximate solution of both the Volterra integral as well as Volterra Integro differential equation of convolution type.

The Laplace transform is very important transform which we used for the solution of the differential as well as the integral equation. In the paper (Marjan Uddin,Suleman Ahmad, 2017) the research used the Laplace transform for the solution of Bagley-Torvik equation. The Laplace transform convert the differential and integral equation in algebraic expression. But the inverse Laplace of every function is not easy to find exactly so in these case we approximate the inverse Laplace by using some suitable numerical method.

2. PROPOSED METHOD

2.1 Linear VIE of First Kind of Convolution Type

The general form of Volterra integral equation is given (Manzhirov, 2008) by

$$av(t) = f(t) + b\int_0^t k(t,x)v(x)dx \tag{1}$$

where a, b are constant and k(t, x) is kernel and v(x) is unknown function.

When we put a=0 and b= -1 then the equation (1) become

$$0 = f(t) - \int_0^t k(t,x)v(x)dx \quad \text{or} \quad f(t) = \int_0^t k(t,x)v(x)dx \tag{2}$$

The VIE of first kind of the convolution type is given by (Manzhirov, 2008)

$$f(t) = \int_0^t k(t-x)v(x)dx \tag{3}$$

Taking the Laplace of both side of equation (3) and utilized the convolution we get the result

$$\pounds\{\mathrm{v(t)*k(t)}\} = \pounds\{\mathrm{f(t)}\}$$

or

$$\pounds\{\mathrm{v(t)}\}.\pounds\{\mathrm{k(t)}\} = \pounds\{\mathrm{f(t)}\} \tag{4}$$

The Laplace transform of function f (t) is defined as

$$\int_0^\infty e^{-st} f(t)dt = \pounds\{\mathrm{f(t)}\} = \mathrm{F(s)}$$

So we have $\pounds\{\mathrm{v(t)}\} = \mathrm{V(s)}$, $\pounds\{\mathrm{k(t)}\} = \mathrm{K(s)}$ equation (4) implies

$$V(s) = \frac{F(s)}{K(s)} \tag{5}$$

The inverse Laplace transform of V(s) is given by

$$v(t) = \frac{1}{2\pi i}\int_{\alpha - i\infty}^{\alpha + i\infty} e^{st} V(s)ds \tag{6}$$

We used parabolic (Trefethen, 2007) or hyperbolic (William Mclean, 2010) contours with parametric equation $s = s(\zeta)$ then $ds = s^{'}(\zeta)d\zeta$

The equation (6) become

$$v(t) = \frac{1}{2\pi i}\int_C e^{s(\zeta)t} V(s(\zeta))s^{'}(\zeta)d\zeta \tag{7}$$

where C represent parabolic contour or hyperbolic contour. Now using the trapezoidal rule with equal step size k > 0 and let $s_j=s(\zeta_j)$ the equation (7)

$$v_N(t) = \frac{k}{2\pi i}\sum_{j=-N}^{j=N} e^{s_j t} V(s_j)s^{'}_j \tag{8}$$

equation (8) represent the approximate solution of the linear VIE of convolution type given in (3).

2.2 Nonlinear VIE of First Kind of Convolution Type

Consider nonlinear VIE of first kind of convolution type

$$f(t) = \int_0^t k(t-x)g(v(x))dx \tag{9}$$

We take $g(v(x)) = u(x)$ then the equation (9) become linear and given by

$$f(t) = \int_0^t k(t-x)u(x)dx \tag{10}$$

We applied the algorithm which we developed for linear case and get a result

$$u_N(t) = \frac{k}{2\pi i}\sum_{j=-N}^{j=N} e^{s_j t} U(s_j)s^{'}_j \tag{11}$$

or $$g(v_N(t)) = \frac{k}{2\pi i}\sum_{j=-N}^{j=N} e^{s_j t} U(s_j)s^{'}_j \tag{12}$$

or $$v_N(t) = g^{-1}\left\{\frac{k}{2\pi i}\sum_{j=-N}^{j=N} e^{s_j t} U(s_j)s^{'}_j\right\} \tag{13}$$

equation (13) gives the numerical solution of the nonlinear VIE of convolution type given in (9).

3. ERROR ANALYSIS AND ERROR BOUND FOR HYPERBOLIC CONTOUR

In the tables $E_{1N}(t)$ represents absolute error between exact and numerical solution using parabolic contour and $E_{2N}(t)$ using hyperbolic contour. The error bound for hyperbolic contour is given in theorem 3.1 in ((William Mclean, 2010) by

$L(p_r N)e^{-\mu N}$, $\check{r} = 2\pi r$, $L(p_r N)=\max(1,\log(1/ p_r N))$.

4. NUMERICAL RESULTS

Example 4.1

Consider the linear VIE of convolution type

$$\int_0^t \cos(t-x)\, u(x)dx = \sin(t) \tag{14}$$

Taking the Laplace of equation (14) and using the convolution theorem we have

$$U(s) = \frac{1}{s} \tag{15}$$

The exact solution of equation (14) is u(t)=1. In Table 1 the error between exact and numerical method using the proposed method and the parabolic and hyperbolic contour is give with the error bound for the hyperbolic contour at 0.1.

Table 1
Example 4.1

N	$E_{1N}(t)$	$E_{2N}(t)$	$L(p_r N)e^{-\mu N}$
32	2.3e-015	2.1e-005	5.9e-003
50	3.4e-012	1.1e-007	9.5e-005
53	2.7e-012	2.0e-009	4.8e-005
60	1.0e-011	6.8e-009	9.7e-006
70	8.8e-011	3.7e-010	9.9e-007
80	1.7e-010	2.3e-011	1.0e-007
90	8.4e-010	1.3e-012	1.0e-008
100	3.0e-010	8.3e-014	1.0e-009
128	4.7e-010	1.2e-016	1.9e012
(Rasheed, An expansion method to treat integrate equation, 2003)	6.6e-014		

Example 4.2

Now consider another VIE of first kind of convolution type which is nonlinear

$$\int_0^t e^{(t-x)} u^2(x)dx = e^{2t} - e^t \tag{16}$$

put $u^2(x) = v(x)$ then equation (16) become

$$\int_0^t e^{(t-x)} v(x)dx = e^{2t} - e^t \tag{17}$$

applying the Laplace transform and convolution theorem we get the result

$$V(s) = \frac{1}{s-2} \tag{18}$$

$$u_N(t) = \sqrt{v_N(t)} \tag{19}$$

The exact solution of equation (16) is $u(t) = e^t$. In Table 2 the error between exact and numerical method using the proposed method and the parabolic and hyperbolic contour is give with the error bound for the hyperbolic contour at 0.1.

Table 2
Example 4.2

N	$E_{1N}(t)$	$E_{2N}(t)$	$L(p_r N)e^{-\mu N}$
32	8.9e-016	9.5e-006	5.9e-003
48	5.6e-015	9.3e-008	1.5e-004
64	7.9e-014	9.8e-010	3.9e-006
80	7.9e-011	1.1e-011	1.0e-007
96	4.3e-013	1.2e-013	2.7e-009
112	4.9e-009	3.9e-015	7.1e-011
128	2.0e-010	2.2e-016	1.8e-012
(Babolian E; Salimi shamloo A, 2008)	3.2e-002		

5. COMMENTS AND CONCLUSION

In the present paper we developed an algorithms for both the linear and nonlinear VIE of first kind of convolution type. The proposed method has high accuracy than the methods discussed by MT Rasheed and E Babolian.

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REFERENCES

1. Babolian, E. and Shamloo, A.S. (2008). Numerical solution of Volterra integral and integro-differential equations of convolution type by using operational matrices of piecewise constant orthogonal functions. *Journal of Computational and Applied Mathematics*, 214(2), 495-508.
2. Babolian, E. and Masouri, Z. (2008). Direct method to solve Volterra integral equation of the first kind using operational matrix with block-pulse functions. *Journal of Computational and Applied Mathematics*, 220(1-2), 51-57.
3. Manzhirov, A.D. (2008). *Handbook of integral equation.* CRC Press.
4. Uddin, M. and Ahmad, S. (2017). On the numerical solution of Bagley-Torvik equation via the Laplace transform. *Tbilisi Mathematical Journal*, 10(1), 279-284.
5. Rashed, M.T. (2003). An expansion method to treat integral equations. *Applied Mathematics and Computation*, 135(1), 65-72.
6. Rashed, M.T. (2004). Lagrange interpolation to compute the numerical solutions of differential, integral and integro-differential equations. *Applied Mathematics and computation*, 151(3), 869-878.

7. Weideman, J. and Trefethen, L. (2007). Parabolic and hyperbolic contours for computing the Bromwich integral. *Mathematics of Computation*, 76(259), 1341-1356.
8. McLean, W. and Thomée, V. (2010). Numerical solution via Laplace transforms of a fractional order evolution equation. *The Journal of Integral Equations and Applications*, 22(1), 57-94.

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A NUMERICAL ALGORITHM FOR VOLTERRA INTEGRO DIFFERENTIAL EQUATION OF FIRST KIND OF CONVOLUTION TYPE

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ABSTRACT

In this paper we develop a numerical algorithm for Volterra integro differential equation of first kind of convolution type in complex plane, first we take Laplace transform of the equations then evaluating the inverse of Laplace transform with the help of Bromwich integrals. Then this integral form of the problem is approximated with the help of equal weight quadrature trapezoidal rule to obtain an accurate result, order of accuracy depends on the selection of different types of contours. Here the optimal parameters are used for construction of Parabolic contours.

By comparison with the methods which were developed in the past we observed that the present algorithm has better accuracy. Examples show the efficiency of the proposed algorithm, the error for each problem is given in the tables.

KEY WORDS

Volterra Integro differential equations, Laplace transformation, Contour of integrations, Convolution.

1. INTRODUCTION

The Volterra integro differential equation of first kind was introduced for the first time by Volterra. Now VIDEs are an important part of mathematics, Abdul Majeed discussed some real-life problems using VIDEs and some analytical and numerical solutions are also highlighted in the book (Wazwaz, 2015), Spread of alcohol abuse is another example from real life using integro differential equations model (French et al, 2010).

But always it is not possible to solve it analytically, so various numerical methods for the solution of VIDEs were developed in the past. During the literature review some efficient numerical methods were found. For example direct method for the solution of VIDEs using operational matrix for integration with block pulse functions (E Babolian, Z Mausori, 2008), Lagrange interpolation for differential, integrals and integro differential equations Rashed, M.T. (2004). is useful method for the approximation of VIDEs. An important method for the numerical solution of VIDEs is Numerical solution of Volterra integrals and integro differential equations of convolution type by using operational matrices of piecewise constant orthogonal functions (E Babolian, A Shamloo, 2008), Elzaki transformations are used for the analytical solution of VIDEs by transforming VIDEs into more simple form (EZAKI, 2011). Homotopy perturbation method and finite difference method are also very important method used for approximating VIDEs (Khan, N., et al. 2014). A numerical solution of VIDEs is converting to power series and transforming power series to pade series (Vanani, S.K. and Aminataei, A. 2011).

Mustafa Elshahed gave a numerical method naming Application of He's Homotopy perturbation method for the solution of VIDEs (Elshahed, 2005). Numerical solution of inverse Laplace transformation is also given in Uddin, M. and Ahmad, S. (2017).

2. PROCEDURE

Volterra integro differential equation of first kind of convolution type is given by

$$\int_0^t k(t-s)v^n(s)ds = f(t) \tag{1}$$

Taking Laplace transform of (1) and applying convolution theorem we have

$$£\{\int_0^t k(t-s)v^n(s)ds\} = £\{f(t)\} \tag{2}$$

$$£\{k(t)\}.£\{v^n(t)\} = £\{f(t)\} \tag{3}$$

$$£\{k(t)\} = K(s), £\{v(t)\} = V(s), £\{f(t)\} = F(s)$$

So, after applying Laplace transforms and convolution theorem we get

$$V(s) = \frac{F(s)}{s^n K(s)} + \frac{s^{n-1}v(0)+s^{n-2}v'(0)+\cdots+v^{(n-1)}(0)}{s^n} \tag{4}$$

The inverse Laplace transform formula for (4) is given by

$$v(t) = \frac{1}{2\pi\iota}\int_{c+\iota\infty}^{c+\iota\infty} V(s)e^{st}ds \tag{5}$$

This formula is also called Bromwich integral formula and where c is any real number.

3. PATH OF INTEGRATION

Now we pick a path or contour for the approximation of line integral given in (5) from c-i∞ to c+i∞ by contour integral, we take the parabolic path (Weideman, J. and Trefethen, L. 2007). with parametric equation is

$$s = \mu((1-c)^2 - u^2) + 2i\mu u(1-c) \quad -\infty<u<\infty \text{ and } c\geq 0 \tag{p}$$

So, using the parabolic contour given with $s = s(u)$ and $V\big(s(u)\big) = V(s)$ the equation (5) becomes

$$v(t) = \frac{1}{2\pi\iota}\oint V(s(u))e^{s(u)t}s'(u)du \tag{6}$$

Now using Trapezoidal rule with k>0 and setting the parameters as $s_j = s(u_j)$, $s' = s'(u_j)$ the equation (7) becomes

$$V(t) = \frac{k}{2\pi\iota}\sum_{j=-n}^{n} V(s)\, e^{ts_j}s'_j \tag{7}$$

4. NUMERICAL RESULTS

Problem 1

$$\int_0^t \cos(t-s)u''(s)ds = 2\sin(t) \tag{8}$$

With initial conditions $(0) = 0$, $u'(0) = 0$ and exact solution of the given problem is $u(t) = t^2$

Now applying Laplace transforms to equation (8) and using convolution theorem we get

$$u(s) = \frac{2}{s^3}$$

The error between exact answer and numerical result using the optimal parameters is given in the table

Table 1

N	Error (using p)
32	$5.249e^{-018}$
48	$1.050e^{-017}$
64	$3.499e^{-017}$
67	$4.762e^{-015}$
96	$3.487e^{-016}$
Method2 Rashed, M.T. (2004).	$5.300e^{-016}$

Table 1 error using (7) and parameters are $t = 0.1$,

$T = 1$, $t_0 = 0.01$ for the path (p) are $c = 0.3$,

$k = \frac{3}{N}$, $\mu = \pi \frac{N}{(12t)}$

Problem 2

$$u(t) = a\sin(t) + 2\int_0^t \sin(t-s)u'(s)ds \tag{9}$$

where 'a' is a constant and initial condition is given by $u(0) = 0$ while the exact solution of the given problem is $u(t) = ate^t$ taking $a = 1$

Now taking Laplace of (9) and using convolution theorem we get

$$u(s) = \frac{a}{(s-1)^2}$$

Error between the exact solution and numerical solution is given in the table

Table 2

N	Error (using p)
25	$7.128e^{-013}$
35	$1.049e^{-014}$
45	$1.282e^{-014}$
55	$8.734e^{-016}$
65	$4.560e^{-013}$

Table 2 error using (7) and parameters are $t = 0.1$,

$T = 1$, $t_0 = 0.01$ for the path (p) are $c = 0.3$, $k = \frac{3}{N}$, $\mu = \pi \frac{N}{(12t)}$

5. COMMENTS AND CONCLUSION

In this paper numerical method is based on Laplace transformation and contour integration, from numerically solved problem we conclude that the above method gives more accurate result than the method discussed in Rashed, M.T. (2004). therefore this method is very valuable for the solution of linear Volterra integro differential equations of first kind of convolution type.

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REFERENCES

1. Babolian, E. and Masouri, Z. (2008). Direct method to solve Volterra integral equation of the first kind using operational matrix with block-pulse functions. *Journal of Computational and Applied Mathematics*, 220(1-2), 51-57.
2. Babolian, E. and Shamloo, A.S. (2008). Numerical solution of Volterra integral and integro-differential equations of convolution type by using operational matrices of piecewise constant orthogonal functions. *Journal of Computational and Applied Mathematics*, 214(2), 495-508.
3. El-Shahed, M. (2005). Application of He's homotopy perturbation method to Volterra's integro-differential equation. *International Journal of Nonlinear Sciences and Numerical Simulation*, 6(2), 163-168.
4. Elzaki, T.M. and Ezaki, S.M. (2011). On the solution of integro-differential equation systems by using Elzaki transform. *Global Journal of Mathematical Sciences: Theory and Practical*, 3(1), 13-23.
5. French, D.A., Teymuroglu, Z., Lewis, T.J. and Braun, R.J. (2010). An integro-differential equation model for the spread of alcohol abuse. *The Journal of Integral Equations and Applications*, 22(3),443-464.

6. Khan, N., Hashmi, M.S., Iqbal, S. and Mahmood, T. (2014). Optimal homotopy asymptotic method for solving Volterra integral equation of first kind. *Alexandria Engineering Journal*, 53(3), 751-755.
7. Rashed, M.T. (2004). Lagrange interpolation to compute the numerical solutions of differential, integral and integro-differential equations. *Applied Mathematics and computation*, 151(3), 869-878.
8. Uddin, M. and Ahmad, S. (2017). On the numerical solution of Bagley-Torvik equation via the Laplace transform. *Tbilisi Mathematical Journal*, 10(1), 279-284.
9. Vanani, S.K. and Aminataei, A. (2011). Numerical solution of Volterra integro-differential equations. *J. of Comp. Analysis and Applications*, 13, 654-662.
10. Wazwaz, A.M. (2015). A first course in integral equations. *World Scientific Publishing Company*.
11. Weideman, J. and Trefethen, L. (2007). Parabolic and hyperbolic contours for computing the Bromwich integral. *Mathematics of Computation*, 76(259), 1341-1356.

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UNEMPLOYMENT; FACTORS AFFECT EDUCATED YOUTH OF MULTAN, PAKISTAN

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ABSTRACT

This study sought to distinguish the significant factors of unemployment among educated youth of Multan, which is an intricate issue. For the evaluation of the factors of unemployment which influenced the educated youth, a structured questionnaire was used to gather the data among 500 educated individuals whose age lies between 18 to 35 years. Binary logistic regression was used to obtain the key determinants of unemployment which highly influenced the educated segment. On the basis of findings, it can be concluded that government should take radical steps to provide more employment opportunities to fresher, built more industries, provide a good quality education and establish many setups of anticorruption departments in Pakistan.

KEYWORDS

Unemployment, Factors, Educated youth, intricate issue, Questionnaire, Logistic regression.

1. INTRODUCTION

The problem of unemployment is worldwide reality. The developing countries as well as developed countries suffer from it. Pakistan is a South Asian developing country. As per UNICEF's 2013 Statistics, Pakistan is the country with one of the largest youth bulge in the world where almost 35% of the population is under the age of 15. This staggering amount of young people on one hand is a huge human resource and full of potential waiting to be tapped and on the other hand unemployment problem is a bomb waiting to explode the youth potential any time (Azhar 2015).

Pakistan Bureau of Statistics reported that, Pakistan's unemployment rate dropped to 5.94% in June 2015, from the previously reported number of 5.96% in June 2014. In latest reports, Pakistan's population reached 199.10 million in June 2017. Monthly Earnings of Pakistan stood at 147.60 USD in June 2015.

Youth under the age of 25 constitutes 63% of the total population of Pakistan. Unemployment is a major factor which affects the economy of Pakistan. In this study we focused on the following factors; High growth of population, poor education systems, poverty, energy crises, High age of retirement, lack of investment, technology, lack of academic-industry linkage, requirement of experienced lot, corruption, political reason and poor planning. Unemployment is one of the most common and chronic problem worldwide. It is concern for individuals as well as global communities.

Research Objectives

Generally this Research aims to explore the current scenario of unemployment and study the (Contributing Factors) towards Unemployment among the educated youth of Multan.

Specifically focus has been given to:

- Explore and identify the determinants (Contributing factors) of unemployment.
- Determine the relationship between contributing factors and Employment status.
- Predict the probability of unemployment among the young educated individuals of Multan.
- Provide relevant recommendations for policy makers and programme managers.

2. REVIEW OF LITERATURE

Bashir, Ahmad, and Hidayat (2015) studied about the causes of unemployment among educated women in Bahawalnagar district of Pakistan. They concluded that age, education, mother education, total employed person at home, mother job status and technical education are reducing unemployment. While the joint family system, number of children, and household sizes are causes of higher unemployment in educated women in Bahawalnagar.

Naureen and Lodhi (2014) had worked on the reasons of unemployment among the educated youth in Karachi of Pakistan. They conclude that inadequate resources, lack of proper planning and lack of employment opportunities are the major reasons behind the unemployment among the educated youth in Karachi.

Msigwa and Kipesha (2013) investigated the factors which determined youth unemployment in Tanzania. They were used secondary data of Tanzania integrated labour force survey (ILFS) of 2006. The youth employment status was categorized into three categories i.e. employed, unemployed, and inactive. After the finding they suggest that the government and policy makers should analyse job market laws. Also government should create interruption for more formal jobs and strengthening job market regulation which are related to youth population. They recommended that the government and policy makers should strengthen the laws and regulation relating to gender balance in job market.

Mahmood et al. (2011) identified the causes of unemployment in educated portion in Peshawar division of Pakistan. This research has been shown the important factors of unemployment among the educated segments in Peshawar division. They concluded that HP, LR, HP*RL, HP*NE.J, NE.J*RB*RL are some important determinants of unemployment rate in Peshawar division.

3. PROPOSED METHODOLOGY

This study used primary data collected by means of questionnaire as our study identifies the factors affects the unemployment status of young educated individuals in Multan. A sample of 500 young educated individuals having high school diploma as minimum qualification and capable of any professional/technical job whether they are employed or unemployed. The age interval in our study is 18-36 years. The descriptive and inferential statistics including frequencies, percentages, chi-square and regression modeling was employed for the exploration and identification of unemployment factors.

4. STATISTICAL DATA ANALYSIS

The total number of 328women and 172 men covered among these, overall 60.8% were unemployed whereas 39.2% were employed. The main demographic factors including age, gender, living area, education, marital status, family type, family size, no. of dependents and economical status.

4.1 Descriptive Analysis

Frequency and percentage distribution of Demographic Factors of the respondents with Unemployment and Employment Status was presented in Table 1.

Table 1
Distribution of Demographic Factors

Demographic Features		Unemployment		Employment	
Variables	**Category**	**n**	**%**	**n**	**%**
Age(years)	18- 23	242	79.6	59	30.1
	24 – 29	61	20.1	105	53.6
	30- 36	1	0.3	32	16.3
Gender	Male	88	28.9	84	42.9
	Female	216	71.1	112	57.1
Living Area	Rural	44	14.5	53	27
	Urban	260	85.5	143	73
Education	High school diploma	3	1.0	7	3.6
	Graduate	104	34.2	47	24.0
	Master	159	52.3	80	40.8
	MPhil/PHD	38	12.5	62	31.6
Marital Status	Single	288	94.7	144	73.5
	Married	16	5.3	52	26.5
Family Type	Nuclear	196	64.5	117	59.7
	Joint	108	35.5	79	40.3
Family Size	2-9	262	86.2	154	78.6
	10-18	41	13.5	24	12.2
	19-27	1	0.3	8	4.1
	28-36	0	0.0	10	5.1
No of Dependents	0-6	248	93.4	177	90.3
	7-13	20	6.6	19	9.7
Sector of job	Public	169	55.6	49	25
	Private	135	44.4	147	75
Duration of Unemployment	2-11 Months	34	11.2	-	-
	1-3 Years	239	78.6	-	-
	>4 Years	31	10.2	-	-
Economic Status	Upper	17	5.6	10	5.1
	Middle	276	90.8	170	86.7
	Lower	11	3.6	16	8.2

Above table revealed the important figures which showed that there is excessive unemployment among the age interval 25-29 years. The haphazard unemployment attributed among females as there is an increase literacy tendency of females in this area.

It was also revealed that unemployment is demonstrated among those respondents who qualified at high school diploma and who were graduated. The contributions of unemployment become increased among those respondents who resided in rural areas.

An increase number of respondents seem to be unemployed, were single. Unemployment observed among the individuals having a large family size i.e. family size 10-36 and 7-13. The 28.3% youth were recorded employed in their past employment status. Individuals who were belonged to the lower economic status were revealed more unemployment.

Frequency and Percentages of the Socio-economic Factors with Unemployment and Employment Status were presented in Table 2.

Table 2
Distribution of the Socio-economic Factors with Unemployment Status and Employment Status

Items	Strongly Agree /Agree			
	Unemployed		Employed	
	f	%	f	%
Increase population	237	77.9	146	74.5
Low quality education	238	78.3	149	76
Poverty	220	72.4	111	56.6
Energy crises	222	73	141	72
High age of retirement	164	54	83	42.3
Lack of investment	194	63.8	125	63.8
Advance technology	123	40.4	87	44.4
Lack of academic industry-linkage	193	63.4	127	64.8
Corruption	258	84.8	179	91.4
Requirement of experienced lot	210	69	119	60.7
Political reasons	213	70.1	137	69.9
Poor planning and management	251	82.6	155	79.1
Poor law implementation	225	74	140	71.5

Table showed the pivotal factors of youth unemployment which extremely governs on educated youth. These are Population growth, Low quality education, Poverty, High age of retirement, Lack of linkage between college/university and industry, Corruption, Requirement of previous job experience, Poor law implementation.

4.2 Statistical Testing Using Chi-Square Test

In the Table 3 Chi-Square analysis of factors versus Employment Status was given.we found that some of the factors are statistically significant which are presented in the table. And these factors are associated with employment status.

Table 3
Chi-Square Analysis of different Factors versus Employment Status

Variable	Chi-square	(df)	p-value
Age	1.350	2	0.00
Gender	10.217	1	0.01
Living Area	12.036	1	0.001
Education	33.211	4	0.00
Marital Status	45.871	1	0.00
Family Size	25.805	3	0.00
Sector of job	4.917	2	0.00
Duration of Unemployment	16.585	2	0.00
Employment History	16.718	1	0.00
Poverty	22.235	4	0.00
Advance technology	14.953	4	0.005

4.3 Statistical Modelling

We identify the key determinants of unemployment among educated youth of Multan through modelling the binary data. **Binary Logistic regression** identifies the statistical significant factors and determines the direction of relationship and its contribution to the dependent variable.

The variable of unemployment coded“0” for “No” while the variable of employment coded“1”for “Yes”. Also the categorical variable coded as 1for the employment and 0 for unemployment status of each variable.

4.3.1 Result of Null Model

The following tables described the results of baseline model (Null Model) a model that does not include Explanatory factors. According to findings the null model is 60.8% accurate and in the Table 9 the constant is significant and its odd ratio is 15.51%.

Table 8
Classification Table

Observed		Predicted		
		Employment Status		Percentage Correct
		Employed	Unemployed	
Employment Status	Employed	0	196	.0
	Unemployed	0	304	100.0
Overall Percentage		-	-	60.8

Table 9
Variables in the Equation

Constant	Estimate	S.E.	Wald	DF	P	OR
	.439	.092	22.957	1	.000	1.551

4.3.2 Result of Full Model

Full model is a model which included all the explanatory variables. The Omnibus Tests of Model Co-efficient table shows the result of full model. The Table 10 presents chi-square test which gives the result of the Likelihood Ratio (LR) test between null model and the full model. When a p-value (sig) of full model is less than from 0.05 then the full model is a significant rather than null model. These indicate that all three tests yield similar conclusions, that is, the final model with explanatory variables was more effective than the null model. So we can say that our full model is significantly better.

Table 10
Tests of Model Coefficients

	Chi-square	DF	P
Likelihood Ratio	225.393	28	.000
Score	225.393	28	.000
Wald	225.393	28	.000

In the model summary it was observed that the -2 Log likelihood statistic is 450.092. The value Cox & Snell R Square is 0.355 and the value of Nagelkerke R Square is 0.482 which shows that the variation of employment is between 35.5% and 48.2% can be explained by the full model.

Table 11 equivalent to the null model but this model includes independent variables. This table correctly specified outcome 79.8% as compared to 60.8% in the base line model.

Table 11
The Classification Table

<table>
<tr><th rowspan="3" colspan="2">Observed</th><th colspan="3">Predicted</th></tr>
<tr><th colspan="2">Present Employment Status</th><th rowspan="2">Percentage Correct</th></tr>
<tr><th>Employed</th><th>Unemployed</th></tr>
<tr><td rowspan="2">Present Employment Status</td><td>Employed</td><td>135</td><td>61</td><td>68.9</td></tr>
<tr><td>Unemployed</td><td>40</td><td>264</td><td>86.8</td></tr>
<tr><td colspan="2">Overall Percentage</td><td></td><td></td><td>79.8</td></tr>
</table>

According to the test of significance of independent variables using Wald test the factors Age, Area, Education, Family size, Employment history, Economical Status, Population growth, Low quality education, Poverty, High age of retirement, Lack of academic-industry linkage, Corruption, Requirement of previous job experience and Poor law implementation were found to be significant predictors of unemployment at 5% level of significance. Thus, the estimated model is given by:

$$\textbf{logit}[\hat{p}] = \boldsymbol{\alpha} + \boldsymbol{\beta_1 AGE_1} + \boldsymbol{\beta_2 AR_2} + \boldsymbol{\beta_3 EDU} + \boldsymbol{\beta_4 FS} + \boldsymbol{\beta_5 EH} + \boldsymbol{\beta_6 ES} + \boldsymbol{\beta_7 PG} + \boldsymbol{\beta_8 LQE} + \boldsymbol{\beta_9 POV} + \boldsymbol{\beta_{10} HAR} + \boldsymbol{\beta_{11} LAIL} + \boldsymbol{\beta_{12} CORR} + \boldsymbol{\beta_{13} ROPJE} + \boldsymbol{\beta_{14} PLI}$$

where $\hat{p}$= predicted probability of unemployment, α = constant, AGE= Age of the respondent, EDR=Education of respondent, FS=Family size of respondent, EH=Employment history of respondent, ES=Economical status of respondent, PG=Population growth, LQE=Low quality education, POV=Poverty, HAR=High age of retirement, LAIL=Lack of academic-industry linkage, CORR=Corruption, ROEL=Requirement of experienced lot, PLI=Poor law implementation.

Hosmer-Lemeshow Goodness-of-fit test tests having chi-square value 5.595 with P-value = 0.692 is larger than 0.05, we do not reject the null hypothesis, and we conclude that the final model is a good fit; its predictive capability is more than the null model.

5. COMMENTS AND CONCLUSION

This study found evidence for the contributing factors that have significant influence on unemployment of educated youth of Multan. Age, Living Area, Education, Family size, Past employment status, Class, Population growth, Low quality education, Poverty, High age of retirement, Lack of academic-industry linkage, Corruption, Requirement of experienced lot and Poor law implementation were found to be important determinants of unemployment among educated youth age (18-36 years).

Unemployment can be overcome by releasing Pakistan from corruption among all the members of the society. Specifically youth unemployment is spreading as there are no new avenues of success and prosperity which guide the youth which field to adopt for the requirement of the country.

In Pakistan there is lack of institution which could provide compatible training skills. Population growth and poverty is a factor behind unemployment nevertheless population increase day by day. Government should make any strategy against poverty and growth of population to control unemployment. There is needed to take corrective measure for the implementation of laws.

Special internships programs should be launched which help the youth to get experience. In Pakistan there is need of job portals which become useful for freshers. And government should take radical steps for high age of retirement this also a demonstrative factor of youth unemployment. Small scale industries should be established so that those labors that were unemployed can play a vital role in society as an employed member. There are some recommendations which should be considered to reduce the youth unemployment from Multan, Pakistan:

- Technical education should be given to students that developed confidence and skills in students via providing chances to utilize their abilities practically.
- Government should take radical steps about the quality of education. Education should be based on practical approach along with theory.
- Government should create strong link between industries and academic sectors and to create connection between course content and job market requirement.
- Government should improve the quality of education rather than quantity.

- Government should take rapid actions like relaxation for industry in trading taxes, making good policies for trading, promoting our industry export and minimizing the imports.

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REFERENCES

1. Azhar, Ibn. (2015). *Utilization of Young Population in Pakistan*. In Pak Tea House, http://pakteahouse.net/2015/06/19/utilization-of-young-population-in-pakistan/
2. Bashir, F., Ahmad, T.I. and Hidayat, T. (2015). Causes of Unemployment among Highly Educated Women in Pakistan: A Case Study of Bahawalnagar District. *Pakistan Journal of Humanities and Social Sciences*, 1(1), 1-10.
3. Mahmood, Z., Akhtar, N., Amin, M. and Idrees, M. (2011). Causes of unemployment among the education segments in Peshawar division, Pakistan: A Statistical Study. *Sarhad Journal of Agriculture*, 27(1), 139-142.
4. Naureen, S. and Lodhi, F.A. (2014). Reasons of Unemployment among the Educated Youth in Pakistan: Some Strategies for Improvement. *Pakistan Perspectives*, 19(2), 127-134.
5. Naureen, S. and Lodhi, F.A. (2014). Reasons of Unemployment among the Educated Youth in Pakistan: Some Strategies for Improvement. *Pakistan Perspectives*, 19(2), 127-134.
6. Robert, M. and Kipesha, E.F. (2013). Determinants of youth unemployment in developing countries: Evidences from Tanzania. *Journal of Economics and Sustainable Development*, 4(14), 67-76.
7. www.pbs.gov.pk

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LEVEL OF CONCEPTUAL UNDERSTANDING OF STATISTICS AMONG RESEARCHERS OF MULTAN

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ABSTRACT

All the developments that have been seen in this modern world are due to research and definitely statistics plays most important role in this regard. The main goal of this study is to check the ability level of conceptual understanding of statistics among researchers. All those researchers of Public and Private educational sectors of Multan were targeted to collect data, who have studied the course of statistics in their academic career and/or they are using statistical analysis in their research. Purposive sampling is used to collect data and 200 respondents were selected as a sample. The importance of statistics in their field, their level of conceptual understanding and other factors affecting their understanding was studied. According to findings, 86% respondents believe that statistics is very important in their field but at the same time they have very low level of conceptual understanding of the said course. Unfortunately, it is found that many of them had not being taught statistical packages so that they are not able to use them properly in their research.

KEYWORDS

Conceptual understanding, Statistics, Researchers, Public and Private educational sectors, Multan, Purposive sampling, low level.

1. INTRODUCTION

Statistics may be defined as the sciences of conducting studies to collect, organize, summarize, analyze, and draw conclusions from data (Bluman, 2009).

"Statistics is a grammar of science"
(Pearson, 1900)

There are five major reasons to study statistics(Garfield & Ben-Zvi, 2007) and the most important is to be able to effectively conduct research as statistics play an important role in research of almost every field because it deals with quantified data. The study of statistics enabled researchers to look at a largest of data and condense it into meaningful information i.e.

Validity: will this study help answer the research question?

Analysis: What analysis, and how should this be interpreted?

Efficiency: Is the experiment the correct size, making best use of recourse?

1.1 Problem of the Statement and Research Objectives

It was observed that researchers who have studied the course of statistics still not able to solve their research problems. On the basis of which, the main objective of this study was to assess the level of conceptual understanding among researchers of Multan. Other objectives include;

- To enhance the importance of statistics in different fields;
- To identify the influence of teaching method on the conceptual understanding of researchers;
- To enhance the application of statistics in research.

2. LITERATURE REVIEW

At the end of 18^{th}, century the level of conceptual understanding and teaching methods of statistics was assessed. Three main factors in teaching method were explored i.e. classification scheme, Newer method of teaching assessment and attitude scale (Jolliffe, 1990).

Later research review of social sciences had been made and result applied to teaching of college level statistics courses, by which assessed statistics educators need to determine what they want to learn students (Garfield, 1995).

Moore (1997) suggested that the new pedagogy and new content of statistics are the demand of subject. As the teaching method made an impact on all level of educational curriculum, use the computer simulation method (CSMs) to teach the concept of statistics. Garfield and Ben-Zvi (2007) overviewed all the research on teaching and learning statistics and conclude that students faced difficulties in learning statistics by traditional teaching methods. Schau and Mattern (1997) suggested changing the teaching methods to assess the concept map.

Budé (2007) explore that the conceptual understanding of statistics is low in the social and health sciences. Jacobbe, Whitaker, Case, and Foti (2014) test the validity of LOCUS (Level of conceptual understanding of statistics) at the college level and found to be appropriate as a placement test and a research instrument as well.

3. RESEARCH METHODOLOGY

Primary data was collected by means of questionnaires. The target population for this study comprised of all faculty members of Public and Private Universities of Multan and professionals and other research scholars, who have studied the course of statistics in their academic career or they are applying statistical analysis in their researches. The purposive sampling was used. Duly filled 200 responses were received.

The Cronbach’s alpha was applied to test the reliability of questionnaire filled from 20 respondents as pre-test. It was found to be 0.762 which is sufficient. Descriptive statistics including frequency tables and inferential techniques includes the ordinal regression and chi square test.

4. EMPIRICAL FINDINGS AND DATA ANALYSIS

The respondents of this study are predominantly females (60%) mostly from different institutions. Other socio-demographic factors involve age group, education level, teaching experience and research experience.

4.1 Learning Experience of the Statistics Course

Table 1 show the learning experience of the course which shows that 97% respondents have studied the course of statistics in their academic career while 54.9% are able to apply their theoretical knowledge into practice. 32.6% respondents have learnt to use the statistical packages while mostly are not able to use them and interpret the results.

Table 1
Learning Experience of the Statistics Course

B	Question	%
B1	Studied the course	97.2%
B2	Importance of course in their field	86%
B3	Good Learning Experience	62%
B4	GPA above 3.50/4.00	70%
B5	Applied practically during study	75.8%
B6	Satisfied from teaching method	61.8%
B7	Able to apply practically	54.9%
B8	Statistical packages included in the course	32.6%
B9	High level of ability to use statistical packages	27.9%
B10	Able to interpret the results of statistical calculations	56.4%
B11	Able to decide appropriate statistical technique	44.1%

4.2 Research Experience Involving Statistical Analysis

Table 2 is showing the descriptive statistics of research experience involving statistical analysis. Most of the respondents have done the research work and applied the statistical analysis but consulting a statistician to apply.

Table 2
Research Experience Involving Statistical Analyses

C	Question	%
C1	Done research work in academic or professional career	92%
C2	Applied statistical analysis in research	85%
C3	Research problems involve statistical analysis	36%
C4	Statistical techniques are most widely used	63%
C5	Able to solve the research problems after learning the course of statistics	41%
C6	Consult a statistician for the statistical analysis	45%
C7	Able to research problems without statistical techniques	15%

Pearson Chi-Square statistic, 43.67, and $p < 0.05$, shows that there is association between ability level and learning of statistical packages in academic career. Chi square 84.08, and $p < 0.05$ also shows that there is association between the two variables(able to solve the research problems and satisfaction from teaching method). Hence teaching method has a great impact on the understanding level.

4.3 Ordinal Regression Results

Ordinal regression was used to check the dependence of conceptual understanding on

- GPA in statistics course (B4)
- Ability to apply theoretical knowledge of statistics into practice (B7)
- Ability to interpret the results of statistical calculations (B10)
- Ability to decide appropriate statistical technique (B11)
- Studied the course of statistics in academic career (B1)
- Satisfied from teaching method (B6)

B3, B4, B5, B7, B8, B10, B11 and C5. Table 3 shows the Model fitting information. The significant chi-square statistic (P-value < 0.05) indicates that the Final model gives a significant improvement over the baseline intercept-only model. This tells us that the model gives better predictions than if we just guessed based on the marginal probabilities for the outcome categories.

Table 3
Model Fitting Information

Model	-2 Log Likelihood	Chi-Square	df	Sig.
Intercept Only	354.541			
Final	281.368	73.173	25	.000

Pearson's chi-square statistic (281.74) for the model suggest the model fit very well as P-value>0.05.

The pseudo R^2 values (e.g. Nagelkerke = 64.5%) indicated that the explanatory variables explains a relatively large proportion of the variation between level of conceptual understanding of respondents.

The Parameter estimates table (Table 4) is the core of the output, telling us specifically about the relationship between our explanatory variables and the outcome. These are the ordered log-odds (logit) regression coefficients. Standard interpretation of the ordered logit coefficient is that for a one unit increase in the predictor, the response variable level is expected to change by its respective regression coefficient in the ordered log-odds scale while the other variables in the model are held constant. Interpretation of the ordered logit estimates is not dependent on the ancillary parameters; the ancillary parameters are used to differentiate the adjacent levels of the response variable. However, since the ordered logit model estimates one equation over all levels of the outcome variable, a concern is whether our one-equation model is valid or if a more flexible model is required.

These are the p-values of the coefficients or the probabilities that, within a given model, the null hypothesis that a particular predictor’s regression coefficient is zero given that the rest of the predictors are in the model. The Wald test statistic for the predictor B4 i.e. high GPA in statistics course is 2.059 with an associated p-value of 0.151. If we set our alpha level to 0.05, we would fail to reject the null hypothesis and conclude that the regression coefficient for high GPA in the course has not been found to be statistically different from zero in estimating level of conceptual understanding given other predictors are in the model. All the other estimates can be interpreted as above. By the Wald statistic and the p-values, we can see that the regression coefficient for some predictors has not been found to be statistically different from zero in estimating level of conceptual understanding given other predictors are in the model except high ability level to apply theoretical knowledge into practice, ability to interpret statistical results, ability to apply appropriate statistical technique are statistically significant.

Table 4
Parameter Estimates

		Estimate	**Std. Error**	**Wald**	**df**	**Sig.**	**95% Confidence Interval**	
							Lower Bound	**Upper Bound**
Threshold	[TEST = 1.00]	-1.309	1.133	1.337	1	.248	-3.529	.910
	[TEST = 2.00]	1.309	1.133	1.337	1	.248	-.910	3.529
Location	[B4=1.00]	2.271	1.954	1.351	1	.245	-1.558	6.101
	[B4=2.00]	1.694	1.443	1.378	1	.240	-1.134	4.522
	[B4=3.00]	2.056	1.433	2.059	1	.151	-.752	4.865
	[B4=4.00]	0[a]	.	.	0	.	.	.
	[B7=1.00]	-3.012	.993	9.194	1	.002	-4.959	-1.065
	[B7=2.00]	-1.490	.759	3.854	1	.050	-2.978	-.002
	[B7=3.00]	-.036	.658	.003	1	.957	-1.325	1.254
	[B7=4.00]	-.363	.592	.376	1	.540	-1.523	.797
	[B7=5.00]	0[a]	.	.	0	.	.	.
	[B10=1.00]	.861	1.504	.328	1	.567	-2.086	3.808
	[B10=2.00]	-.994	.912	1.187	1	.276	-2.781	.794
	[B10=3.00]	-.368	.755	.237	1	.626	-1.847	1.112
	[B10=4.00]	-1.298	.690	3.542	1	.040	-2.650	.054
	[B10=5.00]	0[a]	.	.	0	.	.	.
	[B11=1.00]	-3.361	1.198	7.873	1	.005	-5.708	-1.013
	[B11=2.00]	-1.873	.876	4.577	1	.032	-3.590	-.157
	[B11=3.00]	-2.480	.647	14.689	1	.000	-3.748	-1.212
	[B11=4.00]	-1.863	.610	9.331	1	.002	-3.058	-.668
	[B11=5.00]	0[a]	.	.	0	.	.	.
	[B1=.00]	-2.378	1.466	2.631	1	.105	-5.252	.495
	[B1=1.00]	0[a]	.	.	0	.	.	.
	[B6=1.00]	.479	.831	.333	1	.564	-1.149	2.108
	[B6=2.00]	1.484	.654	5.148	1	.023	.202	2.767
	[B6=3.00]	.564	.567	.989	1	.320	-.548	1.676
	[B6=4.00]	.226	.477	.225	1	.636	-.708	1.160
	[B6=5.00]	0[a]	.	.	0	.	.	.

a. This parameter is set to zero because it is redundant.

For location-only models, the test of parallel lines compares the estimated model with one set of coefficients for all categories to a model with a separate set of coefficients for each category. Here the general model gives a significantly better fit to the data than the ordinal (proportional odds) model (i.e. $p<.05$) so in our case then we are led to reject the assumption of proportional odds.

Table 5
Test of Parallel Lines

Model	-2 Log Likelihood	Chi-Square	df	Sig.
Null Hypothesis	281.368			
General	56.089	225.279	25	.000

5. COMMENTS AND CONCLUSION

The findings show that almost all of the respondents have studied the course of statistics and applying in their research. They accepted the importance of statistics in their field but still they have a low level of conceptual understanding. They are not able to decide which statistical technique should be used in which situation. They have low competence level of using statistical softwares.

It is suggested to include the use of statistical packages into the curriculum and teaching methods should more focus on the practical implications of the said course.

REFERENCES

1. Bluman, A.G. (2009). *Elementary statistics: A step by step approach*: McGraw-Hill Higher Education.
2. Budé, L.M. (2007). *On the improvement of students' conceptual understanding in statistics education*: Maastricht university.
3. Garfield, J. (1995). How students learn statistics. *International Statistical Review/Revue Internationale de Statistique*, 25-34.
4. Garfield, J. and Ben-Zvi, D. (2007). How students learn statistics revisited: A current review of research on teaching and learning statistics. *International Statistical Review*, 75(3), 372-396.
5. Jacobbe, T., Whitaker, D., Case, C. and Foti, S. (2014). *The LOCUS assessment at the college level: conceptual understanding in introductory statistics.* Paper presented at the Sustainability in Statistics Education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9), Flagstaff, Arizona, USA. Voorburg: International Association of Statistics Education.
6. Jolliffe, F.R. (1990). *Assessment of the Understanding of Statistical Concepts* Paper presented at the the third international conference on teaching statistics, Dunedin, New Zealand.
7. Moore, D.S. (1997). New pedagogy and new content: The case of statistics. *International Statistical Review*, 65(2), 123-137.
8. Pearson, K. (1900). *The grammar of science*: Рипол Классик.
9. Schau, C. and Mattern, N. (1997). Assessing students’ connected understanding of statistical relationships. *The Assessment Challenge in Statistics Education*, 91-104.

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ON THE LIFE-DISTRIBUTION OF A PARALLEL SYSTEM CONSISTING OF IDENTICAL COMPONENTS POSSESSING LOG-LOGISTIC LIFE

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ABSTRACT

Reliability is a basic requirement of household or industrial products and engineers have to ensure that a particular product will perform reliably within the desired lifetime of the said product. Evidently, the constituent components must be reliable so that the product as a whole is reliable. This calls for statistical tools to be employed in order to be able to model the life-length of the product on the basis of information regarding the life-distribution(s) of its components. One of the basic configurations for a product is that when the constituent components are assembled in 'parallel'. In this paper, we derive the probability density function of the life-length of a parallel system consisting of *n* identical components the life-lengths of which are log-logistically distributed. The log-logistic distribution belongs to the class of SIA log-symmetric distributions, and one of our basic goals is to determine whether or not the pdf of the system-life is also SIA-log-symmetric. We derive the basic properties of the system-life-distribution including measures of central tendency, dispersion and skewness as well as reliability-related properties including the hazard rate and the reliability function. The shape of the density being moderately positively skewed, it is obvious that this newly derived pdf possesses the potential to act as an appropriate model for various phenomena including those that have nothing to do with a parallel system. We demonstrate this fact by fitting the distribution to a set of data pertaining to the life-lengths of guinea-pigs.

1. INTRODUCTION

In terms of formulating a reliability model one must bear in mind the fact that a majority of modern machines consist of several interconnected components each performing a defined task in order to accomplish the machine's purpose as a whole. One of the simplest ways of creating a system containing a number of components is the Parallel System. In this paper we will focus on a parallel system of components the life-lengths of which are identically distributed according to one of the distributions belonging to the class of SIA Log-Symmetric distributions.

2. SIA LOG-SYMMETRIC DISTRIBUTIONS

By the term 'SIA log-symmetric distribution', we mean the one for which A/X is distributed exactly as X/A where X is a non-negative continuous random variable and A is an arbitrary positive real number representing the median of the distribution of X. The origins of this notion can be traced back to a number of decades. Seshadri (1965) explored the intrinsic nature of this class of distributions for the case A=1 whereas Saunders (1974) discussed this class in general. Jones (2008) adopted the nomenclature "log-symmetric" for such distributions whereas Habibullah and Saunders (2011) adopted the terminology "self-inverse distributions". Habibullah & Fatima (2015) espoused the term "Self-Inverse at A" and the abbreviation SIA for such distributions with median A. Recently, Habibullah (2017) proposed that the two concurrently existing nomenclatures "log-symmetric" and "SIA" be combined in order to obtain the nomenclature "SIA log-symmetric distributions".

3. LOG-LOGISTIC DISTRIBUTION

The log-logistic distribution was derived by Tadikamalla and Johnson (1982) and is one of the distributions belonging to the class of SIA Log-Symmetric distributions. The distribution finds applications in survival analysis, hydrology, economics, networking, etc. The probability density function (PDF) of log-logistic distribution can be written as

$$f(x)=\frac{\left(\frac{\beta}{\mathrm{A}}\right)\left(\frac{x}{\mathrm{A}}\right)^{\beta-1}}{\left(1+\left(\frac{x}{\mathrm{A}}\right)^{\beta}\right)^{2}},\quad 0<x<\infty,\mathrm{A}>0,\beta>0 \tag{3.1}$$

where A is the scale parameter and β is shape parameter.

The cumulative distribution function (CDF) is given as:

$$F(x;\mathrm{A},\beta)=\frac{1}{1+\left(\frac{x}{\mathrm{A}}\right)^{-\beta}}\quad x>0,\ \mathrm{A},\beta>0 \tag{3.2}$$

The shape of the distribution is positively skewed. The skewness decreases as β increases. It is easy to verify that the log-logistic distribution is a member of the class of SIA log-symmetric distributions.

4. PARALLEL SYSTEM

A parallel system is the one where at least one component must perform successfully for the system success.In other words, the system does not fail unless every single component fails. The diagrammatic representation of a parallel system is given in Fig. 1.

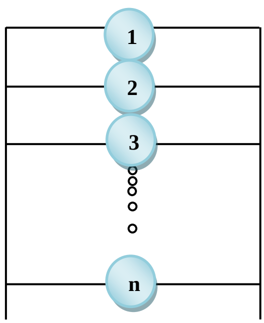

Fig. 1: Diagrammatic Representation of a Parallel System
drawn using Microsoft Word

5. CDF FOR PARALLEL SYSTEM AND ITS GRAPH

For a parallel system, we have

$$F_s(x) = \prod_{i=1}^{n} F_i(x) = F_1(x)F_2(x).....F_n(x)$$

$$= \frac{1}{1+\left(\dfrac{x}{A_1}\right)^{-\beta_1}} \times \frac{1}{1+\left(\dfrac{x}{A_2}\right)^{-\beta_2}} \times ... \times \frac{1}{1+\left(\dfrac{x}{A_n}\right)^{-\beta_n}}$$

In case of *iid* components, the above reduces to

$$F_s(x) = \left(\frac{1}{1+\left(\dfrac{x}{A}\right)^{-\beta}} \right)^n \tag{5.1}$$

Therefore, the System CDF of Log-Logistic in case of iid components is given by (5.1). The graph of the CDF for various values of the parameters are given in Fig. 2.

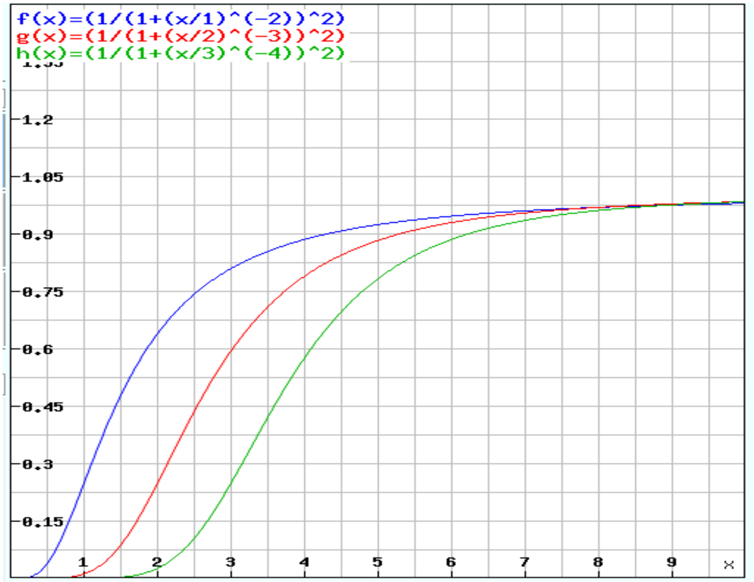

Fig. 2: Graph of CDF of Parallel System comprising components having *iid* log-logistic life-lengths

6. PDF FOR PARALLEL SYSTEM AND ITS GRAPH

Taking the derivative of *F(x)* with respect to *x*, the PDF of the parallel system consisting of n identically Log-logistically distributed components is given by:

$$f_s(x) = \frac{n\beta}{A}\left(1+\left(\frac{x}{A}\right)^{-\beta}\right)^{-n-1}\left(\frac{x}{A}\right)^{-\beta-1} \tag{6.1}$$

The graph of the PDF for various values of the parameters is given in Fig. 3.

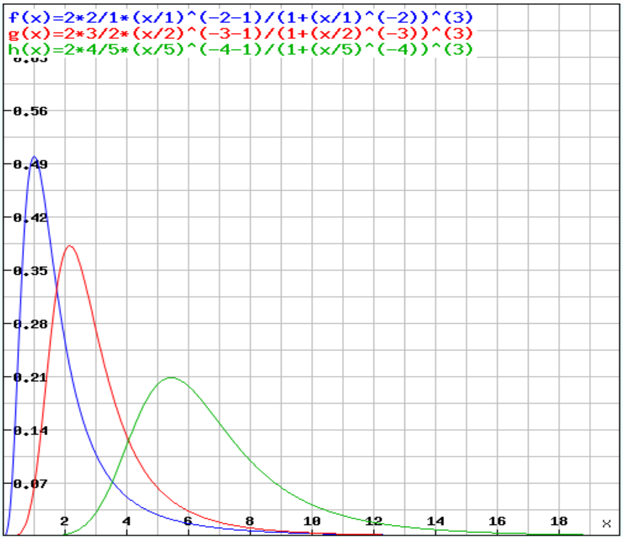

Fig. 3: Graph of Parallel System PDF

We see that the PDF has an attractive, unimodal, positively skewed shape for various values of the parameters which seems to indicate that this density may turn out to be a useful model in a variety of situations including those that have nothing to do with parallel systems.

Remark: It is interesting to note that, contrary to expectations, the PDF of the system is not SIA-log-symmetric.

7. PROPERTIES BASEED ON MOMENTS

In this section, we present algebraic expressions pertaining to the first four moments of the Life-Distribution of a Parallel System consisting of identical log-logistically distributed components.

Mean: The mean of the distribution with A=1 is given by:

$$E(X) = \frac{(\beta+1)\pi}{\beta^2 \sin\left(\frac{\pi}{\beta}\right)} \tag{7.1}$$

Variance: The variance of the distribution for A=1 is given by:

$$Var(X) = \frac{2\pi\beta^2(\beta+2)\sin\left(\frac{\pi}{\beta}\right) - \pi^2(\beta+1)^2\sin\left(\frac{2\pi}{\beta}\right)}{\beta^4\sin\left(\frac{2\pi}{\beta}\right)\sin^2\left(\frac{\pi}{\beta}\right)} \quad (7.2)$$

Higher Moments

The third and fourth moments about originof the distribution for A=1 are given by:

$$E(X^3) = \frac{(3\beta+9)\pi}{\beta^2\sin\left(\frac{3\pi}{\beta}\right)}$$

and

$$E(X^4) = \frac{(4\beta+16)\pi}{\beta^2\sin\left(\frac{4\pi}{\beta}\right)}$$

The third and fourth moments about the mean are related to the raw moments by the equations

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\left(\mu_1'\right)^3$$

and

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' - 6\mu_2'\left(\mu_1'\right)^2 + 2\left(\mu_1'\right)^4$$

As such, we have

$$\mu_3 = \frac{3\pi(\beta+3)}{\sin\left(\frac{3\pi}{\beta}\right)} - \frac{6\pi^2(\beta+1)(\beta+2)}{\beta^4\sin^2\left(\frac{\pi}{\beta}\right)\sin\left(\frac{2\pi}{\beta}\right)} + \frac{5\pi^3(\beta+1)^3}{\beta^6\sin^3\left(\frac{\pi}{\beta}\right)}$$

and

$$\mu_4 = \frac{4\pi(\beta+4)}{\beta^2\sin\left(\frac{4\pi}{\beta}\right)} - \frac{12\pi^2(\beta+1)(\beta+3)}{\beta^4\sin\left(\frac{\pi}{\beta}\right)\sin\left(\frac{3\pi}{\beta}\right)} - \frac{12\pi^3(\beta+1)^2(\beta+2)}{\beta^6\sin^3\left(\frac{\pi}{\beta}\right)\sin\left(\frac{2\pi}{\beta}\right)} + \frac{8\pi^4(\beta+1)^4}{\beta^8\sin^4\left(\frac{\pi}{\beta}\right)}$$

8. GEOMETRIC AND HARMONIC MEAN

The Geometric mean of the life distribution for Parallel System is given by:

$$\text{G.M} = \frac{1}{\beta} \tag{8.1}$$

whereas the Harmonic mean of the life distribution for Parallel System for A=1 is given by:

$$\text{H.M} = -\frac{(\beta-1)\,\pi}{\beta^2 \sin\left(\frac{\pi}{\beta}\right)} \tag{8.2}$$

9. LOWER AND UPPER QUARTILES

In this section, we derive the algebraic expressions of the quartiles of the distribution. The first quartile is given by:

$$Q_1 = \frac{\text{A}}{\left((4)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} \tag{9.1}$$

whereas the third quartile is given by:

$$Q_3 = \frac{\text{A}}{\left(\left(\frac{4}{3}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} \tag{9.2}$$

10. MEASURES OF CENTER, SPREAD AND SKEWNESS BASED ON QUARTILES

Measures of central tendency, dispersion and skewness of the life-distribution of the parallel system based on quartiles of the distribution are given below:

Mid-Quartile Range: The MQR is given by

$$\text{MQR} = \frac{Q_3 + Q_1}{2}$$

or

$$\text{MQR} = \frac{\text{A}}{2}\left(\frac{1}{\left(\left(\frac{4}{3}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} + \frac{1}{\left((4)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}}\right) \tag{10.1}$$

Inter-Quartile Range: The IQR is given by:

$$\mathrm{IQR} = Q_3 - Q_1$$

or

$$\mathrm{IQR} = \mathrm{A}\left(\frac{1}{\left(\left(\frac{4}{3}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} - \frac{1}{\left((4)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}}\right) \tag{10.2}$$

Bowley's Coefficient of Skewness: The formula of Bowley's coefficient of skewness is

$$\mathrm{Sk}_{\text{Bowley's}} = \frac{Q_1 - 2Q_2 + Q_3}{Q_3 - Q_1}$$

As such, the Bowley's Coefficient of Skewness for the parallel system life-distribution is given by:

$$\mathrm{Sk}_{\text{Bowley's}} = \frac{\left(\frac{1}{\left((4)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} - \frac{2}{\left(\left(\frac{4}{2}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} + \frac{1}{\left(\left(\frac{4}{3}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}}\right)}{\left(\frac{1}{\left(\left(\frac{4}{3}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} - \frac{1}{\left((4)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}}\right)} \tag{10.3}$$

11. FIRST AND NINTH DECILES

In this section, we present the deciles for the life-distribution of the Parallel System:

The first decile is given by:

$$D_1 = \frac{A}{\left((10)^{\frac{1}{n}} - 1 \right)^{\frac{1}{\beta}}} \tag{11.1}$$

whereas the ninth decile is:

$$D_9 = \frac{A}{\left(\left(\frac{10}{9} \right)^{\frac{1}{n}} - 1 \right)^{\frac{1}{\beta}}} \tag{11.2}$$

As such, measures of central tendency and dispersion of the life-distribution of the parallel system based on deciles of the distribution come out to be

Mid-Decile Range

$$\text{MDR} = \frac{A}{2} \left(\frac{1}{\left(\left(\frac{10}{9} \right)^{\frac{1}{n}} - 1 \right)^{\frac{1}{\beta}}} + \frac{1}{\left((10)^{\frac{1}{n}} - 1 \right)^{\frac{1}{\beta}}} \right) \tag{11.3}$$

Inter-Decile Range

$$\text{IDR} = A \left(\frac{1}{\left(\left(\frac{10}{9} \right)^{\frac{1}{n}} - 1 \right)^{\frac{1}{\beta}}} - \frac{1}{\left((10)^{\frac{1}{n}} - 1 \right)^{\frac{1}{\beta}}} \right) \tag{11.4}$$

12. PERCENTILES

In this section, we derive the algebraic expressions of two of the percentiles of the distribution as well as that of the percentile coefficient of kurtosis.

The Tenth Percentile is given by:

$$P_{10} = \frac{A}{\left(\left(\frac{100}{10}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} \tag{12.1}$$

whereas the Ninetieth Percentile is given by:

$$P_{90} = \frac{A}{\left(\left(\frac{100}{90}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} \tag{12.2}$$

The formula of the percentile coefficient of kurtosis K is

$$K = \frac{Q_3 - Q_1}{2\left(P_{90} - P_{10}\right)}$$

Hence, the percentile coefficient of kurtosis for the life-distribution of the Parallel System is given by:

$$\mathrm{K} = \frac{\left(\frac{1}{\left(\left(\frac{4}{3}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} - \frac{1}{\left(\left(\frac{4}{1}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}}\right)}{\left(2\left(\frac{1}{\left(\left(\frac{100}{90}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}} - \frac{1}{\left(\left(\frac{100}{10}\right)^{\frac{1}{n}} - 1\right)^{\frac{1}{\beta}}}\right)\right)} \tag{12.3}$$

13. RELIABILITY-RELATED PROPERTIES

In this section we derived the reliability related properties of Life- Distribution for Parallel System:

Reliability Function: The Reliability Function is given by

$$R(x) = 1 - F(x)$$

$$R_S(x) = 1 - \frac{1}{\left(1 + \left(\frac{x}{A}\right)^{-\beta}\right)^n} \tag{13.1}$$

The graph of the reliability function is given in Fig. 4.

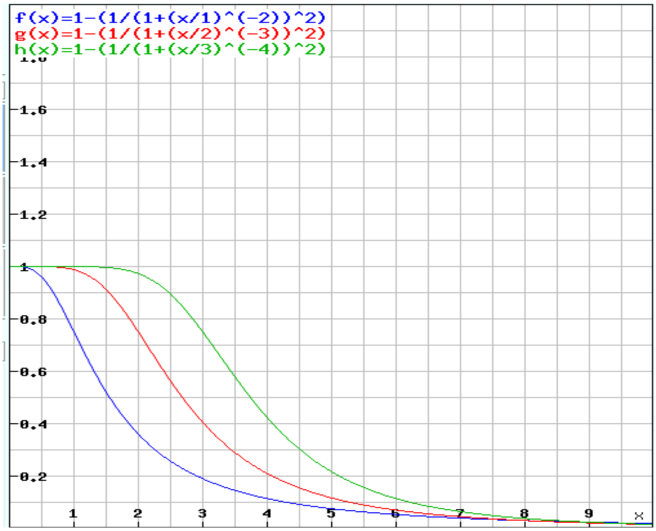

Fig. 4: Graph of Reliability Function

Hazard Rate: The system hazard rate is given by

$$h_s(x) = \frac{f_s(x)}{1 - F_s(x)}$$

or

$$h_s(x) = \frac{\left(\frac{n\beta}{A}\frac{\left(\frac{x}{A}\right)^{-\beta-1}}{\left(1 + \left(\frac{x}{A}\right)^{-\beta}\right)^{n+1}}\right)}{\left(1 - \frac{1}{\left(1 + \left(\frac{x}{A}\right)^{-\beta}\right)^n}\right)} \tag{13.2}$$

The graph of the hazard rate is given in Fig. 5.

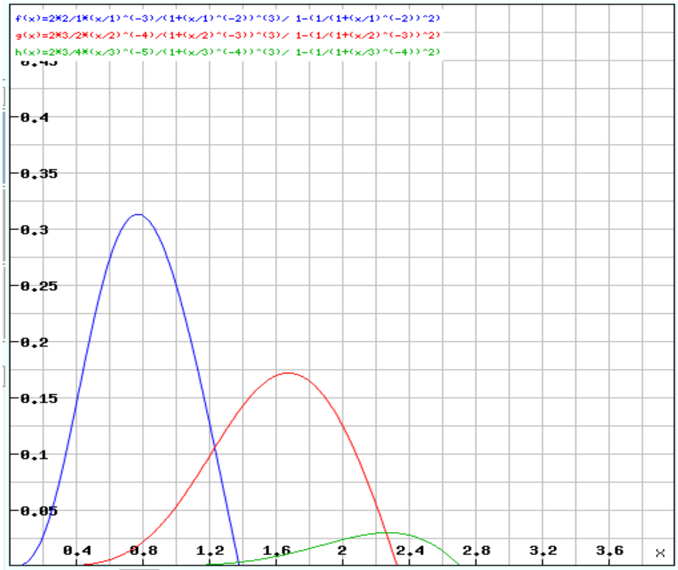

Fig. 5: Graph of Hazard Rate

Reverse Hazard Rate: The Reverse Hazard Rate is given by:

$$\text{R.H.R} = \frac{f_s(x)}{F_s(x)}$$

or

$$\text{R.H.R} = \frac{\left(\dfrac{n\beta}{\text{A}} \dfrac{\left(\dfrac{x}{\text{A}}\right)^{-\beta-1}}{\left(1+\left(\dfrac{x}{\text{A}}\right)^{-\beta}\right)^{n+1}} \right)}{\left(\dfrac{1}{\left(1+\left(\dfrac{x}{\text{A}}\right)^{-\beta}\right)^{n}} \right)} \tag{13.3}$$

The graph of the reversed hazard rate is given in Fig. 6.

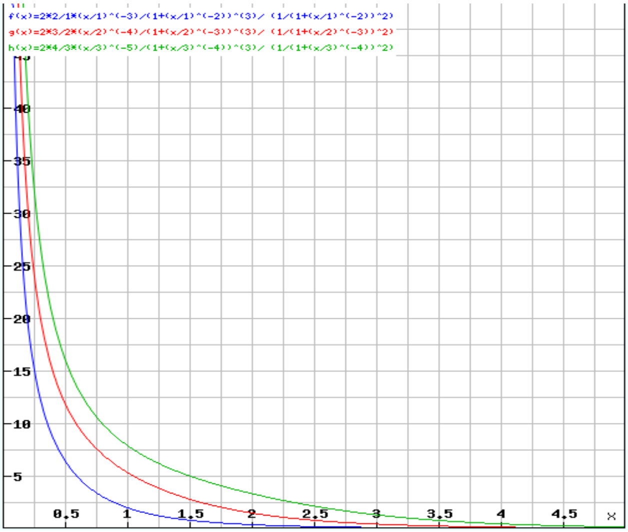

Fig. 6: Graph of Reverse Hazard Rate

14. DISTRIBUTIONS OF ORDER STATISTICS

In this section we focus on order statistics of the life-distribution for the parallel system.

The probability density function of rth order statistic is:

$$g_r(y) = \frac{n!}{(r-1)!(n-r)!}\left[F(x)\right]^{r-1}\left[1-F(x)\right]^{n-r} f(x)$$

For the distribution of the first order statistic, we put r=1to obtain

$$g_r(y) = \frac{n\beta\left(\frac{x}{\mathrm{A}}\right)^{-\beta-1} n!}{A(n-1)!\left(1+\left(\frac{x}{\mathrm{A}}\right)^{-\beta}\right)^{n+1}}\left[1-\left(\frac{1}{1+\left(\frac{x}{A}\right)^{-\beta}}\right)^{n}\right]^{n-r} \tag{14.1}$$

For the distribution of the nth order statistic, we put r=n thus obtaining

$$g_r(y) = \frac{n\beta\left(\frac{x}{\mathrm{A}}\right)^{-\beta-1} n!}{A(n-1)!\left(1+\left(\frac{x}{\mathrm{A}}\right)^{-\beta}\right)^{n+1}}\left[\left(\frac{1}{1+\left(\frac{x}{A}\right)^{-\beta}}\right)^{n}\right]^{n-1} \tag{14.2}$$

For the distribution of the sample median, we put $r = (n+1)/2$ (assuming that n is odd). This yields

$$g_r(y) = \frac{n\beta\left(\frac{x}{A}\right)^{-\beta-1} n!}{A\left(1+\left(\frac{x}{A}\right)^{-\beta}\right)^{n+1}\left(\frac{n-1}{2}\right)!\left(\frac{n+1}{2}\right)!}\left[\left(\frac{1}{1+\left(\frac{x}{A}\right)^{-\beta}}\right)^{n}\right]^{\frac{n-1}{2}}\left[1-\left(\frac{1}{1+\left(\frac{x}{A}\right)^{-\beta}}\right)^{n}\right]^{\frac{n+1}{2}} \tag{14.3}$$

15. APPLICATION

In this section, we consider the data-set analyzed by Bjerkedal (1960), Gupta et al. (1997), and Kundu et al. (2008) that reperesents the life-lengths of guinea pigs that have been injected with different doses of tubercle bacilli. The 72 observations are as follows:

12, 15, 22, 24, 24, 32, 32, 33, 34, 38, 38, 43, 44, 48, 52, 53, 54, 54, 55, 56, 57, 58, 58, 59, 60, 60, 60, 60, 61, 62, 63, 65, 65, 67, 68, 70, 70, 72, 73, 75, 76, 76, 81, 83, 84, 85, 87, 91, 95, 96, 98, 99, 109, 110, 121, 127, 129, 131, 143, 146, 146, 175, 175, 211, 233, 258, 258, 263, 297, 341, 341, 376.

The histogram of this data-set and the shape of PDF (6.1) are contained in Fig. 7 and 8 respectively. The positively skewed shape of the histogram being similar to the shape of the distribution for some values of β, we decide to fit PDF (6.1) to the data-set.

Having fitted the distribution, we apply the Kolmogorov Smirnov Test of goodness of fit. We find that, the computed value of the KS-Statistic D is 0.0983, whereas the Kolmogorov Smirnov table shows the critical value Dn,α=D72,0.05=0.160. As our calculated value is less than the tabulated value, we conclude that PDF (6.1) provides a good fit to the data.

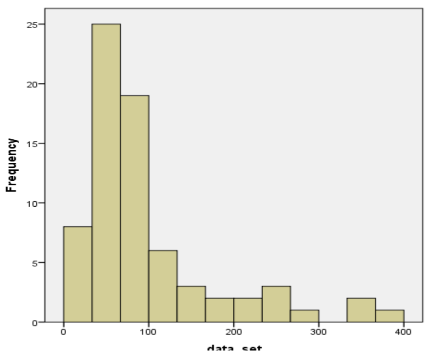

Fig. 7: Histogram of the data set

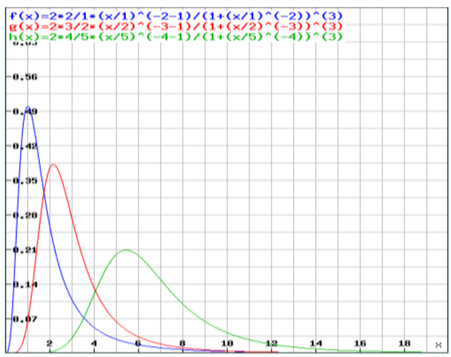

Fig. 8: PDF of Parallel System

16. COMMENTS AND CONCLUSION

In this paper, we have derived the life-distribution of a parallel system consisting of identical components the life-lengths of which are following the log-logistic distribution.

We have derived the basic properties of the life-distribution including measures of central tendency, dispersion, skewness and kurtosis as well as the reliability-related properties.

The shape of the density being moderately positively skewed, this newly derived probability density function seems to possess the potential to act as an appropriate model for various phenomena including to those that have nothing to do with parallel systems.

We demonstrate this by fitting the distribution to a data-set taken from the literature.

REFERENCES

1. Habibullah, S.N. (2017). On the Power Generalization of Log-Symmetric SIA Distributions. *Journal of ISOSS*, 3(1), 45-54.
2. Habibullah, S.N. and Fatima, S.S. (2015). On a Newly Developed Estimator for More Accurate Modeling with an Application to Civil Engineering, Proceeding of the *12th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP12) organized by CERRA* (Vancouver, BC, Canada, July 12-15, 2015). Sponsoring Agency: Higher Education Commission, Pakistan.
3. Habibullah, S.N. and Saunders, S.C. (2011). A Role of Self Inversion, *Proc. of International Conference on Advanced Modeling and Simulation (ICAMS)* published by Department of Mechanical Engineering, College of Electrical and Mechanical Engineering, National University of Science and Technology (NUST), Islamabad, Pakistan, ISBN 978-869-8535-11-7.
4. Jones, M.C. (2008). On reciprocal symmetry. *Journal of Statistical Planning and Inference*, 138(10), 3039-3043.
5. Saunders, S.C. (1974). A family of random variables closed under reciprocation. *J. Amer. Statist. Assoc*., 69(346), 533-539.
6. Seshadri, V. (1965). On Random Variables which have the same Distribution as their Reciprocals. *Can. Math. Bull*., 8(6), 819-824.
7. Tadikamalla, P.R. and Johnson, N.L. (1982). Systems of frequency curves generated by transformations of logistic variables. *Biometrika*, 69(2), 461-465.

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ON THE LIFE-DISTRIBUTION OF A SERIES SYSTEMS CONSISTING OF IDENTICAL COMPONENTS POSSESSING LOGNORMAL LIFE

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ABSTRACT

Reliability is a fundamental requirement of people or technological products and engineers have to ensure that a particular output will perform reliably within the desired lifetime of the circulating product. Evidently, the constituent components must be reliable so that the product as a whole is reliable. This calls for statistical tools to be employed in order to be able to model the life-length of the product on the basis of information regarding the life-distribution(s) of its components. One of the basic configurations for a product is that when the constituent components are assembled in 'Series'. In this paper, we derive the probability density function of the life-length of a Series system consisting of *n* identical components the life-lengths of which are lognormally distributed. The lognormally distributed belongs to the class of SIA log-symmetric distributions, and one of our basic goals is to determine whether or not the pdf of the system-life is also SIA-log-symmetric. We derive the basic properties of the system-life-distribution including measures of central tendency, dispersion and skewness as well as reliability-related properties including the hazard rate and the reliability function. The shape of the density being an exponential-type, it is obvious that this newly derived pdf possesses the potential to act as an appropriate model for various phenomena including those that have nothing to do with a Series system. We demonstrate this fact by fitting the distribution to a set of data on waiting times (in seconds), between 64 successive eruptions of the Kiama Blow hole.

1. INTRODUCTION

Reliability theory is based on probability distributions that are used for modeling life-lengths of components and systems. This paper relates to the lognormal distribution which is often used for reliability-related analyses and is a member of the class of SIA log-symmetric distributions. As a huge majority of machinery and other items in this world consist of not just one but a number of components, naturally, the question arise: What will be the situation if a number of lognormally distributed components are assembled as a Series System? As such, in this paper, we derive the probability density function of a Series System consisting of lognormally distributed components. Next, we derive the fundamental properties of the life-distribution of the System including the reliability function, hazard rate and reverse hazard rate.

2. SIA DISTRIBUTIONS

Habibullah and Fatima (2015) have introducedthe term "Self-inverse at A (SIA)" whichimplies that, for any non-negative randomvariable Y with median A, the probabilitydistribution of Y/A is identical to that of A/Y.

The class of SIA distributions contains probability models that are extensively used in engineering and Reliability studies such as the lognormal distribution and the Birnbaum Saunders distribution**.**

3. LOGNORMAL DISTRIBUTION

In probability theory, lognormal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. In other words, if the random variable *X* is log-normally distributed, then *Y* = ln(*X*) has a normal distribution. Stated a little differently, if *Y* has a normal distribution, then, *X* = exp(*Y*) has a lognormal distribution

Hence, the Probability Density Function of SIA-Log-Normal distribution can be re-written as:

$$f(x)=\frac{e^{\left(-\frac{(\ln x-\ln A)^2}{2\sigma^2}\right)}}{x\sigma\sqrt{2\pi}}\;,\;\; x>0,\ A>0,\ \sigma>0 \tag{3.1}$$

Where, A is the median (location parameter) and σ> 0 is the scale parameter of the distribution.

The Cumulative Distribution Function (CDF) can be re-written as**:**

$$\mathrm{F(x)}=\frac{1}{2}+\frac{1}{2}\mathrm{erf}\left(\frac{\ln \mathrm{x}-\ln \mathrm{A}}{\sigma\sqrt{2}}\right) \tag{3.2}$$

The shape of the distribution is positively skewed. The skewness decreases as σ increases. It is easy to verify that the lognormal distribution is a member of the class of SIA log-symmetric distributions.

4. SERIES SYSTEM

This is a system in which all the components are in series and they all have to work for the system to work. Even if one component fails, the system fails.

We can understand clearly by looking at Fig. 1 that if any one of the component will fails e.g. if component '3' fails, automatically the system will stop working and the whole system will shut down.

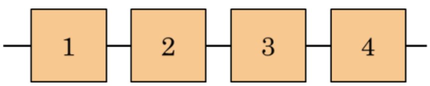

Fig. 1: Diagrammatic Representation of a Series System drawn using Microsoft Word

5. CDF OF SERIES SYSTEM AND ITS GRAPH

The reliability of a series system is given by

$$R_s(x) = \prod_{i=1}^{n} R_i(x)$$

implying that

$$F_s(x) = 1 - \prod_{i=1}^{n}\left[1 - F_i(x)\right]$$

Here, we have

$$F_s(x) = 1 - \prod_{i=1}^{n}\left[1 - \left(\frac{1}{2} + \frac{erf\left(\frac{\ln x - \ln A_i}{\sqrt{2}\sigma_i}\right)}{2}\right)\right] \Rightarrow 1 - \prod_{i=1}^{n}\left[1 - \frac{1}{2} - \frac{erf\left(\frac{\ln x - \ln A_i}{\sqrt{2}\sigma_i}\right)}{2}\right] \Rightarrow 1 - \prod_{i=1}^{n}\left[\frac{1}{2} - \frac{erf\left(\frac{\ln x - \ln A_i}{\sqrt{2}\sigma_i}\right)}{2}\right]$$

Because of identically independently distributed (*iid*) components we have

$$\mathrm{F_s}(x) = 1 - \left[\frac{1}{2} - \frac{1}{2} erf\left(\frac{\ln x - \ln A}{\sigma\sqrt{2}}\right)\right]^n \tag{5.1}$$

The graph of the CDF for various values of the parameters is given in Fig. 2.

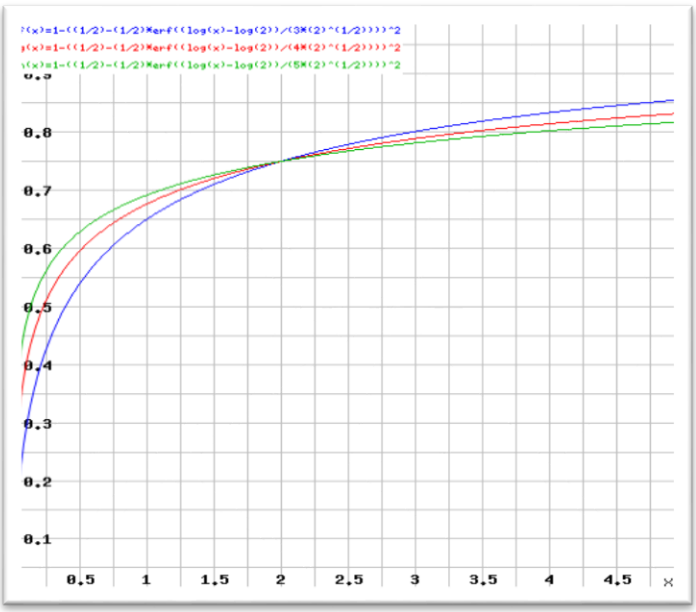

Fig 2: Graph of CDF of Parallel System Comprising Components having *iid* lognormal life-lengths

6. PDF OF SERIES SYSTEM AND ITS GRAPH

By taking the derivative of the CDF derive above with respect to x we obtain PDF of the series system consisting of lognormal distribution.

$$f_s(x) = \frac{ne^{-\frac{(\ln x - \ln a)^2}{2\sigma^2}}}{x\sigma\sqrt{2\pi}} \left[\frac{1}{2} - \frac{1}{2} erf\left(\frac{\ln x - \ln A}{\sigma\sqrt{2}}\right)\right]^{n-1} \quad (6.1)$$

The graph of the probability density function is given in Fig 3:

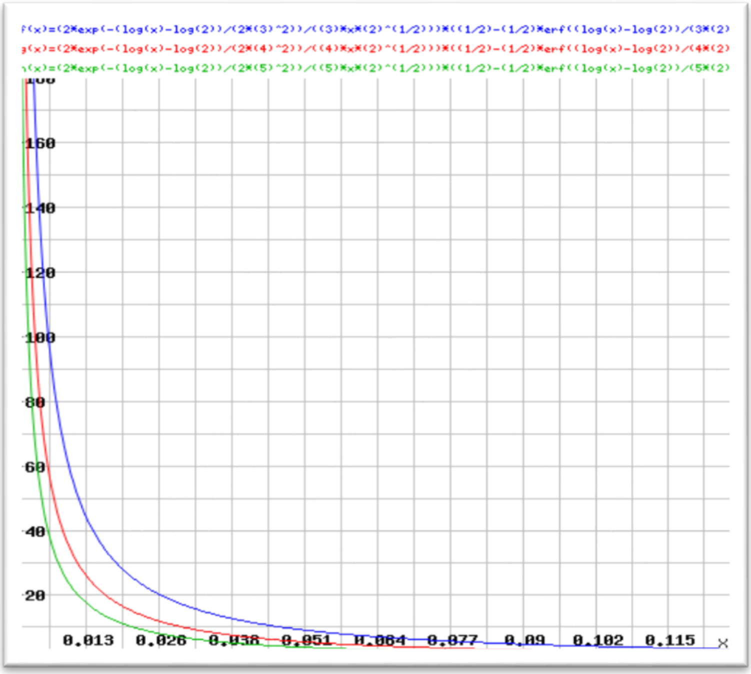

Fig. 3: Graph of the Series System PDF

We see that the PDF has obtain an exponential-type shape for various values of the parameters which seems to indicate that this density may turn out to be a useful model in a variety of situations including those that have nothing to do with parallel systems.

Remark: It is interesting to note that, contrary to expectations, the PDF of the system is not SIA-log-symmetric.

7. QUARTILES

In this section, we present algebraic expressions pertaining to the first and third quartile of the Life-Distribution of a Series System consisting of identical lognnormally distributed components.

The lower and upper quartiles of the Series System are as follows.

The first quartile is given as:

$$Q_1 = Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \tag{7.1}$$

and the second quartile i.e. the median is given by

$$Q_2 = Ae^{\sigma\sqrt{2}erf^{-1}\left[\left[-\frac{2^n}{2(-1)^n}+(-2)^n\right]^{\frac{1}{n}}+1\right]} \tag{7.2}$$

whereas the expression of the third quartile is:

$$Q_3 = Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{3.2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \tag{7.3}$$

8. MEASURES OF CENTER, SPREAD AND SKEWNESS BASEED ON QUARTILES

Measures of central tendency, dispersion and skewness of the life-distribution of the series system based on quartiles of the distribution are given below:

The Mid-Quartile Range is given by

$$MQR = \frac{Ae^{\sigma\sqrt{2}}}{2}\left[e^{erf^{-1}\left[\left(-\frac{3.2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} + e^{erf^{-1}\left[\left(-\frac{2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}\right] \tag{8.1}$$

The Inter-Quartile Range is given by

$$IQR = Ae^{\sigma\sqrt{2}}\left[e^{erf^{-1}\left[\left(-\frac{3.2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} - e^{erf^{-1}\left[\left(-\frac{2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}\right] \tag{8.2}$$

and the Bowley's coefficient of skewness for the life-distribution of the parallel system is given by:

$$SK_{Bowley}=\frac{Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}-2Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{2^n}{(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}+Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{3.2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}}{Ae^{\sigma\sqrt{2}}\left[e^{erf^{-1}\left[\left(-\frac{3.2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}-e^{erf^{-1}\left[\left(-\frac{2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}\right]} \tag{8.3}$$

9. PERCENTILES

In this section, we present algebraic expressions pertaining to the first and ninth percentile of the Life-Distribution of a Series System consisting of identical lognnormally distributed components the tenth percentile is the same as the first decile whereas the ninetieth percentile is the same as the ninth decile. As such, the percentile coefficient of kurtosis comes out to be

$$Kurtosis=\frac{e^{erf^{-1}\left[\left(\frac{3.2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}-e^{erf^{-1}\left[\left(\frac{2^n}{4(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}}{2\left(e^{erf^{-1}\left[\left(\frac{9.2^n}{10(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}-e^{erf^{-1}\left[\left(\frac{2^n}{10(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]}\right)} \tag{9.1}$$

10. FIRST AND NINTH DECILES

In this section, we present algebraic expressions pertaining to the first and ninth decile of the life-distribution of a Series System consisting of identical lognnormally distributed components

The first decile is given as:

$$D_1=Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{2^n}{10(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \tag{10.1}$$

and the ninth decile is given as:

$$D_9=Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{9.2^n}{10(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \tag{10.2}$$

As such, measures of central tendency and dispersion of the life-distribution of the series system based on deciles of the distribution come out to be

$$MDR = \frac{Ae^{\sigma\sqrt{2}}}{2}\left(e^{erf^{-1}\left[\left(-\frac{9.2^n}{10(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} - e^{erf^{-1}\left[\left(-\frac{2^n}{10(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \right) \tag{10.3}$$

and

$$IDR = Ae^{\sigma\sqrt{2}}\left(e^{erf^{-1}\left[\left(-\frac{9.2^n}{10(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} + e^{erf^{-1}\left[\left(-\frac{2^n}{10(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \right) \tag{10.4}$$

respectively.

11. QUINTILES

The first and fourth quintiles of series system consisting of lognormally life-distribution are given below:

The first quintile is given as:

$$Qn_1 = Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{2^n}{5(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \tag{11.1}$$

and the Fourth Quintile is given as:

$$Qn_4 = Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{4.2^n}{5(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \tag{11.2}$$

12. OCTILES

The first and seventh octile of series system consisting of lognormally life-distribution are presented below:

The First Octile is given as:

$$O_1 = Ae^{\sigma\sqrt{2}erf^{-1}\left[\left(-\frac{2^n}{8(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \tag{12.1}$$

and the Seventh octile is given as:

$$O_7 = Ae^{\sigma\sqrt{2}\,erf^{-1}\left[\left(-\frac{7.2^n}{8(-1)^n}+(-2)^n\right)^{\frac{1}{n}}+1\right]} \quad (12.2)$$

13. RELIABILITY-RELATED PROPERTIES

In this section we derived the reliability related properties of Life- Distribution for series System:

The reliability function is given as:

$$R_s(x) = \left[\frac{1}{2} - \frac{1}{2} erf\left(\frac{\ln x - \ln A_i}{\sigma_i\sqrt{2}}\right)\right]^n \quad (13.1)$$

The graph of the reliability function is given in Fig. 4.

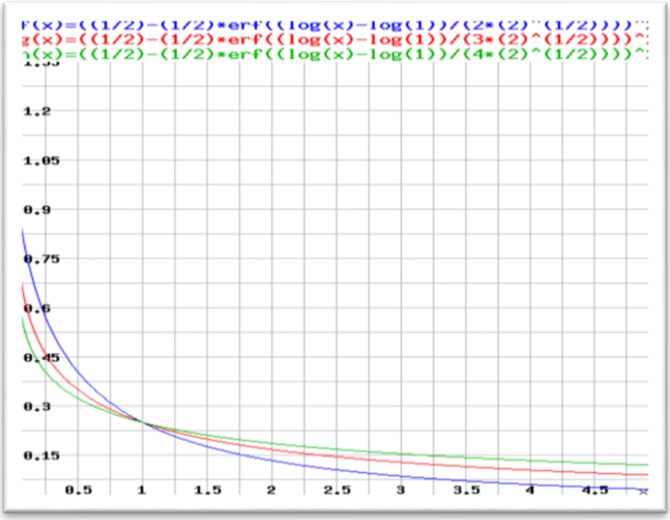

Fig. 4: Graph of Reliability Function

The System Hazard rate is given as:

$$h_s(x) = \frac{f_s(\mathrm{x})}{1-F_S(\mathrm{x})} = \frac{\frac{\sqrt{2}}{x\sigma\sqrt{\pi}} e^{\frac{-(\ln x - \ln A)^2}{2\sigma^2}}\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x - \ln A}{\sigma\sqrt{2}}\right)\right]}{\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x - \ln A}{\sigma\sqrt{2}}\right)\right]^2} \quad (13.2)$$

and the reversed hazard rate is given as:

$$R.H.R = \frac{\frac{\sqrt{2}}{x\sigma\sqrt{\pi}} e^{\frac{-(\ln x - \ln A)^2}{2\sigma^2}} \left[\frac{1}{2} - \frac{1}{2} erf\left(\frac{\ln x - \ln A}{\sigma\sqrt{2}}\right)\right]}{1 - \left[\frac{1}{2} - \frac{1}{2} erf\left(\frac{\ln x - \ln A}{\sigma\sqrt{2}}\right)\right]^2} \tag{13.3}$$

The graph of the hazard rate is given in Fig. 5.

14. DISTRIBUTIONS OF ORDER STATISTICS

In this section we focus on order statistics of the life-distribution for the series system.

The probability density function of r^{th} order statistic is:

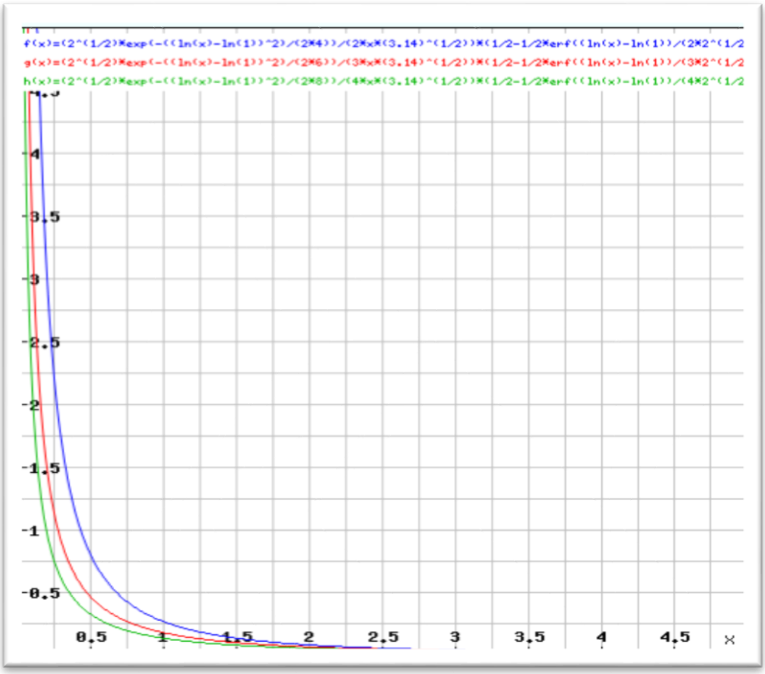

Fig.5: Graph of the Hazard Rate

$$g_r(y) = \frac{n!}{(r-1)!(n-r)!}\left[F(x)\right]^{r-1}\left[1-F(x)\right]^{n-r} f(x)$$

For the distribution of the first order statistic, we put r=1to obtain

$$g_r(y)=\frac{ne^{-\frac{(\ln x-\ln A)^2}{2\sigma^2}}n!}{\sigma x\sqrt{2\pi}(r-1)!(n-r)!}\left[1-\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]^n\right]^{r-1}$$
$$\times\left[-\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]^n\right]^{n-r}\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]^{n-1} \tag{14.1}$$

For the distribution of the last order statistic, we put r=n to obtain

$$g_r(y)=\frac{ne^{-\frac{(\ln x-\ln A)^2}{2\sigma^2}}n!}{\sigma x\sqrt{2\pi}(r-1)!}\left[1-\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]^n\right]^{n-1}$$
$$\times\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]^{n-1} \tag{14.2}$$

For the distribution of the first order statistic, we put r = (n+1)/2 to obtain

$$g_r(y)=\frac{ne^{-\frac{(\ln x-\ln A)^2}{2\sigma^2}}n!}{\sigma x\sqrt{2\pi}\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)!}\left[1-\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]^n\right]^{\frac{n-1}{2}}$$
$$\times\left[-\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]^n\right]^{\frac{n+1}{2}}\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]^{n-1} \tag{14.3}$$

15. RECORD VALUES

In statistics, a record value or record statistic is the largest or smallest value obtained from a sequence of random variables. The theory is closely related to that used in order statistics. The term was first introduced by K. N. Chandler in 1952.

For this distribution the upper record values is given by

$$f_n(x)=\frac{\left[R(x)\right]^{n-1}}{\Gamma(n)}f(x)$$

As suc, we have

$$f_n(x)=\frac{ne^{-\frac{(\ln x-\ln \mathrm{A})^2}{2\sigma^2}}}{\Gamma(n)\sigma x\sqrt{2\pi}}\left[n\ln\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]\right]^{n-1}$$
$$\times\left[\frac{1}{2}-\frac{1}{2}erf\left(\frac{\ln x-\ln A}{\sigma\sqrt{2}}\right)\right]^{n-1} \tag{15.1}$$

16. APPLICATION

In this section, this data set consists of the waiting times (in seconds), between 64 successive eruptions of the Kiama Blow hole. These values were recorded with the aid of digital watch on July 12, 1998 by Jim Irish and has been referenced by several authors including da Silva, Thiago, Maciel, Campos, & Cordeiro(2013) and Pinho, Cordeiro & Nobre(2015). The actual data are:

83, 51, 87, 60, 28, 95, 8, 27, 15, 10, 18, 16, 29, 54, 91, 8, 17, 55, 10, 35, 47, 77, 36, 17, 21, 36, 18, 40, 10, 7, 34, 27, 28,56, 8, 25, 68, 146, 89, 18, 73, 69, 9, 37, 10, 82, 29, 8, 60, 61, 61, 18, 169, 25, 8, 26, 11, 83, 11, 42, 17, 14, 9, 12.

The histogram of this data-set and the shape of pdf (6.1) are contained in Fig. 6 and 7 respectively. The exponentiated type shape of the histogram being similar to the shape of the distribution for some values of σ, we decide to fit pdf (6.1) to the data-set.

Having fitted the distribution, we apply the Kolmogorov Smirnov Test of goodness of fit. We find that, the computed value of the KS-Statistic D is 0.0983, whereas the Kolmogorov Smirnov table shows the critical value $D_{n,\alpha}=D_{64,0.05}=0.1697$. As our calculated value is less than the tabulated value, we conclude that pdf (6.1) provides a good fit to the data.

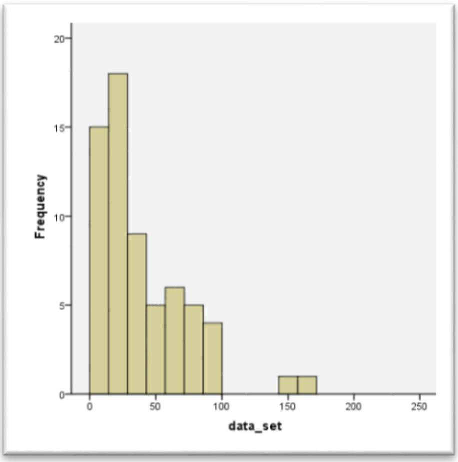

Fig. 6: Histogram of the Data Set

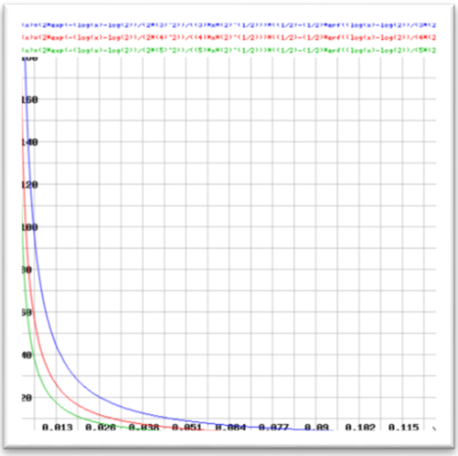

Fig. 7: PDF of Series System

17. COMMENTS AND CONCLUSION

In this paper, we have explored the life-length properties of a series system consisting of *n* identical components which possesses the same lognormal distribution. We have derived the basic properties of the life-distribution of the system including measures of central tendency, dispersion and skewness as well as the reliability-related properties i.e. the hazard rate and the reliability function**.** The shape of the density being 'exponential-type', this newly derived probability density function seems to possess the potential to act as an appropriate model for various phenomena including to those that have nothing to do with series systems. We have demonstrated this by fitting the distribution to a data-set taken from the literature.

REFERENCES

1. Chandler, K.N. (1952). The Distribution and Frequency of Record Values. *Journal of the Royal Statistical Society*, 14(2), 220-228.
2. Habibullah, S.N. and Fatima, S.S. (2015), On a Newly Developed Estimator for More Accurate Modeling with an Application to Civil Engineering. Proceedings of the *12th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP12) organized by CERRA* (Vancouver, BC, Canada, July 12-15, 2015)**.** Sponsoring Agency: Higher Education Commission, Pakistan.
3. Toulias, T.L. and Kitsos, C.P. (2013). On the Generalized Lognormal Distribution. *Journal of Statistics Applications & Probability*. 2013, 1-5.
4. Pinho, L.G.B., Cordeiro, G.M. and Nobre, J.S. (2015). The Harris extended exponential distribution. *Communications in Statistics-Theory and Methods*, 44(16), 3486-3502.
5. Pohjanpalo, H. (1978). System Identifiability based on the power series expansion of the solution. *Mathematical Biosciences*, 42(1-2), 21-23.
6. Silva, G.O., Ortega, E.M.M. and Cordeiro, G.M. (2010). The beta modified Weibull distribution. *Lifetime Data Analysis*, 16, 409-430.
7. Xavier, K. and Habibullah, S.N. (2016). On A Decile Based SIA-Estimator of the Shape Parameter of the Log-Logistic Distribution. *Proceedings of the 14th International Conference on Statistical Sciences*, Karachi, Pakistan. 29, 419-426.

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PRODUCTION AND PRODUCTIVITY IN OIC: A COMPARISON

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ABSTRACT

The aim of this paper is to assess the structural changes in value added production and labor engagement as well as the productivity in OIC group on the basis of a uniform econometric framework. Three previous studies pertaining to the same areas are particularly compared with. Chenery and Syrquin methodology for the purpose of analysis of structural change and underlying stylized facts in the world second largest inter-governmental organization OIC-Organization of Islamic Co-operation is adopted. Semi-logarithmic model with time dummies reveals the dependence of the value added production with labor engaged therein on income per capita and population size in OIC. Period 1980-2012 covers many economic shocks like oil price boom and financial meltdown. Effects of time shifts for the variables studied are portrayed in time dummies. Value added production and labor engagement with productivity depicts some different production patterns in OIC compared with the normal patterns of the world. Lesser share of GDP in value added agriculture accompanied with comparatively lesser levels of productivity in rich OIC countries hints towards the low technological change in this sector pointing to the food security issue in this group. Falling manufacturing sector is indicative of the natural resource curse as industry sector posts growing posture with level of development. This study done on aggregate level at economic and geographical division provides a starting point for subsector and country level analysis and inferences for framing policies of the countries relevant to the group.

1. INTRODUCTION

Patterns of development mainly portray the stylized facts underlying the structural change in the economic and social indicators. Industrialization being at the heart of the structural change is discussed with rising level of development. GNI per capita is used to indicate the level of development. Simon Kuznets'[1] approach to the evaluation of development patterns is largely inductive. Mainly advanced countries are discussed individually or in groups and comparable measures of development patterns are measured. Kuznets also showed that that the long-term experience of industrialized economies could also be observed in cross-country comparisons for a given period. Chenery developed the deductive approach, form general to specific, of comparative analysis both over time and countries. Kuznets holds responsible the transnational factors to the basis for comparative analysis. Chenery renamed this one as universal factors- "those potentially common to the world[2]".

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[1] Kuznets (1956-67) is a series of 10 articles on different aspects of national account

[2] Syrquin (1988) discusses these factors

Patterns of development developed by Chenery and Syrquin (1975) present average relations portraying the expected transformation from low income economy (agrarian) to an industrial one with high per capita income. In general the path of transformation may be different, reflecting variations, in preliminary conditions like size and resource, in the historical environment for example world markets and wars and in the economic policies prevalent in the period. This study goes on the steps of Chenery to explore the development patterns in the world's second largest inter-governmental organization, after The UN, OIC- Organization of Islamic Co-operation.

Patterns of development in value added Production and labor allocation in different sector and its effects on productivity in the OIC group as whole is evaluated and compared with the global trends. Usage of a uniform econometric framework for evaluation provides a consistent description of the processes considered in the study as well as the interrelated structural change. It also enables the identification and comparison of patterns of development among countries following various economic development strategies in this group.

OIC is regarded as developing group as mainly the countries are in poor and middle income range, only three countries (Saudi Arabia, Kuwait and UAE) in this study go beyond 13000 per capita. These high income countries are endowed with natural resource of oil and are fuel exporting. High dependence on natural resources causes the manufacturing sector come down hindering the economic growth. Shift of focus to natural resource extraction and exports may dwindle other sectors of economy. The productivity that is sensitive to the mobility of resources between sectors of economy is affected in this way. Keeping this in view the development patterns in OIC are expected to show varied trends with increasing level of income.

The economy for the purpose of this study is divided into three sectors viz. agriculture, industry and services with industry into its subsector manufacturing to peep deep into the industry sector. ISIC rev 3 is used for the division of activities.

The organization of the paper is formal, following is the OIC in inter-country comparative framework. Next is methodology, econometric procedure and data, elaborating respective details. Results and discussion section follows conclusion.

2. OIC IN INTER-COUNTRY COMPARATIVE FRAMEWORK

Chenery and his associates founded the edifice of patterns of development on Kuznets (1956-67), Clark (1940) and Lewis (1954) work. Kuznets in his seminal work dealt with some aspects of the national accounts, Clark went through the labor divisions. Lewis analyzed the transformation of economy from agrarian to modern sector of manufacturing and industry. Syrquin (1988) includes Fisher (1935,1939) and others in this list but mostly these studies used single process with different frameworks. The need to have a uniform framework analyzing the economic and social indicators to chalk out the stylized facts underlying these patterns compelled Chenery and his associates in the World Bank towards the study on patterns of development for 1950-70 on a uniform framework. Chenery and Syrquin (1975) contains a comprehensive list of indicators for which an attempt is made to explore the stylized facts underlying the patterns of development among countries and over period of time. Syrquin and Chenery (1989) extends the period of Chenery and Syrquin (1975) to 1983 studying long run transformation in allocation of resources and labor allocation. Furthermore Khan and Khan (2016) extends the previous two studies up to 2012 for global patterns of structural change. These three studies will be referred to as 'The Studies' throughout this paper.

Studies conducted for OIC as group on different aspects of production mainly are of descriptive nature. In literature I found SESRIC (2007 and 2012) that provide details on the subdivisions of OIC group regarding production activities. Both these studies mostly use simple averages to compare different subdivisions of OIC at different time periods. These studies analyze data for five year period. Individual countries are also dealt in this way. SESRIC (2014) exclusively deals with different aspects of food security and issues related to agriculture. Some studies like Akkemik (2006) and Shantakumar (1977) use the approach developed by Chenery for inter-country comparative analysis, but that too for an association of countries that include some OIC members or single country in OIC.

This study takes a glimpse of the OIC as a group with the help of a uniform econometric framework. This study uses the model that has the capability to simultaneously explore the dependence of the production variables on income per capita and population size and time shifts in the relationship. The development patterns that associate the changes in the variables of interest with the level of development (GNI per capita for this study) and underlying 'stylized facts' are discussed for the OIC. Cross-section data from 1980 to 2012 covering major economic events like oil price boom and financial meltdown provide picture of long run transformation in OIC countries.

3. METHODOLOGY, ECONOMETRIC PROCEDURE AND DATA

Based on the econometric framework for inter-country comparative analysis the methodology of Chenery and Syrquin (1975 & 1989) is used. The following models explore the relationship between the production sectors and income per capita and population size. Squared terms of these two explanatory variables are used to capture the non-linearities present in the data and this has been done evolutionarily since the adoption of the Houthakker (1957) model.

$$X = \alpha + \beta_1 lnY + \beta_2 \left(lnY\right)^2 + Y_1 lnN + Y_2 \left(lnN\right)^2 + \sum \delta iTi \tag{1}$$

where X is the dependent variable denoting value added production as share of GDP (Va, Vi, Vm and Vs) and labor allocation as share of total work force (La, Li and Ls). Subscripts a, i, m and s denote agriculture, industry, manufacturing and services respectively.

Y and N show GNI per capita (US$2005) and mid-year population in millions of inhabitants respectively, whereas Ti s are time dummies,

α, β's, γ's, and δ's, are the parameters to be estimated.

Model 1 is used to estimate the regression along with the time trend in the various time periods incorporated with the help of OLS method. Time dummies are incorporated to capture the time shift in the economic variables and are assigned value one for the years 1980-1988, 1989-1997, and 1998-2005 in T_1, T_2 and T_3 and zero otherwise respectively. Base period corresponds to 2006-2012. Normal values for this study are taken for a country of size N=20 million at various levels of development. Both the models are run on full and reduced samples.

For relative productivity of labor value added production share is divided by the respected share of labor engaged. The period of this study is 1980-2012. Cross-section data for OIC countries on the variables for production in different sectors are collected. For Table 1a: Descriptive Statistics (Full Sample): Value Added Production in OIC 57 OIC countries only 48[3] fit.

Table 1a
Descriptive Statistics (Full Sample):
Value Added Production in OIC

	Max/min*	Mean	SD	n
Y	47330/135	1929.97	4121.42	
N	246.9/0.6	29.62	45.94	
Va	0.620/0.008	0.23	0.12	1061
Y	47330/160	2107.70	4686.78	
N	246.9/0.6	31.86	48.49	
Vm	0.402/0.026	0.16	0.07	892
Y	47330/135	1959.81	4190.30	
N	246.9/0.6	29.70	46.10	
Vi	0.755/0.069	0.29	0.12	1053
Y	47040/135	1913.93	3948.32	
N	246.9/0.6	29.69	46.08	
Vs	0.808/0.128	0.48	0.11	1055

Notes to Table 1a: *Va: Sierra Leone/UAE; Vm: Cameroon/Sudan; Vi: Libya/Sierra Leone Vs: Lebanon/Sierra Leone, N: Indonesia/Gambia, The, Y: UAE/Mozambique (for all except Vm), for Vm Y: min is Mali One obs. Included (except Vm): Libya, Niger, Saudi Arabia; Kuwait not included; (in Vm): Guinea, Guinea Bissau, Iraq, Kuwait, Libya, Saudi Arabia on the criterion of Kuznets[4]of one million population or more and the availability of data[5] on the variables. Table A6 shows the list of 32 OIC countries for which data is available for both value added production and labor allocation in the sectors of production. Two types of samples are run.

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[3] Excluded countries: Bahrain, Brunei-Darussalam, Comoros, Djibouti, Guyana, Maldives, Qatar, and Suriname are excluded for less than one million population in 2005. Palestine is not included as not listed at WDI.

[4] Kuznets (1971:105)

[5] This study bases on the year 2005 for exclusion on population size criterion.

Table 1b
Descriptive Statistics (Full sample): Labor Allocation

	Max/min*	Mean	SD	n
N	246.9/1	49.0	60.3	
La	0.888/0.017	0.4	0.2	359
Y	44230/249	2571.4	4013.7	
N	246.9/1	49.0	60.3	
Li	0.431/0.021	0.2	0.1	359
Y	44230/249	2571.4	4013.7	
N	246.9/1	49.0	60.3	
Ls	0.805/0.075	0.4	0.1	359

Notes to Table 1b: *La: Burkina Faso/Jordan; Li: Mauritius/Chad; Ls: Jordan/Burkina Faso; N: Indonesia/Gambia (for Li & Ls) and Indonesia/Gabon; Y: UAE/Chad

One obs. Included countries: Benin, Chad, Kuwait, Lebanon, Niger, Nigeria, Togo; Two obs. Included countries: Gabon, Mali, Senegal, Sierra Leone; Saudi Arabia not included. for regression, full and reduced. Reduced sample does not include the income and size upper outliers (UAE and Indonesia) and Kuwait and Saudi Arabia (scanty data available on the variables under consideration). The reduced sample comprises countries with moderate population size and income up to 7000 GNI per capita (US$2005) to portray a picture of developing income group as OIC is termed.

Data on all variables were collected from the World Bank's official website[6]. ISIC Rev 3 is used to differentiate between different sectors of production. Agricultural value added sector corresponds to the categories A and B or 1-5 divisions that include agriculture, hunting, forestry and fishing. Industry corresponds to top level C to F or 10-45 divisions comprising mining and quarrying, manufacturing, electricity, gas and water supply and construction categories. Services is residual from GDP that is GDP minus agriculture and industry. In ISIC rev 3 this sector corresponds to top level G to Q categories. Reference year is taken as 2005.

Tables 1a & b portray the descriptive statistics regarding production and labor respectively. About three times more observations are there in Value added production than in labor allocation. The standard deviation for income per capita (Y) is more or less the same with mean around 2000 and 2500 in value added production and labor allocation respectively.

In value added production Sierra Leone has the least services and industry sector portion in its GDP and it comes out to be with highest agricultural share in GDP (62 percent of GDP) in the OIC countries. The value added manufacturing sector in Cameroon, industry in Libya and services in Lebanon lead the OIC group. Burkina Faso is an OIC country that has maximum labor engaged in agriculture sector and minimum in services sector whereas is Jordan agriculture labor in the least and services labor is maximum up to the level of 80 percent of GDP. Mauritius comes out to be the industry leader in OIC group with maximum 43 percent of labor involved in these activities.

[6] data.worldbank.org/data-catalog/world-development-indicators

4. RESULTS AND DISCUSSION

Figure 1 based on Table (A4) shows that the share of value added industry in OIC goes upward with manufacturing portraying a sharp decline. The sharp declining agriculture sector with comparatively stagnant service sector helps the rise in industry that is other than manufacturing. This pattern is generally followed by the labor allocation with some modifications in the magnitude.

Figure 3 tells that relative labor productivity in industry goes upward beyond 4000 GNI per capita (US$2005). After this point labor engagement in industry becomes stagnant but the value added production goes on rising with the level of development. The difference between value added industry and manufacturing widens as the level of development goes beyond 1000 GNI. In OIC countries with higher level of income are FEC-fuel exporting countries. This phenomenon of declining of manufacturing at the cost of rising of other sectors of industry (mainly mining and quarrying) is indicator of Dutch disease in OIC countries.

Table 2
Basic Regression OIC (Full Sample):
Value Added Production and Labor in Different Sectors

Dep Var and t-ratio	const	lnY	$(\ln Y)^2$	lnN	$(\ln N)^2$	T_1	T_2	T_3	R^2	R^2
Va	1.4055	-0.2535	0.0113	0.0166	-0.0042	0.0223	0.0329	0.0146	0.6678	0.0051
t	17.4694	-11.2527	7.2465	3.1115	-4.251	3.1944	5.0639	2.2872		
Vm	-0.2552	0.1139	-0.008	0.0202	-0.0049	0.014	0.0225	0.0105	0.0782	0.0005
t	-3.056	4.9321	-5.112	3.6026	-4.8636	1.7513	3.3013	1.5961		
Vi	-0.3488	0.107	-0.0027	0.0039	0.0023	-0.0007	0.0069	0.0142	0.3818	0.0091
t	-3.287	3.6052	-1.3162	0.5535	1.731	-0.0783	0.7933	1.6667		
Vs	-0.0489	0.1443	-0.0084	-0.0221	0.0022	-0.0198	-0.0385	-0.0272	0.1135	0.0100
t	-0.4393	4.6295	-3.9169	-2.9492	1.614	-2.0242	-4.2107	-3.0381		
La	2.258	-0.424	0.0212	-0.0055	0.0038	0.0299	0.0202	0.0303	0.4382	0.0192
t	6.7921	-4.7546	3.5529	-0.278	1.1522	1.115	0.9526	1.5728		
Li	-0.7685	0.2135	-0.0109	-0.0056	0.001	0.0287	0.0372	0.0026	0.3838	0.0041
t	-4.976	5.1522	-3.9219	-0.614	0.6913	2.3057	3.7735	0.2938		
Ls	-0.6365	0.2438	-0.0122	0.0066	-0.0037	-0.0621	-0.0632	-0.0303	0.4004	0.0106
t	-2.5782	3.6816	-2.7482	0.454	-1.5455	-3.122	-4.0098	-2.1187		

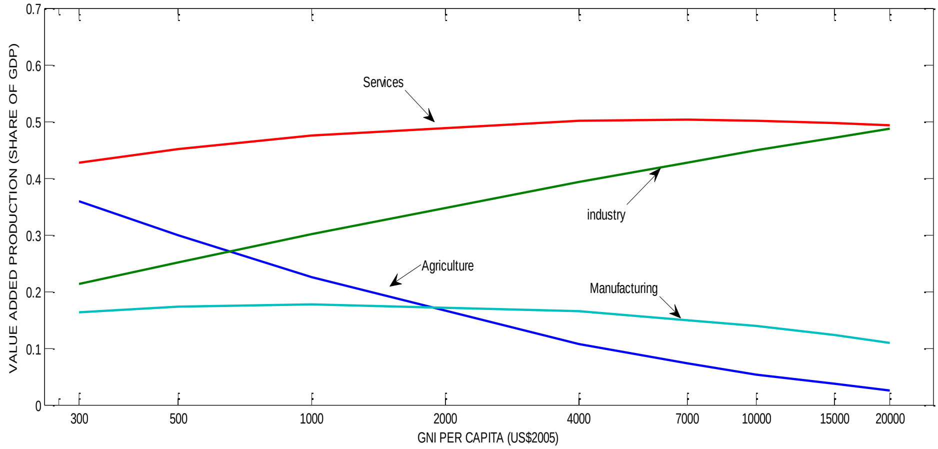

Figure 1: OIC: Value Added Production in Different Sectors 1980-2012

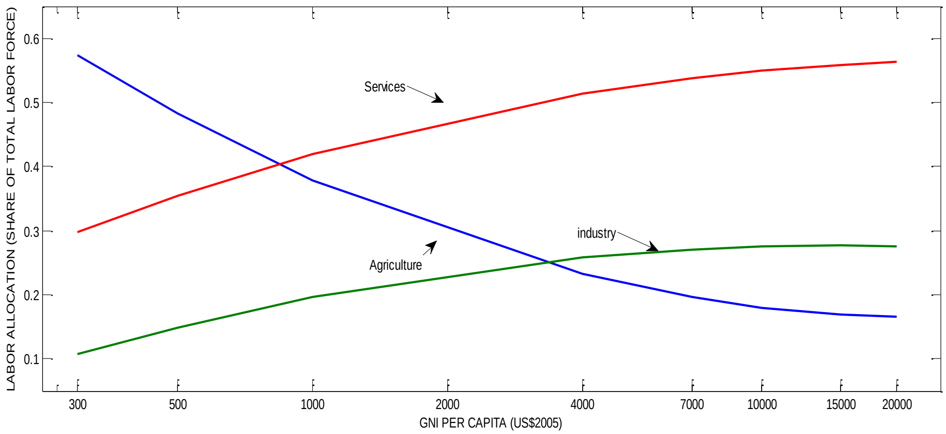

Figure 2: OIC: Labor Allocation in Different Sectors of Production 1980-2012

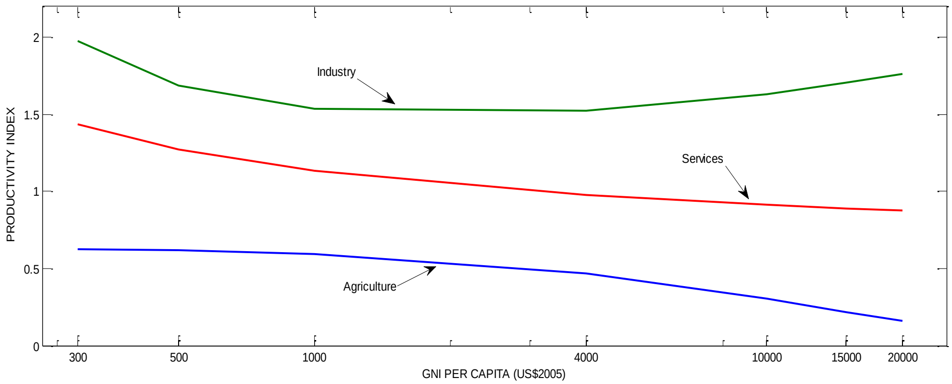

Figure 3: OIC: Average Relative Labor Productivity 1980-2012

Agriculture sector goes down in both labor and value added, but engagement of more labor than the output in terms of value added production fetches the productivity down for the rich economies of OIC group. This trend of productivity indicates the low levels of technology employed in production activities in rich OIC countries. Many of the rich OIC members are leading food importers as the picture portrayed by the OIC is of a net importer of food (SESRIC2014). The declining agriculture sector surpasses the rising industry at 26 percent of the GDP at about 600 GNI per capita (US$2005) level of income. This confirms the benchmark observed in the literature (Chenery and Syrquin (1975 and 1989)).

Value Added Production in OIC

Table 2 shows that in OIC value added production with respect to income is far more pronounced than size effects and time shifts. Model is robust as shown by the low mse values in Table 2 but except for the agriculture sector in OIC the explanatory variables could not explain the dependent variable very much, as shown by R^2 . Time dummies reflect stagnant behavior of relationship with respect to time.

Within industry sector manufacturing could not appreciate beyond 1000 level of income but industry that comprises manufacturing and other subsectors increases throughout the transition widening the gap between the two as the level of development increases (see Figure 1). Agriculture declines sharply till 4000 level of development. For higher level of development the decline in agriculture and services sector is offset by the increasing industry throughout the transition.

The widening gap between industry and manufacturing for both samples with respect to development shows that the other sub sectors of industry crowd out manufacturing. Dutch disease, that shows the decline of manufacturing due to natural resource discovery, is prevalent in OIC. Manufacturing diminishes after 1000 GNI per capita. For the full sample the manufacturing sector dwarfs to only one fifth of the industry. Full sample includes Saudi Arabia, Kuwait and UAE (all above 13000 GNI per capita level of income) and these are tagged as fuel exporting country (FEC) in IMF Outlook (2007). In literature on natural resource curse (Sachs, J. D. 2001; Auty 1990, and Sachs and Warner 1995, 1999) the lowering of manufacturing sector is held responsible for the hindrance to growth. In OIC manufacturing sector moves in a relatively narrower band of 5 percent (16 to 11 percent) whereas the industry moves in a wider band of 21 to 49 percent of GDP throughout the transition (Figure 1 & Table A4). Rowthron (1994) cross-sectional study on seventy countries portrays an inverse U-shaped curve for manufacturing sector in both value added production and labor engaged. This pattern of industry in OIC is contrasted with the global pattern in this sector. In 'The Studies' ,for the rich countries the value added industry declines beyond certain point and this is happening earlier in the transition as the time moves away from the post war era.

Value added agriculture in OIC countries moves downward with the level of development. Total decline throughout the transition is 33 percent, but it is more prominent from 300 to 4000 GNI per capita 15 percent of GDP from the initial income level (Figure 1). Then the decline slows down gradually with income level beyond 4000 per capita and only 8 percent of GDP decrease in agriculture up to 20000 GNI per capita is noted. Meager levels of agricultural production (5-3 percent at 10000-20000 per capita) in rich OIC are noted.

The declining trend in agriculture sector in OIC is in line with the global patterns. ‘The Studies’ show similar trends. One important benchmark that is confirmed in this study is the intersection of declining agriculture and rising industry value added at 26 percent of GDP. For OIC it happens at about 600 GNI per capita (US$2005). In post war era this appears at 300 level of income (US$ 1964) (Chenery (1975)) which turns out to be at 1500 (US$ 1980) in Syrquin (1989) and at 750 GNI per capita (US$2005) in Khan (2016) for global pattern. SESRIC (2007 and 2012) post a grim picture of agriculture in OIC especially at higher levels of income per capita. SESRIC (2014) regards this group of countries as net importer of food products with Saudi Arabia and UAE, the income leaders in our study, as leading food importers. This study also confirms the low share of high income OIC members in agriculture sector.

For rich countries in OIC value added services posts a behavior like one at the onset of the post war era. Value added services as share of GDP increases with income per capita from 43 percent of GDP at 300 level of development to 50 percent at 4000 income level. It stalls at this share of output for 10000 GNI per capita and then shows a decline of just one percent of GDP at 49 percent for the final level of development in Figure 1. The reduced sample shows an increase of 2 percent over the full sample (52 percent of GDP) at 4000 and 6000 level of development. The decrease in both value added agriculture and services at the higher level of development has allowed the industry in OIC to expand throughout the transition generally.

Chenery (1975) observes the declining trend for services sector for rich countries as for the OIC in this study. Fuchs (1969) concludes that for advanced countries the phenomenon of declining services sector is a sign of comparatively low rates of technological progress in this sector and greater difficulty for substitution of labor for capital. The patterns of production in services in OIC are similar to those at the onset of the post war era. With the ease in the substitution of capital with labor, the services sector as shown in the subsequent global patterns (Syrquin (1989)), divergence in the value added production and labor engaged may vanish in OIC with both increasing for the higher levels of development.

Labor Allocation

Comparing Figures 1 and 2 it is noticed that the trend of labor allocation in OIC generally follows the behavior of the value added production in the group. Perusing Figure 2 it is revealed that labor in agriculture falls from 57 percent of GDP at initial income level to 23 percent at 4000 GNI per capita (US$2005) and thereafter comes down very slowly declining only 6 percentage points (16 percent of total labor force at 20000 GNI per capita) for the richest countries in OIC, almost steady after 10000 level of development.

Labor industry in OIC remains stagnant after 4000 GNI per capita (US$2005). It starts initially engaging 11 percent of workforce at 300 level of development. At 4000 income level industry sector could absorb only a quarter of the labor force. In OIC the countries above 7000 level of income are fuel exporting. Industry is dominant sector in these countries with pronounced mining and quarrying sector. Lesser use of labor fetching more value added production hints towards capital intensive industry in rich OIC countries.

After 4000 level of development mark the services sector could only provide half of the jobs in OIC countries and becomes stagnant afterwards in the transition. This is contrasting with the engagement of labor globally in this sector of production where the labor allocation is increasing from the initial to final level of income. As discussed above the patterns of services sector resemble the one at the onset of post war era.

Labor allocation beyond 4000 level of development moves in a narrow band comparative to the low and middle income countries. Reduction of the sample size affects the behavior of employment in OIC, showing stagnation beyond 4000 level of income. The patterns of employment look similar to that of Chenery and Syrquin (1975). Keeping in view trend of value added production similar to that at the onset of post war era; it is pertinent to look for the behavior of productivity in OIC that is discussed below.

Relative Labor Productivity

Productivity in agriculture sector in OIC comes down steadily with increasing level of development. The decline is steep for the middle to high income group of countries. This is in contrast of the normal behavior of the rich countries of the world. Chenery and Syrquin (1975) and Syrquin and Chenery (1989) for the period 1950-83 show upward trend of relative labor productivity in world richest economies. An explanation given by Chenery and Syrquin(1975) for this upward trend of rich countries is the technological advancement in the agricultural arena, is not apparent in OIC pattern as the productivity in agriculture sector decreases 30 percent (0.46 to 0.15) beyond 4000 level of income as shown in Figure 3 (Table A4). Low labor productivity along with a very low share of value added production in agriculture sector post a picture of OIC as of a net importer of food product that poses the danger of national food insecurity in the group including the rich countries in OIC. Shifting the focus towards the technological change in agricultural sector would help out nations of OIC grow economically with better food security conditions in the group.

In contrast to the agriculture sector the productivity in industry goes up after declining for the middle income range and increases steadily for the richest countries. Figure 3 shows that the decrease in productivity in industry is apparent till 1000 GNI per capita and higher income countries of OIC till 4000 GNI per capita remain stagnant but afterwards it records 15 percent increment on productivity in rich OIC countries. Rich OIC countries are mainly fuel exporting like Saudi Arabia, Kuwait and UAE. The rich countries of the world as a whole depend less on mining and quarrying sector[7] whereas OIC group's rich countries are mainly dependent on other than manufacturing in industry sector that includes mining and quarrying also. It is evident that manufacturing beyond 1000 level of development decreases in contrast to the increasing industry sector

[7] Syrquin (1989: 20) reveals that in fact the mining subsector of industry starts diminishing from 8percent at 1000 GNP per capita (US$ 1980) to only one percent for the richest countries in the world.

throughout the transition. Relative labor productivity in industry sector in the leading slot for this study compared to the services sector in 1950-70 post war era.

In OIC relative labor productivity in services sector follows the general pattern of declining nature with increasing level of development, but after 4000 GNI per capita it moves in a very narrow band of 10 percent from 0.98 to 0.88. The pattern is similar to the global[8] one but with narrower range of productivity beyond 4000 level of development.

Baumol law (Baumol 1967) explains declining services productivity in industrialized countries with rising share of value added services and holds responsible labor intensive services for the slowdown of productivity in services. This argument is contrasted by the findings of Szirmai (2009) that many services for industry sector have experienced significant productivity improvements through the diffusion of information and communication technologies (ICT). The net effect of rising share of services on labor productivity thus requires deeper and disaggregated analysis of the sector.

Comparison of expected[9] and actual relative labor productivity in different sectors in Table 3 reveals that of all countries in the study Bangladesh and Sierra Leone find special place as they post below expectation and above expectation productivity respectively in all sectors of production under consideration. Lebanon leads the agriculture sector but is a laggard in industry productivity. Mozambique and Burkina Faso go ahead of all OIC members in industry and services. Azerbaijan and Gabon are well above the expectations as per their income and population size in industry but the relative labor productivity in services sector in these countries drops these countries down in the list of the OIC group. Agriculture productivity difference in Azerbaijan, Mozambique and Kazakhstan is at the peak of the OIC group. Mozambique records more than the expected productivity in industry with Burkina Faso and Azerbaijan and Gabon next to it. The countries with least productivity in industry are Niger and Lebanon.

As pointed out in Ark and McGuckin (1999) the policy analyst must look at the range of issues as no two countries productivity can be compared on the same footings. Like if one has relatively low productivity due to the greater use of low-skilled labor then the other’s increase of labor participation might reduce the productivity while increasing per capita income. It may also be the comparatively higher substitution of capital for labor that has increased the level of productivity.

Whatever be the issue this measure provides a beneficial and important benchmark analysis. But policy makers should take care in devising the policy in this respect.

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[8] In Chenery (1975) and Syrquin (1989) the productivity in services sector declines up to 30 and 50 percent (author’s calculations) respectively from middle to rich income countries.

[9] Expected productivities are calculated with the help of estimated models for value added production and labor (appearing in table A2 for full sample) for the corresponding income and size of respective country for 2005 or noted year in table 3.

Table 3
Productivity Difference Between Actual* and Expected**

Country	Y	N	Agriculture	Industry	Services
Albania	2670	3.2	0.18	-0.20	-0.97
Algeria[b]	2800	33.5	-0.01	-0.57	0.36
Azerbaijan	1380	8.4	0.36	-3.81	0.54
Bangladesh	440	143.1	0.09	0.06	0.09
Benin[a]	530	7.7	-0.11	0.12	0.08
Burkina Faso	410	13.4	0.18	-4.08	-2.21
Cameroon	890	18.1	0.25	-0.61	-0.51
Egypt, Arab Rep.	1250	71.8	0.01	-0.06	0.12
Gabon	5590	1.4	0.23	-3.80	0.47
Indonesia	1210	224.5	0.08	-0.80	0.20
Iran, Islamic Rep.	2690	70.2	-0.01	0.11	0.05
Jordan	2390	5.4	-0.33	0.04	0.13
Kazakhstan	3400	15.1	0.30	-0.73	-0.08
Kyrgyz Republic	460	5.2	-0.18	0.29	0.25
Lebanon[d]	6560	4.2	-0.53	0.67	-0.13
Malaysia	5310	25.8	-0.16	0.00	0.14
Mali[c]	450	12.3	0.08	-2.62	-0.09
Mauritius	5050	1.2	-0.17	0.53	-0.14
Morocco	1920	30.1	0.20	0.08	-0.57
Mozambique[a]	270	19.9	0.29	-5.39	-1.46
Niger	260	13.2	0.21	0.67	-0.45
Nigeria[b]	710	136	-0.29	-1.91	0.77
Pakistan	710	158	-0.03	0.43	-0.05
Sierra Leone[b]	310	4.9	-0.10	-0.20	-0.01
Syrian Arab Republic[b]	1410	17.3	-0.37	0.35	0.14
Tajikistan[b]	310	6.7	0.26	-0.19	-0.24
Togo[c]	380	5.7	-0.01	-1.06	0.12
Tunisia	3060	10	0.00	0.56	-0.24
Turkey	7040	67.7	-0.12	0.50	-0.36
Uganda	300	28.7	0.24	-3.54	-0.62
United Arab Emirates	44230	4.1	-0.21	0.51	0.06
Yemen, Rep.[b]	730	19.6	0.23	-1.19	0.37

* Actual productivity corresponds to the year 2005 except for indicated years; a=2003, b=2004, c=2006, d=2009.

** expected productivity corresponds to the year 2005 (or indicated year) for that particular country’s income and size using the estimated models for value added production and labor in Table A2

5. CONCLUSION

Agriculture sector's downward trend with respect to the income per capita is in line with the global pattern in this regard. But the declining productivity for rich OIC members poses a question mark on the technological access to these countries. The downing of manufacturing and upward trend in industry has its peculiar repercussions on the growth of the OIC countries.

Further investigation as regards to the industry sector in relation with the manufacturing one is needed to ascertain that whether there is 'natural resource curse' crowding out the growth of OIC. The declining trend of manufacturing at early stage of development in OIC requires in depth investigation as to what is crowding it out. It may be a topic of future research on OIC. Declining value added production in services sector for the middle to upper income countries with more labor engaged to produce it reducing the productivity of services beyond a certain level of development is in line with the global trend.

As a matter of fact this study provides the basis for further exploration on the topic. These results depict the general behavior of only half of the OIC countries as rest of the half countries could not be included in the study. Labor data for OIC compared to the value added production is in lesser quantity. Better data coverage would certainly enhance the reliability of results.

REFERENCES

1. Akkemik, K.A. (2006). Patterns of industrialisation, structural changes and productivity in Turkish manufacturing (1970-2000). *Journal of Economic Cooperation*, 27(1), 71-88.
2. Ark, B.V. and R.H. McGuckin. (1999). International comparisons of labor productivity and per capita income. *Monthly Labor Review,* 33-41. http://www.bls.gov/mlr/1999/07/art3full.pdf
3. Auty, R.M. (1990). *Resource-based industrialization: Sowing the oil in eight developing countries.* Oxford University Press, USA.
4. Baumol, W.J. (1967). Macroeconomic of unbalanced growth: the anatomy of urban crises. *The American Economic Review.*57(3), 415-126.
5. Chenery, H.B. and Syrquin, M. (1975). *Patterns of Development, 1950-70.* London: OUP.
6. Clark, C. (1940). *The Conditions of Economic Progress.* London: Macmillan.
7. Fisher, A.G.B. (1935). *The Clash of Progress and Security*. London: Macmillan.
8. Fisher, A.G.B. (1939). Production, primary, secondary, and tertiary. *Economic Record*, 15(1), 24-38.
9. Fuchs, V.R. (1969). *Production, and productivity in the services industry.* New York: National Bureau of Economic Research, distributed by Columbia University Press.
10. Houthakker, H.S. (1957). An international comparison of household expenditure patterns: commemorating the centenary of Engel's law. *Econometrica,* 25(4), 532-551.
11. IMF (2007). *World Economic Outlook (WEO) Database*. http://www.imf.org/external/pubs/ft/weo/2006/01/data/index.htm

12. Lewis, W.A. (1954). Economic Development with Unlimited Supplies of Labor. *Manchester School of Economics and Social Studies*, 22, 139-191.
13. Kuznets, S. (1956-67). Quantitative aspects of economic growth of nations. *Economic Development and Cultural Change.* (A series of ten articles)
14. Kuznets, S. (1971). *Economic Growth of Nations: Total Output and Production Structure.* Cambridge, MA, Harvard University Press.
15. Rowthron, R. (1994). Korea at the cross-roads. Working Paper, No. 11. ESRC Centre for Business Research, Cambridge University.
16. Sachs, J.D. and A.M. Warner (1995). *Natural resource abundance and economic growth.* National Bureau of Economic Research Working Paper No. 5398, Cambridge, MA.
17. Sachs, J.D. and A.M. Warner (1999). The big push, natural resource booms and growth. *Journal of Development Economics,* 59(1), 43-76
18. Sachs, J.D. and A.M. Warner (2001). The curse of natural resources. *European Economic Review*, 45(4-6), 827-38. http://mais611.wiki.usfca.edu/file/view/Sachs+and+Warner+Curse+of+Natural+Resources.pdf
19. Shantakumar, G. (1977). Post war development patterns of the ASEAN economies. *The Malayan Economic Review*, XXII (1), April: 88-105.
20. Statistical, Economic and Social Research and Training Centre for Islamic Countries (SESRIC). (2007). *Annual economic report on the OIC countries 2007*. Ankara. http://www.sesrtcic.org/files/article/263.pdf
21. Statistical, Economic and Social Research and Training Centre for Islamic Countries (SESRIC). (2012). *Annual economic report on the OIC countries 2012*. Ankara. http://www.sesrtcic.org/files/article/454.pdf
22. Statistical, Economic and Social Research and Training Centre for Islamic Countries (SESRIC). (2014). *Agriculture and Food Security for OIC Member Countries 2014*. Ankara. http://www.sesrtcic.org/files/article/483.pdf
23. Syrquin, M. and Chenerym H.B. (1988). Patterns of Structural Change. In H.B. Chenery and T.N. Srinivasin (eds). *Handbook of Development of Economics I.* Elsevier Publisher B.V. 201-271.
24. Syrquin, M. and H.B. Chenery, (1989). Patterns of Development 1950-83. World Bank Discussion Paper: 41. Washington D.C. The World Bank. http://www-wds.worldbank.org/external/default/WDSContentServer/IW3P/IB/2000/07/07/000178830_98101903571311/Rendered/PDF/multi_page.pdf
25. Szirmai, A. (2009). *Industrialization as an engine of growth in developing countries*, 1950-2005. http://www.merit.unu.edu/about/profile.php?id=752&stage=2#pub
26. *World Bank data* available at: data.worldbank.org

APPENDICES

Table A1
Basic Regression OIC (Reduced sample*):
Value Added Production and Labor in Different Sectors

Dep Var and t-ratio	const	lnY	$(\ln Y)^2$	lnN	$(\ln N)^2$	T_1	T_2	T_3	R^2	mse
Va	1.4329	-0.2498	0.0108	0.0006	-0.0016	0.0108	0.0289	0.0033	0.6842	0.0054
t	11.3884	-6.7344	4.0625	0.078	-1.2152	1.3129	3.8402	0.445		
Vi	-0.426	0.1358	-0.0052	0.0089	0.0001	0.0154	0.0146	0.0197	0.3173	0.0094
t	-2.5677	2.7746	-1.4955	0.9089	0.0592	1.4214	1.4546	1.9935		
Vm	-0.2066	0.1171	-0.0087	0.0033	-0.0031	0.0083	0.019	0.0112	0.0776	0.0053
t	-1.4733	2.8197	-2.9575	0.3987	-2.125	0.8445	2.3637	1.4255		
Vs	0.0167	0.1068	-0.005	-0.0115	0.002	-0.0246	-0.0418	-0.0213	0.1759	0.0098
t	0.0984	2.1329	-1.3969	-1.1513	1.1076	-2.2119	-4.0816	-2.0998		
La	3.5599	-0.8053	0.0478	0.0333	-0.0041	0.0346	0.0237	0.0406	0.412	0.0203
t	6.7359	-5.4542	4.7238	1.3956	-0.9531	1.1688	1.0143	1.9275		
Li	-1.1134	0.3186	-0.0182	-0.0349	0.0073	0.0332	0.0373	-0.0026	0.4107	0.0041
t	-4.7053	4.8201	-4.0098	-3.268	3.7608	2.5023	3.5673	-0.2743		
Ls	-1.5833	0.5191	-0.0315	-0.0014	-0.0023	-0.0745	-0.0696	-0.0365	0.3711	0.0113
t	-4.0124	4.709	-4.1743	-0.0812	-0.73	-3.3715	-3.9903	-2.3226		

* Not included: Indonesia and UAE (both are size and Income outliers respectively) and Kuwait and Saudi Arabia.

Table A2
Regression for Normal Variation in OIC (full sample): Production and Labor

Dependent variable and t-ratios	Const	lnY	$(\ln Y)^2$	lnN	$(\ln N)^2$	R^2	mse
Va	1.4265	-0.2512	0.011	0.0151	-0.0041	0.6593	0.0053
	17.5851	-11.0346	6.9773	2.8118	-4.1177		
Vm	-0.234	0.1136	-0.0081	0.0189	-0.0048	0.0666	0.005
	-2.8023	4.8953	-5.142	3.393	-4.7807		
Vi	-0.3411	0.1061	-0.0026	0.0052	0.002	0.3793	0.0091
	-3.2193	3.5745	-1.2857	0.7376	1.5749		
Vs	-0.0724	0.1419	-0.0081	-0.022	0.0023	0.0977	0.0102
	-0.646	4.5234	-3.7519	-2.9389	1.7074		
La	2.3326	-0.4343	0.0216	-0.0066	0.0039	0.4339	0.0191
	7.0899	-4.8847	3.6276	-0.3343	1.1893		
Li	-0.7324	0.2118	-0.011	-0.0075	0.0014	0.3469	0.0043
	-4.672	4.9996	-3.8883	-0.8004	0.9261		
Ls	-0.7473	0.2551	-0.0123	0.0098	-0.0043	0.3682	0.011
	-2.9914	3.7778	-2.729	0.6611	-1.745		

Table A3
Regression for Normal Variation in OIC (reduced sample): Production and Labor

Dependent variable and t-ratios	Const	lnY	$(\ln Y)^2$	lnN	$(\ln N)^2$	R^2	mse
Va	1.4265	-0.2512	0.011	0.0151	-0.0041	0.6593	0.0053
t	17.5851	-11.0346	6.9773	2.8118	-4.1177		
Vm	-0.234	0.1136	-0.0081	0.0189	-0.0048	0.0666	0.005
t	-2.8023	4.8953	-5.142	3.393	-4.7807		
Vi	-0.3411	0.1061	-0.0026	0.0052	0.002	0.3793	0.0091
t	-3.2193	3.5745	-1.2857	0.7376	1.5749		
Vs	-0.0724	0.1419	-0.0081	-0.022	0.0023	0.0977	0.0102
t	-0.646	4.5234	-3.7519	-2.9389	1.7074		
La	2.3326	-0.4343	0.0216	-0.0066	0.0039	0.4339	0.0191
t	7.0899	-4.8847	3.6276	-0.3343	1.1893		
Li	-0.7324	0.2118	-0.011	-0.0075	0.0014	0.3469	0.0043
t	-4.672	4.9996	-3.8883	-0.8004	0.9261		
Ls	-0.7473	0.2551	-0.0123	0.0098	-0.0043	0.3682	0.011
t	-2.9914	3.7778	-2.729	0.6611	-1.745		

Table A4
Normal Variation in Production and Labor with Productivity at Different Levels of Development in OIC (Full Sample)

GNI per capita	Va	Vi	Vm	Vs	La	Li	Ls	Pa	Pi	Ps
300	0.360	0.213	0.164	0.428	0.573	0.108	0.298	0.628	1.974	1.435
500	0.299	0.251	0.173	0.451	0.483	0.149	0.354	0.618	1.686	1.276
1000	0.225	0.301	0.178	0.476	0.378	0.196	0.419	0.593	1.538	1.137
4000	0.108	0.394	0.165	0.502	0.232	0.258	0.513	0.467	1.527	0.978
7000	0.073	0.428	0.150	0.504	0.196	0.271	0.538	0.374	1.581	0.937
10000	0.054	0.449	0.139	0.502	0.180	0.275	0.550	0.302	1.631	0.914
15000	0.037	0.472	0.123	0.498	0.169	0.277	0.559	0.216	1.704	0.890
20000	0.026	0.488	0.110	0.493	0.165	0.276	0.563	0.158	1.766	0.875

Table A5
Normal variation in Production and Labor with Productivity at Different Levels of development in OIC (Reduced Sample)

GNI per capita	Va	Vi	Vm	Vs	La	Li	Ls	Pa	Pi	Ps
300	0.358	0.218	0.169	0.421	0.616	0.087	0.280	0.581	2.505	1.503
500	0.295	0.256	0.175	0.445	0.494	0.140	0.354	0.598	1.829	1.257
1000	0.219	0.302	0.176	0.474	0.368	0.195	0.431	0.596	1.549	1.101
2000	0.153	0.342	0.168	0.500	0.287	0.231	0.480	0.534	1.483	1.041
4000	0.097	0.377	0.152	0.521	0.250	0.247	0.502	0.388	1.524	1.037
6000	0.097	0.377	0.152	0.521	0.250	0.247	0.502	0.388	1.524	1.037

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DISCRIMINATING BETWEEN WEIBULL, BURR TYPE XII AND GENERALIZED EXPONENTIAL DISTRIBUTIONS WITH PROPORTIONAL HAZARDS

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ABSTRACT

Weibull, Burr type XII and generalized exponential distributions are three non-negative skewed distributions that are often used for modeling failure time data. The paper deals with some diagnostic tools of goodness of fit of these distributions with proportional hazards model of random censorship. We consider maximum likelihood ratio test, minimum Kolmogorov distance and Kaplan-Meier survival curve to discriminate among these models. Two real data sets are used for comparison purposes.

1. INTRODUCTION

There are several censoring mechanisms that are used in failure time studies to reduce the experimental time and cost. The most popular among these are the right censoring schemes because of their crucial importance in reliability and life testing experiments. Of the three types of right censoring we deal with random censoring in this paper. In random censoring the time of censoring is not fixed but depends on other random factors which are modeled by an independent random variable known as censoring time variable. In medical investigations, for example, the patients often enter into the study after some surgical operation and hence the enrolment process is random. Therefore, the censoring time that is the lapse between an individual's entry into the study and the termination of the study, is random. A nice account of censoring schemes can be seen in Miller (1981), and Kalbfleisch and Prentice (2002).

In the analysis of failure time data, an investigator is frequently faced with the problem of choosing an underlying parametric model. The lack of the general theory for the choice of an underlying parametric distribution has resulted in the usage of distributions which are not necessarily robust to the possible alternative distributions. The exponential distribution is often used in life testing and reliability studies because of its simplicity and its inherent relation to the well-defined theory of Poisson processes. However, the investigations by Hager, Bain and Antle (1971) concerning the robustness of the assumed exponential distribution when the true underlying distribution belongs to the Weibull family have shown that these widely used statistical techniques are very sensitive to departures from exponentiality. Recently, the Weibull, Burr type XII and generalized exponential (GE) distributions have emerged as the most popular distributions in failure time studies. The two parameters of these distributions denote the shape and scale parameters and because of shape and scale parameters these have a quite flexibility for modeling any general lifetime data. Unlike the exponential distribution which has only a constant failure rate, these distributions can have increasing, decreasing as well as constant failure rates.

The rest of the paper is organized as follows. In the next section, we define the model and its assumptions. Then we derive the models assuming the Weibull, Burr type XII and GE distributions in Sections 3-5. The proposed model fitness measures are discussed in Section 6. Finally, two real data illustrations are presented in Section 7.

2. THE MODEL AND ASSUMPTIONS

Let $X_1,...,X_n$ be independent and identically distributed (i.i.d.) random variables with distribution function $F_X(t)$ and density function $f_X(t)$. Consider another sequence, independent of $\{X_i\}$, $T_1,...,T_n$ of i.i.d. random variables with distribution function $F_T(t)$ and density function $f_T(t)$. In the terminology of life-testing experiments, X_i 's are the true life times of n individuals censored by T_i 's from the right so that we observe i.i.d. random pairs $(Y_1,D_1),...,(Y_n,D_n)$, where $Y_i=\min(X_i,T_i)$ and $D_i=\mathrm{I}(X_i\le T_i)$ is the indicator of non-censored observation for $1\le i\le n$. Thus the observed $Y_i's$ constitute a random sample from $F_Y(t)$, where $1-F_Y(t)=(1-F_X(t))(1-F_T(t))$. This is the usual model of random censorship studied by Kaplan and Meier (1958). Under this model, Kaplan and Meier introduced their historic product limit estimator of the survival function given by

$$S(y)=\prod_{i:Y_i\le t}\left(\frac{n-R_i}{n-R_i+1}\right)^{d_i}.$$

Now it is simple to show that the joint density function of Y and D is

$$f_{Y,D}(y,d)=\{f_X(y)(1-F_T(y))\}^d\{f_T(y)(1-F_X(y))\}^{1-d}. \tag{1}$$

In some situations the censoring time variable T is related to survival time variable X in terms of distribution functions. Koziol and Green (1976) introduced a special model assuming

$$1-F_T(y)=\{1-F_X(y)\}^{\beta}, \tag{2}$$

for some positive constant β. In case of relation (2), the joint density in (1) reduces to

$$f(y,d)=f_X(y)\{1-F_X(y)\}^{\beta}\beta^{1-d}. \tag{3}$$

2.1 Weibull Model

The density and distribution functions of the Weibull distribution are

$$f_X(x;\theta,\lambda)=\theta\lambda x^{\theta-1}e^{-\lambda x^{\theta}};\quad x>0,\ \theta,\lambda>0 \tag{4}$$

$$F_X\left(x;\theta,\lambda\right)=1-e^{-\lambda x^{\theta}}. \tag{5}$$

Using (4) and (5) in (3), the joint density of Y and D can be written as

$$f_{Y,D}\left(y,d;\theta,\lambda,\beta\right)=\theta\lambda y^{\theta-1}e^{-(1+\beta)\lambda y^{\theta}}\beta^{1-d}. \tag{6}$$

For an observed sample $\left(y_1,d_1\right),...,\left(y_n,d_n\right)$, the likelihood function can be written as

$$l\left(\boldsymbol{\theta}\right)=\theta^{n}\lambda^{n}\beta^{n-I}e^{-(1+\beta)\lambda\sum_{i=1}^{n}y_i^{\theta}}\prod_{i=1}^{n}y_i^{\theta-1}, \text{where } \boldsymbol{\theta}=\left(\theta,\lambda,\beta\right) \text{ and } I=\sum_{i=1}^{n}d_i\ .$$

The corresponding log-likelihood function, say $L\left(\boldsymbol{\theta}\right)$, is

$$L\left(\boldsymbol{\theta}\right)=n\ln\theta+n\ln\lambda-\left(1+\beta\right)\lambda\sum_{i=1}^{n}y_i^{\theta}+\left(n-I\right)\ln\beta+\left(\theta-1\right)\sum_{i=1}^{n}y_i. \tag{7}$$

2.2 Burr Type XII Model

The density and distribution functions of Burr type XII distribution are

$$f_X\left(x;\theta,\lambda\right)=\theta\lambda x^{\lambda-1}\left(1+x^{\lambda}\right)^{-\theta-1} \quad ;\ x>0,\ \theta,\lambda>0 \tag{8}$$

$$F_X\left(x;\theta,\lambda\right)=1-\left(1+x^{\lambda}\right)^{-\theta}. \tag{9}$$

Using (8) and (9) in (3), we have the joint density of Y and D as

$$f_{Y,D}\left(y,d;\theta,\lambda,\beta\right)=\theta\lambda y^{\lambda-1}\left(1+y^{\lambda}\right)^{-\theta(1+\beta)-1}\beta^{1-d}; y>0, d=0,1. \tag{10}$$

For an observed sample $\left(y_1,d_1\right),...,\left(y_n,d_n\right)$, the likelihood function is

$$l\left(\boldsymbol{\theta}\right)=\theta^{n}\lambda^{n}\prod_{i=1}^{n}y_i^{\lambda-1}\prod_{i=1}^{n}\left(1+y_i^{\lambda}\right)^{-\theta(1+\beta)-1}\beta^{n-I}.$$

The corresponding log-likelihood function is

$$L\left(\boldsymbol{\theta}\right)=n\ln\theta\lambda+\left(\lambda-1\right)\sum_{i=1}^{n}\ln y_i+\left(n-I\right)\ln\beta-\left(\theta+\theta\beta+1\right)\sum_{i=1}^{n}\ln\left(1+y_i^{\lambda}\right). \tag{11}$$

2.3 Generalized Exponential Model

The density and distribution functions of GE distribution are

$$f_X\left(x;\theta,\lambda\right)=\theta\lambda\left(1-e^{-\lambda x}\right)^{\theta-1}e^{-\lambda x};\quad x>0,\lambda>0,\theta>0 \tag{12}$$

$$F\left(x;\theta,\lambda\right)=\left(1-e^{-\lambda x}\right)^{\theta}. \tag{13}$$

Now using (12) and (13) in (3), the joint density function of Y and D is

$$f_{Y,D}\left(y,d;\theta,\lambda,\beta\right)=\theta\lambda\left(1-\left(1-e^{-\lambda y}\right)^{\theta}\right)^{\beta}\left(1-e^{-\lambda y}\right)^{\theta-1}e^{-\lambda y}\beta^{1-d}. \tag{14}$$

For an observed sample $\left(y_1,d_1\right),\ldots,\left(y_n,d_n\right)$, the likelihood function is

$$l\left(\boldsymbol{\theta}\right)=\theta^{n}\lambda^{n}\beta^{n-I}e^{-\lambda\sum_{i=1}^{n}y_i}\prod_{i=1}^{n}\left(1-\left(1-e^{-\lambda y_i}\right)^{\theta}\right)^{\beta}\left(1-e^{-\lambda y_i}\right)^{\theta-1}.$$

The corresponding log-likelihood function is

$$\begin{aligned}L\left(\boldsymbol{\theta}\right)=n\ln\theta+n\ln\lambda+\left(n-I\right)\ln\beta-\lambda\sum_{i=1}^{n}y_i+\left(\theta-1\right)\sum_{i=1}^{n}\ln\left(1-e^{-\lambda y_i}\right)\\+\beta\sum_{i=1}^{n}\ln\left(1-\left(1-e^{-\lambda y_i}\right)^{\theta}\right).\end{aligned} \tag{15}$$

3. MODEL FITNESS MEASURES

Here we use the likelihood ratio test (LRT) and Kolmogorov distance (KD) criteria to discriminate between the three proposed models.

3.1 LRT Criterion

Assume that we observe a sample $\left(y_1,d_1\right),\ldots,\left(y_n,d_n\right)$ that is supposed to belong to one of the two models, say, Weibull or GE. The LRT statistic, also called Cox statistic, in this case will be

$$T=\ln\left(\frac{l_{Weibull}\left(\hat{\boldsymbol{\theta}}\right)}{l_{GE}\left(\hat{\boldsymbol{\theta}}\right)}\right)=L_{Weibull}\left(\hat{\boldsymbol{\theta}}\right)-L_{GE}\left(\hat{\boldsymbol{\theta}}\right), \tag{16}$$

where $L\left(\hat{\boldsymbol{\theta}}\right)$ is the log-likelihood function evaluated at the maximum likelihood (ML) estimate $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$. The decision rule is then to choose Weibull model if $T>0$ otherwise choose GE model. Similarly, the Cox statistic and decision rule for comparing other models can be defined. For the ML estimation of unknown parameters of the models under discussion, the readers are referred to Danish and Aslam (2013), Danish and Aslam (2017) and Danish et al. (2018).

3.2 KD Criterion

The KD for a sample from the assumed Weibull model, for example, can be defined as

$$\mathrm{KD}_{Weibull}=\operatorname*{Sup}_{\mathrm{y}}\left|\tilde{\mathrm{F}}_{\mathrm{n}}\left(\mathrm{y}\right)-\hat{F}_{Weibull}\left(y;\hat{\boldsymbol{\theta}}\right)\right|, \tag{17}$$

where $\hat{F}_{Weibull}\left(y;\hat{\boldsymbol{\theta}}\right)$ is the estimated Weibull distribution function and $\tilde{F}_n\left(y\right)$ is empirical distribution function of the data at hand. Similarly, the KD criteria can be defined for other models. The model with minimum value of the KD is considered as a better model.

4. REAL DATA EXAMPLES

The first data set belongs to survival times (in months) of 24 patients with Dukes' C colorectal cancer reported by McIllmurray and Turkie (1987) as 3+, 6, 6, 6, 6, 8, 8, 12, 12, 12+, 12+, 16+, 18+, 18+, 20, 22+, 24, 28+, 28+, 28+, 30, 30+, 33+, 42. The observations with '+' indicate the censored survival times. This data will be denoted by Duke Data. The second data set belongs to liver study conducted by Mayo Clinic from 1974 to 1984. The survival times (in days) of 36 patients reported by Fleming and Harrington (1991) are 400, 77, 859, 71, 1037, 1427, 733, 334, 41, 51, 549, 1170, 890, 1413, 853, 216, 1882^+, 1067^+, 131, 223, 1827, 2540, 1297, 264, 797, 930, 1329^+, 264, 1350, 1191, 130, 943, 974, 790, 1765^+, 1320^+. This data will be denoted by Liver Data.

4.1 Comparison through Duke Data

For each model, we compute the ML estimates, log-likelihood functions and KD statistics for the Duke Data. The results are reported in Table 1. Since the value of the log-likelihood function corresponding to the Weibull model is greater than the other competitive models, we can say that the Weibull model fits this data set better than Burr and GE models. Based on the KD criterion, the Burr model fits the data quite better than the other two models. As the KD value for the GE model is smaller than that of the Weibull model, we may also conclude that GE model is better than the Weibull model. Figure 1 shows the survival functions of Weibull, Burr type XII and GE models fitted to the Kaplan-Meier survival curve of the Duke Data. The fitted survival functions provide a good summary of the Kaplan-Meier (K-M) survival curve even for such a small data set. The survival function of the Burr type XII distribution provides a relatively better fit to the Kaplan-Meier survival curve than the survival functions of Weibull and GE distributions.

Table 1
The Values of the ML Estimates, Estimated Log-Likelihood Functions and KD Statistics for the Duke Data

Model	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\beta}$	**KD**	$L\left(\hat{\theta},\hat{\lambda},\hat{\beta}\right)$
Weibull	1.8095	0.1395	1.0000	0.3435	– 49.61
Burr type XII	2.8224	0.2931	1.0000	0.2771	– 52.78
GE	2.4341	0.5841	1.0331	0.3233	– 49.91

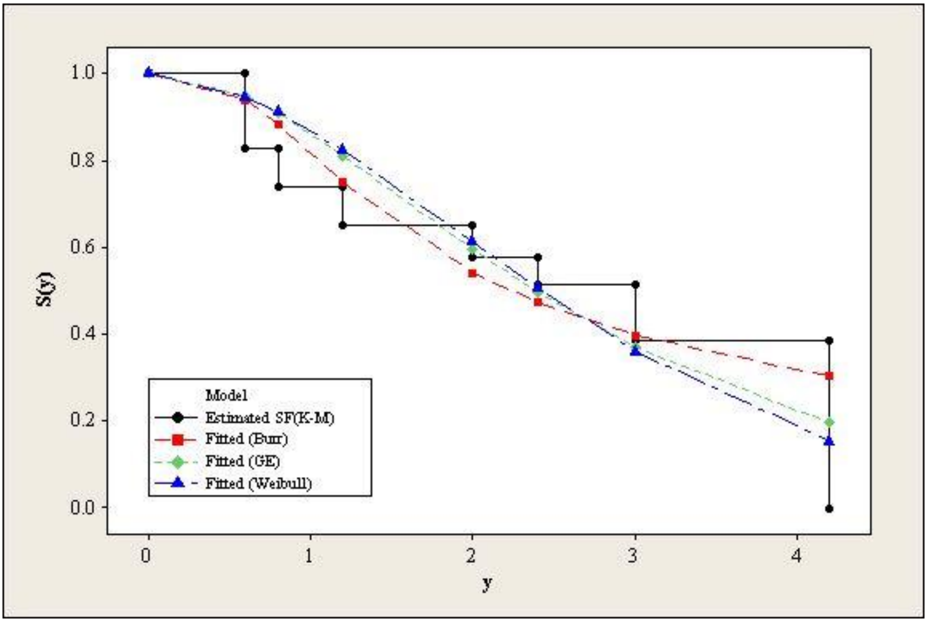

Figure 1: The Survival Functions Fitted to the K-M Survival Curve of the Duke Data

4.2 Comparison through Liver Data

For Liver Data too, we compute the ML estimates, the log-likelihood functions and the KD statistics. The results are reported in Table 2. Again the Weibull model fits this data better than the other two models based on the LRT rule. The KD criterion suggests the Burr type XII and the GE models more plausible than the Weibull model. Figures 2 shows the survival functions of Weibull, Burr type XII and GE models fitted to the Kaplan-Meier survival curve of the Liver Data. It is clear that the fitted survival functions provide a very close, but smoothed, summary of the Kaplan-Meier survival curve. However, it is very difficult to discriminate among the fit of these models.

Table 2
The Values of the ML Estimates, Estimated Log-Likelihood Functions and KD Statistics for the Liver Data

Model	$\hat{\theta}$	$\hat{\lambda}$	$\hat{\beta}$	**KD**	$L\left(\hat{\theta},\hat{\lambda},\hat{\beta}\right)$
Weibull	1.2965	0.9465	0.1527	0.1455	–43.72
Burr type XII	1.4904	1.5108	0.1613	0.1273	–46.98
GE	1.3286	1.2072	0.1628	0.1195	–44.46

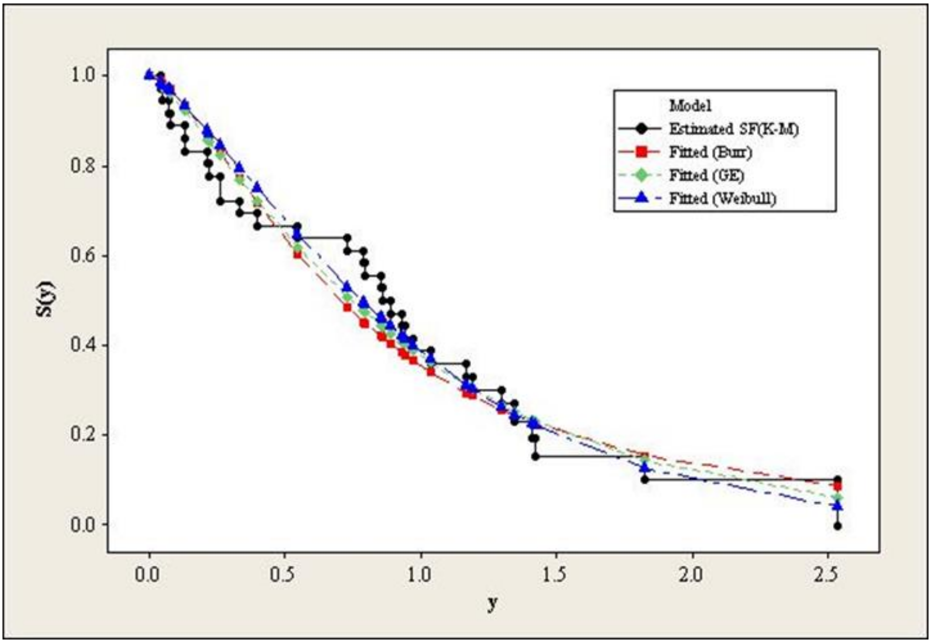

Figure 2: The Survival Functions Fitted to the K-M Survival Curve of the Liver Data

5. CONCLUSION

Weibull, Burr type XII and generalized exponential distributions are often used for modeling failure time data. This paper presents diagnostic tools based on the Kaplan-Meier estimator of the survival function, the maximum likelihood ratio test and the minimum Kolmogorov distance statistic to discriminate among these distributions in the proportional hazards model of random censorship. Two real data sets are considered for the application of this discrimination procedure. It is observed that none of the three models might work better than the others in all cases. Indeed, in some cases the Weibull model may best fit the data, whereas in other circumstances the Burr type XII or generalized exponential model may provide best fit.

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REFERENCES

1. Danish, M.Y., Arshad, I.A. and Aslam, M. (2018). Bayesian inference for the randomly censored Burr type XII distribution. *Journal of Applied Statistics*, 45(2), 270-281.

2. Danish, M.Y. and Arshad, I.A. (2017). Survival analysis with parametric proportional hazards model of random censorship. *American Journal of Mathematical and Management Sciences*, 36(1), 66-83.
3. Danish, M.Y. and Aslam, M. (2013a). Bayesian estimation for randomly censored generalized exponential distribution under asymmetric loss functions. *Journal of Applied Statistics*, 40(5), 1106-1119.
4. Fleming, T.R. and Harrington, D.P. (1991). *Counting processes and survival analysis*. New York: Wiley.
5. Hager, H.W., Bain, L.J. and Antle, C.E. (1971). Reliability estimation for the generalized gamma distribution and robustness of the Weibull model. *Technometrics*, 65, 1601-1609.
6. Kalbfleisch, J.D. and Prentice, R.L. (2002). *The statistical analysis of failure time data*. New York: Wiley.
7. Kaplan, E.L. and Meier, P. (1958). Nonparametric estimation from incomplete observations. *JASA*, 53, 457-481.
8. Koziol, J.A. and Green, S.B. (1976). A Cramer-von Mises statistic for random censored data. *Biometrika*, 63, 465-474.
9. McIllmurray, M.B. and Turkie, W. (1987). Controlled trial of γ linolenic acid in Dukes' C colorectal cancer. *British Medical Journal*, 294, 1260.
10. Miller, R.G. (1981). *Survival analysis*. New York: Wiley.

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AN OVERVIEW OF HEALTH INFORMATION SYSTEM IN PAKISTAN

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ABSTRACT

Health plan in Pakistan need seen natural advancement Furthermore dates once again of the medieval, accepted wellness forethought, wellness for the greater part approach, elementary quill wellbeing guardianship methodology What's more health association fortifying approach for better health conclusions. The primary item of health framework is change for health, equitability to measurable conveyance from claiming hazard. Furthermore, responsiveness of the non-medical necessities of the populace. With diminishing consumption for health care, blasting private health segment Furthermore thriving pharmaceutical industry, legislature might best decrease calamitous health consumptions toward poor people, furthermore devastated through an efficient, effective, approachable. Furthermore, responsive state funded health framework. Bury sectoral collaboration, private region engagement, social protection, fair dissemination of resources, people driven health policy, health worth of effort energy development, evidence -based health data framework Also nature power about fundamental drugs will reinforce health framework over Pakistan.

INTRODUCTION

An example of a country that exhibits bottlenecks between health and development is Pakistan, which is the 6th most populous country with a total population of 191.71 million, growth rate of 1.91% per annum. (Ministry of Finance, Gov't of Pakistan, 2015) A majority of the population lives in rural areas. However, due to fast pace development, urban cities such as Karachi and Lahore have emerged, causing major shifts in society and culture. By 2015, the yearly development rate of city areas is 3.1%, and 37% of the total population resides in these city areas. (Ministry of Finance, Government of Pakistan, 2015) Pakistan, a democratic state, four states, namely Punjab, Baluchistan, Khyber Pakhtunkhwa, Sindh, and the state Gilgit-Baltistan.

Although its annual per-capita income is $1,512, Pakistan is poor and comparably weak compared with other countries. The country's ranking is based on its Human Development Index (HDI) which put it at 110 out of 186 countries. Moreover, 55 per cent of females aged 15 years and below are uneducated, placing the country 123rd out of 186 countries in the Gender Inequality Index (0.567). Even with 36% of the population is aged below 15 years old, the average life expectancy is 63 years old. Finally, only 48% of Pakistanis had access to sanitation. Given these facts, the paper aims at reviewing the development of Pakistan's healthcare delivery system in light of the necessary elements to strengthen the health care systems (Syed Fawad, et al., 2016).

The decentralization of political and administrative authorities of 135 districts in Pakistan took place in 2001, placing them under the power of the local government. This restructuring of power was undertaken with a vision to empower local authorities and increase the efficacy. However, the system failed because of animosities between the states and districts. The changes took place at a discontinuous pace, reaching midpoint between 2002 to 2009. In July 2009, three provincial governments declared their respective plans to revert to the administrative system prior to 2001 (Shaikh, Naeem, Nafees, Zahidie, Fatima, Kazi, 2012), aiming at implementing authority-related reforms, the country's Ministry of Health was abolished on 30 June 2011. Health-related national duties were subsequently distributed to seven other ministries. Moreover, in order to improve the delivery of service and expand the health care facilities at the community level, the health sector was decentralized. This allowed for more health-related allocation of resources and authority at the provincial level. By 2012, the government re-established the Ministry of National Regulations and Services. Its mandates were further expanded when it became the Ministry of Health Service Regulations and Coordination (Ministry of National health Pakistan, 2015).

The Public Health Care System

Figure 1 shows the community health distribution system, which serves as the combined health compound overseen administratively by the district. The health care system is provided by the Ministry through a three-tiered system, along with other various public health programs. The Primary Health system includes BHUs and RHCs, while the secondary health care is composed of two levels referral for the services that care for acute, in-patient, and ambulatory patients at the Tehsil/Taluka headquarter hospitals (THQ) and District headquarter hospitals (DHQs). These are usually aided by teaching hospitals (tertiary care). Although health care for mothers and children are part of the integrated health system, there are a limited number of health care units, including the BHUs and RHCs. These institutions are able to provide basic obstetric care through community outreach programs wherein lady health workers and community midwives deliver the needed medical services (Syed Fawad, et al., 2016).

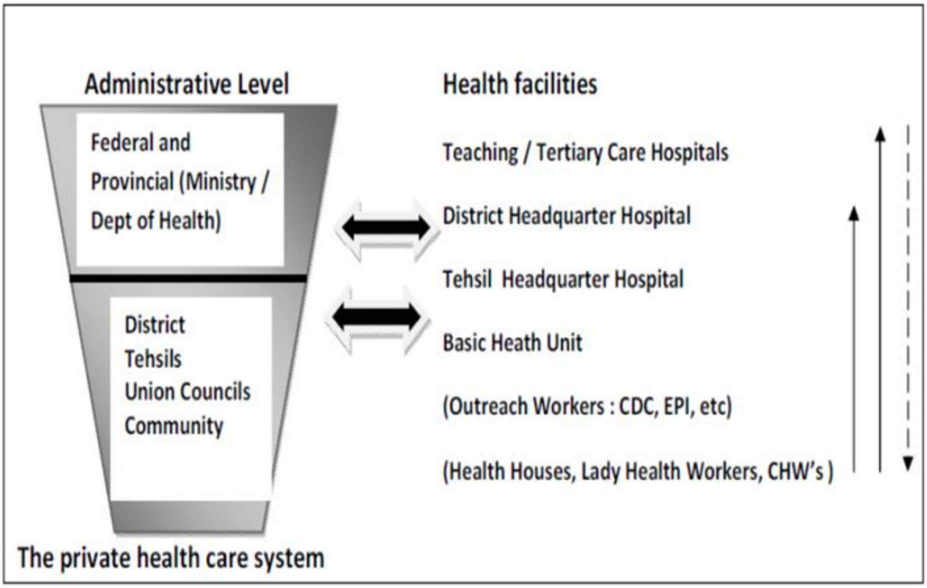

Figure 1: Illustration of Public Health Care Delivery in Pakistan (Source: Health Systems Profile–Pakistan, WHO).

The Private Health Care System

In a Pakistani community, the doctors, traditional healers, pharmacists, drug vendors, nurses and shopkeepers, unqualified practitioners, and laboratory technicians are those that provide private health care (Ahmed, Nisar, 2010). The total cost for health care services by the private sector in Pakistan is about 70%, of which 98% is paid by the individual (Statistics Division Gov't of Pakistan, 2011-12).

Health Information Systems with the objective of assisting the middle-level and senior level managers in undertaking an evidence-based decision-making, the Federal Ministry of Health established a basic health service unit called the Health Management Information System (HMIS) (Ali M, Horiloshi Y, 2002, Qazi MS, Ali M, 2009). During this time, the HMIS was restricted only to first level health care. (Qazi MS, Ali M, Kuroiwa C, 2008) However, due to the existence of a variety of health care information systems, there was a need to establish a better coordination and eliminate duplication. The HIMS was limited to data management and does not provide needed action such as, patient information, hospital management, laboratory management, etc. (Qazi MS, Ali M, 2009).

District Health Information System

Japan International Cooperation Agency (JICA) did a study for the Pakistani government on the need to develop a Management Information in the Health Sector from 2004 to 2007 (Qazi MS, Ali M, 2009). The study led to the launching of the District Health Information System (DHIS), which was comparatively more effective than the previous system, which was limited to data collection from secondary hospitals and health care units at other levels. The National Action Plan (NAP) for the DHIS was approved for the nationwide implementation (Fawad, et al., 2016).

Health Information System (HIS) is mainly concerned with the health information of the community. Its main target is to ensure the effective uses of resources to improve the health service facilities of the community. The HIS collects the data, analyzes it and converts the data into information which is useful in managing the Health Information System. To manage the system, the data must be reliable, accurate and timely (Fozia Anwar, Seema Rizvi, Shafaat A. Khan, Ramesh Kumar, 2015). The HIS requires for its management from various sources, various types of data. Data collection includes disease surveillance, facility surveys and routine reporting to manage the health service statistics. These systems collect, analyze and convert data into useful information which is helpful in the health system management (Garrib, Herbst, lamini, Mckenzie, Stoors, Govender, and Rohde, 2008).

The DHIS is being synchronized at the national and provincial levels for three years now, focusing mainly on the aspects wherein there is community-based knowledge on the District Health facilities. The main focus is on the design, structure, and workings of the DHIS components. There are certain districts that need information on their performances in the health sectors; for these, annual reviews and issues that need further attention are emphasized. Moreover, there are district level improvements of the HIS that reflects the provision of health facilities. Such as planning and management of healthcare services (Moazzam Ali, Yoichi Horikosh, 2002, Fozia Anwar, et al., 2015).

The standardization of data collection enhanced the effectiveness of healthcare service (Odhiambo-Otiene, 2005). This, however, also revealed that standards are being utilized based on their purpose and design. (Mitchell E, Sullivari F, 2001) Such misuse, which can lead to poor data, and it can lead to major inefficiencies (Fozia Anwar, et al., 2015). It is for this reason that a shared data collection and an analysis should be put in place at the national and provincial levels.

Data for the implementation of the DHIS in different provinces has shown improvements in data collection tools used in the regular documentation and monthly gathering of the District Headquarter Hospitals (DHQ) in every district one hospital with good health facility, Taluka Headquarters Hospitals (THQ) with basic health facility, and the patient linked information. These efforts for the data collection aided in the effortless and consistent data flow. Computers needed by each DHQ and THQ were provided, and were used for proper data collection, analysis, presentation and storage. Computers are also being regularly used in the DHIS and the HIS. Nonetheless, there are some districts that require help in addressing their respective hardware and software problems (Fozia, et al., 2015).

In the course of the system's implementation, progress has been noted particularly in the completeness and quality of data that were collected (HISP, 2010, Abou Zahr, Boerma, 2005). However, there are cracks and areas for improvement in the system, such as the reported delays in the data submission because no forms were delivered; some of the data were unreliable; the inadequate grasp of the indicators, poor feedback system; and inability of managers to maintain data summaries (Victora, Black, Boema, Bryce., 2011, Center for Global Development, 2006).

Grasping the value of HMIS, Pakistan's public health sector started data gathering at the national level in the early 90s. In 2004, it was transformed into the DHIS because the district health management gave emphasis to gauge service performance, efficient logistic management and future health planning rather than HIS improvement itself (Moazzam, 2002). The HMIS was changed into DHIS in 2004 because District Health Management is more important on account of measuring services performance and future health planning. For this purpose, the process headed to amend the indicators for data collection, tools and software. In 2006, the DHIS was enhanced, and was implemented at the district level. This was supposed to be undertaken by the provincial health departments; however, the specific requirements vary in each province. The system, despite its setbacks, had a good point. By 2007, the Punjab health department implemented this system at all its districts. Data reporting structure in Pakistan is as follows: The Basic Health Units (BHU) and Rural Health center (RHC) report to Tehsil Head Quarter (THQ), which subsequently reports to the District Health Quarter (DHQ), and then forwarded the same to the provincial level (Fozia, et al., 2015).

To date, the DHIS is the only regular data source for government-managed hospitals which is the public sector. This is the reason this research has significant value for supervision, monitoring and evaluation of this system from all health service units as it focuses on private sector. It is necessary to note that the DHIS is not an Electronic Health/Medical Record (EHR/EMR) system that is geared to clinical management of patients. The DHIS is basically an action-oriented HMIS built to reform and support the health sector, localize the decision-making, and ensure the effective use of information. The objective of Shaw 2005 research was to explore the operational structure of system for reports generating, data processing and health sector. It is therefore imperative to sustain the DHIS culture in data collection, the analysis undertaken, and the dissemination of the information to enhance public health service delivery (Shaw, 2005). As this research is based on HIS in private sector hospitals, therefore DHIS not be considered.

CONCLUSION

On requesting to accomplish in general change done health, honesty done danger measurable conveyance Also financing. Furthermore, responsiveness of the non-medical needs of the customers by those health plan over Pakistan, it may be basic will make solid inter-sector office, standards. Furthermore, standard spot setting to health tending conveyance, timber certification in the pharmaceutical innovation What's more additional coordinated effort for the private health awareness segment. Will furnish guiding, furthermore correct oversight a solid stewardship work will be fundamental. Universe private organization might fortify those stewardship part of those administration as far as bringing great legislation. Furthermore, Push more responsiveness On Pakistan health framework.

REFERENCES

1. Abouzahr, C. and Boerma, T. (2005). Health information systems: the foundations of public health. *Bulletin of the World Health Organization*, 83(8), 578-583.
2. Ahmed, F. and Nisar, N. (2010). Public-private partnership scenario in the health care system of Pakistan. *East Mediterr Health J.*, 16(8), 910-2.
3. Ali, M. and Horikoshi, Y. (2002). Situation analysis of health management information system in Pakistan. *Pak. J. Med. Res.*, 41(2), 64-9.
4. Fozia, A., Seema, R., Khan, S.A. and Ramesh, K. (2015). Case study of district health information system performance from Punjab province of Pakistan. *Pakistan Journal of Public Health*, 5(3), 3-7.
5. Garrib, A., Herbst, K., Dlamini, L., McKenzie, A., Stoops, N., Govender, T. and Rohde, J. (2008). An evaluation of the district health information system in rural South Africa. *South African Medical Journal*, 98(7), 549-552.
6. Health Information Systems Program. East London: HISP SA. Accessed 2 March 2010. Available from: www.hisp.org.
7. Mashhadi, S.F., Hamid, S., Roshan, R. and Fawad, A. (2016). Healthcare in Pakistan-A Systems Perspective. *Pakistan Armed Forces Medical Journal*, 66(1), 136-142.
8. Mitchell, E. and Sullivan, F. (2001). A descriptive feast but an evaluative famine: systematic review of published articles on primary care computing during 1980-97. *BMJ: British Medical Journal*, 322(7281), 279-128.
9. Odhiambo-Otieno, G.W. (2005). Evaluation of existing district health management information systems: a case study of the district health systems in Kenya. *International Journal of Medical Informatics*, 74(9), 733-744.
10. Qazi, M.S. and Ali, M. (2009). Pakistan's Health Management Information System: Health Managers' Perspectives. *JPMA. The Journal of the Pakistan Medical Association*, 59(1), 10-14.
11. Shaikh, S., Naeem, I., Nafees, A., Zahidie, A., Fatmi, Z. and Kazi, A. (2012). Experience of devolution in district health system of Pakistan: perspectives regarding needed reforms. *JPMA-Journal of the Pakistan Medical Association*, 62(1), 28-32.
12. Victora, C.G., Black, R.E., Boerma, J.T. and Bryce, J. (2011). Measuring Impact in the Millennium Development Goal era and beyond: a new approach to large-scale effectiveness evaluations. *The Lancet*, 377(9759), 85-95.

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AN EXPONENTIAL ESTIMATOR IN PRESENCE OF NON-RESPONSE

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ABSTRACT

In this paper, we propose an exponential estimator for population mean of variable y, with two auxiliary variables using two phase sampling scheme with sub sampling techniques in presence of non-response. The two situations with known population means of auxiliary variables, incomplete information on study variables and incomplete/ complete information on auxiliary variables have been considered. Expressions for bias and mean square error have been derived. The proposed estimator has been compared with usual unbiased estimator and some existing estimators. An empirical study has also been carried out.

KEY WORDS

Non-Response, Exponential Estimator, Two Phase Sampling, Sub Sampling Technique.

1. INTRODUCTION

In sample surveys specially relating to human populations, problem of non-response often exists. Unsuccessfulness in getting information from one or more sampling unit(s) on study variable or auxiliary variable(s) or both is called non-response. Reduction in effective sample size is a major consequence of non-response due to which further problems arises like increase in variance, bias and estimation of sampling error etc. Survey statisticians have made various suggestions to overcome this problem by taking different measures into consideration. Hansen and Hurwitz (1946) were the pioneers who addressed this problem by introducing sub-sampling technique. Many eminent survey statisticians have utilized this technique for further advancement in estimation of population parameters like Cochran (1977), Rao (1986, 1990), Khare and Srivastava (1993, 1995,1997,2010), Tripathi and Khare (1997), Tabasum and Khan (2004, 2006) Sodipo and Obisesan (2007), Singh and Kumar (2008, 2009, 2011), Sing et al. (2010), Ismail et al. (2011, 2015), Kumar and Bhougal (2011), Singh et al. (2009), Chanu and B. K. Singh (2015), and Pal and Singh (2016).

Suppose a sample of 'n' units is drawn from a population of size 'N' by simple random sample without replacement. From the 'n' sample units, 'r_1' sample units respond and 'r_2' are non-respondents. The population is divided in respondent and non-respondent groups containing 'N_1' and 'N_2' units corresponding to sample respondents and non-respondents. From 'r_2' non-respondents units, a sub-sample of size 'k' $\left(k = \frac{r_2}{h},\ h > 1\right)$ is drawn and information about study variable from these 'k' units is obtained.

Hansen and Hurwitz (1946), proposed the following unbiased estimator of population mean by using sub-sampling technique to deal with the problem of non-response:

$$t_0 = \bar{y}^* = \left(\frac{r_1}{n}\right)\bar{y}_{r_1 1} + \left(\frac{r_2}{n}\right)\bar{y}_{k2}, \tag{1.1}$$

where $\bar{y}_{r_1 1} = \frac{\sum_{i=1}^{r_1} y_i}{r_1}$ mean of responding part of sample and $\bar{y}_{k2} = \frac{\sum_{i=1}^{k} y_i}{k}$ mean of sub sample taken from non-responding part of sample. The estimator (1.1) is unbiased with variance:

$$\text{Var}\left(t_0\right) = \bar{Y}^2\left(\lambda C_y^2 + \theta C_{y(2)}^2\right), \tag{1.2}$$

with $\bar{Y} = \frac{\sum_{i=1}^{N_1} Y_i}{N}$, $\bar{Y}_2 = \frac{\sum_{i=1}^{N_2} Y_i}{N_2}$ $\lambda = \frac{1}{n} - \frac{1}{N}$, $\theta = \frac{W_2\left(h-1\right)}{n}$, $W_2 = \frac{N_2}{N}$,

$$C_y = \frac{S_y}{\bar{Y}}\ \ C_{y(2)} = \frac{S_{y(2)}}{\bar{Y}}\ \ S_y^2 = \frac{\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2}{N-1},\ S_{y(2)}^2 = \frac{\sum_{i=1}^{N_2}\left(y_i - \bar{Y}_2\right)^2}{N_2 - 1},$$

2. TWO-PHASE SAMPLING PROCEDURE IN THE PRESENCE OF NON-RESPONSE

The two-phase sampling in the presence of non-response using sub sampling technique is as follows:-

In first phase, a large sample of size n_1 units is drawn using simple random sampling without replacement and information on auxiliary variable(s) is collected.

In second phase, from 'n_1' first phase sample units, another sample of size n_2 using simple random sampling without replacement is drawn. From these n_2 units, r_1 units respond and r_2 units do not. Information on study variable 'y' is collected from these responding units. Then a sub sample of size k $\left(k = \frac{r_2}{h},\ h > 1\right)$ is selected from the r_2

non-responding units using simple random sampling without replacement and information on study variable is also collected from these selected units.

Several Statisticians have proposed estimators of population mean using two-phase sampling scheme with sub-sampling techniques taking auxiliary variable(s) into consideration in the presence of non-response. Some of them have suggested exponential estimators under the following two situations.

Situation I: Known mean(s) of auxiliary variable(s) and incomplete information on both study and auxiliary variable(s).

Situation II: Known mean(s) of auxiliary variable(s) and incomplete information on study variable but complete information on auxiliary variable(s).

Singh et al. (2009) considered the following estimators under above mentioned situation I.

$$t_1 = \bar{y}^* \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right) \tag{2.1}$$

$$t_2 = \bar{y}^* \exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right) \tag{2.2}$$

The expressions for bias are given below.

$$B(t_1) = \lambda_2 \bar{Y}\left(\frac{3}{8}C_x^2 - \frac{1}{2}\rho_{yx}C_xC_y\right) + \theta\bar{Y}\left(\frac{3}{8}C_{x(2)}^2 - \frac{1}{2}\rho_{yx(2)}C_{x(2)}C_{y(2)}\right), \tag{2.3}$$

$$B(t_2) = \lambda_2 \bar{Y}\left(\frac{-1}{8}C_x^2 + \frac{1}{2}\rho_{yx}C_xC_y\right) + \theta\bar{Y}\left(\frac{-1}{8}C_{x(2)}^2 + \frac{1}{2}\rho_{yx(2)}C_{x(2)}C_{y(2)}\right), \tag{2.4}$$

and the expressions for mean square error are as under:

$$MSE(t_1) = \lambda_2 \bar{Y}^2\left(C_y^2 + \frac{1}{4}C_x^2 - \rho_{yx}C_xC_y\right) + \theta\bar{Y}^2\left(C_{y(2)}^2 + \frac{1}{4}C_{x(2)}^2 - \rho_{yx(2)}C_{x(2)}C_{y(2)}\right), \tag{2.5}$$

$$MSE(t_2) = \lambda_2 \bar{Y}^2\left(C_y^2 + \frac{1}{4}C_x^2 + \rho_{yx}C_xC_y\right) + \theta\bar{Y}^2\left(C_{y(2)}^2 + \frac{1}{4}C_{x(2)}^2 + \rho_{yx(2)}C_{x(2)}C_{y(2)}\right). \tag{2.6}$$

Kumar and Bhougal (2011) proposed following ratio-product type exponential estimator of population mean in presence of non-response considering situation I *i.e.* known population mean of auxiliary variable with incomplete information on both study as well as auxiliary variable.

$$t_3 = \bar{y}^*\left(\alpha \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right) + (1-\alpha)\exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right)\right). \tag{2.7}$$

where α is suitably chosen constant.

The expression for bias of (2.7) is

$$B(t_3) = \frac{1}{4}\bar{Y}\left(\lambda_2\left(1+2(1-2\alpha)k_{yx}\right)C_x^2 + \theta\left(1+2(1-2\alpha)k_{yx(2)}\right)C_{x(2)}^2\right), \tag{2.8}$$

where $k_{yx} = \rho_{yx}C_y / C_x, \quad k_{yx(2)} = \rho_{yx(2)}C_{y(2)} / C_{x(2)}.$

The minimum mean square error of (2.7) is

$$MSE(t_3)_{opt} = \bar{Y}^2\left(\lambda_2\left(C_y^2 + \frac{B}{A}\left(\frac{B}{A} - 2k_{yx}\right)C_x^2\right) + \theta\left(C_{y(2)}^2 + \frac{B}{A}\left(\frac{B}{A} - 2k_{yx(2)}\right)C_{x(2)}^2\right)\right), \tag{2.9}$$

where $A = \lambda_2 C_x^2 + \theta C_{x(2)}^2, \quad B = \lambda_2 k_{yx} C_x^2 + \theta k_{yx(2)} C_{x(2)}^2.$

Chanu and Singh (2015) proposed the following estimator under the above mentioned situation I.

$$t_4 = \bar{y}^*\left(\alpha_2 \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right) + (1-\alpha_2)\exp\left(\frac{\bar{x}^\sigma - \bar{X}}{\bar{x}^\sigma + \bar{X}}\right)\right), \tag{2.10}$$

where $x^\sigma = \dfrac{N\bar{X} - n\bar{x}}{N-n}$ and α_2 is suitably chosen constant.

The bias and mean square errors of (2.10) is

$$B(t_4) = -\bar{Y}\left(\frac{-g}{2}\left(\lambda_2\rho_{yx}C_xC_y + \theta\rho_{yx(2)}C_{x(2)}C_{y(2)}\right) + \frac{g^2}{8}\left(\lambda_2 C_x^2 + \theta C_{x(2)}^2\right)\right.$$
$$\left. + \alpha_2\left(\frac{g-1}{2}\left(\lambda_2\rho_{yx}C_xC_y + \theta\rho_{yx(2)}C_{x(2)}C_{y(2)}\right) + \frac{3+g^2}{8}\left(\lambda_2 C_x^2 + \theta C_{x(2)}^2\right)\right)\right), \tag{2.11}$$

$$MSE(t_4)_{opt} = \bar{Y}^2\left(\lambda_2 C_y^2 + \theta C_{y(2)}^2 - \frac{\left(\lambda_2\rho_{yx}C_xC_y + \theta\rho_{yx(2)}C_{x(2)}C_{y(2)}\right)^2}{\lambda_2 C_x^2 + \theta C_{x(2)}^2}\right), \tag{2.12}$$

where $g = \dfrac{n}{N-n}$ and $\alpha_2 = \dfrac{g}{g-1} - \dfrac{2}{g-1}\dfrac{\lambda_2\rho_{yx}C_xC_y + \theta\rho_{yx(2)}C_{x(2)}C_{y(2)}}{\lambda_2 C_x^2 + \theta C_{x(2)}^2}.$

Pal and Singh (2016) proposed the following two-parameter ratio estimator for population mean in the presence of non-response under Situation I.

$$t_5 = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*} \right)^{\alpha} \exp\left(\frac{\delta\left(\bar{X} - \bar{x}^*\right)}{\left(\bar{X} + \bar{x}^*\right)} \right), \tag{2.13}$$

where α and δ are suitably chosen constants.

The bias and minimum mean square error of (2.13) is

$$B(t_5) = \frac{\bar{Y}(\delta + 2\alpha)}{8} \left(\lambda_2 C_x^2 + \theta C_{x(2)}^2 \right)\left(\delta + 2\alpha - 4R + 2\right), \tag{2.14}$$

$$MSE(t_5)_{opt} = \bar{Y}^2 \left(\left(\lambda_2 C_y^2 + \theta C_{y(2)}^2 \right) - \frac{\left(\lambda_2 C C_x^2 + \theta C_{(2)} C_{x(2)}^2 \right)^2}{\lambda_2 C_x^2 + \theta C_{x(2)}^2} \right), \tag{2.15}$$

where $R = \dfrac{\lambda_2 C C_x^2 + \theta C_{(2)} C_{x(2)}^2}{\lambda_2 C_x^2 + \theta C_{x(2)}^2}$, $C = \rho_{yx} C_y / C_x$, and $C_{(2)} = \rho_{yx(2)} C_{y(2)} / C_{x(2)}$.

Singh et al. (2009) proposed following ratio and product type exponential estimators of population mean in the presence of non-response considering situation II *i-e* known population mean of auxiliary variable with incomplete information on study and complete information auxiliary variable.

$$t_6 = \bar{y}^* \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right), \tag{2.16}$$

$$t_7 = \bar{y}^* \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right). \tag{2.17}$$

The expression for bias and mean square error of (2.16) and (2.17) are

$$B(t_6) = \lambda_2 \bar{Y} \left(\frac{3}{8} C_x^2 - \frac{1}{2} \rho_{yx} C_x C_y \right), \tag{2.18}$$

$$B(t_7) = \lambda_2 \bar{Y} \left(\frac{-1}{8} C_x^2 + \frac{1}{2} \rho_{yx} C_x C_y \right), \tag{2.19}$$

and the expressions for mean square error are as under:

$$MSE(t_6) = \lambda_2 \bar{Y}^2 \left(C_y^2 + \frac{1}{4} C_x^2 - \rho_{yx} C_x C_y \right) + \theta \bar{Y}^2 C_{y(2)}^2, \tag{2.20}$$

$$MSE(t_7) = \lambda_2 \bar{Y}^2 \left(C_y^2 + \frac{1}{4} C_x^2 + \rho_{yx} C_x C_y \right) + \theta \bar{Y}^2 C_y^2. \tag{2.21}$$

Chanu and Singh (2015) proposed the following estimator under the above mentioned situation II.

$$t_8 = \bar{y}^*\left(\alpha_1 \exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)+\left(1-\alpha_1\right)\exp\left(\frac{\bar{x}^{\sigma}-\bar{X}}{\bar{x}^{\sigma}+\bar{X}}\right)\right), \tag{2.22}$$

where $x^{\sigma} = \dfrac{N\bar{X}-n\bar{x}}{N-n}$ and α_1 is suitably chosen constant.

The bias and mean square errors of (2.22) is

$$B\left(t_8\right)=\bar{Y}\left(\frac{\lambda_2 g\rho_{yx}C_xC_y}{2}+\frac{\lambda_2 g^2C_x^2}{8}+-\alpha_1\left(\frac{\lambda_2\left(g-1\right)\rho_{yx}C_xC_y}{2}+\frac{\lambda_2\left(3+g^2\right)C_x^2}{8}\right)\right), \tag{2.23}$$

$$MSE\left(t_8\right)_{opt} = \lambda_2\bar{Y}^2C_y^2\left(1-\rho_{xy}^2\right)+\theta\bar{Y}^2C_{y(2)}^2 \tag{2.24}$$

where $g = \dfrac{n}{N-n}$, $\alpha_1 = \dfrac{g-2k_{yx}}{g-1}$ and $k_{yx} = \rho_{yx}C_y / C_x$.

Pal and Singh (2016) proposed the following two-parameter ratio estimator for population mean in the presence of non-response under Situation II.

$$t_9 = \bar{y}^*\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha_1}\exp\left(\frac{\delta_1\left(\bar{X}-\bar{x}\right)}{\left(\bar{X}+\bar{x}\right)}\right), \tag{2.25}$$

where α_1 and δ_1 are suitably chosen constants.

The bias and minimum mean square error of (2.13) is

$$B\left(t_9\right)=\frac{\bar{Y}\left(\delta_1+2\alpha_1\right)}{8}\lambda_2C_x^2\left(\delta_1+2\alpha_1-4C+2\right), \tag{2.26}$$

$$MSE\left(t_9\right)_{opt} = \lambda_2\bar{Y}^2C_y^2\left(1-\rho_{xy}^2\right)+\theta\bar{Y}^2C_{y(2)}^2 \tag{2.27}$$

where $C = \dfrac{\delta_1+2\alpha_1}{2}$.

3. THE PROPOSED EXPONENTIAL ESTIMATOR UNDER SITUATION I

The proposed exponential estimator for population mean using two auxiliary variables in the presence of non-response under situation 1 is

$$t_{ne1} = \bar{y}^* \left[\alpha \exp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*} \right) + (1-\alpha) \exp\left(\frac{\bar{Z} - \bar{z}^*}{\bar{Z} + \bar{z}^*} \right) \right]. \quad (3.1)$$

To find out bias and mean square error of (3.1) we describe some well-known relations as:

$$\bar{y}^* = \bar{Y} + \bar{e}_y^*, \ \bar{x}^* = \bar{X} + \bar{e}_x^* \text{ and } \bar{z}^* = \bar{Z} + \bar{e}_z^*.$$

Such that

$$E\left(\bar{e}_y^*\right) = E\left(\bar{e}_x^*\right) = E\left(\bar{e}_z^*\right) = 0$$

and

$$\left.\begin{aligned}
&E\left(\bar{e}_y^{*2}\right) = \bar{Y}^2\left(\lambda_2 C_y^2 + \theta C_{y(2)}^2\right), \ E\left(\bar{e}_x^{*2}\right) = \bar{X}^2\left(\lambda_2 C_x^2 + \theta C_{x(2)}^2\right),\\
&E\left(\bar{e}_z^{*2}\right) = \bar{Z}^2\left(\lambda_2 C_z^2 + \theta C_{z(2)}^2\right), \ E\left(\bar{e}_x^2\right) = \lambda_2 \bar{X}^2 C_x^2,\\
&E\left(\bar{e}_z^2\right) = \lambda_2 \bar{Z}^2 C_z^2, \qquad E\left(\bar{e}_y^* \bar{e}_x\right) = \bar{Y}\bar{X}\left(\lambda_2 \rho_{xy} C_y C_x + \theta \rho_{xy(2)} C_{y(2)} C_{x(2)}\right),\\
&E\left(\bar{e}_y \bar{e}_z\right) = \lambda_2 \bar{Y}\bar{Z} \rho_{yz} C_y C_z, \qquad E\left(\bar{e}_x \bar{e}_z\right) = \lambda_2 \bar{Y}\bar{Z} \rho_{xz} C_x C_z.
\end{aligned}\right\} \quad (3.2)$$

Using the above relations (3.1) may be written as

$$t_{ne1} = \left(\bar{Y} + \bar{e}_y^*\right)\left[\alpha \exp\left(\frac{-\bar{e}_x^*}{2\bar{X} + \bar{e}_x^*} \right) + (1-\alpha) \exp\left(\frac{-\bar{e}_z^*}{2\bar{Z} + \bar{e}_z^*} \right) \right],$$

or

$$t_{ne1} = \left(\bar{Y} + \bar{e}_y^*\right)\left[\alpha \exp\left(\frac{-\bar{e}_x^*}{2\bar{X}} \right)\left(1 + \frac{\bar{e}_x^*}{2\bar{X}}\right)^{-1} + (1-\alpha) \exp\left(\frac{-\bar{e}_z^*}{2\bar{Z}} \right)\left(1 + \frac{\bar{e}_z^*}{2\bar{Z}}\right)^{-1} \right]. \quad (3.3)$$

Expanding the right hand side of (3.3) and neglecting third and higher order terms we have

$$t_{ne1} \approx \left(\bar{Y} + \bar{e}_y^*\right)\left[\alpha\left(1 - \frac{\bar{e}_x^*}{2\bar{X}} + \frac{3\bar{e}_x^{*2}}{8\bar{X}^2}\right) + (1-\alpha)\left(1 - \frac{\bar{e}_z^*}{2\bar{Z}} + \frac{3\bar{e}_z^{*2}}{8\bar{Z}^2}\right) \right],$$

or

$$\begin{aligned}
t_{ne1} - \bar{Y} \approx\ & \bar{e}_y^* - \frac{\bar{Y}\bar{e}_z^*}{2\bar{Z}} + \frac{3\bar{Y}\bar{e}_z^{*2}}{8\bar{Z}^2} - \alpha\bar{Y}\left(\frac{\bar{e}_x^*}{2\bar{X}} - \frac{\bar{e}_z^*}{2\bar{Z}} - \frac{3\bar{e}_x^{*2}}{8\bar{X}^2} + \frac{3\bar{e}_z^{*2}}{8\bar{Z}^2} \right)\\
& - \frac{\bar{e}_y^*\bar{e}_z^*}{2\bar{Z}} - \alpha\left(\frac{\bar{e}_y^*\bar{e}_x^*}{2\bar{X}} - \frac{\bar{e}_y^*\bar{e}_z^*}{2\bar{Z}} \right).
\end{aligned} \quad (3.4)$$

Applying expectation on both sides of (3.4) we get the expression for bias of (3.1)

$$B(t_{ne1}) \approx \lambda_2 \bar{Y}\left(\frac{3}{8}\left(\alpha C_x^2 + (1-\alpha) C_z^2\right) - \frac{1}{2}\left(\alpha \rho_{xy} C_x C_y + (1-\alpha)\rho_{yz} C_y C_z\right)\right)$$
$$+\theta \bar{Y}\left(\frac{3}{8}\left(\alpha C_{x(2)}^2 + (1-\alpha) C_{z(2)}^2\right)\right.$$
$$\left. -\frac{1}{2}\left(\alpha \rho_{xy(2)} C_{x(2)} C_{y(2)} + (1-\alpha)\rho_{yz(2)} C_{y(2)} C_{z(2)}\right)\right). \qquad (3.5)$$

Squaring both sides of (3.4) neglecting second and higher order terms we get

$$\left(t_{ne1} - \bar{Y}\right)^2 \approx \left(\bar{e}_y^* - \alpha \frac{\bar{Y}\bar{e}_x^*}{2\bar{X}} - (1-\alpha)\frac{\bar{Y}\bar{e}_z^*}{2\bar{Z}}\right)^2. \qquad (3.6)$$

Applying expectation on both sides of (3.6) and simplifying we get

$$MSE(t_{ne1}) \approx \lambda_2 \bar{Y}^2 \left(C_y^2 + \frac{1}{4}\alpha^2 C_x^2 + \frac{1}{4}(1-\alpha)^2 C_z^2 - \alpha \rho_{xy} C_x C_y\right.$$
$$\left. -(1-\alpha)\rho_{yz} C_y C_z + \frac{1}{2}\alpha(1-\alpha)\rho_{xz} C_x C_z\right)$$
$$+\theta \bar{Y}^2 \left(C_{y(2)}^2 + \frac{1}{4}\alpha^2 C_{x(2)}^2 + \frac{1}{4}(1-\alpha)^2 C_{z(2)}^2 - \alpha \rho_{xy(2)} C_{x(2)} C_{y(2)}\right.$$
$$\left. -(1-\alpha)\rho_{yz(2)} C_{y(2)} C_{z(2)} + \frac{1}{2}\alpha(1-\alpha)\rho_{xz(2)} C_{x(2)} C_{z(2)}\right). \qquad (3.7)$$

Now for optimum value of α taking derivative with respect to α and equating to zero, we get

$$\alpha = \frac{\lambda_2\left(2C_y\left(\rho_{xy}C_x - \rho_{yz}C_z\right) + C_z\left(C_z - \rho_{xz}C_x\right)\right) + \theta\left(2C_{y(2)}\left(\rho_{xy(2)}C_{x(2)} - \rho_{yz(2)}C_{z(2)}\right) + C_{z(2)}\left(C_{z(2)} - \rho_{xz(2)}C_{x(2)}\right)\right)}{\lambda_2\left(C_x\left(C_x - \rho_{xz}C_z\right) + C_z\left(C_z - \rho_{xz}C_x\right)\right) + \theta\left(C_{x(2)}\left(C_{x(2)} - \rho_{xz(2)}C_{z(2)}\right) + C_{z(2)}\left(C_{z(2)} - \rho_{xz(2)}C_{x(2)}\right)\right)},$$

or

$$\alpha = \frac{\lambda_2(2A + C) + \theta\left(2A_{(2)} + C_{(2)}\right)}{\lambda_2(B + C) + \theta\left(B_{(2)} + C_{(2)}\right)}, \qquad (3.8)$$

where

$$A = C_y\left(\rho_{xy}C_x - \rho_{yz}C_z\right), \qquad A_{(2)} = C_{y(2)}\left(\rho_{xy(2)}C_{x(2)} - \rho_{yz(2)}C_{z(2)}\right),$$

$$B = C_x\left(C_x - \rho_{xz}C_z\right), \qquad B_{(2)} = C_{x(2)}\left(C_{x(2)} - \rho_{xz(2)}C_{z(2)}\right),$$

$$C = C_z\left(C_z - \rho_{xz}C_x\right), \qquad C_{(2)} = C_{z(2)}\left(C_{z(2)} - \rho_{xz(2)}C_{x(2)}\right).$$

Putting (3.8) in (3.7) we get minimum mean square error of (3.1) as

$$MSE\left(t_{ne1}\right)_{opt} \approx Var\left(t_0\right) + \frac{1}{4}\bar{Y}^2\left[\lambda_2 D + \theta D_{(2)} - \frac{\left(\lambda_2\left(2A+C\right)+\theta\left(2A_{(2)}+C_{(2)}\right)\right)^2}{\lambda_2\left(B+C\right)+\theta\left(B_{(2)}+C_{(2)}\right)}\right], \qquad (3.9)$$

where

$$D = C_z^2 - 4\rho_{yz}C_yC_z, \qquad D_{(2)} = C_{z(2)}^2 - 4\rho_{yz(2)}C_{y(2)}C_{z(2)}.$$

4. THE PROPOSED EXPONENTIAL ESTIMATOR UNDER SITUATION II

The proposed exponential estimator for population mean using two auxiliary variables in the presence of non-response under situation II is

$$t_{ne2} = \bar{y}^*\left[\alpha\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) + \left(1-\alpha\right)\exp\left(\frac{\bar{Z}-\bar{z}}{\bar{Z}+\bar{z}}\right)\right]. \qquad (4.1)$$

To find out bias and mean square error of (4.1) we describe some well-known relations as:

$$\bar{y}^* = \bar{Y} + \bar{e}_y^*, \quad \bar{x} = \bar{X} + \bar{e}_x \text{ and } \bar{z} = \bar{Z} + \bar{e}_z$$

Such that

$$E\left(\bar{e}_y^*\right) = E\left(\bar{e}_x\right) = E\left(\bar{e}_z\right) = 0$$

and

$$\left.\begin{aligned} E\left(\bar{e}_y^{*2}\right) &= \lambda_2 S_y^2 + \theta S_{y(2)}^2,\ E\left(\bar{e}_x^2\right) = \lambda_2 S_x^2, \\ E\left(\bar{e}_z^2\right) &= \lambda_2 S_z^2,\ E\left(\bar{e}_y^*\bar{e}_x\right) = \lambda_2 S_{xy},\ E\left(\bar{e}_x\bar{e}_z\right) = \lambda_2 S_{xz} \end{aligned}\right\} \qquad (4.2)$$

Using the above relations (4.1) may be written as

$$t_{ne2} = \left(\bar{Y} + \bar{e}_y^*\right)\left[\alpha\exp\left(\frac{-\bar{e}_x}{2\bar{X}+\bar{e}_x}\right) + \left(1-\alpha\right)\exp\left(\frac{-\bar{e}_z}{2\bar{Z}+\bar{e}_z}\right)\right],$$

or

$$t_{ne2}=\left(\bar{Y}+\bar{e}_y^*\right)\left[\alpha\exp\left(\frac{-\bar{e}_x}{2\bar{X}}\right)\left(1+\frac{\bar{e}_x}{2\bar{X}}\right)^{-1}+(1-\alpha)\exp\left(\frac{-\bar{e}_z}{2\bar{Z}}\right)\left(1+\frac{\bar{e}_z}{2\bar{Z}}\right)\right]. \quad (4.3)$$

Expanding the right hand side of (4.3) and neglecting third and higher order terms we have

$$t_{ne2}\approx\left(\bar{Y}+\bar{e}_y^*\right)\left[\alpha\left(1-\frac{\bar{e}_x}{2\bar{X}}+\frac{3\bar{e}_x^2}{8\bar{X}^2}\right)+(1-\alpha)\left(1-\frac{\bar{e}_z}{2\bar{Z}}+\frac{3\bar{e}_z^2}{8\bar{Z}^2}\right)\right],$$

or

$$\begin{aligned}t_{ne2}-\bar{Y}\approx\bar{e}_y^*-\frac{\bar{Y}\bar{e}_z}{2\bar{Z}}+\frac{3\bar{Y}\bar{e}_z^2}{8\bar{Z}^2}-\alpha\bar{Y}\left(\frac{\bar{e}_x}{2\bar{X}}-\frac{\bar{e}_z}{2\bar{Z}}-\frac{3\bar{e}_x^2}{8\bar{X}^2}+\frac{3\bar{e}_z^2}{8\bar{Z}^2}\right)\\ -\frac{\bar{e}_y^*\bar{e}_z}{2\bar{Z}}-\alpha\left(\frac{\bar{e}_y^*\bar{e}_x}{2\bar{X}}-\frac{\bar{e}_y^*\bar{e}_z}{2\bar{Z}}\right)\end{aligned} \quad (4.4)$$

Applying expectation on both sides of (4.4) we get the expression for bias of (4.1)

$$B\left(t_{ne2}\right)\approx\lambda_2\bar{Y}\left(\frac{3}{8}\left(\alpha C_x^2+(1-\alpha)C_z^2\right)-\frac{1}{2}\left(\alpha\rho_{xy}C_xC_y+(1-\alpha)\rho_{yz}C_yC_z\right)\right) \quad (4.5)$$

Squaring both sides of (4.4) neglecting second and higher order terms we get

$$\left(t_{ne2}-\bar{Y}\right)^2\approx\left(\bar{e}_y^*-\alpha\frac{\bar{Y}\bar{e}_x}{2\bar{X}}-(1-\alpha)\frac{\bar{Y}\bar{e}_z}{2\bar{Z}}\right)^2 \quad (4.6)$$

Applying expectation on both sides of (4.6) and simplifying we get

$$\begin{aligned}MSE\left(t_{ne2}\right)\approx\lambda_2\bar{Y}^2\Bigg(C_y^2+\frac{1}{4}\alpha^2C_x^2+\frac{1}{4}(1-\alpha)^2C_z^2-\alpha\rho_{xy}C_xC_y\\ -(1-\alpha)\rho_{yz}C_yC_z+\frac{1}{2}\alpha(1-\alpha)\rho_{xz}C_xC_z\Bigg)+\theta\bar{Y}^2C_{y(2)}^2\end{aligned} \quad (4.7)$$

Now for Optimum value of α taking derivative with respect to α and equating to zero, we get

$$\alpha=\frac{2C_y\left(\rho_{xy}C_x-\rho_{yz}C_z\right)+C_z\left(C_z-\rho_{xz}C_x\right)}{C_x\left(C_x-\rho_{xz}C_z\right)+C_z\left(C_z-\rho_{xz}C_x\right)}$$

or

$$\alpha=\frac{2A+C}{B+C}. \quad (4.8)$$

here

$$A = C_y\left(\rho_{xy}C_x - \rho_{yz}C_z\right), \qquad B = C_x\left(C_x - \rho_{xz}C_z\right), \qquad C = C_z\left(C_z - \rho_{xz}C_x\right).$$

Putting (4.8) in (4.7) we get minimum mean square error of (4.1) as

$$MSE\left(t_{ne2}\right)_{opt} \approx Var(t_0) + \frac{1}{4}\lambda_2\bar{Y}^2\left(D - \frac{\left(2A+C\right)^2}{B+C}\right) \tag{4.9}$$

where

$$D = C_z^2 - 4\rho_{yz}C_yC_z.$$

5. EFFICIENCY COMPARISON

The proposed estimators are compared with some existing exponential estimators in presence of non-response. The proposed estimators are found to be more efficient under following conditions.

Situation I

$$Var\left(t_0\right) - MSE\left(t_{ne1}\right)_{opt} \geq 0 \qquad \text{if}$$

$$\rho_{yz} \geq \frac{1}{4}.\frac{C_z}{C_y} \quad \& \quad \rho_{yz(2)} \geq \frac{1}{4}.\frac{C_{z(2)}}{C_{y(2)}}$$

$$MSE\left(t_1\right) - MSE\left(t_{ne1}\right) \geq 0 \qquad \text{if}$$

$$\frac{C_x^2 - C_z^2}{4C_y} \geq \rho_{yz}C_z - \rho_{yx}C_x \qquad \& \qquad \frac{C_{x(2)}^2 - C_{z(2)}^2}{4C_{y(2)}} \geq \rho_{yz(2)}C_{z(2)} - \rho_{yx(2)}C_{x(2)}$$

$$MSE\left(t_2\right) - MSE\left(t_{ne1}\right) \geq 0 \qquad \text{if}$$

$$\frac{C_x^2 - C_z^2}{4C_y} \geq -\rho_{yz}C_z - \rho_{yx}C_x \quad \& \quad \frac{C_{x(2)}^2 - C_{z(2)}^2}{4C_{y(2)}} \geq -\rho_{yz(2)}C_{z(2)} - \rho_{yx(2)}C_{x(2)}$$

As minimum/optimum MSE of t_3, t_4, and t_5 are same. So

$$MSE\left(t_3, t_4, t_5\right)_{opt} - MSE\left(t_{ne1}\right) \geq 0 \qquad \text{if}$$

$$G^2 < H\left(\lambda_2 D + \theta D_{(2)}\right)$$

where $\quad G = \lambda_2\rho_{yx}C_yC_x + \theta\rho_{yx(2)}C_{y(2)}C_{x(2)}, \quad H = \lambda_2 C_x^2 + \theta C_{x(2)}^2$

Situation II

$$Var\left(t_0\right) - MSE\left(t_{ne2}\right) \geq 0 \qquad \text{if}$$

$$\rho_{yz} \geq \frac{1}{4}.\frac{C_z}{C_y}$$

$$MSE\left(t_6\right) - MSE\left(t_{ne2}\right) \geq 0 \qquad \text{if}$$

$$\frac{C_x^2 - C_z^2}{4C_y} \geq \rho_{yz}C_z - \rho_{yx}C_x$$

$$MSE\left(t_7\right) - MSE\left(t_{ne2}\right) \geq 0 \qquad \text{if}$$

$$\frac{C_x^2 - C_z^2}{4C_y} \geq -\rho_{yz}C_z - \rho_{yx}C_x$$

As minimum/optimum MSE of t_8, and t_9 are same. So

$$MSE\left(t_{8,}t_9\right)_{opt} - MSE\left(t_{ne2}\right) \geq 0 \qquad \text{if}$$

$$\rho_{yx}^2 < \frac{D}{4C_y^2}$$

The proposed estimators are more efficient if the given conditions are fulfilled.

6. EMPIRICAL COMPARISON BETWEEN PROPOSED ESTIMATORS WITH SOME OF THE EXISTING ESTIMATORS

In this section, numerical comparison has been made among the proposed exponential estimators, usual unbiased estimator and some existing estimators of population mean of study variable in presence of non-response. Empirical study has been carried out using the data considered by Khare and Sinha (2007). The description of the data is as follows:-

The data on physical growth of upper socioeconomic group of 95 school children of Varanasi under an ICMR study, Department of pediatrics, B.H.U., during 1983-1984 has been taken under study. The first 25% units have been considered as non-responding units. Let us consider the study and auxiliary variables as follows:

y: weight in kg of the children,
x: skull circumference in cm of the children,
z: chest circumference in cm of the children.

$\bar{Y} = 19.4968$,	$C_y = 0.15613$,	$C_{y(2)} = 0.12075$,
$\bar{X} = 51.1726$,	$C_x = 0.03006$,	$C_{x(2)} = 0.02478$,
$\bar{Z} = 55.8611$,	$\bar{Z} = 55.8611$,	$C_{z(2)} = 0.05402$,
$\rho_{yx} = 0.328$,	$\rho_{yx(2)} = 0.477$,	$\rho_{yz(2)} = 0.729$,
$\rho_{yz} = 0.846$,	$\rho_{xz} = 0.297$,	$\rho_{xz(2)} = 0.570$

$N = 95$, $n_1 = 35$, $n_2 = 24$ $W_1 = 0.75$, $W_2 = 0.25$.

Table 1
Percent Relative Efficiencies (PREs) of Estimators with Respect to t_0 Considering Situation I for Different Values of h

Estimators	1/h			
	1/2	**1/3**	**1/4**	**1/5**
t_0	100	100	100	100
t_1	106.3224	106.775	107.1169	107.3843
t_2	92.72829	92.36116	92.08774	91.87621
t_3, t_4, t_5	114.335	116.1003	117.5027	118.6418
t_{ne1}	**218.3771**	**211.6934**	**207.0563**	**203.6523**

Table 2
Percent Relative Efficiencies (PREs) of Estimators with Respect to t_0 Considering Situation II for Different Values of h

Estimators	1/h			
	1/2	**1/3**	**1/4**	**1/5**
t_0	100	100	100	100
t_6	104.701	104.0024	103.4845	103.0853
t_7	94.30903	95.08227	95.67052	96.13308
t_8, t_9	109.8475	108.3232	107.2075	106.3556
t_{ne2}	**188.3714**	**167.2507**	**154.2782**	**145.5012**

7. COMMENTS AND CONCLUSION

Mathematical comparison made with usual unbiased estimator and some other exponential estimators cited in section 2 shows that our proposed estimators are more efficient under the conditions given in section 5. Furthermore, results of empirical study, presented in Table 1 & Table 2 revealed that our proposed estimators are more efficient. The percentage relative efficiency of our proposed estimators increases as the sub sampling fraction increases.

REFERENCES

1. Chanu, W.W. and Singh B.K. (2015). Improved Exponential Ratio cum Exponential Dual to Ratio Estimator of Finite Population Mean in Presence of Non-Response. *J. Stat. Appl. Pro.*, 4(1), 103-111.
2. Cochran, W.G. (1977). *Sampling Techniques*. 3rd ed., John Wiley and Sons, New York.
3. Hansen, M.H. and Hurwitz, W.N. (1946). The problem of non-response in sample surveys. *J. Amer. Statist. Assoc*. 41, 517-529.
4. Ismail, M., Shahbaz, M.Q. and Hanif, M. (2011). A General class of estimator of population mean in the presence of non-response. *Pak. J. Statist.*, 27(4), 467-476.
5. Ismail, M., Hanif, M. and Shahbaz, M.Q. (2015). Generalized Estimators for Population Mean in the Presence of Non-Response for Two-Phase Sampling. *Pak. J. Statist.*, 31(3), 295-306.

6. Khare, B.B. and Srivastava, S. (1993). Estimation of population mean using auxiliary character in presence of non-response. *Nat. Acad. Sci. Lett. India*, 16, 111-114.
7. Khare, B.B. and Srivastava, S. (1995). Study of conventional and alternative two-phase sampling ratio, product and regression estimators in presence of non-response. *Proc. Indian. Nat. Sci. Acad.*, 65, 195-203.
8. Khare, B.B. and Srivastava, S. (1997). Transformed ratio type estimators for the population mean in presence of non-response. *Comm. Statist. Theory Methods*, 26, 1779-1791.
9. Khare, B.B. and Srivastava, S. (2010). Generalized two phase estimators for the population mean in the presence of non-response. *Aligarh J. Statist.*, 30, 39-54.
10. Kumar, S. and Bhougal, S. (2011). Estimation of the Population Mean in Presence of Non-Response. *Communications of the Korean Statistical Society*, 18(4), 537-548.
11. Pal, S.K. and H.P. Singh, H.P. (2016). Finite Population Mean Estimation through a Two-Parameter Ratio Estimator Using Auxiliary Information in Presence of Non-Response. *JAMSI*, 12(2), 5-39.
12. Rao, P.S.R.S. (1986). Ratio estimation with sub sampling the non-respondents. *Surv. Methodology*, 12(2), 217-230.
13. Rao, P.S.R.S. (1990). Regression estimators with sub sampling of non-respondents, In-*Data Quality Control.* Theory and Pragmatics, (Gunar E. Liepins and V.R.R. Uppuluri, eds) Marcel Dekker, New York, 191-208.
14. Singh, H.P. and Kumar, S. (2008). Estimation of mean in presence of non-response using two-phase sampling scheme. *Statist. Pap*. DOI 10.1007/s00362-008-040-5.
15. Singh, H.P. and Kumar, S. (2009). A general procedure for estimating the population mean in presence of non-response under double sampling using auxiliary information. *SORT*, 33(1), 71-83.
16. Singh, H.P. and Kumar, S. (2011). Combination of regression and ratio estimate in presence of non- response. *Braz. J. Statist. Assoc.*, 25(2), 205-217.
17. Singh, H.P., Kumar, S. and Kozak, M. (2010). Improved Estimation of Finite-Population Mean Using Sub-Sampling to Deal with Non-response in Two-Phase Sampling Scheme. *Communications in Statistics - Theory and Methods*, 39(5), 791-802.
18. Singh, R., Kumar, M., Chaudhary M.K. and Smarandache, F. (2009). Estimation of mean in presence of non-response using exponential estimator. *arXiv preprint arXiv:0906.2462*.
19. Sodipo, A.A. and Obisesan, K.O. (2007). Estimation of the population mean using difference cum ratio estimator with full response on the auxiliary character. *Research Journal of Applied Sciences* 2, 769-772.
20. Tabasum, R. and Khan, I.A. (2004). Double sampling for ratio estimation with non-response. *J. Ind. Soc. Agricult. Statist*., 58(3), 300-306.
21. Tabasum, R. and Khan, I.A. (2006). Double sampling ratio estimator for the population mean in presence of non-response. *Assam Statist. Rev*., 20(1), 73-83.
22. Tripathi, T.P. and Khare, B.B. (1997). Estimation of mean vector in the presence of nonresponse. *Commun. Statist. Theor. Meth*., 26(9), 2255-2269.

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FORECASTING OF PAKISTAN'S INFLATION RATE: A COMPARISON OF SOME TIME SERIES METHODOLOGIES

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ABSTRACT

Inflation is an important economic indicator of Pakistan. Box-Jenkins methodology, Neural Networking, and Fourier analysis have been utilized for forecasting of Pakistan's Inflation rate. Data have been collected from the site of Pakistan Bureau of Statistics ranging from January 2008 to May 2017. On the basis of accuracy measures, models have been compared and found that Neural Networking is more suitable to forecast inflation rate of Pakistan.

KEY WORDS

Inflation, Periodogram, Fourier analysis, Neural Networking, ARIMA.

1. INTRODUCTION

Inflation is a state in which price level of goods and services increases and purchasing power decreases. There are many elements cause rise in inflation such as increase in population, increase in demand, lack of supply, development expenditures and constantly low production due to some social, political, climatic, national or international situations. Increase in inflation rate gives birth to unemployment, poverty, unfair distribution, poor labour force and social evils. Therefore, it is important to determine inflation rate in advance to control it as much as possible.

There are many researchers who tries to develop an efficient procedure to forecast the inflation rate and other economic indicators. Etuk (2012) presented time series analysis of Nigerian monthly inflation rates and fitted a multiplicative seasonal autoregressive integrated moving model. He showed that the model was adequate and forecasts were closely related with observations. Raza, Javed and Naqvi (2013) discussed the impact of inflation on economic growth of Pakistan. They suggested that government should maintain inflation in single digit which is favorable for economic growth. Ekpenyong et al. (2014) proposed the application of Periodogram analysis and Fourier analysis to model inflation rates in Nigeria. The main objectives of the study was to identify inflation cycles and fit appropriate model to forecast future values. Konarasinghe and Abeynayake (2015) proposed Fourier transformation on model fitting for Sri Lankan share market. They also analyzed the monthly returns by Auto-Regressive Integrated

Moving Average model and concluded that Fourier transformation along with multiple regression was suitable procedure.

Jere and Siyanga (2016) utilized Holt's exponential smoothing and ARIMA models to forecast inflation rate of Zambia. They showed that the choice of Holt's exponential smoothing was as good as an ARIMA model. Konarasinghe, Abeynayake and Gunaratne (2016) developed a model named as circular model based on Fourier transformation to forecast Returns of Sri Lankan Share Market. Thomson and Vuuren (2016) proposed Fourier transformation analysis to determine the duration of South African business cycle which was measured by using log changes in nominal GDP (Gross Domestic Product). Three dominant cycles have been used to forecast log monthly nominal GDP and the forecasts were compared to historical data. They found that Fourier analysis was more effective in estimating the business cycle length as well as in determining the position of economy in business cycle. Iqbal et al. (2018) utilized Fourier transformation and ARIMA model to predict the Pakistan's inflation rate. Results revealed that Fourier transformation has been more suitable to predict inflation rate in Pakistan.

2. METHODOLOGY

The main objective of this study is to develop a significant model to forecast monthly inflation rates in Pakistan. For this purpose, monthly data on CPI inflation rates from January, 2008 to May, 2017 have been collected from Pakistan Bureau of Statistics (PBS). Box-Jenkins methodology, Neural Networking, and Fourier analysis have been utilized for this purpose. By using accuracy measures (Root Mean square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE)), all proposed models have been compared.

3. RESULTS AND DISCUSSION

Box-Jenkins Methodology

Time series plot of original inflation rates in Pakistan is developed.

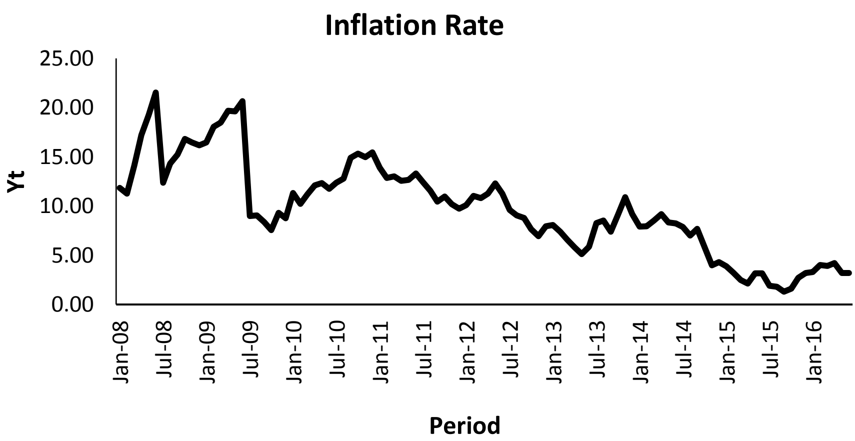

Figure 1: Original Series of Inflation Rates

Figure 1 shows that there is trend, seasonality and cyclical variations in the series but the length of cycle is not confirm. Therefore, Time series of inflation rates is non-stationary. To confirm this, Augmented Dickey-Fuller (ADF) test has been utilized. Results of ADF show that, time series has been stationary after applying first differencing transformation (see Table 1).

Table 1
Augmented Dickey-Fuller Test

	Test Statistic	**Sig.**
Level	-1.181013	0.216
First difference	-11.30547	0.00

Correlogram of first differenced series has been constructed to decide the values of parameters p and q (See Figure 2). Correlogram suggested that possible value of p is 12 or 16 while value of q is 16. From the possible values of p and q, two ARIMA models have been proposed. The forecasting errors of proposed models have been given in Table 2.

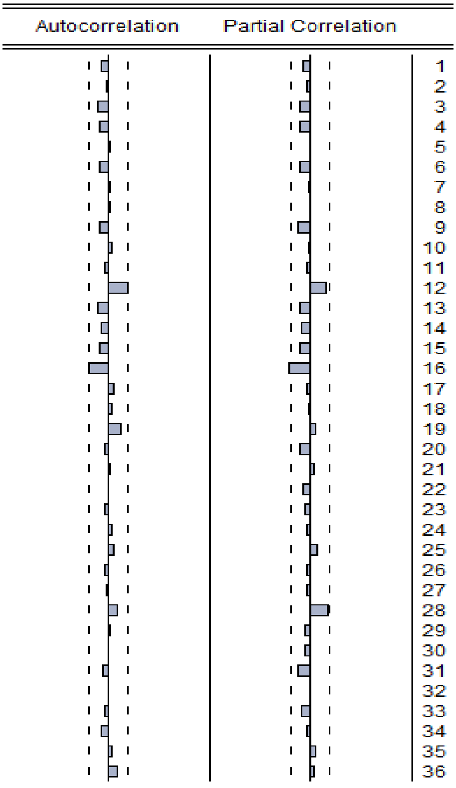

Figure 2: Correlogram

Results in Table 2 reveal that ARIMA (12, 1, 16) has less forecasting errors as compare to ARIMA (16, 1, 16). Therefore, from Box-Jenkins Methodology our suggested model has been ARIMA (12, 1, 16).

Table 2
Forecasting Errors of ARIMA Models

Model	RMSE	MSE	MAPE
(12,1,16)	2.52	6.36	34%
(16,1,16)	3.03	9.18	40%

Fourier Transformation

Fourier transformation has been utilized to transform time series into frequency domain series for determining the importance of each frequency in the original series. The general Fourier series model is written as:

$$IR_t = T_t + \sum_{i=1}^{k}(\alpha_i cos_i wt + \beta_i sin_i wt) + \varepsilon_t \quad (1)$$

Where T_t is trend component, k = n/2 (in case of even number) and k = (n-1)/2 (in case of odd number), w is angular frequency (= 2πf), α_i and β_i are coefficients, and ε_t is error term. The coefficients will have been calculated as:

$$\alpha_i = \frac{2}{N}\sum_{t=1}^{N}\left(RF_t - \hat{T}_t\right) cosw_i t \quad (2)$$

$$\beta_i = \frac{2}{N}\sum_{t=1}^{N}\left(RF_t - \hat{T}_t\right) sinw_i t \quad (3)$$

Fitting trend model

The estimated trend equation for inflation rate of Pakistan is:

$$\hat{T}_t = 16.927 - 0.1387t \quad (4)$$

Table 3
Summary of trend model

Predictor	coef	SE coef	t	P
Constant	16.9270	0.4992	33.91	0.000
T	-0.1387	0.0084	-16.48	0.000
S = 2.50222 R-Sq = 73.1% R-Sq(adj) = 72.8%				

Table 3 shows that both the parameters in the trend model are significant.

Periodogram Analysis

Periodogram has been utilized to partition the total variance of inflation rate into component variances because period in cycles are not exactly known. The estimated frequencies (freq), periods (p), and their intensities (pdg) are given in table 4. Results showed that, period and frequency corresponding to largest intensity are n = 34, f = 0.02941. Here frequency is just the inverse of period. Thus the period of cycle or long term inflation cycle is 34 months.

Table 4
Frequencies, Periods and Intensities

Freq	P	PDG	Freq	P	PDG
0		0	0.254902	3.923077	2.469466
0.009804	102	17.82893	0.264706	3.777778	2.571481
0.019608	51	0.283872	0.27451	3.642857	4.361273
0.029412	34	151.7146	0.284314	3.517241	3.985825
0.039216	25.5	5.061672	0.294118	3.4	1.131958
0.04902	20.4	143.5882	0.303922	3.290323	0.115284
0.058824	17	24.33793	0.313725	3.1875	0.848861
0.068627	14.57143	6.144949	0.323529	3.090909	4.099034
0.078431	12.75	13.55495	0.333333	3	4.243186
0.088235	11.33333	17.27606	0.343137	2.914286	3.418537
0.098039	10.2	27.54437	0.352941	2.833333	1.415018
0.107843	9.272727	63.83039	0.362745	2.756757	1.373335
0.117647	8.5	3.69523	0.372549	2.684211	1.026038
0.127451	7.846154	1.240284	0.382353	2.615385	1.869164
0.137255	7.285714	6.553623	0.392157	2.55	3.213267
0.147059	6.8	4.288788	0.401961	2.487805	3.215139
0.156863	6.375	17.12196	0.411765	2.428571	2.99018
0.166667	6	23.81014	0.421569	2.372093	2.466381
0.176471	5.666667	14.43552	0.431373	2.318182	2.288237
0.186275	5.368421	4.424012	0.441176	2.266667	0.927199
0.196078	5.1	3.554464	0.45098	2.217391	0.390481
0.205882	4.857143	3.682635	0.460784	2.170213	0.329865
0.215686	4.636364	0.798868	0.470588	2.125	1.620553
0.22549	4.434783	6.765822	0.480392	2.081633	3.01465
0.235294	4.25	2.518789	0.490196	2.04	3.122662
0.245098	4.08	3.450255	0.5	2	4.191004

Significance test of periodic component

Table 5
Output of Significance test

	G statistic	Critical value
Fisher Test	0.2415	0.126

As Fisher test statistic value is greater than the critical value so the null hypothesis of white noise is rejected and the selected period is significant.

Estimation of seasonal component

Since the number of observations are even therefore,

$$K = \frac{n}{2} = \frac{34}{2} = 17$$

and

$$w = 2 \times \pi \times 0.02941$$

Hence

$$Y_t = \sum_{i=1}^{17} [\alpha_i \cos iwt + \beta_i \sin iwt] \quad (5)$$

The parameters are estimated by least square method.

Table 6
Parameter Estimation in Seasonal Component

Predictor	Coef	SE coef	t	p	Predictor	Coef	SE coef	t	p
coswt	0.2011	0.3326	0.60	0.547	sin9wt	-0.1804	0.3326	-0.54	0.589
sinwt	1.7138	0.3325	5.15	0.000	cos10wt	0.0979	0.3326	0.29	0.769
cos2wt	0.5942	0.3326	1.79	0.078	sin10wt	-0.1098	0.3327	-0.33	0.742
sin2wt	-0.3602	0.3325	-1.08	0.283	cos11wt	0.1520	0.3326	0.46	0.649
cos3wt	-0.5690	0.3326	-1.71	0.092	sin11wt	-0.2342	0.3328	-0.70	0.484
sin3wt	0.0970	0.3325	0.29	0.771	cos12wt	0.1696	0.3326	0.51	0.612
cos4wt	0.2673	0.3326	0.80	0.424	sin12wt	0.0322	0.3330	0.10	0.923
sin4wt	-0.0619	0.3325	-0.19	0.853	cos13wt	0.1959	0.3326	0.59	0.558
cos5wt	-0.1403	0.3326	-0.42	0.674	sin13wt	-0.0045	0.3332	-0.01	0.989
sin5wt	-0.2481	0.3325	-0.75	0.458	cos14wt	-0.1704	0.3326	-0.51	0.610
cos6wt	0.4640	0.3326	1.39	0.168	sin14wt	0.1847	0.3339	0.55	0.582
sin6wt	-0.2661	0.3325	-0.80	0.426	cos15wt	0.1041	0.3326	0.31	0.755
cos7wt	0.0541	0.3326	0.16	0.871	sin15wt	-0.0645	0.3356	-0.19	0.848
sin7wt	-0.2593	0.3326	-0.78	0.438	cos16wt	-0.0459	0.3325	-0.14	0.891
cos8wt	0.2135	0.3326	0.64	0.523	sin16wt	-0.1154	0.3450	-0.33	0.739
sin8wt	0.0808	0.3326	0.24	0.809	cos17wt	0.4074	0.4954	0.82	0.414
cos9wt	-0.1208	0.3326	-0.36	0.717	sin17wt	28.55	47.13	0.61	0.547

Table 6 indicates that sinwt is significant at 5% level of significance and sinwt, cos2wt, cos3wt are significant at 10% level of significance. So the estimated seasonal models are given by:

$$\Delta\hat{Y}_t = 1.7138sinwt \text{ (at 5\%)} \tag{6}$$

$$\Delta\hat{Y}_t = 1.7138sinwt + 0.5942cos2wt - 0.5690cos3wt \text{ (at 10\%)} \tag{7}$$

Since $\Delta\hat{Y}_t = Y_t - \hat{T}$,

By exploring the randomness of error term with the help of ACF and PACF it is found that error term is not random. So, first order autoregressive is used.

Table 7
Summary of First Order Autoregressive

Predictor	Coef	SE coef	t	P
ε_{t-1}	0.65893	0.07242	9.1	0
s = 1.41653				

Table 7 presents that the parameter estimate of error term is significant in model. Thus,

$$\hat{\varepsilon}_t = 0.658822\varepsilon_{t-1} \tag{8}$$

General Fourier Series Model

The general models consists on estimated trend, estimated seasonal component and error component has been written as:

$$\hat{Y}_t = 16.927 - 0.1387t + 1.7138sinwt + 0.65893\varepsilon_{t-1} \tag{9}$$

$$\hat{Y}_t = 16.927 - 0.1387t + 1.1738sinwt + 0.5942cos2wt - 0.5690cos3wt + 0.65893\varepsilon_{t-1} \tag{10}$$

Model (9) and model (10) are found to be overall significant. The Residuals of both models are also found to be independently and normally distributed.

Table 8
Summary of Fourier Series Models

Model	RMSE	MSE	MAPE
(9)	1.68	2.82	15%
(10)	1.57	2.46	13%

Table 8 shows that RMSE, MSE and MAPE of model 10 has been smaller than model 9. Therefore, from Fourier series models our suggested model is 10.

Figure 3: Plot of actual and estimates

Figure 3 reveals that estimated values of both models are very close to actual values.

Neural Networking

Back Propagation NN with bipolar sigmoid function, 12 input layer neurons, 12 hidden layer neurons, and 1 output layer neuron has been utilized. In this method, there is no parametric model to represent the time series. The NN is applied for prediction of monthly inflation rate and plot of the original and its prediction is given in figure 4.

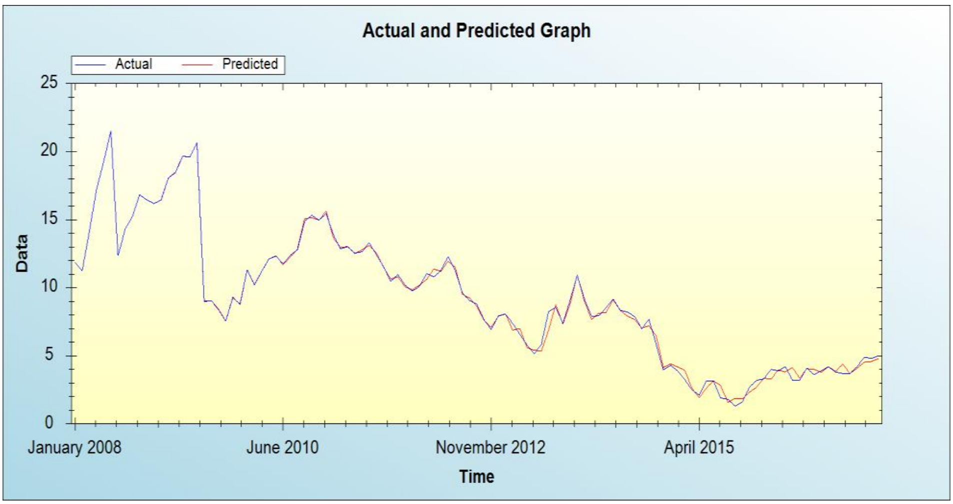

Figure 4: Actual and predicted Inflation Rate

The forecasted errors of neural networking method has also been quantified in Table 9.

Table 9
Forecasting Error

RMSE	MSE	MAE
0.315	0.099	0.209

From the output of Table 2, 8, and 9, it can be seen that forecasting errors in neural networking have been least as compare to Fourier series models and Box-Jenkins models. Therefore, neural networking has been utilized for forecasting of monthly inflation rate (see Table 10).

Table 10
Forecasted Values of Inflation Rate

Month	Inflation Rate	Month	Inflation Rate
Jun 2017	4.9292	Apr 2018	5.5453
Jul 2017	4.8422	May 2018	5.5895
Aug 2017	5.1746	Jun 2018	5.7662
Sep 2017	4.9977	Jul 2018	6.0149
Oct 2017	4.9049	Aug 2018	5.9587
Nov 2017	5.0330	Sep 2018	6.1389
Dec 2017	4.8546	Oct 2018	6.1165
Jan 2018	5.0548	Nov 2018	5.9815
Feb 2018	5.1364	Dec 2018	6.0878
Mar 2018	5.1913		

4. COMMENTS AND CONCLUSION

The main objective of this study is to develop a significant model to forecast monthly inflation rates in Pakistan. For this purpose, monthly data on CPI inflation rates from January, 2008 to May, 2017 have been collected from Pakistan Bureau of Statistics (PBS). By utilizing Box-Jenkins methodology, Neural Networking, and Fourier analysis different models have been proposed. It has been observed that accuracy measures RMSE, MAE, MAPE for NN are least among all proposed methodologies. Therefore, it is recommended to policy makers for utilizing NN for prediction of inflation rate.

REFERENCES

1. Ekpenyong, E.J., Omekara, C.O. and Ekerete, M.P. (2014). Modeling inflation rates using periodogram and Fourier series analysis methods: The Nigerian case. *International Journal of African and Asian Studies. Nigeria*. 4, 49-62.
2. Etuk, E.H. (2012). Predicting Inflation Rates of Nigeria Using A Seasonal Box-Jenkins Model. *Journal of Statistical and Econometric Methods,* 3, 27-37.
3. Iqbal, A., Ahmad, B., Iqbal, K. and Ali, A. (2018). Fourier Transformation on Model Fitting for Pakistan Inflation Rate. *Business and Economic Research*, 8(1), 84-94
4. Jere, S. and Siyanga, M. (2016). Forecasting Inflation Rate of Zambia Using Holt's Exponential Smoothing. *Open Journal of Statistics*, 6, 363-372.
5. Konarasinghe, W.G.S. and Abeynayake, N.R. (2015). Fourier Transformation on Model fitting for Sri Lankan Share Market Returns. *Global Journal for Research Analysis*, 4(1), 159-161.
6. Konarasinghe, W.G.S., Abeynayake, N.R. and Gunaratne, L. (2016). Circular Model on Forecasting Returns of Sri Lankan Share Market. *International Journal of Novel Research in Physics Chemistry and Mathematics*, 3(1), 49-56.
7. Raza, S.H., Javed, M.R. and Naqvi, S.A. (2013). Economic Growth And Inflation: A Time Series Analysis Of Pakistan. *International Journal of Innovative Research and Development*, 2(6), 689-703.
8. Thomson, D. and Vuuren, G.V. (2016). Forecasting The South African Business Cycle Using Fourier Analysis. *International Business and Economics Research Journal*, 15(4), 175-192.

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CHALLENGES OF ISLAMIC MARKETING – NAVIGATING BETWEEN TRADITIONAL AND CONTEMPORARY BOUNDARIES

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ABSTRACT

This paper is written with the purpose to discover the fundamentals of a market approach oriented from Islamic cultural with respect to its probable enactment by both the Muslims and non-Muslim firms. The paper is based on the analysis of available literature on the branding and marketing strategies for Islamic societies. Various approaches focused on associated market (from a traditional viewpoint) are debated with reference to their likely application by Muslims and non-Muslim firms, in particular in the Pakistani background.

The theoretic grounds and the results of present inquiry are meant to limit the view point and thought process of European and Western business community about Islamic marketing philosophies. At this stage of study, the paper does not include any empirical examination. The paper carries only conceptual arguments. Along with a proposed quadrate which is finally authenticate these arguments.

Making choices and arriving with a verdict is not an easy task for Western businesses especially, when we talk about marketing mix from Islamic point of view. The foremost challenge is how effectively the needs and wants of Muslim customers are responded. Keep following Islamic philosophies and customs without discriminating in Muslim and non-Muslim clienteles. The study emphasizes the significance of growing the edges of recent work on Islamic marketing and its related market strategies along with references to theoretical grounds of "traditional" advertising approaches through the worth making goal. In a nutshell, the paper interrogates the significance of the customary contrast between localization and global standardization

KEYWORDS

Muslim firms, Islam, National cultures, Marketing strategy, Islamic marketing/ branding, Market orientation, Islamic cultural values

1. INTRODUCTION

Marketing in the start was perceived as a merely tool to attain organizational objective. But now with the evolution of time it has become the fundamental measure of each business which is not only useful for the business but to the stake holders as well. It is a process which creates, communicate and deliver value to customers.

According to Kotler, organizations should continuously remain involved in R&D, innovative and learning efforts to stimulate buyers for the firms' output. The process should also be focused on adding values to the existing products, making the prices competitive, improving distribution channels and communication network. Businesses should adopt themselves according to the changing needs of customers.

With the process of evolution, the trend is now changing and the focus is shifting towards Customer relationship – the organizations are now striving more and more to create and maintain healthy and long-lasting relationship with their customers. Customer Relation Management (CRM) is a process that moves away from conventional Business model, aiming in internal affairs and is continually evolving. It is an approach taken by companies toward their customers with action to thoughtfully invest in resources such as individuals, expertise and business processes.

2. ISLAM AND MARKETING

It is proven that Islam is such a religion which guides us throughout our life in all phases of worldly affairs and so is the case with trade, business, banking and marketing.

The Holy Prophet S.A.W.W. said: 'The truthful merchant [is rewarded by being ranked] on the Day of Resurrection with prophets, veracious souls, martyrs and pious people.' (Tirmidhi, No: 1130).

Surprisingly, this idea has been presented by Islam 1400 years ago. Our studies of various books which were written on topics like Islam and Business, the Islamic ways of business etc. one will wonder to notice the emphasis given by this wonderful region on avoiding loss and precluding greed by adopting ethical business manners.

Following these can help us to elevate not only our behavioral standards but living standards as well, of traders and consumers alike.

The concept of marketing in Islam syndicates the benefits of both Social marketing and Relationship marketing. The concept does not only stress the impression of exclusively fulfilling the customers' wants and needs. Rather it contains severe constraints which were defined by Quran and Sunnah and must be followed

3. WHAT IS ISLAMIC MARKETING

The field of Islamic marketing is an emerging field which has a great potential to serve not only 1.7 billion Muslims of the world but the overall population. Unfortunately, researchers did not work as mush on the concept of Islamic marketing therefore we have no significant definition present of the concept but some experts have laid the foundation.

The identification and application of value maximization strategies processes for the stakeholders' welfare in particular and the general public is general are guided by the teachings of Quran and Sunnah (Hussnain, 2011).

Later in his research, based on the Islamic law and ethics, he has enlisted the following are the accepted and unaccepted manners of Marketing:

- ✓ Balancing supply and demand ratio to avoid too much loos and too much profit for either
- ✓ Avoiding dealing in socially unethical business products
- ✓ Return of goods sold to parties or their withdrawal from contracts based on solid compassionate ground may be accepted
- ✓ Prices should be control and unanimous for all.

The disapproved ones are as follows:

- ✓ Overemphasis over the product or services is not allowed
- ✓ Swearing truly about the sale or purchase is prohibited.
- ✓ Not attending to the less privileged or financial weak customers
- ✓ Offering discounts to buyers when they finalize deal with others
- ✓ Intervening in competitor's dealings when you are not part of the deal

4. PRINCIPLES OF ISLAMIC MARKETING

In Muslim society, Marketing should be religion-based (Alserhan, 2010) in which the ways and means of advertisement should take guidance from the Sharia framework. This also include the practices carried out inside the business entities independent of the religion of the owners, if the customers or consumers are Muslims (Alserhan, 2010). Lives of all Muslims are directed by the Islamic Sharia.

Companies trying to engage in business with Muslim consumers must be aware of these fundamental beliefs that act as a driving factor for the Muslim consumers. In case of MNC they should now learn to be culturally diverse and in addition they should not only enforce their own culture; rather they should take care for the Muslims employees, shareholders, stakeholder and customers. The religion possesses great power of accommodating and joining, therefore more Muslim consumers can be added to their customers base if their businesses will ethically attract the Muslims in the marketplace (Rice, 1999).

Even though maximization of profit is not the conclusive goal of trade in Islam, Islam agrees to take revenues and carry out the trade. It also does not target to eradicate all alterations in wealth and income whose end results in numerous communal and financial classes.

5. ISLAM AND THE MARKETING MIX

Islam is usually considered as a complete code of conduct in life; a worldview that unites the material and metaphysical and provides consistency and structure to individual survival (Izetbegovic, 1993). According to Abul Hassan and Abdul Latif (2008) claim that the principles of Islamic marketing which are based on justice and equity differs from worldly consciences on various terms. They debated the three features of market consciences or principles from the Islamic viewpoint. Table 1 shows a gist of the three viewpoints.

Table 1
Ethics of Islamic Market

- First, Islam being a complete code of conduct provides sufficient grounds for marketers to adopt Sharia compliant advertisements and leave no space for executives to argue otherwise.
- Second, the foremost difference is their inspiring characteristic of completeness and stiff
- Third, the Islamic teachings dictates enhancement in value of goods to benefit the society instead of the selfish pursuit of profit enhancement.
- The above principles are sufficient to guide the marketers to adopt ethical ways of advertisements and pave way for Muslim customers to respond and take part in selling-buying activities.

Source: Abul Hassan and Abdul Latiff (2008).

6. THE FIVE PS OF THE MARKETING MIX FROM ISLAMIC PERSPECTIVE

Abul Hassan and Abdul Latiff (2008), analyze the marketing principles based on the five ‘Ps’ concept in the Islamic marketing perspective and Islamic principles and morals. The essential five ‘Ps’ are; price, place, product, people and promotion.

PRODUCT

The first ‘P’ is product. According to Islamic view the product should be Halal. This means that all the inputs used to produce the product and the process applied should be compliance to sharia law. For example, in ready to eat food products the core ingredient is the meat, the animal should be slaughtered according to laws prescribed by Quran and Sunnah, further whole processing should be done in ways that no prohibited item should be used or include in the final product. To stimulate a company’s products, implementation of Shariah principles in marketing can work wonders. It is because of ultra-sensitive of Muslims towards their religion and its relevance with the products.

PRICE

We often observe that firms price the products in order to exploit and manipulate the general public. In this way the businesses in fact deceive the customers with the impression of offering discounts (Abul Hassan and Abdul Latiff, 2008). This act is being opposed by Islam and don’t allow organization to play with the minds of their consumers (Abul Hassan and Abdul Latiff, 2008).

PROMOTION RULES

Abul Hassan and Abdul Latiff (2008) emphasize that there is no chance in Islam for organization to justify any vague way of promotion or marketing. The holy Quran convicts all forms and figures of wrong claims, groundless blame, fabrication and false testimony. It is unethical, in terms of Islamic marketing, for the sales person or

Relationship Executives to promote their products and its feature and qualities which they lack same in reality. Furthermore, demonstrating deceit of any quality of a product to stimulate or sell it is strictly forbidden or banned in the Islam. Samad (2012) states that the publicity that delivers the information of truthful product distinction and supports in growing the moral ethics is welcome.

Thus, in the capacity of product promotions, Islamic advertising morals and ethics follows three rules shown in Table 2.

Table 2
Three Essential Rules in Islamic Market Ethics for Product Promotion

(i)	Incorrect and ambiguous publicity should be prevented;
(ii)	Refusal of high gravity operations, or deceptive sales strategies; and,
(iii)	Deception sales raises or manipulation are not allowed.
	Source: Abul Hassan and Abdul Latiff (2008).

PLACE

Based on the enlightened guidelines of Islamic society, the main purpose of establishing distribution channels must be for betterment and welfare of the society and should raise the living standard of people. Maximizing the profits should not be the sole purpose of product placement (Abul Hassan and Abdul Latiff, 2008).

PEOPLE

Under the aspirations of Islam, stress is always on the importance of independent and free decision on the customer's share. (Abul Hassan and Abdul Latiff, 2008) uphold that "large society should flourish honesty, should be free from intimidate information of marketing. Customer has a right to obtain such information and is representative of the status given to him/her by Islam, in addition one has the engrained privileges of his/her wealth that he/she spends in purchasing goods and services. It's the marketer's duty, to not to select any system of force and essentially, all in all situations, have a worry for the well-informed and knowledgeable dependability and an advanced grade of understanding of the customers and consumers to certify that the hard-received money of customers is used".

Islamic business practices and viewpoints signify an unusual means and ways business is directed these days. Islam makes it obligatory for traders to ensure balance between commercialism and humanitarianism as well as between profits and social integrity. This underlying approach to conduct business not merely provide a solution to the problems such as exploiting customers, negligent environment destruction and corporate governance, it sorts each effort to stimulate positive features of business such as honest behavior, rational revenue, reasonable struggle, business partnership, collaboration, least wages for workforces and basic consumerism values for instance the right of purchasers to return the purchased goods.

7. ISLAMIC MARKETING PRACTICES IN CONTEXT OF MUSLIM COUNTRIES

The recent findings from various researches reveal that Muslim consumer is now more particular towards their religious pride, economic advancement and international citizenship. The Muslim world is now getting more diverse in terms of social, ethnic, economic and political aspects which make it essential to know what determines their conduct in numerous economies (Ogilvy and Mather, 2010). As printed in a report by Ogilvy and Mather (2010) additionally elucidates the Muslim consumer's opinions and inclinations cannot be stereotype to many brands and their marketing communications. E.g., although certifying halal is important criteria, but now is no longer enough to influence the new Muslim consumer about the reliability of the product agreements with Islamic principles and values.

8. THEORETICAL BACKGROUND OF THE RESEARCH

The theoretical background of the research focuses merely on Islamic Marketing and Advertising in Pakistani context.

More than a decade ago, in mature markets weak economic growth rates impacting negatively to the consumption, has opened the race to the emerging markets as the volume drivers. Besides this, the academic and the managerial attention given to the Asian markets i.e. China and India, as well as South American markets especially Brazil, for the solution of the global economic crisis, research interest in Muslim majority markets is on rise. The Macroeconomic indicators emphasizes this growing interest i.e. 23% of the world's global population is Muslim, and it is expected that it will reach 33% by 2030 (Pew Forum, 2011). The growth of the Muslim population globally presents opportunities and challenges for tapping the growing Shariah compliant market expected at $2.3 trillion dollars annually (Al Bayan, 2010).

9. ISLAMIC MARKETING PRACTICES IN PAKISTAN

Mr. Jim O'Neill, a British economist discloses that Pakistan, being the atomic power, central Muslim State, possessing great importance in the Muslim economies, having potentials can become number 18th in the largest economies of the world by 2050.

In Pakistan, which is a Muslim-majority country, the concept of Islamic marketing is somewhat still unknown and even more hardly understood by marketing practitioners. Marketers show moderate to low level of interest due to the above reason. Pakistani marketers must be aware of this concept not just because it has strong cultural influence but as a stand-alone concept as well.

Following we are some of the most prominent firms which are engaged in Islamic marketing practices:

- Shan Foods Ltd,
- Junaid Jamshed
- Meezan Bank
- Shahid Afridi Men's garments
- Al-Baraka bank,
- Cobra Insecticides.

A bit more flexible impacts of Islamic marketing are evident in the case of ICON garment, Rooh-e-Afza, Hamdard and UBL Ameen.

Insights about Islamic marketing diverse from covert to overt appearances of it in advertisements. Executive instantly associate Islamic Advertising with apparent appearances such as “women head covered with scarves”, however less executives had supposed the concealed appearances such as “no human figures in ads, use of animations, no materialism, no fraud, no over promising, no delusions, etc.”

In the countries under the influence of western cultures use women in their advertisement just for the sake of attraction and increasing glamour by exposing their private body parts unnecessarily. On the contract, the Holy Qura’an forbids such things and dictates that keep your private parts covered and don’t expose them to other for any reason and promises that same is in their best interest. The Holy Qura’an especially warns the women to control their looks and guard their private parts. (Quran, 24:31, 32). This does not mean that Islam prohibits ladies from taking part in businesses rather they are informed about their decency and importance while precluding emotional inspirations. Women in Islam are not allowed to expose any part of their body except face, hands and feet (Al-Olayan and Karande, 2000). Exposing any part of body except the naturally revealed one is considered as stimulating and hence incorrect for the civic exhibition (Chachula, 2009).

Nakedness of ladies’ private body parts in the ads is a grave matter that now govern the entire world inclusive of Muslim countries. The nature of advertisement having ladies exposure varies from country to country. For example, advertisement for an item done in Saudi Arabia will be quite different for the advertisement of same item in Dubai, Qatar and Bahrain. Dubia, Qatar and Bahrain will follow the pattern of Western Styles while Saudi will never due to its strict culture on women affairs (Al-Olayan and Karande, 2000).

10. BRIDGING THE GAP BETWEEN TRADITIONAL MARKETING STRATEGIES AND ISLAMIC CULTURAL VALUES

Surprisingly, the value creation objectives are not mentioned in any of the Islamic marketing or advertising research. Even the most recent developed work regarding a more strategic framework is not mentioned. Although, the marketing philosophy has been supporting this notion since decades (Drucker, 1954; Holbrook, 2001). It’s been perceived that value creation is not a western concept but that collectively transforms cultures, plus economic and religious contexts. Maybe, it is in the new 2005 definitions of the AMA (American Marketing Association) that one should look for conceptual grounding that would bring the disciplines collectively rather than producing with Islamic marketing and branding yet another panacea (Cova, 2008).

Provided that the selected research ground is Muslim majority markets, and in light of the limitation of prior research, it’s believed necessary to study them within an oriented frame that would help in decision making in the light of product strategy. In that, the economic value creation of firms is professed as a method implicating an innovative combination and an exchange of multiple resources (Nambisan, 2002). This idea contributes to separate the value of a product as professed by clients in light of the competitive offers, from the method of generating that value, which is internal to the firm, and its assessment in terms of performance. Researcher takes the stand of seeing that the adoption of a market orientation strategy by firms is one of the key elements in this process. This direction is classified as an intangible resource (Srivastava, 1998), offers a continual cultural structure permissive the leveraging of consumer perceptive (generically labeled as "demand"). The competitive activities, with intend of reinforcing the primary capabilities that constantly create a better consumer perceived value (Slater and Narver, 1994).

11. CONCLUSION

Within a different cultural setting, marketing strategies need to be evaluated and implemented differently by (western) multinational corporations. Consequently, this research would like to ponder upon the issue whether this dichotomy is still relevant because western brands will incline towards a form of Islamization and "acculturation" and said local brands to a certain degree of Westernization. The phenomenon of convergence/divergence might change practices from globalization to fusion marketing (Sheth, 2011). At this level, the first step in this regard could be briefed as follows: what does it mean to be market oriented for western firms operating on Islamic markets? This query updates the debate between global standardization and localization that has endured for many years (Birnik and Bowman, 2007; Coskun and Jacobs, 1994; Okazaki, 2005).

In this framework, can religion be a segmentation criterion and drive market-oriented strategies of western firms? Religion is one of the characteristics that influence the 138 culture of a given group, other characteristics which are language, institutions, nationality, customs, etc. Hence, religion and the degree of religiosity can have impact on to varying degrees individual consumption habits, hence brand choices, and consumer behavior, plus the weight of influence of faith in way of life. In these circumstances, we assume that using the tribal metaphor might extend the conceptual foundations of Islamic marketing and branding in considering Muslim targets (as well as non-Muslim targets) as consumer tribes (Cova, 2007) whose behavior is part of a social context which is ruled by Shariah rules. The fundamental rationale is that 'the link is more important than the good itself', which highlights the emotional and symbolic associations between brands and consumers but also among consumers themselves. Therefore, marketers who understand the structure and philosophy of a sub-culture of consumption can be greatly lucrative by serving its needs with not only products and services but also with all of the flow-ons (Cova, 1996).

12. FUTURE IMPLICATIONS

There is a dire requirement to advance general guidelines for attaining marketing objectives and corporate goals in an ethically acceptable way. Ethical conduct in marketing and other advertising and promotional actions must be based on a strong basis of Islamic doctrines and commands. Corporate culture that draws upon Islamic norms of social behavior has to be developed. For example, attempts are made by multinational corporations to abolish indigenous industries by dumping their products in the markets of less developed economies, including some Muslim countries. Such diplomacies cannot be justified as fair promotion activities in a competitive world. Therefore, if a corporation condemns a competitors' products and promotes itself through unfair practices, then it is the responsibility of its professional management, the government, shari'ah enforcement agencies and authorities, and public interest groups to control these practices in order to pave the way for the development of an Islamic society based on the principles of fairness, equity, and justice. Hence, a council of Muslim marketers, advertisers. Corporate leaders and religious scholars/experts should be fashioned to develop an Islamic code of conduct for marketing and set rules and principles that define the expectations of an Islamic society of the business world. Taking into consideration the code of ethics adopted by the International Association of Business Communicators, it is suggested that the following issues be shielded by the code:

1) Maintaining the reliability and self-respect of their profession by inspiring the practice of frank, truthful and appropriate and well-timed communication;
2) Refrain from using any information that has been produced or properly attained from other firms without the permission of the latter (PR personnel should be able to distinguish carefully between emulation and plagiarism);
3) Abiding by laws, regulations, and Islamic standards governing their professional activities;
4) Not overlooking any unlawful, unethical or un-Islamic action related to, their corporate activities;
5) Respecting and protecting the secrecy and right of all individual's, employees, clients, and customer's privacy.

Promotional activities can play a vital role in creating awareness about halal and haram products among the Muslim segment of consumers in Muslim as well as non-Muslim societies. In this respect, a segmentation strategy can be followed by marketers to classify products into halal and haram products (as is done in the case of meat products in Malaysia). While, care must be taken that no contradictions or controversies are produced regarding the beliefs of non-Muslims. Cosmetics, toiletries, clothing, novelty items, and other consumer objects and household products should be advertised while taking keen care of special product features, needs, and expectations of Muslim consumers. Promotions directed at the non-Muslim consumer fragment of the population should be designed and launched with prominence on scientific rationale and ethical explanations.

13. RESEARCH PERSPECTIVE

In relations of methodological choices, we chose "quadrats" as units of analyses (Deshpande´, 1993). Hence, our sample will be consisting of following four types of firms practicing Islamic marketing for their products and services:

(1) A local brand
(2) An international brand
(3) A local brand having no separate strategies for religion-based marketing
(4) An international brand that does not make any difference between Muslim and non-Muslim groups.

As underlined by Deshpande´ (1993), this sort of method is time-consuming and tremendously expensive to be completed successfully. Thus, we will only focus on four market segments: haircare, hair coloring, make-up, and fashion. In a context such as that culturally diverse country in terms of religious practices, this research takes on particular significance and interest is thereby strengthened.

REFERENCES

1. Al, Bayan (2010). The global halal market worth 2.1 trillion US dollars. *Al Bayan*, No. 10955, June 16, p. 17 (translated from Arabic).
2. Holbrook, M.B. (2001). Marketing Behavior and Executive Action. *ACR News*, (Winter), 37-8.
3. Al-Olayan, F.S. and Karande, K. (2000). A content analysis of magazine advertisements from the United States and the Arab world. *Journal of Advertising*, 29(3), 69-82.
4. Alserhan, B. (2010). Islamic branding: a conceptualization of related terms. *Journal of Brand Management*, 18(1), 34-49.
5. Badot, O. and Cova, B. (2008). The myopia of new marketing panaceas: the case for rebuilding our discipline. *Journal of Marketing Management*, 24(1-2), 205-219
6. Birnik, A. and Bowman, C. (2007). Marketing mix standardization in multinational corporations: 139 a review of the evidence. *International Journal of Management Reviews*, 9(4), 303-24.
7. Chachuła, G., Kucharski, H., Lubaś, A., Małachowska, K. and Martinovski, B. (2009). The influence of selected aspects of Islam on advertising efforts. T*he 16th NIC Conference on Intercultural Communication*, University of Borås, Sweden.
8. Cova, B. (1996). The postmodern explained to managers: implications for marketing. *Business Horizons*, 39(6), 15-27.
9. Cova, B., Kozinets, R.V. and Shankar, A. (2007). *Consumer Tribes*, Butterworth-Heinemann, Oxford.
10. Deshpande, R., Farley, J.U. and Webster, F.E. (1993). Corporate culture, customer orientation and innovativeness in Japanese firms: a quadrad analysis. *Journal of Marketing*, 57(1), 23-37.
11. Drucker, P.F. (1954). *The Practice of Management*, Harper and Row, New York, NY.
12. Han, J.K., Kim, N. and Srivastava, R.K. (1998). Market orientation and organizational performance: is innovation a missing link?. *The Journal of Marketing*, 62(4), 30-45.
13. Hassan, A., Chachi, A. and Abdul Latiff, S. (2008). Islamic marketing ethics and its impact on customer satisfaction in the Islamic banking industry. *Islamic Economics*, 21(1), 27-46.
14. Hussnain, S. (2011). What Is Islamic Marketing? *Global Journal of Management and Business Research*, 11(11), 101-103.
15. Izetbegovic, A.A. (1993). *Islam between east and west* (3rd edition). Plainfield, IN: American Trust Publications.
16. Kotler, P. and Levy, S.J. (1969). Broadening the concept of marketing. *The Journal of Marketing*, 33(1), 10-15.
17. Nambisan, S. (2002). Information systems as a reference discipline for new product development. *Mis Quarterly*, 27(1), 1-18.
18. Ogilvy and Mather (2010). *Brands, Islam and the new Muslim consumer*, viewed 17 June 2012, http://www.ogilvy.com/News/Press-Releases/May-2010-The-Global-Rise-of-the- New-Muslim-Consumer.aspx
19. Ogilvy Noor (2010). Ogilvy & Mather's research defines the global rise of the 'New Muslim Consumer', available at: www.ogilvy.com/News/Press-Releases/May-2010-The-GlobalRise-of-the-New-Muslim-Consumer.aspx

20. Okazaki, S. (2005). Searching the web for global brands: how American brands standardize their web sites in Europe. *European Journal of Marketing*, 39(1/2), 87-109.
21. O'Neill, J. (2001). *Building better global economic BRICs* (pp. 1-15). New York: Goldman Sachs.
22. Pew Forum (2011). The future of the global Muslim population – projections for 2010-2030. available at: http://pewresearch.org/pubs/1872/muslim-population-projections-worldwidefast-growth\
23. Rice, G. (1984). Marketing management in Egypt. *Management Decision*, 22(4), 3-13.
24. Samad, A. (2008). Market Analysis from an Islamic Perspective and the Contribution of Muslim Scholars. *Journal of Islamic Economics, Banking and Finance*, 4(3), 55-68.
25. Schein, E.H. (1992). *Organizational Culture and Leadership*, Jossey-Bass, *San Francisco, CA.*
26. Sheth, J.N. (2011). Impact of emerging markets on marketing: rethinking existing perspectives and practices. *Journal of Marketing*, 75(4), 166-82.
27. Slater, S.F. and Narver, J.C. (1994). Market orientation, customer value, and superior performance. *Business horizons*, 37(2), 22-28.
28. Temporal, P. (2011). *Islamic Branding and Marketing: Creating a Global Islamic Business,* Wiley, Singapore.
29. Tournois, L. and Aoun, I. (2012). From traditional to Islamic marketing strategies: Conceptual issues and implications for an exploratory study in Lebanon. *Education, Business and Society: Contemporary Middle Eastern Issues*, 5(2), 134-140.
30. Usmani, M.I.A. (2006). Meezan Bank guide to Islamic Banking, Darul Ishat Karachi.
31. Waller, D.S. and Shyan Fam, K. (2000). Cultural values and advertising in Malaysia: views from the industry. *Asia Pacific Journal of Marketing and Logistics*, 12(1), 3-16.

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CHARACTERIZATION OF SOME BETA-G DISTRIBUTIONS

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ABSTRACT

We propose the methods of characterizing some members of the Beta-G family of distributions (that is, Burr XII (BBXII) of Cordeiro et al. (2011), beta Inverse Weibull (BIW) distribution of Khan (2010), beta modified Weibull (BMW) distribution of Silva et al. (2009) and beta Weibull (BW) distribution of Lee et al. (2007)). These methods are based on the use of truncated moments.

KEY WORDS

Univariate continuous distributions; Characterizations; Truncated moments.

1. INTRODUCTION

Cordeiro (2011) introduced BBXII distribution with probability density function (pdf) f(x) given by

$$f(x)=\frac{ck\ x^{c-1}}{\lambda^{c}B(a,b)}\left[1+\left(\frac{x}{\lambda}\right)^{c}\right]^{-kb-1}\left[1-\left\{1+\left(\frac{x}{\lambda}\right)^{c}\right\}^{-k}\right]^{a-1},\quad x>0 \tag{1.1}$$

where all parameters α, λ, c, a, b >0.

Corresponding cumulative distribution function (cdf) for x>0 is

$$F(x)=\frac{1}{B(a,b)}\int_{0}^{\left\{1+\left(\frac{x}{\lambda}\right)^{c}\right\}^{-k}} w^{a-1}(1-w)^{b-1}\,dw \tag{1.2}$$

Khan (2010) proposed beta Inverse Weibull (BIW) distribution with pdf and cdf given, respectively, by

$$f(x)=\frac{\beta x^{-\beta-1}}{\eta B(a,b)}\exp\left(-\frac{1}{\eta}\left(\frac{1}{x}\right)^{\beta}\right)\left\{1-\exp\left(-\frac{1}{\eta}\left(\frac{1}{x}\right)^{\beta}\right)\right\}^{b-1},\quad x>0 \tag{1.3}$$

where all parameters β, η, a, b >0. and

$$F(x)=\frac{1}{B(a,b)}\int_{0}^{1-\exp\left(-\frac{1}{\eta}\left(\frac{1}{x}\right)^{\beta}\right)} w^{a-1}(1-w)^{b-1}dw \tag{1.4}$$

Silva et al. (2009) proposed beta modified Weibull (BMW) distribution with pdf and cdf given, respectively, by

$$f(x)=\frac{\alpha(\gamma+\lambda x)x^{\gamma-1}}{B(a,b)}e^{\lambda x}\left(1-\exp\left(-\alpha x^{\gamma}e^{\lambda x}\right)\right)^{a-1}\exp\left(-\alpha b x^{\gamma}e^{\lambda x}\right),\ \ x>0 \tag{1.5}$$

where all parameters α, λ, γ, a, b >0.

Thus cumulative distribution function (cdf) of the beta modified Weibull distribution is given by

$$F(x)=\frac{1}{B(a,b)}\int_{0}^{1-\exp\left(-\alpha x^{\gamma}e^{\lambda x}\right)} w^{a-1}(1-w)^{b-1}dw \tag{1.6}$$

Lee et al (2007) introduced beta Weibull (BW) distribution

$$f(x)=\frac{\gamma x^{\gamma-1}}{B(a,b)}\left(1-e^{-(\lambda x)^{\gamma}}\right)^{a-1}e^{-b(\lambda x)^{\gamma}},\quad x>0 \tag{1.7}$$

where all parameters α, λ, γ, a, b >0.

Thus cumulative distribution function (cdf) of the beta Weibull distribution is given by

$$F(x)=\frac{1}{B(a,b)}\int_{0}^{1-e^{-(\lambda x)^{\gamma}}} w^{a-1}(1-w)^{b-1}dw \tag{1.8}$$

2. CHARACTERIZATIONS

Characterization of a distribution is theoretically a unique way of identifying it. In this paper we use of truncated moments in proposing the characterization of the above members of Beta-G distributions. We present here characterizations for four probability distributions based on truncated moments.

2.1 Characterizations based on Two Truncated Moments

In this subsection, characterizations of BBXII, BIW, BMW and BW distributions based on the ratio of two truncated moments are presented. For characterization of a distribution, we employ a theorem of Glänzel (1987).

Proposition 1:

Let X: $\Omega \rightarrow (0,\infty)$ be distributed as Eq.(1.1)

$$q_1(x) = \left(1+\left(\frac{x}{\lambda}\right)^c\right)^{kb-k}\left\{1-\left(1+\left(\frac{x}{\lambda}\right)^c\right)^{-k}\right\}^{1-a}$$

$$q_2(x) = q_1(x)\left(1+\left(\frac{x}{\lambda}\right)^c\right)^{-k} \quad \text{for } x>0. \tag{2.1}$$

The random variable X follows BBXIID if and only if the function η, defined using theorem presented by Glänzel (1987), is of the form

$$\eta(x) = \frac{1}{2}\left(1+\left(\frac{x}{\lambda}\right)^c\right)^{-k}. \tag{2.2}$$

Corollary 1:

Let $X: \Omega \rightarrow (0,\infty)$ be a continuous random variable and let $q_2(x)$ be as in Proposition 1. The random variable X has pdf (1.1) if and only if there exist functions $q_1(x)$ and $\eta(x)$ defined in theorem (Glänzel (1987)) satisfying the following differential equation.

$$\frac{\acute{\eta}(x)q_1(x)}{\eta(x)q_1(x)-q_2(x)} = \frac{\frac{k}{\lambda^c}c\,x^{(c-1)}}{\left(1+\left(\frac{x}{\lambda}\right)^c\right)} \tag{2.3}$$

The general solution of the above differential equation is

$$\eta(x) = \left(1+\left(\frac{x}{\lambda}\right)^c\right)^{-1}\left[-\int_0^x \frac{c\,x^{c-1}q_2(x)}{\lambda^c\,q_1(x)}dx + D\right]$$

where D is a constant. We like to point out that one set of functions satisfying the above differential equation is given (Glänzel (1987)) with D=1/2.

Proposition 2:

Let X: $\Omega \rightarrow (0,\infty)$ be distributed as Eq.(1.3)

$$q_1(x) = \left(1-exp\left(\frac{-1}{\eta}\left(\frac{1}{x}\right)^\beta\right)\right)^{1-b} exp\left(\frac{-(1-a)}{\eta}\left(\frac{1}{x}\right)^\beta\right)$$

$$q_2(x) = q_1(x)\,exp\left(\frac{-1}{\eta}\left(\frac{1}{x}\right)^\beta\right),\quad x>0 \tag{2.4}$$

The random variable X follows BIW if and only if the function η is of the form

$$\eta(x) = \frac{1}{2}\left(1+exp\left(\frac{-1}{\eta}\left(\frac{1}{x}\right)^\beta\right)\right). \tag{2.5}$$

Corollary 2:

Let $X: \Omega \to (0, \infty)$ be a continuous random variable and let $q_2(x)$ be as in Proposition 2. The random variable X has pdf (1.3) if and only if there exist functions $q_1(x)$ *and* $\eta(x)$ satisfying the following differential equation.

$$\frac{\acute{\eta}(x) q_1(x)}{\eta(x) q_1(x) - q_2(x)} = \frac{\frac{\beta}{\eta}\ x^{-\beta-1} exp\left(\frac{-1}{\eta}\left(\frac{1}{x}\right)^{\beta}\right)}{\left\{1 - exp\left(\frac{-1}{\eta}\left(\frac{1}{x}\right)^{\beta}\right)\right\}} \tag{2.6}$$

The general solution of the above differential equation is

$$\eta(x) = \left(1 - exp\left(\frac{-1}{\eta}\left(\frac{1}{u}\right)^{\beta}\right)\right)^{-1} \left[-\int_0^x \frac{\beta}{\eta}\ x^{-\beta-1} exp\left(\frac{-1}{\eta}\left(\frac{1}{x}\right)^{\beta}\right) \frac{q_2(x)}{q_1(x)} dx + D\right]$$

where D is a constant. We like to point out that one set of functions satisfying the above differential equation is given (Glänzel (1987)) with D=1/2.

Proposition 3:

Let $X: \Omega \to (0, \infty)$ be distributed as Eq.(1.5)

$$q_1(x) = exp\left(-\alpha(1-b) x^{\gamma} e^{\lambda x}\right)$$

$$q_2(x) = q_1(x)\ \left(1 - exp\left(-\alpha x^{\gamma} e^{\lambda x}\right)\right)^{a}, \ x>0. \tag{2.7}$$

The random variable X follows BIW if and only if the function η is of the form

$$\eta(x) = \frac{1}{2}\left(1 + \left(1 - exp\left(-\alpha x^{\gamma} e^{\lambda x}\right)\right)^{a}\right). \tag{2.8}$$

Corollary 3:

Let $X: \Omega \to (0, \infty)$ be a continuous random variable and let $q_2(x)$ be as in Proposition 3. The random variable X has pdf (1.5) if and only if there exist functions $q_1(x)$ *and* $\eta(x)$ satisfying the following differential equation.

$$\frac{\acute{\eta}(x) q_1(x)}{\eta(x) q_1(x) - q_2(x)} = \frac{a\ \alpha x^{\gamma-1} e^{\lambda x} (\gamma + \lambda x) exp\left(-\alpha x^{\gamma} e^{\lambda x}\right) \left(1 - exp\left(-\alpha x^{\gamma} e^{\lambda x}\right)\right)^{a-1}}{\left(1 - \left(1 - exp\left(-\alpha x^{\gamma} e^{\lambda x}\right)\right)^{a}\right)} \tag{2.9}$$

The general solution of the above differential equation is

$$\eta(x) = \left(1 - \left(1 - exp\left(-\alpha x^{\gamma} e^{\lambda x}\right)\right)^{a}\right)^{-1} \left[-\int_0^x a\ \alpha x^{\gamma-1} e^{\lambda x} (\gamma + \lambda x) exp\left(-\alpha x^{\gamma} e^{\lambda x}\right) \left(1 - exp\left(-\alpha x^{\gamma} e^{\lambda x}\right)\right)^{a-1} \frac{q_2(x)}{q_1(x)} dx + D\right]$$

where D is a constant. We like to point out that one set of functions satisfying the above differential equation is given (Glänzel (1987)) with D=1.

Proposition 4:

Let X: $\Omega \to (0,\infty)$ be distributed as Eq.(1.7)

$$q_1(x) = \left(1 - e^{-(\lambda x)^\gamma}\right)^{1-a}\left\{e^{-(\lambda x)^\gamma}\right\}^{1-b}$$

$$q_2(x) = q_1(x)\left(1 - e^{-(\lambda x)^\gamma}\right), \text{for } x > 0 \tag{2.10}$$

The random variable X follows BWD if and only if the function η is of the form

$$\eta(x) = 1 - \tfrac{1}{2}e^{-(\lambda x)^\gamma}. \tag{2.11}$$

Corollary 4:

Let X: $\Omega \to (0,\infty)$ be a continuous random variable and let $q_2(x)$ be as in Proposition 4. The random variable X has pdf (1.7) if and only if there exist functions $q_1(x)$ *and* $\eta(x)$ defined in Theorem1 satisfying the following differential equation.

$$\frac{\acute{\eta}(x)q_1(x)}{\eta(x)q_1(x) - q_2(x)} = \frac{a\,\alpha x^{\gamma-1}e^{\lambda x}(\gamma+\lambda x)exp\left(-\alpha x^\gamma e^{\lambda x}\right)\left(1 - exp\left(-\alpha x^\gamma e^{\lambda x}\right)\right)^{a-1}}{\left(1-\left(1-exp(-\alpha x^\gamma e^{\lambda x})\right)^a\right)} \tag{2.12}$$

The general solution of the above differential equation is

$$\eta(x) = \left(1-\left(1-exp\left(-\alpha x^\gamma e^{\lambda x}\right)\right)^a\right)^{-1}\left[-\int_0^x a\,\alpha x^{\gamma-1}e^{\lambda x}(\gamma + \lambda x)exp\left(-\alpha x^\gamma e^{\lambda x}\right)\left(1-exp\left(-\alpha x^\gamma e^{\lambda x}\right)\right)^{a-1}\frac{q_2(x)}{q_1(x)}dx + D\right]$$

where D is a constant. We like to point out that one set of functions satisfying the above differential equation is given (Glänzel (1987)) with D=1.

3. CONCLUSION

The present work deals with the characterizations of some beta-G univariate continuous distributions based on ratio of truncated moments. Characterization of the distribution helps the investigator in selection the appropriate model for fitting the particular data. We certainly hope that the content of this work will be useful to the investigators who are interested to know whether the chosen distributions are right.

REFERENCES

1. Glänzel, W. (1987). A characterization theorem based on truncated moments and its application to some distribution families. *In Mathematical Statistics and Probability Theory*. Springer, Dordrecht. 75-84.

2. Glanzel W. (1990). Some consequences of a characterization theorem based on truncated moments. *Statistics*, 21(4), 613-618.
3. Lee, C., Famoye, F. and Olumolade, O. (2007). Beta-Weibull distribution: some properties and applications to censored data. *Journal of Modern Applied Statistical Methods*, 6(1), 173-186.
4. Khan, M.S. (2010). The beta inverse Weibull distribution. *International Transactions in Mathematical Sciences and Computer*, 3(1), 113-119.
5. Silva, G.O., Ortega, E.M. and Cordeiro, G.M. (2010). The beta modified Weibull distribution. *Lifetime Data Analysis*, 16(3), 409-430.
6. Paranaíba, P.F., Ortega, E.M., Cordeiro, G.M. and Pescim, R.R. (2011). The beta Burr XII distribution with application to lifetime data. *Computational Statistics & Data Analysis*, 55(2), 1118-1136.

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COMPARISON OF BAYESIAN AND NON-BAYESIAN ESTIMATIONS FOR TYPE-II CENSORED GENERALIZED RAYLEIGH DISTRIBUTION

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ABSTRACT

In this paper, we compare Bayesian and non-Bayesian estimations for the unknown parameters of Generalized Rayleigh distribution under Type-II censoring schemes. First we deal with non-Bayesian method namely maximum likelihood estimation along with their asymptotic confidence intervals with a given coverage probability. Further we consider the Bayesian estimates of unknown parameters under different loss functions. As Bayes estimators cannot be obtained in nice closed form. We use Lindley's approximation. Monte Carlo simulation study is carried out to compare different methods and the performance of the estimates is judged by the mean squared error values. All the numerically computations are performed in R software. Finally, a real life data set analysis is performed for the illustration purpose.

KEYWORDS

Generalized Rayleigh distribution; Type-II censoring; Bayesian and non-Bayesian estimations; Symmetric and asymmetric loss functions; Lindley's approximation.

1. INTRODUCTION

Twelve different forms of cumulative distribution functions for modeling lifetime data were introduced by Burr (1942). Among these distributions, Burr type X distribution has the significant interest from lots of researchers. Many researchers examined the single parameter Burr type X model by putting scale parameter $\lambda = 1$. Recently, the single parameter distribution of the extended Burr type X by Surles and Padgett (2001) named as the generalized Rayleigh (GR) distribution making its more applicable for lifetime data.

Moreover, the two parameters GR distribution is a particular member of Generalized Weibull distribution, originally proposed by Mudholkar and Srivastava (1993). Kundu and Raqab (2005) have discussed the different techniques of estimation of the parameters and further properties of GR distribution. Aludaat et al. (2008) used the Bayesian and non-Bayesian methods to estimate the parameters of Burr type X distribution for Group data.

The probability density function (pdf) of GR distribution for a random variable X is:

$$f(x;\alpha,\lambda)=2\alpha\lambda^2 xe^{-(\lambda x)^2}\left(1-e^{-(\lambda x)^2}\right)^{\alpha-1};\ x,\alpha,\lambda>0, \tag{1}$$

and the corresponding cumulative distribution function (cdf)

$$F(x;\alpha,\lambda)=\left(1-e^{-(\lambda x)^2}\right)^{\alpha};\ x,\alpha,\lambda>0. \tag{2}$$

Also, the reliability and hazard functions of GR distribution with $\alpha > 0$ and $\lambda > 0$ are given

$$R(x;\alpha,\lambda)=1-\left(1-e^{-(\lambda x)^2}\right)^{\alpha}. \tag{3}$$

and

$$H(x;\alpha,\lambda)=\frac{2\alpha\lambda^2 xe^{-(\lambda x)^2}\left(1-e^{-(\lambda x)^2}\right)^{\alpha}}{1-\left(1-e^{-(\lambda x)^2}\right)^{\alpha}}. \tag{4}$$

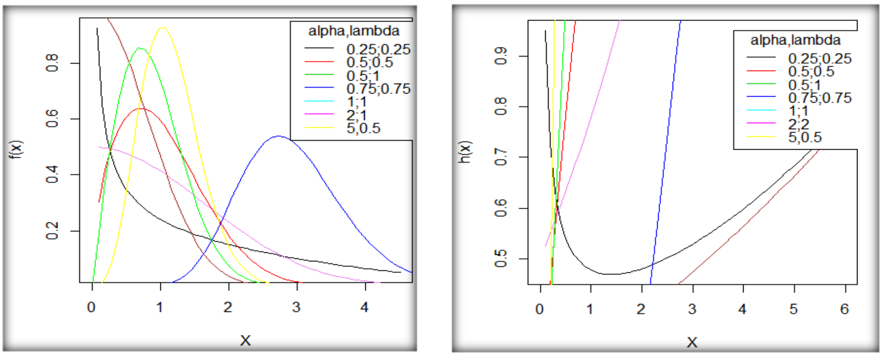

Figure 1: Shapes of Density Functions and Hazard Functions of GR Distribution

From Figure 1, we see that for $\alpha \leq 0.5$, the pdf of GR distribution is a decreasing function and it is unimodal right skewed for $\alpha > 0.5$. The hazard functions of GR distribution can be either bathtub type or an increasing function, depending on the shape parameter α. For $\alpha \leq 0.5$, the hazard function of GR distribution is a bathtub type and for $\alpha > 0.5$ it has an increasing hazard function.

The aim of this paper is to derive the non-Bayesian method namely maximum likelihood estimation and then Bayes estimates of α and λ under different loss functions; squared error (SE) loss function and LINEX loss function based on Type-II censoring scheme and compare their performance using extensive computer simulations. It may be cited that though in the frequentist setup the comparison of the different estimators and various confidence intervals of the GR parameters have been obtained. We used more flexible priors and it will be explained in details in the later section. The asymptotic confidence intervals for the parameters are also derived from the Fisher Information matrix.

In this paper, we consider both the informative and non-informative priors in different Loss functions to compute the Bayes estimators of GR parameters. As Bayes estimators cannot be expressed in a closed form. Thus the different numerical approximation procedures are used. Here we use the Lindley's approximation.

The rest of the paper is organized as follows: In section 2, we briefly describe the Bayesian and non-Bayesian methods of estimation. Section 3 provides the simulation and numerical comparisons of the different Bayes estimates and MLE. One real data set has been presented in section 4. Finally conclusion of the paper is provided in section 5.

2. METHODS OF ESTIMATION

Here we briefly describe the non-Bayesian and Bayesian methods of estimation under Type-II censoring scheme.

2.1 Maximum Likelihood Estimation

Let $X_1 < X_2 < ... < X_r$ denotes type-II censored observations from a sample of r failure units under consideration and the other $(n-r)$ items are functioning till the end of experiment and they are censored. Assume that the data follows two parameters GR distribution. The likelihood function will be:

$$L(\mathbf{x};\alpha,\lambda) \propto \left(\prod_{i=1}^{r} f(x_i)\right)\left\{1-F\left(x_r\right)\right\}^{n-r}. \tag{5}$$

The likelihood function of GR distribution under Type-II censored sample

$$L(\mathbf{x};\alpha,\lambda) = \frac{n!}{(n-r)!}\left(2\alpha\lambda^2\right)^r \prod_{i=1}^{r} x_i \prod_{i=1}^{r} e^{-(\lambda x_i)^2} \prod_{i=1}^{r}\left(1-e^{-(\lambda x_i)^2}\right)^{\alpha-1}\left\{1-\left(1-e^{-(\lambda x_r)^2}\right)^{\alpha}\right\}^{n-r}. \tag{6}$$

The log likelihood function of equation is

$$L(\mathbf{x};\alpha,\lambda) = \ln n! - \ln(n-r)! + r\ln 2 + r\ln\alpha + 2r\ln\lambda + \sum_{i=1}^{r}\ln x_i - \lambda^2\sum_{i=1}^{r} x_i^2$$
$$+ (\alpha-1)\sum_{i=1}^{r}\ln\left(1-e^{-(\lambda x_i)^2}\right) + (n-r)\ln\left\{1-\left(1-e^{-(\lambda x_r)^2}\right)^{\alpha}\right\}. \tag{7}$$

To obtain the MLE of α and λ, we can maximize equation directly w.r.t α and λ

$$\frac{\partial L(\mathbf{x};\alpha,\lambda)}{\partial\alpha}=\frac{r}{\alpha}+\sum_{i=1}^{r}\ln\left(1-e^{-(\lambda x_i)^2}\right)-\frac{(n-r)\left(1-e^{-(\lambda x_r)^2}\right)^{\alpha}\ln\left(1-e^{-(\lambda x_r)^2}\right)}{1-\left(1-e^{-(\lambda x_r)^2}\right)^{\alpha}}=0, \tag{8}$$

$$\frac{\partial L(\mathbf{x};\alpha,\lambda)}{\partial\lambda}=\frac{2r}{\lambda}-2\lambda\sum_{i=1}^{r}x_i^2+2\lambda(\alpha-1)\sum_{i=1}^{r}\frac{x_i^2e^{-(\lambda x_i)^2}}{(1-e^{-(\lambda x_i)^2})}-\frac{2(n-r)\,\alpha\lambda x_r^2\left(1-e^{-(\lambda x_r)^2}\right)^{\alpha}e^{-(\lambda x_r)^2}}{\left(1-e^{-(\lambda x_r)^2}\right)\left\{1-\left(1-e^{-(\lambda x_r)^2}\right)^{\alpha}\right\}}=0. \tag{9}$$

The above equations are not in closed form. Thus MLE can be obtained by any iterative method such as Newton Raphson method for numerical solution.

To derive the $100(1-\zeta/2)\%$ confidence intervals for the parameters α and λ respectively,

$$\hat{\alpha}\pm Z_{\zeta/2}\sqrt{\mathrm{var}(\hat{\alpha})} \text{ and } \hat{\lambda}\pm Z_{\zeta/2}\sqrt{\mathrm{var}(\hat{\lambda})}.$$

2.2 Bayesian Estimation

In this section, we compute the Bayesian estimation of GR distribution under SE and LINEX loss functions. When both parameters are unknown, we assume the following independent gamma prior distributions for α and λ,

$$g_1(\alpha)\ \propto\ \alpha^{a_1-1}e^{-b_1\alpha};\quad \alpha>0,\quad \text{and}\quad g_2(\lambda)\ \propto\ \lambda^{a_2-1}e^{-b_2\lambda};\quad \lambda>0.$$

Here all the hyper-parameters a_1,a_2,b_1,b_2 are assumed to be known and non-negative. Our prior assumptions of independent gamma distributions are not unreasoning; many authors have used the independent gamma priors for the scale and the shape parameters of two-parameter lifetime distributions (Berger and Sun, 1993; Kundu, 2008; Shrestha and Kumar, 2014). It is to be noted that the non-informative priors for the scale and the shape parameters are the special cases of these independent gamma priors.

The joint posterior density function of α and λ can be written as;

$$p(\alpha,\lambda\,|\,data)=\frac{L(data\,|\,\alpha,\lambda)g_1(\alpha)g_2(\lambda)}{\int_0^\infty\int_0^\infty L(data\,|\,\alpha,\lambda)g_1(\alpha)g_2(\lambda)d\alpha d\lambda}. \tag{10}$$

For Bayesian estimation, the following loss functions are considered.

Squared Error Loss Function

SELF is the most familiar and commonly used symmetric loss function. It was proposed by Legendre (1805) and Gauss (1810). Mathematically, defined as $L(\theta,\theta^*)=(\theta-\theta^*)^2$. The Bayes estimator under SELF is given by: $\theta^*=E_{\theta|x}(\theta)$.

Therefore the Bayes estimator of any function of α and λ, say u (α, λ) using SELF for GR distribution is

$$\hat{u}\left(\alpha,\lambda\right) = E_{\alpha,\lambda|\,data}(u(\alpha,\lambda)) = \frac{\int_0^\infty\int_0^\infty u(\alpha,\lambda)\; L(data\,|\,\alpha,\lambda)\; g_1(\alpha)\, g_2(\lambda)\; d\alpha d\lambda}{\int_0^\infty\int_0^\infty L(data\,|\,\alpha,\lambda)\; g_1(\alpha) g_2(\lambda)\; d\alpha d\lambda}. \tag{11}$$

It is clear that the above equation (11) cannot be written in close form. It is not possible to compute analytically. So we use Lindley's approximation to compute Bayesian estimators.

2.3 Lindley's Approximation

According to Lindley (1980), a solution for the ratio of integrals which produce a single numerical result of the form as:

$$I(X) = E(\alpha,\lambda\,|\,\mathbf{x}) = \frac{\iint u(\alpha,\lambda) e^{L(\alpha,\lambda)+G(\alpha,\lambda)} d(\alpha,\lambda)}{\iint e^{L(\alpha,\lambda)+G(\alpha,\lambda)} d(\alpha,\lambda)}. \tag{12}$$

where, $u\left(\alpha,\lambda\right) =$ is a function of α and λ only,

$L\left(\alpha,\lambda\right) =$ Log-Likelihood function and $G\left(\alpha,\lambda\right) =$ Log of joint prior density.

According to Lindley's approximation if maximum likelihood estimates of the parameters are available and n is sufficiently large then the above ratio integral can be approximated as:

$$\begin{aligned} I(X) = \quad & u\left(\hat{\alpha},\hat{\lambda}\right) + \frac{1}{2}\begin{Bmatrix} \left(\hat{u}_{\lambda\lambda} + 2\hat{u}_{\lambda}\hat{\rho}_{\lambda}\right)\hat{\sigma}_{\lambda\lambda} + \left(\hat{u}_{\alpha\alpha} + 2\hat{u}_{\alpha}\hat{\rho}_{\lambda}\right)\hat{\sigma}_{\alpha\lambda} \\ + \left(\hat{u}_{\lambda\alpha} + 2\hat{u}_{\lambda}\hat{\rho}_{\alpha}\right)\hat{\sigma}_{\lambda\alpha} + \left(\hat{u}_{\alpha\alpha} + 2\hat{u}_{\alpha}\hat{\rho}_{\alpha}\right)\hat{\sigma}_{\alpha\alpha} \end{Bmatrix} \\ & + \frac{1}{2}\begin{Bmatrix} \left(\hat{u}_{\lambda}\hat{\sigma}_{\lambda\lambda} + \hat{u}_{\alpha}\hat{\sigma}_{\lambda\alpha}\right)\left(\hat{L}_{\lambda\lambda\lambda}\hat{\sigma}_{\lambda\lambda} + \hat{L}_{\lambda\alpha\lambda}\hat{\sigma}_{\lambda\alpha} + \hat{L}_{\alpha\lambda\lambda}\hat{\sigma}_{\alpha\lambda} + \hat{L}_{\alpha\alpha\lambda}\hat{\sigma}_{\alpha\alpha}\right) + \\ \left(\hat{u}_{\lambda}\hat{\sigma}_{\alpha\lambda} + \hat{u}_{\alpha}\hat{\sigma}_{\alpha\alpha}\right)\left(\hat{L}_{\alpha\lambda\lambda}\hat{\sigma}_{\lambda\lambda} + \hat{L}_{\lambda\alpha\alpha}\hat{\sigma}_{\lambda\alpha} + \hat{L}_{\alpha\lambda\alpha}\hat{\sigma}_{\alpha\lambda} + \hat{L}_{\alpha\alpha\alpha}\hat{\sigma}_{\alpha\alpha}\right) \end{Bmatrix}. \end{aligned} \tag{13}$$

The Bayes estimator of α under SELF is given by:

$$\alpha^*_{SELF} = \hat{\alpha}_{MLE} + \hat{\sigma}_{\alpha\alpha}\left(\frac{a_1 - 1}{\hat{\alpha}} - b_1\right) + 0.5\left(\hat{\sigma}_{\alpha\alpha}\hat{L}_{\alpha\lambda\lambda}\hat{\sigma}_{\lambda\lambda} + \hat{L}_{\alpha\alpha\alpha}\hat{\sigma}_{\alpha\alpha}\hat{\sigma}_{\alpha\alpha}\right). \tag{14}$$

Similarly, The Bayes estimator of λ under SELF is given by:

$$\lambda^*_{SELF} = \hat{\lambda}_{MLE} + \hat{\sigma}_{\lambda\lambda}\left(\frac{a_2 - 1}{\hat{\lambda}} - b_2\right) + 0.5\left(\hat{\sigma}_{\lambda\lambda}\hat{L}_{\lambda\lambda\lambda}\hat{\sigma}_{\lambda\lambda} + \hat{L}_{\alpha\alpha\lambda}\hat{\sigma}_{\alpha\alpha}\hat{\sigma}_{\lambda\lambda}\right). \tag{15}$$

LINEX Loss Function:

Varian (1975) established linearly-exponential loss function (LINEX) and its further properties are investigated by Zellner (1986). Mathematically as;

$$L(\theta,\theta^*) = e^{c(\theta^*-\theta)} - c(\theta^*-\theta) - 1.$$

The Bayes estimator under the LINEX Loss Function is given by the expression:

$$\theta^* = -\left(\frac{1}{c}\right)\ln\left\{E_{\theta|x}\left(e^{-c\theta}\right)\right\}.$$

Therefore the Bayes estimator of α and λ, using LLF for BX distribution is,

$$\hat{u}(\alpha,\lambda) = E_{\alpha,\lambda|\,data}(u(e^{-c\alpha},e^{-c\lambda})) = \frac{\int_0^\infty\int_0^\infty u(e^{-c\alpha},e^{-c\lambda})\,L(data\,|\,\alpha,\lambda)\,g_1(\alpha)\,g_2(\lambda)\,d\alpha d\lambda}{\int_0^\infty\int_0^\infty L(data\,|\,\alpha,\lambda)\,g_1(\alpha)\,g_2(\lambda)\,d\alpha d\lambda}. \quad (16)$$

The Bayes estimator of α under LLF is given by:

$$\alpha^*_{LINEX} = \hat{\alpha}_{MLE} - \frac{1}{c}\ln\left[1 - c\left\{\begin{array}{l}\left(\frac{a_1-1}{\hat{\alpha}} - b_1\right)\hat{\sigma}_{\alpha\alpha} - \frac{c}{2}\hat{\sigma}_{\alpha\alpha} \\ +\frac{1}{2}\left(\hat{L}_{\alpha\lambda\lambda}\hat{\sigma}_{\lambda\lambda}\hat{\sigma}_{\alpha\alpha} + \hat{L}_{\alpha\alpha\alpha}\hat{\sigma}^2_{\alpha\alpha}\right)\end{array}\right\}\right]. \quad (17)$$

The Bayes estimator of λ under LLF is given by:

$$\lambda^*_{LINEX} = \hat{\lambda}_{MLE} - \frac{1}{c}\ln\left[1 - c\left\{\begin{array}{l}\left(\frac{a_2-1}{\hat{\lambda}} - b_2\right)\hat{\sigma}_{\lambda\lambda} - \frac{c}{2}\hat{\sigma}_{\lambda\lambda} \\ +\frac{1}{2}\left(\hat{L}_{\lambda\lambda\lambda}\hat{\sigma}_{\lambda\lambda}\hat{\sigma}_{\lambda\lambda} + \hat{L}_{\alpha\alpha\lambda}\hat{\sigma}_{\alpha\alpha}\hat{\sigma}_{\lambda\lambda}\right)\end{array}\right\}\right]. \quad (18)$$

3. SIMULATION STUDY

In this section, Monte Carlo simulation study is performed to obtain the Bayes estimates for both parameters of the GR distribution. The Bayes estimates are computed under different loss functions; SELF and LLF for type-II censoring schemes. We have taken in case of 10% and 20% censored sampling schemes. For all the censoring schemes, we have used $\alpha = 0.5$ and $\lambda = 1$. First we considered the non-informative prior for both α and λ, i.e. $a_1 = a_2 = b_1 = b_2 = 0$. In this case the priors becomes improper. We call this prior as Prior-I. We have taken one informative priors, namely Prior-II: $a_1 = a_2 = 1, b_1 = b_2 = 2$. In case we have computed the average Bayes estimates, mean square error (MSE), average confidence intervals and coverage probabilities based on k=1000 data generation replications. For comparison purpose we have computed MLE and confidence intervals based on Fisher information matrix. The results are shown in Tables 1 to 3. Table 1 represent the average estimates and the mean squared of the MLE and different Bayes estimators when Prior-I has been used. In Table 2 we present the average estimates and MSE of different Bayes estimators when Prior-II has been used. In Table 3 we report the average confidence interval lengths based on MLE were used.

Some points are quite clear from the results. It is observed that for comparison the Bayesian method perform better than non-Bayesian method. It is also to be noted that the performance of estimators become better and converge to the true parameter values as sample size is increased. The MSE of all estimates decrease with increasing sample size. It is obvious that with the increase in censoring rate, the MSE of estimates have been increased. The performance of the LLF is better than SELF. The performance of all the Bayes estimators becomes better for Prior-II than Prior-I.

Comparing the confidence lengths, it is clear that as sample increases the bandwidth decreases.

4. REAL LIFE DATA ANALYSIS

Following data represent tests on endurance of deep groove ball bearings. The data represent the number of million revolutions before failure for each of the 23 ball bearings in the life test (Lawless; 1982, p.228). Raqab and Kundu (2001) have analyzed this data set and generalized Rayleigh distribution works affectively. Caroni (2002) also pointed out that this data contains censored points.

The data set are 17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.80, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40.

The data follows the BX distribution, as the goodness of fit for data is tested by Kolmogorov-Smirnov test applied in “R” and the output shown as:

One-sample Kolmogorov-Smirnov test

Data: x
D = 0.1572, p-value = 0.6208
Alternative hypothesis: two-sided

The output indicates that data set follows the GR distribution. So we conclude that the fit is good. First we compute the MLE of α and λ then we find the Bayes estimates with respect to different loss functions, namely SELF and LINEX Loss function. For LINEX loss function we take c=1.5 and all the results are presents in Table 4 and Table 5.

All the estimates are quite close to each other and Bayesian estimates are quite different than MLE. In fact, it is observed that the Bayes estimates provide a much better performance than MLE. The performance of all the Bayes estimators becomes better for Prior-II than Prior-I in real data analysis.

5. CONCLUSION

In this paper we have considered the comparison of non-Bayesian and Bayesian estimations of a two-parameter Generalized Rayleigh distribution when the data are Type-II censored. We have compared the MLE with different Bayesian estimators obtained by using different loss functions, by computer simulations in terms of average estimates and MSEs for different censoring schemes. We have also computed the confidence intervals based on Fisher information matrix. It is observed that the informative prior executes super performance than non-informative prior. If we use informative prior, Bayesian inference has clear advantage over the non-Bayesian one. It may be cited though we have provided the results mainly for Type-II censored samples, but our method can be extended for other censoring schemes as Type-I censoring, Hybrid or Progressive Type-I or Type-II censoring also. More work is desired along these directions.

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REFERENCES

1. Aludaat, K.M., Alodat, M.T. and Alodat, T.T. (2008). Parameter estimation of Burr type X distribution for grouped data. *Applied Mathematical Sciences*, 2(9), 415-423.
2. Berger, J.O. and Sun, D. (1993). Bayesian analysis for the Poly-Weibull distribution. *Journal of American Statistical Association*, 88, 1412-1418.
3. Burr, l.W. (1942). Cumulative frequency functions, *Annals of Mathematical* Statistics, 13, 215-232.
4. Caroni, C. (2002). The correct "Ball Bearings" data. *Lifetime Data Analysis*, 8, 395-399.
5. Gauss, C.F. (1810). Method des Moindres Carres Memoire sur la Combination des Observations. Translated by J. Bertrand. (1955). Mallet-Bachelier, Paris.
6. Kundu, D. and Raqab, M.Z. (2005). Generalized Rayleigh distribution: different methods of estimation, *Computational Statistics and Data Analysis*, 49, 187-200.
7. Kundu, D. and Gupta, R.D. (2008). Generalized exponential distribution: Bayesian estimation. *Computation Statistics and Data Analysis*, 52, 1873-1883.
8. Lawless, J.F. (1982). *Statistical Models and Methods for Lifetime data*, John Wiley & Sons, New York.
9. Legendre, A. (1805). Nouvelles Methodes Pour la Determination des Orbites des Cometes, Courcier, City Paris.
10. Lindley, D.V. (1980). Approximate Bayesian method. *Trabajos de Esta*, 31, 223-237.
11. Mudholkar, G.S. and Srivastava, D.K. (1993). Exponentiated Weibull family for analyzing bathtub failure data, *IEEE Transactions on Reliability*, 42, 299-302.
12. Raqab, M.Z. and Kundu, D. (2006). Burr type X distribution: revisited. *Journal of Probability and Statistical Sciences*, 4(2), 179-193.
13. Shrestha, S.K. and Kumar, V. (2014). Bayesian Analysis for the Generalized Rayleigh Distribution. *Journal of Statisatika and Mathematika*, 9(3), 118-131.
14. Surles, J.G. and Padgett, W.J. (2001). Inference for reliability and stress-strength for a scaled Burr X distribution. *Lifetime Data Analysis*, 7, 187-200.
15. Varian, H.R. (1975). A Bayesian Approach to Real estate assessment, In: Stephen, E.F., Zellner, A. (Eds.), Studies in Bayesian Econometrics and Statistics in Honor of Leonard J. Savage, North-Holland, Amsterdam, 195-208.
16. Zellner, A. (1986). Bayesian Estimation and prediction using asymmetric loss Function. *Journal of American Statistics Association*, 81, 446-451.

APPENDIX

Table 1
MLE and Lindley Bayes Estimates with Respect to Different Loss Functions when Prior-I is used. True $\alpha = 0.5$ and $\lambda = 1$

Scheme			MLE	SELF	LLF
20	10%	$\hat{\alpha}$	0.65301 (0.03878)	0.64567 (0.03653)	0.63486 (0.03302)
		$\hat{\lambda}$	1.39513 (0.12710)	1.38793 (0.12353)	1.37127 (0.11519)
	20%	$\hat{\alpha}$	0.71793 (0.06550)	0.71101 (0.06427)	0.69384 (0.05509)
		$\hat{\lambda}$	1.70383 (0.33302)	1.69512 (0.32576)	1.66787 (0.30276)
40	10%	$\hat{\alpha}$	0.60527 (0.01720)	0.60246 (0.01666)	0.59803 (0.01583)
		$\hat{\lambda}$	1.35927 (0.08579)	1.35564 (0.08430)	1.34767 (0.08109)
	20%	$\hat{\alpha}$	0.65247 (0.02872)	0.64907 (0.02782)	0.64302 (0.02622)
		$\hat{\lambda}$	1.65889 (0.25491)	1.65407 (0.25142)	1.64092 (0.24196)
80	10%	$\hat{\alpha}$	0.58227 (0.00733)	0.58106 (0.00719)	0.57905 (0.00697)
		$\hat{\lambda}$	1.33685 (0.06659)	1.33503 (0.06594)	1.33117 (0.06455)
	20%	$\hat{\alpha}$	0.61971 (0.01261)	0.61822 (0.01238)	0.61561 (0.01197)
		$\hat{\lambda}$	1.62560 (0.21309)	1.62311 (0.21146)	1.61674 (0.20728)

Table 2
Lindley Bayes Estimates with Respect to Different Loss Functions when Prior-II is used. True $\alpha = 0.5$ and $\lambda = 1$

Scheme			SELF	LLF
20	10%	$\hat{\alpha}$	0.61829 (0.02801)	0.61785 (0.02815)
		$\hat{\lambda}$	1.34881 (0.10441)	1.34680 (0.10373)
	20%	$\hat{\alpha}$	0.67057 (0.04516)	0.66975 (0.04523)
		$\hat{\lambda}$	1.63244 (0.27397)	1.63003 (0.27283)
40	10%	$\hat{\alpha}$	0.59163 (0.01467)	0.59095 (0.01457)
		$\hat{\lambda}$	1.33706 (0.07692)	1.33564 (0.07639)
	20%	$\hat{\alpha}$	0.63453 (0.02405)	0.63359 (0.02386)
		$\hat{\lambda}$	1.62403 (0.23006)	1.62178 (0.22857)
80	10%	$\hat{\alpha}$	0.57622 (0.00667)	0.57585 (0.00663)
		$\hat{\lambda}$	1.32604 (0.06273)	1.32525 (0.06246)
	20%	$\hat{\alpha}$	0.61198 (0.01142)	0.61150 (0.01135)
		$\hat{\lambda}$	1.60859 (0.20200)	1.60726 (0.20115)

Note: For each scheme, first entry represents average estimates and MSE within Brackets.

Table 3
Confidence Intervals, Bandwidths Based on MLE and the Associated Coverage Probabilities. True $\alpha = 0.5$ and $\lambda = 1$

95% CIs and CPs for α						95% CIs and CPs for λ			
Scheme		**LL** (α)	**UL** (α)	**BW**	**CPs**	**LL** (λ)	**UL** (λ)	**BW**	**CPs**
20	10%	0.25889	0.89417	0.63528	0.965	0.66637	1.50240	0.83603	0.914
	20%	0.24798	0.92214	0.67416	0.962	0.62979	1.59013	0.96034	0.909
40	10%	0.32927	0.73767	0.40840	0.950	0.75037	1.32347	0.57311	0.917
	20%	0.32328	0.75535	0.43207	0.949	0.72036	1.38370	0.66333	0.931
80	10%	0.37585	0.65284	0.27699	0.951	0.80451	1.20636	0.40185	0.929
	20%	0.37002	0.65837	0.28835	0.956	0.77641	1.23195	0.45554	0.919

Table 4
MLE and Bayes Estimates with Respect to Different Loss Functions When Prior-I is used.

Estimates		**MLE**	**Lindley Bayes Estimates**	
			SELF	**LLF**
10%	$\hat{\alpha}$	1.95066	1.91456	1.84184
	$\hat{\lambda}$	0.01852	0.01846	0.01848
20%	$\hat{\alpha}$	2.21545	2.17826	2.06717
	$\hat{\lambda}$	0.02089	0.02084	0.02085

Table 5
MLE and Bayes Estimates with Respect to Different Loss Functions When Prior-II is Used

Estimates		**MLE**	**Lindley Bayes Estimates**	
			SELF	**LLF**
10%	$\hat{\alpha}$	1.95066	1.72628	1.74294
	$\hat{\lambda}$	0.01852	0.01846	0.01848
20%	$\hat{\alpha}$	2.21545	1.89572	1.93122
	$\hat{\lambda}$	0.02089	0.02083	0.02085

Table 6
Confidence Intervals, Bandwidths based on MLE and the Associated Coverage Probabilities

95% CIs and CPs for α					95% CIs and CPs for λ		
Scheme		**LL** (α)	**UL** (α)	**BW**	**LL** (λ)	**UL** (λ)	**BW**
23	10%	0.76032	3.14101	2.38069	0.01418	0.02287	0.00870
	20%	0.76565	3.66524	2.89959	0.01584	0.02594	0.01010

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ON THE DISTRIBUTION OF THE MEDIAN OF A SAMPLE FROM A SELF-INVERSE PROBABILITY MODEL

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ABSTRACT

Self-Inverse distributions are those for which every pair of upper and lower quantiles is related to the median in an interesting way. Denoting the median by A, Self-Inverse at A (SIA) distributions are those for which, for every positive real number q less than unity, the q^{th} and $(1-q)^{th}$ quantiles X_q and X_{1-q} fulfill the equation $X_{1-q} / \mathrm{A} = \mathrm{A} / X_q$. The case Self-Inverse at Unity refers to the situation when every upper quantile is the reciprocal of the corresponding lower quantile. In this short paper, we focus on the sampling distribution of the median of a sample drawn from an SIA distribution and determine whether or not this distribution too is self-inverse.

1. INTRODUCTION

An interesting class of continuous distributions defined on the positive half-line are those that are known as Self-Inverse at A (SIA). Denoting the random variable by X, for each distribution belonging to this class, the distribution of X / A is identical to the distribution of A/X where A is the median of the random variable X. Consequently, for every positive real number q less than unity, the q^{th} and $(1-q)^{th}$ quantiles X_q and X_{1-q} fulfill the equation $X_{1-q} / \mathrm{A} = \mathrm{A} / X_q$. The situation of distributions Self-Inverse at Unity (SIU) is obvious; letting A=1, every upper quantile is the **reciprocal** of the corresponding lower quantile.

This class of distributions has been considered, among others, by Seshadri (1965), Saunders (1974), Jones (2008) and Habibullah et al. (2010) and various properties common to all such distributions have been derived. Habibullah and Saunders (2011) adopted the nomenclature ‘self-inverse’ for such distributions and proposed an estimator of the cdf that performs better than the well-known ecdf when sampling from a self-inverse distribution.

Whereas Jones (2008) used the term ‘log-symmetric’ for such distributions, Habibullah and Fatima (2015) adopted the terminology ‘Self-Inverse at A (SIA)’ where A is the median of the distribution and Habibullah (2017) proposed that, merging the two nomenclatures, these distributions be referred to as ‘SIA log-symmetric distributions’.

Some well-known distributions belonging to this class are the lognormal, log-logistic and Birnbaum Saunders distributions. During the past few years, a number of papers have appeared presenting estimators of distribution parameters that are based on the self-inversion property and the sampling distributions of which are narrower than those of the corresponding well-known method of moments estimators. The superiority of these estimators over the moment estimators in terms of efficiency is a good sign as these are likely to yield better-fitting models when applied to real data.

However, not much work seems to have been carried out on the distributions of order statistics from SIA log-symmetric distributions. In this brief paper, we focus on the sampling distribution of the median of a random sample drawn from an SIA log-symmetric distribution. The objective of the study is to determine whether or not this distribution too is self-inverse.

2. FUNCTIONAL EQUATIONS OF AN SIA DISTRIBUTION

As indicated by Jones (2008), the functional equation for the pdf of an SIA log-symmetric distribution is given by

$$f(\mathrm{A}x) = \frac{1}{x^2} f\left(\frac{\mathrm{A}}{x}\right), 0 < x < \infty, \mathrm{A} > 0 \tag{2.1}$$

Whereas the functional equation of the cdf is

$$F(\mathrm{A}x) = 1 - F\left(\frac{\mathrm{A}}{x}\right), 0 < x < \infty, \mathrm{A} > 0 \tag{2.2}$$

These equations can be utilized to check whether or not any particular distribution is SIA log-symmetric.

3. SAMPLING DISTRIBUTION OF THE MEDIAN

We know that, for a random sample of size n drawn from a probability distribution $f(x)$, the pdf of the k^{th} order statistic is given by

$$f_{k:n}(x) = \frac{n!}{(k-1)!(n-k)!}\left[F(x)\right]^{k-1}\left[1-F(x)\right]^{n-k} f(x)$$

where $f(x)$ is a probability density function.

If n is odd, then, substituting $k = (n+1)/2$, the *pdf* of the median is given by

$$f_{\tilde{X}}(x) = \frac{n!}{\left[\left(\frac{n-1}{2}\right)!\right]^2}\left[F(x)\right]^{\frac{n-1}{2}}\left[1-F(x)\right]^{\frac{n-1}{2}} f(x)\,. \tag{3.1}$$

We now present the following theorem:

Theorem 1:

For a random sample of size $n\,(n = 2m+1, m \in \mathbb{N})$ drawn from an SIA log-symmetric distribution $f(x)$, the sampling distribution of the sample median is SIA log-symmetric.

Proof:

From (2.1) and (2.2), it is easy to see that, for the pdf (3.1) to be SIA log-symmetric, we need to show that

$$f_{\tilde{X}}(\mathrm{A}x) = \frac{1}{x^2} f_{\tilde{X}}\left(\frac{\mathrm{A}}{x}\right) \tag{3.2}$$

Considering the R.H.S. of the equation (3.2), we have:

$$\begin{aligned}\frac{1}{x^2} f_{\tilde{X}}\left(\frac{\mathrm{A}}{x}\right) &= \frac{1}{x^2}\frac{n!}{\left[\left(\frac{n-1}{2}\right)!\right]^2}\left[F\left(\frac{\mathrm{A}}{x}\right)\right]^{\frac{n-1}{2}}\left[1-F\left(\frac{\mathrm{A}}{x}\right)\right]^{\frac{n-1}{2}} f\left(\frac{\mathrm{A}}{x}\right) \\ &= \frac{n!}{\left[\left(\frac{n-1}{2}\right)!\right]^2}\left[1-F(\mathrm{A}x)\right]^{\frac{n-1}{2}}\left[F(\mathrm{A}x)\right]^{\frac{n-1}{2}} f(\mathrm{A}x)\end{aligned} \tag{3.3}$$

as the parent distribution is SIA.

Replacing x by Ax in the LHS of (3.1), we have

$$f_{\tilde{X}}(\mathrm{A}x) = \frac{n!}{\left[\left(\frac{n-1}{2}\right)!\right]^2}\left[F(\mathrm{A}x)\right]^{\frac{n-1}{2}}\left[1-F(\mathrm{A}x)\right]^{\frac{n-1}{2}} f(\mathrm{A}x)$$

which is the same as (3.3). Hence the result.

4. COMMENTS AND CONCLUSION

The simple proof of Theorem 1 implies that the sampling distribution of the median of a random sample of size $n\,(n = 2m+1, m \in \mathbb{N})$ drawn from an SIA log-symmetric distribution possesses all the properties shared by every distribution belonging to the class of SIA log-symmetric distributions. This has implications for further interesting theoretical results and may lead to new insight with regard to modeling real data.

REFERENCES

1. Habibullah, S.N. (2017). On the Power Generalization of Log-Symmetric SIA Distributions. *Journal of ISOSS*, 3(1), 45-54.
2. Habibullah, S.N. and Fatima, S.S. (2015). On a Newly Developed Estimator for More Accurate Modeling with an Application to Civil Engineering. Proceedings of the *12th International Conference on Applications of Statistics and Probability in Civil Engineering* (ICASP12) organized by CERRA (Vancouver, BC, Canada, July 12-15, 2015). Sponsoring Agency: Higher Education Commission, Pakistan.
3. Habibullah, S.N. and Saunders, S.C. (2011). A Role for Self-Inversion, Proceedings of *International Conference on Advanced Modeling and Simulation* (ICAMS, Nov 28-30, 2011) published by Department of Mechanical Engineering, College of Electrical and Mechanical Engineering, National University of Science and Technology (NUST), Islamabad, Pakistan, © 2011, ISBN 978-869-8535-11-7.
4. Habibullah, S.N., Memon, A.Z. and Ahmad, M. (2010). On a Class of Distributions Closed under Inversion, Lambert Academic Publishing (LAP), ISBN 978-3-8383-4868-1.
5. Jones, M.C. (2008). On reciprocal symmetry. *Journal of Statistical Planning and Inference*, 138(10), 3039-3043.
6. Saunders, S.C. (1974). A family of random variables closed under reciprocation. *J. Amer. Statist. Assoc*., 69(346), 533-539.
7. Seshadri, V. (1965). On random variables which have the same distribution as their reciprocals. *Can. Math. Bull*., 8(6), 819-824.

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SAMPLE BASED CENSUSES IN PAKISTAN

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ABSTRACT

It is a common perception that census is a complete count and survey is a sample based activity in the field of statistical science. But practically many censuses are conducted on sample basis in many countries throughout the globe. In this article we briefly explained the different censuses conducted on sample basis in Pakistan. We also tried to highlight the difference between a census and a survey which is practiced in the real life in Pakistan as well as rest of the world.

KEYWORDS

Census, Survey, Sample based census, Agricultural census, Livestock census, Agricultural machinery census, Sample design.

1. INTRODUCTION

It is common perception that census is an exercise of data collection which is conducted to collect information from each and every population unit. While survey is an exercise in which the required information is collected from few population units selected for the purpose by applying some sampling techniques. In practice there are many censuses in the world which are conducted on sample basis particularly in the field of agriculture. In addition to many other countries of the globe, Pakistan is also practicing sample based censuses of agriculture since 1972 when the very first sample based census of agriculture and the second agricultural census of Pakistan was conducted.

After first population census of Pakistan in 1951, it was felt that agricultural census should also be conducted in order to get the data regarding agriculture sector so that agriculture resources could be analyzed to meet the needs of food and fiber for the people of Pakistan. The first census of agriculture was conducted in Pakistan in 1960 on complete count basis in which the data were collected from revenue records and were compiled to generate the tabulations. The results of that census were not appreciated and then the sample design for agricultural census of Pakistan was developed and implemented in 1972. Afterwards the agricultural censuses of Pakistan were conducted on sample basis in 1980, 1990, 2000 and 2010. In addition to the agricultural censuses, the livestock censuses in 1976, 1986, 1996 and 2006 while agricultural machinery censuses in 1968, 1975, 1984, 1994 and 2004 were also conducted on sample basis. Even population census of Pakistan had been conducted having big count and sample count strategies. The population censuses in Pakistan were conducted in 1951, 1961, 1972, 1981, 1998 and 2017. Although these censuses were conducted on sample basis but these are called and published as censuses.

In this study we describe the sampling techniques used in different censuses of Pakistan which are applied in real life. After introduction, sample design of agricultural census of Pakistan is explained along with designs of livestock and agricultural machinery censuses in Pakistan. Then the difference between census and survey is explained in the light of practical aspects. However, a research proposal is also given in the light of merger of three major censuses of Pakistan related to agriculture sector, before the references.

There are many other countries in the world which are conducting different censuses on sample basis but those are not named surveys but are called censuses. However, this article cannot explain all the sample based censuses of the world and is restricted to the practices adapted in Pakistan in the field of agriculture for data collection.

2. AGRICULTURAL CENSUS OF PAKISTAN

Agriculture is a major contributor in the economy of Pakistan and is playing multidimensional role in the country. It is fulfilling the basic needs of food and fiber of the populace, supplying raw material to the industrial sector, earning foreign exchange through exports, providing employment opportunities directly or indirectly to the people of Pakistan. According to the Pakistan Economic Survey 2016-17, agriculture is contributing 19.5 % share in the GDP and is employing 42.3 % of the labour force in Pakistan. Labour force survey of Pakistan (2014-15) revealed that total labour force in Pakistan is 61.04 million, out of that 57.42 million are employed. It shows that out of 61.04 million, there are 42.3 % i.e. 25.82 million people working in agriculture. As a whole agriculture is playing important role in the national development for food security, employment, industry, business and poverty reduction.

It is well known reality that data on agriculture provides the basis for planning regarding land management, cropping pattern, important inputs for crops and modern farming practices for cultivation. In order to provide the data on agriculture, decennial agricultural censuses had been conducted in Pakistan since 1960 and since 1972 these censuses had been conducted on sample basis rather on complete count basis. There are so many other countries in the world which are also conducting agricultural censuses on sample basis e.g. Botswana, Nepal, Philippines, Colombia, Tanzania, Gambia, Malawi, Uganda, Bangladesh, India and many others. The sample designs adapted for all agricultural censuses of Pakistan were almost the same except few minor changes or modifications. The overall sample design is presented here.

Sample Design for Agricultural Census

Government of Pakistan (2012) uncovers in the report of Agricultural Census 2010 that this census was conducted on sample basis throughout the country. Prior to go to the sample design, it is necessary to explain the settled and unsettled areas of Pakistan.

Settled Areas: are those areas of the country where settlement has been made through measurement of land and the whole land of those areas has been divided into Mouzas. A Mouza is a demarcated territorial unit for which separate revenue record including a cadastral map is maintained by the provincial revenue departments. One Mouza / Deh may contain one or more population settlements or may have no settlement (be-chiragh). Whole of Punjab excluding some areas of Cholistan, D.G. Khan and Rajanpur districts, whole of Sindh, partially Khyber Pakhtunkhwa and Balochistan provinces are settled.

Unsettled Areas: are those areas where settlement of land could not be made so far and land records are not properly maintained due to non-availability of land measurement. In unsettled areas, a village or Basti or Killi means a chunk of houses known by a certain name.

Mouza is a basic and gross root level revenue estate in Pakistan. One, two or three Mouzas, depending upon the size of Mouza, comprised a Patwar Circle and on similar pattern Patwar Circles make a Kanungo Circle. Then each Tehsil has one or more Kanungo Circles and each district of the country has one or more Tehsils.

For agricultural census, the whole country was divided into four different parts.

Part-1: Rural settled areas of three provinces i.e. Khyber Pakhtunkhwa, Punjab and Sindh. In these areas, a three stage weighted, stratified and systematic cluster sampling approach was used.

Part 2: Rural settled areas of Balochistan and all rural areas of Azad Jammu & Kashmir. In these areas, a two stage weighted and stratified sampling technique was used.

Part 3: Urban areas throughout the country were divided into blocks comprising 200 to 250 households in each block, and a two stage sample design, using stratified and systematic sampling, was used.

Part 4: In rest of the country, comprising unsettled rural areas of Punjab, Balochistan and Khyber Pakhtunkhwa provinces and tribal areas (Agencies and FRs) including Gilgit Baltistan, a single stage stratified and systematic sampling was used.

Mainly each district of the country was a stratum, wherein, rural and urban areas were two sub-strata for sampling purpose. The sample size of Mouzas or villages from each district was decided on the basis of coefficient of variation of the last census, availability of resources / enumerators and ground realities. The sample size of urban blocks was decided on the basis of population of that city or urban unit. The sample of Mouzas, villages, Killies, Basties and urban blocks was selected at district level.

For three stage sample design used for areas mention in part-1, at first stage Patwar Circles, for second stage Mouzas and in third stage households were selected. For first and second stages, selection was made using probability proportional to size (PPS) technique. However, for third stage selection of clusters of households (approximately 30 households in one cluster) was made systematically. For areas mentioned in part 2, PPS was used to select Mouzas while complete enumeration of all households in a selected Mouza was done. The urban blocks as mentioned in part-3, were selected systematically and every 5^{th} household in the selected block was enumerated. For unsettled areas mentioned in part-4, selection of Mouzas was done systematically and complete count of households in selected Mouzas was adopted.

Sampling Frames

Different sampling frames were prepared for each stage of selection as:

i) **Mouza Lists:** For each district, Mouza and village lists were updated through conducting Mouza Census 2008. These lists were arranged for sampling purpose having rural, partly urban and Bechiragh (unpopulated) Mouzas arranged by Patwar Circles and Kanungo Circles. The total area of each Mouza, showing cultivated area and number of households was also included for sampling purpose. These lists were used for first and second stages of selection.

ii) **List of Urban Blocks:** For urban areas of each district, a complete list of urban blocks prepared by the then Federal Bureau of Statistics (FBS), was utilized.

iii) **List of National Certainty Holdings (NCHs):** Lists of NCHs were prepared throughout the country at district level in all the Mouzas / Dehs / villages comprising government agriculture & livestock farms, all private farms having 100 acres or more agriculture land owned or operated, located at one or more than one places throughout the country, for complete count.

iv) **Complete List of Households:** List of all households located in the boundary of each selected Mouza showing farm area, comprising their area owned and rented-in.

v) **List of Mouza Certainty Holdings (MCHs):** In a selected Mouza, list of households having 20 acres or more farm land, livestock holders having 25 or more cattle and / or buffaloes; 25 or more camels; 50 or more sheep and / or goats was prepared and termed them as MCHs, for complete count at Mouza level. In selected urban blocks, households having / operating agricultural land or having any number of livestock were also treated as MCH. The nomads / gipsy / migratory people, if found in the boundary of selected Mouza / urban block during household listing or enumeration, were also treated as MCH.

Sample Selection:

In order to reduce the variance, all the NCHs and MCHs were enumerated on 100 % count basis. However, selection procedures adopted for Mouzas, urban blocks and common households at various sampling stages are given below:

First Stage: Selection of Patwar Circles

Predetermined number of Patwar Circles, keeping in view the availability of enumerators, effective training and supervision, and the variance of main study variables, were selected in the first stage. Every Mouza was assigned a weight / measure of size (MS) as geometric mean of its cultivated area and number of households. The probability of selection was calculated as MS of that Mouza divided by total MS of the district multiplied by the number of Patwar Circles to be selected. Then the probability of selection of every Patwar Circle was calculated by taking sum of probabilities of all the Mouzas within that Patwar Circle. The Patwar Circles were arranged according to their cultivated area within each Kanungo Circle. Patwar Circles having greater than 0.890 probability were selected with certainty while having less than 0.020 probability were merged with nearest above non-certainty, non-merged Patwar Circle(s) of that Kanungo Circle, to make the probability greater than 0.019. This method was repeated till no certainty Patwar Circle was found at district level. Then for non-certainty Patwar Circles, moving cumulative probabilities were calculated and a random number between zero and one was found. A Patwar Circle having cumulative probability equal to or greater than that random number was selected as first one, by adding one to that random number as second, by adding two as third and so on, till to arrive the required number of selected Patwar Circles from the district, in addition to the certainty Patwar Circles.

Second Stage: Selection of Mouzas

Two Mouzas from each selected Patwar Circle were selected in order to make equal workload for every Patwar. The Mouzas of all Patwar Circles selected in the first stage, were arranged in serpentine fashion, i.e., within the odd-numbered Kanungos, in descending order of cultivated area of the Mouzas, and within even numbered Kanungos in ascending order of cultivated area of the Mouzas. Selection of Mouzas at second stage was just like selection of Patwar Circles in the first stage.

Sub Stage: Sector Selection for Big Mouzas Only

All big Mouzas having 900 or more households were divided into sectors for easement of the enumeration work, comprising approximately 300–500 households in a sector. One sector from each big Mouza was selected using simple random sampling. For big Mouzas it may be treated as another stage of selection.

Third Stage: Selection of Clusters of Households

First of all list of all the households in a selected Mouza was prepared by the enumerators. Then clusters of approximately 30 households were selected systematically for detailed interviews, having minimum two clusters from each Mouza.

The ultimate sample of households for the census includes:

a. Households in the selected clusters.
b. Every 5^{th} HH of urban blocks.
c. All HHs in selected villages where two stage or single stage selection was made.
d. The NCH and MCH households.

3. LIVESTOCK CENSUS OF PAKISTAN

The Pakistan Economic Survey 2016-17 highlighted that share of overall agriculture sector in the GDP of Pakistan is 19.5 % and livestock has 58.33 % share in the agriculture which was near about less than 50 % ten years ago. This year by year increasing share of livestock in agriculture indicates that livestock is dominating in the agriculture sector. It reveals that livestock is contributing more than half in the overall agriculture sector which is an indicator that livestock is getting more importance as compared to crops in the agriculture. It is evident that planning of this more important part of agriculture also needs data for livestock population as well as other characteristics of that population. The livestock census provides these data for policy makers, planners and implementation agencies in order to boost the livestock. After every agricultural census, the livestock census was also conducted on sample basis. Discussion about all the aspects of livestock census is not the scope this article, however, the sample design of this census is presented here.

Pakistan livestock census (2006) report uncovers that livestock censuses also have been conducted on sample basis in Pakistan. Just like agricultural census different sample designs were used for different parts of the country on the basis of local conditions, the values of coefficient of variation for different study variables and information available for sample selection. The details of sample design are discussed here.

Sample Design for Livestock Census

Livestock census of Pakistan was not based on complete but was conducted on sample basis. Generally two stage weighted sample design was used but specifically different designs were used for different areas of the country. First we elaborate the sampling frames prepared for selection of population units for detailed study.

Sampling Frames

i) Complete list of Mouzas / Dehs along with number of households for settled areas of the country.

ii) For unsettled areas list of villages / Basties / Killies.

iii) For urban areas, complete list of blocks for small and medium cities.

iv) For big urban areas, complete list of blocks segregated into four categories having livestock concentration i.e. cattle colonies, having livestock in many places, having livestock in few places, and rare activity of livestock.

v) Complete list of National Certainty Holdings (NCHs) including all Government, semi Government, Army, livestock and dairy farms, private livestock holders having 50 or more heads of cattle / buffaloes or both, 200 (500 in case of Balochistan) or more heads of sheep / goats or both and 50 or more heads of camels.

vi) From selected Mouzas, all Mouza Certainty Holdings (MCHs) having 25 or more cattle / buffaloes or both, 50 (100 in case of Balochistan) or more sheep / goats or both OR 25 or more camels.

Sample Selection

All the NCHs and MCHs were enumerated on 100 % count basis throughout the country. However, selection of Mouzas and households from selected Mouzas and urban blocks was not same for all the areas of country. Mouzas / Dehs / villages / Basties / Killies were selected at district level for each district separately. Total number of Mouzas to be selected for each district were determined on the basis of livestock population, coefficient of variation for different study variables in the last census and availability of enumeration force in that district. The Mouzas in first stage and households in second stage from selected Mouzas were selected.

First Stage: In rural settled areas of the country, probability proportional to size (PPS) sample design was used for selection of Mouzas. The number of households of each Mouza were used as weight / measure of size (MS) for that Mouza. The MS of each Mouza divided by total MS of the district multiplied with predetermined number of Mouzas to be selected, provided probabilities of selection for each Mouza. Then the selection criteria was adopted just like explained for agricultural census. In unsettled rural areas and urban areas, systematic sampling was used to select villages / Basties / Killies and blocks.

Second Stage: In selected Mouzas, list of all the households was prepared by the enumerators and then a systematic random sample of HHs was selected in settled areas of the country. Rate of selection was determined on the basis of total number of HHs in the Mouza. For small and medium urban areas every 5^{th} HH was selected for detailed interview of the HH.

In unsettled and big urban areas of the country, all the HHs in every selected village / Basti / Killi and block were enumerated for detailed interview. All the households in Tharparkar district of Sindh and greater Cholistan areas of Punjab were enumerated on 100 % count basis.

4. AGRICULTURAL MACHINERY CENSUS OF PAKISTAN

Progress of agriculture sector depends upon availability of resources like inputs for crop production, machinery usage, irrigation, fodder for livestock etc. Use of agricultural machinery has become basic need for agriculture with the passage of time. Machinery has become necessity for land levelling, ploughing, sowing, planting, hoeing, spraying, harvesting, threshing, crushing, packing, weighing, transportation to the market and from the market etc. Machinery is also playing vital role in livestock farming for fodder cutting, chopping, distribution, watering, treatment, cleaning, milking etc. Therefore tractors, tractor drawn implements, tubewells and other water lifting machines, modern irrigation systems like sprinkler irrigation and drip irrigation systems have become part of agriculture now. In order to know the actual number of agricultural machinery, the census of this machinery have been conducting in Pakistan after every ten years. Like agricultural and livestock censuses, the agricultural machinery censuses were also conducted on sample basis.

Sample Design for Agricultural Machinery Census

All the privately owned tractors, bulldozers, combine harvesters, tubewells, wells with pump, lift pumps and submersible pumps which were used or specified for use for agriculture purposes, were counted and their list was prepared throughout the country at Tehsil level by recording some information about the machinery. However, detailed questionnaire for owner of the machinery was filled on sample basis.

Generally systematic random sampling was used for selection of machinery and to conduct the detailed interview of the owner of selected machine. Every 10^{th} machine was selected with random start in those Tehsils where sampling was done. For each Tehsil sample was selected independently. Sampling was used in whole of Punjab except Cholistan and De-Excluded areas of D.G.Khan and Rajanpur districts, whole of Sindh except Tharparkar district and settled areas of Khyber Pakhtunkhwa. In unsettled areas of Khyber Pakhtunkhwa province, Cholistan and De-Excluded areas of D.G.Khan and Rajanpur districts, Tharparkar district and whole of Balochistan province, the detailed interviews were conducted on 100 % count basis. Information about public sector machinery used for agriculture purpose was collected through official correspondence with government departments, agencies and organizations. While information from private machine owners was collected through face to face interview basis.

5. DIFFERENCE BETWEEN CENSUS AND SURVEY

It is general perception that a census is conducted on complete count basis in which each and every population unit is enumerated and required information is collected. However, a survey is conducted on the basis of sample and a representative sample is selected in order to collect the required information form selected units. While practically many censuses are conducted in the world on sample basis, as discussed above for Pakistan, particularly agricultural censuses, but in spite of sample selected for interviews, are called censuses. In order to learn the definition of census and survey, the general perception is correct. However, in real life practice the difference between a census and a survey is little different and may be explained as below.

i) A census could be a complete count as well as a sample count, however, a survey is always a sample count only.

ii) If a census is sample based, it will has a large enough sample size while a survey has a small sample size as compared to that of census. The agricultural census of Pakistan has round about one million households (HHs) as sample, while the largest sample size of any survey in Pakistan is that of Pakistan Social and Living Standards Measurement Survey (PSLM) i.e. approximately 80000 HHs at country level, otherwise all other surveys have less than this sample size.

iii) Census always has small questionnaire while survey always has large questionnaire. There are many surveys having booklet as one questionnaire while there is no example of any census having long questionnaire comprising a booklet.

iv) Census is an overall structural study of any field of life while survey is a detailed study of a particular problem of that field.

v) Census covers usually more items with less detail of every item while survey covers usually less items with more detail of every item.

vi) Sample of a census may be selected at the lowest administrative level e.g. at Tehsil level or may be at Union Council level, while sample of a survey is usually selected at the highest administrative level i.e. district or province level.

vii) Tabulations are usually generated at the lowest level for a census while for a survey may be at highest level, the level at which the sample was selected.

viii) Usually data tabulations comprising estimates in numbers, are generated for the census and not for the survey.

ix) Surveys usually presented results in percentages while censuses generated estimates in numbers.

6. MERGER OF THREE CENSUSES

It is well known phenomena that agricultural land, livestock and agricultural machinery belongs to the same individual in most of the cases particularly with reference to Pakistan. There may be very few cases that a person who is growing crops but do not have machinery or livestock, OR who has livestock but do not have operated land or machinery, OR has machinery but do not have land or livestock. All these three important segments of agriculture sector are linked with each other in one or another way. This phenomena shows that when we conduct agricultural census, we contact a person who may have two or three segments, to conduct livestock census we may contact the same guy second time, and when we conduct machinery census again we may enquire the same individual. This real life situation leads us to think over it that we contacted the same person three times in order to almost get the same information as we may usually have many same questions about agricultural land, machinery and livestock. Therefore, we must think to contact a person once and only once to get detailed information about agricultural land, crops, machinery and livestock. Having this strategy, we may merge three previous censuses, i.e. agricultural, livestock and agricultural machinery censuses, into one census and name it as agricultural census of Pakistan.

7. FUTURE RECOMMENDATION

The previous three censuses have different three sample designs according to their independent requirements. For merged agricultural census a new sample design is required to fulfill the requirements of data collection for important study variables from previous three censuses. Therefore, it is a good, challenging and need based research problem for researchers to propose a sample design for merged agricultural census for Pakistan.

REFERENCES

1. Bangladesh Bureau of Statistics (2010). *Census of Agriculture 2008, National Series*, Volume-1, Government of the People's Republic of Bangladesh, Dhaka, Bangladesh.
2. Central Statistics Office, Republic of Botswana (2007). *2004 Botswana Agricultural Census Report*, Ministry of Finance and Development Planning, Gaborone.

3. *Colombia Agricultural Census 1988* Main Results.
4. Government of Gambia (2002). *Agricultural Census of the Gambia 2001-02*, Vol. 1, Agricultural Statistics and Resources Economics Unit, Department of Planning, Department of State for Agriculture, Banjul, The Gambia.
5. Government of India (2014). *Agriculture Census 2010-11, All India Report on Number and Area of Operational Holdings*, Agriculture Census Division, Department of Agriculture & Cooperation, Ministry of Agriculture, Government of India.
6. Government of Nepal (2013). *National Sample Census of Agriculture Nepal 2011/12*, National Report, National Planning Commission Secretariat, Central Bureau of Statistics, Kathmandu, Nepal.
7. Government of Pakistan (2004). *Pakistan Agricultural Machinery Census (2004), All Pakistan Report.* Statistics Division, Agricultural Census Organization, Lahore.
8. Government of Pakistan (2006). *Pakistan Livestock Census (2006)*. Statistics Division, Agricultural Census Organization, Lahore.
9. Government of Pakistan (2012). *Agricultural Census 2010, Pakistan Report*. Statistics Division, Agricultural Census Organization.
10. Government of Pakistan (2014). *Pakistan Social and Living Standards Measurement Survey (2014-15)*. Statistics Division, Pakistan Bureau of Statistics, Islamabad.
11. Government of Pakistan (2015). *Labour Force Survey (2014-15)*, Statistics Division, Pakistan Bureau of Statistics, Islamabad.
12. Government of Pakistan (2016). *Pakistan Economic Survey 2016-17*. Economic Advisor's Wing, Finance Division, Islamabad.
13. National Statistical Office (2010). Government of Malawi, *National Census of Agriculture and Livestock 2006-07*, Main Report, Zomba, Malawi.
14. National Statistics Office, Philippines (2002). *Philippines Census of Agriculture*, National Economic and Development Authority, report generated on February 24, 2016.
15. Uganda Bureau of Statistics (2010). *Uganda Census of Agriculture 2008-09*, Vol. II, Methodology Report.
16. Uganda Bureau of Statistics (2010). *Uganda Census of Agriculture 2008-09*, Vol. IV, Crop Area and Production Report.
17. United Republic of Tanzania (2010). *National Sample Census of Agriculture 2007-08, Preliminary Report*, National Bureau of Statistics, and the Office of the Chief Government Statistician, Zanzibar.
18. United Republic of Tanzania (2011). *National Sample Census of Agriculture 2007-08*, Volume-I: Technical and Operation Report, National Bureau of Statistics, and the Office of the Chief Government Statistician, Zanzibar.
19. United Republic of Tanzania (2012). *National Sample Census of Agriculture 2007-08, Small Holder Agriculture, Vol. II: Crop Sector-National Report*. The National Bureau of Statistics and the Office of the Chief Government Statistician, Zanzibar.

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ROLE OF STORM WATER IN PROMOTING URBAN FORESTRY AND APPRAISING ITS ECONOMIC AND ENVIRONMENTAL VALUES

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ABSTRACT

An extensive study was conducted from 2012-2015 to determine the potentials of Lahore metropolitan in sustaining vigorous sizeable Urban Forest. It was concluded that with a little planning and consistent efforts 132 Km^2 Urban Forestry of 2.5 time larger size than a local reputed public productive forest (Chhanga Manga) can be established. It was discovered that with few distinctive forest management features an annual production of 2374 tons of wood can be easily obtained. This will be 25 times more than Chhanga Manga Forest, Lahore. Urban Forest is expected to sequestrate 119 tons of carbon each year and will thus make city environment much pleasant.

1. INTRODUCTION

Urban Forest has been varyingly defined by different authorities. Dr. Irshad Ahmed (2015) defined Urban Forest as all trees within a city, ranging from individual public and private trees to forested parks. "Urban Forestry is the management of trees for their contribution to the physiological, sociological, and economic well-being of urban society. Urban Forestry deals with woodlands, groups of trees, and individual trees, where people live - it is multifaceted, for urban areas include a great variety of habitats (streets, parks, derelict corners, etc.) where trees bestow a great variety of benefits and advantages "Denne, pers. comm. (adapted from Grey and Deneke, 1986)."

According to Miller, (1988:28) Urban Forestry is "an integrated, city wide approach to the planting, care and management of trees in the city to secure multiple environmental and social benefits for urban dwellers."

2. ROLE OF URBAN FORESTRY

Literature analysis shows that the Urban Forest is not an independent factor. It has countless social, economic, cultural and environmental roles. One of these roles is ecological welfare. The ecological role is linked to nature, environment and their interaction (Adams, 2005). Numerous monetary advantages are power efficiency and increased rates of estate (Donovan & Butry, 2010), plus control of pollution. These forests are crucial for saving wildlife, stopping erosion of land, prevention of wastage of rain water and saving the environment (Conine, et al. 2004; Gobster & Westphal, 2004; Kong, Yin, & Nakagoshi, 2007; Yuan & Bauer, 2007).

3. FOREST IN LAHORE

Pakistan is in the midst of extremely serious complex of multiple problems of diverse nature which appear to be unresolvable. The climate of Pakistan is varied. It is characterized by unpredictable weather features. Precipitation ranges from 50 mm in western parts to about 1500mm in Himalayan region.

Haphazard urban development has consumed most of the irrigated lands and precious water in and around Lahore City. Lahore Metropolitan has four seasons and overall dominated by semi-arid to semi humid climate. Lahore mainly receives 65% of its rainfall during the monsoon seasons from June till September, and in winter season from December till February. The highest-ever annual rainfall in Lahore was recorded in 2011 when 1,570 millimeters (62") of rainfall was recorded. The highest rainfall in the city recorded during 24 hours is 221 millimeters (9 in), which occurred on 13 August 2008 (Lahore climate data). The storm and rain water majorly goes waste either by evaporation or it is drained into the rivers and drains. This is an arid or semi-arid region where most of the land receives less than 250mm of annual precipitation. This is a main feature of summer monsoon (Mahmood Iqbal Sheikh, 1993). The level of Urban Forestry is even less than 1% in Lahore. The major part of plantation of Lahore is for ornamental purpose and is not production oriented. The Government is reluctant to initiate Urban Forestry in Lahore because of lack of awareness of the potentials of Lahore Metropolitan in sustaining the huge and vigorous Urban Forest.

Research has shown tall trees as more beneficial for the environment. Their high canopies absorb pollutant gases emitting from industry and road traffic. In the recent afforestation derives have replaced tall trees of the past with small ornamental plants. The pollution controlling capability of tall trees is hundred times more than the in-fashioned small-sized trees. These ornamental plants are small sized and do not play an effective role in reducing air pollution. This is because they cannot interact with the gases in the atmosphere. (The Environment Protection Department, Government of the Punjab, 2013)

Siris, Peepal, Shisham, Bargad, Bakain, Toot, Neem, Alstonia, Samadar Phal, Kachnar, Bottle Brush, Toon, Amaltas, GulMohar, Gul-e-Nishtar, Jaman, Gulnar, Beeri Patta, Gul-e-Fanoos, Magnolia, Sukh Chain, Poplar, Putajan, Sagwan, Arujun, Bahera, Harar and Beri are the local preferred tress that act as a pollution curer. Different species have found that these species are purifiers of the environment. They do so by releasing more oxygen and, absorbing carbon dioxide. These minimize noise pollution and regulate extreme temperature fluctuations. According to a vegetation survey conducted along Lahore-Islamabad motorway (M-2) following findings were obtained.

- The diversity along the roads was high (i.e. 227 species).
- Frequent and dominant species were less in number.
- The nutrient status of land was ideal.
- Further research and evaluation is required for species diversity and habitat conservation.

4. MATERIALS AND METHODS

4.1 Quantifying Storm Water and its Frequency

Procurement and analysis of relevant metrological data for determining severity and frequency of storms.

4.2 Determining Water Holding Capacity and Rate of Percolation

Obtaining relevant data related to rate of water absorption, water holding capacity and rate of percolation etc. from Govt. departments/ Authorities / Commissions/ International research organizations.

4.3 Determining Major Land Uses

Use of latest satellite imagery for determining areas under tarmac/ concrete/ buildings, areas under woody vegetation and additional areas available for urban forestry and its ground trothing to verify satellite imagery information.

4.4 Determining Water Requirement of Woody Vegetation

Collection of the data for water requirement of woody vegetation from Forest Dept. i.e. how much water(data of water) these require i.e. utilization per year or determining the need of the most luxurious forest i.e. water requirement of various trees or of mixed woody vegetation or of mixed trees and grasslands in subtropics and

A) Percentage of water intercepted by woody vegetation and used/evaporated,
B) Percentage of water intercepted by litter,
C) Percentage reaching soil,
D) Percentage absorbed by roots,
E) Percentage retained by soil.

4.5 Directing Surface Runoff Water to Nearby Vegetated Areas

By creating vegetated areas/patches of various sizes/shapes, all such areas or patches must be at least 2' below general land surface and all physical obstruction in the way of flowing water shall be removed. In case wherever vegetative patches/areas are not available / possible sinking wells shall be installed.

4.6 Resolving Management Issues

Sunken beds of variable shapes (circular, square rectangular or any other shape etc.) should be established and for patch size like 0.5 ha, 1 ha, 2 ha etc. these sunken beds should be 2' deeper than the ground level. Numerous sinking wells are proposed which will accelerate percolation. These comprise of PVC pipes of 1' diameter, filled with loose gravel and topped with foam filter which will be replaced periodically. These are usually 50-70' deep. Selecting suitable species and planting these timely is important.

4.7 Estimating Carbon Sequestration

In Urban Forestry the amount of carbon removed from the air is very crucial. It is stored in the wood in relation to its quantity released into the atmosphere. Biomass production and carbon sequestration rate are determined along with estimating its potential value for carbon credits under UNFCC mechanism. The relevant basic data pertaining to suitable woody species and biomass production/carbon sequestration is obtained from Forest Department and concerned research organization and from international literature.

5. RESULTS AND DISCUSSION

5.1 Total Geographical Area of Lahore Metropolitan

The total geographical area of Lahore metropolitan is estimated to be 930 km^2 which is divided into following components

(1) Old Lahore	540 km^2
(2) New Lahore	350 km^2
(3) Drains Banks	10 km^2
(4) River Bank	30 km^2
Total:	**930 km^2**

Lahore is second largest city of Pakistan and is rapidly expanding in area and population. It is Capital of Punjab and has an elaborate intra-city road system along with a wonderful Ring Road. The city receives full attention of Provincial Administration and receives liberal grants for development. Most importantly, this city has innumerable educational institutions parks, playgrounds, stadiums open spaces and boulevards. It may be noticed that all above mentioned sites/locations are ideal for tree planting.

5.2 Major Land Uses

5.2.1 Old Lahore Areas

Latest satellite imageries and various maps were consulted and numerous visits were conducted to evaluate land use pattern for old Lahore as realistic as possible which is as follows:

(a) Buildings, pavements and tarmac areas 40%
(b) Parks, Green belts and woody vegetation 10%
(c) Blanks and farm areas 50%
(d) Additional area from above land uses which is 10% (available for tree plantation)

Although while casually going through old Lahore areas one does not apparently notice areas available for additional tree planting but when one looks at various sites carefully one does recognize and discover a large number of small pockets, strips, areas and locations of variable sizes which are lying blank and are potential areas for plantation of trees.

5.2.2 New Lahore Areas

A Similar exercise was conducted for new Lahore which comprises of Bahria Town, DHA, New Airport, Thokar Niaz Baig, WAPDA Town, State life Housing Society, Electrical/Mechanical Engineers society, Izmir society, Army Welfare Society, Askari 11 LDA city. Avenue -I (LDA Society) Shahpur Kanjra, Nespak Society, Velencia Society, Nashman-a-Iqbal Society where following major land uses were recognized

(a)	Buildings pavements and tarmac areas	25%
(b)	Parks, Green Belts and woody vegetation	20%
(c)	Blanks and farm area	55%

Area that can be easily made available for additional planting is estimated to be 15%. Since new Lahore is better planned and more open city and more or less according to International town planning principles, it has more parks, green belts, woody and vegetated areas than old Lahore. Furthermore there are more additional opportunities and locations available for tree planting.

5.2.3 Drains

There are 21 drains of variable length in Lahore area having a total length of 106 km. The width of the drain banks varies from 2m to 15m. The total area of drain is estimated to be 10km^2 which is all available for tree planting. It is equal to 1.07% of all available area.

Lahore drains are perennial and drain out chemical effluents of industrial nature as well as domestic sewerage. These are semi ideal places for tree planting. These drains are like an unpleasant scar in Lahore landscape and are source of foul and pungent smell. It is therefore mandatory for Lahore Administration to immediately plant suitable bushy and flowering trees to provide necessary cover to the ugly sites as well as a source to produce some fragrance.

5.2.4 River Bank

There is about 20 km length of river bank with variable width of bank meandering from 0.5 km to 2.0 km. The total area associated with river is estimated to be 30 km^2. Half of it is immediately available for tree planting which is 1.6% of the total. River Ravi has in the recent past become an ephemeral river and has become a big natural drain for Lahore during most past of the year. Ravi bank (Bela) had thick tree vegetation in the past which has disappeared because of various reasons. It is quite possible to rebuild an eastern Ravi river bank by planting trees. The entire project of tree planting along river bank must be undertaken in different phases. It is believed that half of the eastern bank (15km^2) can be planted to begin with by installing shallow tube wells for initial irrigations.

5.3 Water Requirements for Woody Vegetation

The satellite imagery of the natural forest from near Lahore area indicates clearly that Lahore climate though semiarid can support and sustain a medium to tall forest of moderate density. The vegetation in question is a result of years of slow growth coupled with effective protection. The imagery provides the clue that if one can augment natural rainfall with canal irrigation or rain harvesting, vigorous fast growing forest can be easily established.

Since Lahore area and nearby irrigated plantations such as Chhanga Manga receives 600mm of natural rain water. This little water is insufficient to support a reasonably vigorous and profitable trees growth. The Punjab Forest Department has thus arranged about 600mm of additional canal water during summers for undertaking commercial forestry. This total of 1200mm is considered reasonable water requirement for sustaining a healthy forest growth.

6. CONCLUSION

1. About 10 to 15% city area 107 km^2 is available for Urban Forestry in Lahore.
2. Out of total drain banks (10 km^2) half of eastern bank of river Ravi (15km^2) is also readily available for Urban Forestry.
3. In total about 33 million trees can be easily planted over 132 km^2 area
4. Maximum total water requirement for sustaining reasonable Urban Forest growth is about 1200-1400 mm. Out of this more than half is easily available from natural rains and the other half from diverting various other sources such as treated sewerage water.
5. Area that can be available for Urban Forest in Lahore Metropolitan is 132 km^2 which is as large as about 2.5 times larger than Chhanga Manga Forest.
6. Soils of Lahore are deep alluvial and can easily hold 1200–1600 mm water and are nearly ideal for Urban Forestry.
7. About 1200 mm of additional water can be easily made available by facilitating the flow of harvested rain water from adjoining buildings, paved and tarmac areas.
8. About 600 mm of partially treated domestic sewerage can be made available with little effort by locally storing/fermenting in septic tanks and periodically pumping it out for irrigation.
9. There are about 2 dozen trees species which are suitable for Urban Forestry are fast growing which can fetch good prices from local markets.
10. It is estimated that total growing stock is 21583 tons from Lahore metropolitan. This growing stock is as big as 10 times more than nearby reputed public productive forest (Chhanga Manga).
11. Annual wood production is estimated to be 2374 tons which is 25 times more than a nearby reputed public productive park (Chhanga Manga).
12. Urban Forest of Lahore Metropolitan is estimated to sequestrate 119 tons of carbon annually thus making Lahore environment pleasant and claiming carbon credits under Kyoto Protocol..
13. Urban Forestry of Lahore Metropolitan is expected to make a handsome profit from 200% to 400% depending on quality of management.

7. RECOMMENDATIONS

1. A vigorous tree planting campaign should be undertaken for a number of years to plant urban trees to cover 132 km^2 areas available by various relevant authorities.
2. About 24 species of trees have been recommended for this purpose that are fast-growing and fetch good prices from local market.

3. A few special silvicultural techniques need to be adopted to ensure sapling establishment, promote growth and to increase profit.
   a) Planting large sized saplings
   b) Staking and fencing of selected areas where required.
   c) Having short rotations of 4 to 8 years
   d) Pruning of lower branches of selected trees if necessary.
4. In order to make additional water (1200 mm or more) available to trees, the level of all green belts and other planting sites must be lowered by 0.5 to 0.75 m than surrounding roads and pavements.
5. In Order to make additional 600mm nutrient rich, partially treated, domestic sewerage water available to plants septic tanks of appropriate size should be built in each residential area colony/town. The sewerage should be fermented there and be used for irrigating trees by a fleet of tankers.

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REFERENCES

1. Adams, L.W. (2005). Urban wildlife ecology and conservation: A brief history of the discipline. *Urban Ecosystems*, 8(2), 139-156.
2. Conine, A., Xiang, W.N., Young, J. and Whitley, D. (2004). Planning for multi-purpose greenways in Concord, North Carolina. *Landscape and Urban Planning*, 68, 271-287.
3. Donovan, G.H. and Butry, D.T. (2010). Trees in the city: Valuing trees in Portland, Oregon. *Landscape and Urban Planning*, 94(2), 77 83.
4. Gobster, P.H. and Westphal, L.M. (2004). The human dimensions of urban greenways: Planning for recreation and related experiences. *Landscape and Urban Planning*, 68, 147-165
5. Kong, F.H., Yin, H.W. and Nakagoshi, N. (2007).Using GIS and landscape metrics in the hedonic price modeling of the amenity value of urban green space: A case study in Jinan City, China. *Landscape and Urban Planning*, 79, 240-252.
6. Yuan, F. and Bauer, M.E. (2007). Comparison of impervious surface area and normalized difference vegetation index as indicators of surface urban heat island effects in Landsat imagery. *Remote Sensing of Environment*, 106, 375-386.

TABLES

Table 1
Percentage of Geographical Area of Lahore Metropolitan

Type	Percentage
Old Lahore	58%
New Lahore	38%
Drains Banks	1%
River Bank	3%
Total	**100%**

Table 2
Geographical Area of Lahore Metropolitan

Type	Area (kms^2)
Old Lahore	540
New Lahore	350
Drains Banks	10
River Bank	30

Table 3
Percentage of Geographical Area of Lahore Metropolitan

Major Land Uses	Old Lahore Areas Percentage	New Lahore Areas Percentage
Buildings, Pavements and tarmac areas	40%	25%
Parks, Green belts and woody vegetation	10%	20%
Blank and farm areas	50%	55%
Total	**100**	**100**

Table 4
Percentage Distribution of Major Land Uses w.r.t Old and New Lahore

Water requirements for Woody Vegetation	Frequency
Chhanga Manga Receives Natural Rain Water	600mm
Chhanga Manga Receives Water from Punjab Forest Department	600mm

Table 5
Frequency Distribution of Water Requirements for Woody Vegetation along drains

Sr.	Name of Drain	From	To	Length (km)
1	Lower Chotta Ravi Drain	Band Road	River Ravi Road	0.8
2	Shahdara Drain	National Bank Shahdara	River Ravi Sagian	4.21
3	Shalimar Escape Channel Drain	GT Road WAPDA Colony	River Ravi	10.00
4	Upper Chotta Ravi Drain	China Scheme	Shad Bagh Disposal Station	4.26
5	Cantt Drain	Dhobi Ghat Railway Line	River Ravi	15.39
6	Sattukatla Drain	Fruit & Vegetable Market at Ferozpur Road	Hadyara Drain	17.68
7	City Drain	Bhatti Gate	Dehli Gate	3.02
8	Kharak Drain	Neelam Block Allama Iqbal town	Cantt Drain Sabzazar	5.60
9	College Road Township Drain	Akbar Chowk	SattuKatla Drain	3.96
10	General Hospital Drain	Chungi Amer Sadhu	SattuKatla Drain F&V Market	3.35
11	Moulana Shaukat Ali Road Drain	Barket Market Town	Akbar Chowk	2.43
12	Industrial Area Drain	Industrial Estate	SattuKatla Drain Qadri Chowk	2.43
13	Gulshan-e-Ravi	Bund Road	Toward Ravi	4.12
14	Main Out Fall	Bund Road	Toward Ravi	2.20
15	Shad Bagh	Bund Road	Toward Ravi	2.43
16	Mehmood Booti	Ring Road	Toward Ravi	5.29
17	Multan Road	Ring Road	Toward Ravi	2.25
18	Nishter Colony	Nishter Colony	Hadyara Drain	4.35
19	Shoukat Khanam	Khiaban-e-Jinnah	SattuKatla Drain Near Tariq Garden	3.00
20	Journalist Colony	G.T Road	Journalist Colony	8.00
21	Fazal Park Shahdara	Fazal Park Disposal	River Ravi	1.07

Table 6
Monthly Meteorological Observation for the Year 2011-12

Month	Rain Fall (cm)	Average Minimum Temperature	Average Maximum Temperature (^{0}C)	Average Mean Temperature (^{0}C)	Average Relative Humidity (%)	Total pan Evaporation (cm)
Jul-11	20.55	30.3	41.6	36	72	20.5
Aug-11	37.8	30	38	33	73	37.81
Sep-11	6.18	24	34	28	66	**
Oct-11	0	26	32	29	63	**
Nov-11	0	20	28	24	62	**
Dec-11	0	9	25	17	56	4.6
Jan-12	1.51	5	22	13	41	3.6
Feb-12	0.83	8	16	16	41	5.28
Mar-12	0	13	28	20	43	**
Apr-12	3.04	20	34	27	43	**
May-12	0	27	40	34	31	**
Jun-12	0	29.1	41.6	35.3	35	**

Table 7
Meteorological Observation for the Year 2011-12

Month	Rain Fall (cm)	Average Minimum Temperature	Average Maximum Temperature (^{0}C)	Average Mean Temperature (^{0}C)	Average Relative Humidity (%)	Total pan Evaporation (cm)
Jul-12	3.13	28.1	39.5	33.8	35	**
Aug-12	9.71	28	38	33	78	**
Sep-12	19.85	22.62	34.75	33	76	**
Oct-12	2	18	32	25	56	5
Nov-12	0	12	24	18	52	4.6
Dec-12	2.51	6.5	20.5	13.5	51	3
Jan-13	0	6	19	12.5	49	4.3
Feb-13	7	8	22	15	40	**
Mar-13	1.77	13	26	19.5	42	**
Apr-13	0.4	18	33	25.5	38	**

Table 8
Meteorological Observation for the Year 2012-13

Month	Rain Fall (cm)	Average Minimum Temperature	Average Maximum Temperature (^{0}C)
May-13	1.2	25.1	40.3
Jun-13	136.0	26.5	38.7
Jul-13	242.2	25.2	35.3
Aug-13	352.3	24.8	32.0
Sep-13	30.7	24.4	35.1
Oct-13	18.4	20.5	32.1
Nov-13	4.5	11.3	26.5
Dec-13	7.0	7.3	20.9
Jan-14	4.0	6.2	19.2
Feb-14	22.6	8.1	20.0
Mar-14	32.3	12.9	25.1
Apr-14	64.8	17.9	32.1
May-14	29.3	*	*
Jun-14	50.1	*	*
Jul-14	31.7	*	*
Aug-14	54.5	*	*

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ROLE OF LEGISLATION FOR SUSTAINABLE ENVIRONMENTAL MANAGEMENT TO PREVENT & CONTROL DENGUE FEVER EPIDEMIC: CASE STUDY ON DENGUE REGULATIONS FOR EPIDEMIC CONTROL IN CANTONMENT TOWN, LAHORE, 2012-2017

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ABSTRACT

Dengue Fever (DF) outbreaks are increasingly common globally and put great burden on economy and health services. The incidence of disease has been on high level in the last decade in Pakistan. The emerging and re-emerging behavior of Vector-Borne Diseases (VBDs) poses a serious health problem. The approach of Integrated Vector Management (IVM) strategy for combating VBDs transmission involves entomological knowledge, technical, infrastructure capacity, systems facilitating stakeholder collaboration, policy and Legislative framework is recommended by the World Health Organization (WHO).

The research work is carried out to study impact of enforcement of dengue regulations during 2012 to 2017 in Cantonment Town, Lahore. The correlation of independent variables of Notices, FIRs impact was studied with dependent variables i.e. No. of Patients and Larva found. The study shows that legislation has impact on substantial control in spread of the disease. The linear regression model curve between enforcement variables and disease burden control indicated that a dramatic increase in the number and frequency of outbreaks followed in 2016 due to unattended surveillance and legal actions in past years. The study indicated the correlation of disease with income levels, age group, private businesses and with education level of surveyed population. Generally younger people with age up to 35 had better knowledge of laws about mode of disease transmission and good practices about monitoring for potential breeding sites and breeding container elimination. It is found that legislation also plays a pivotal role in changing people behavior to adopt vector control measures.

KEY WORDS

Dengue Fever Epidemic, Integrated Vector Management (IVM), Dengue Regulations

1. INTRODUCTION

Dengue Fever (DF) is a vector- borne disease caused due to bite of mosquitoes known as *Aedes agypti* and *Aedes albopictus*. There're more than 3500 mosquito species among which anopheles, aedes and culex are most commonly found in the living environment (Seufi and Galal, 2010). Aedes mosquitoes are highly invasive and can survive almost any climatic conditions. Epidemics like dengue fever are spreading currently in sub-continent due to climatic and weather season changes (Haque et al., 2012). The breeding places for *Aedes aegytpi* and *Aedes albopictus* are found in natural and man-made environment. Sustainable environmental management plays vital role in control of VBDs. Climate change is also major cause of spread of the public health diseases. Malaria, dengue, polio, naegleria and Congo viruses or such other epidemics require proper environmental management to contain these diseases. Internationally health regulations are made as legislative frame work to control the disease and mobilize community. The study looks into the legislation made in Punjab, Pakistan and its impact on disease and vector control in Cantonment Town, Lahore.

2. LITERATURE REVIEW

Dengue virus is flavi-virus which is spread up due to bite of Aedes mosquito. Governments have to strengthen their vector control program elements based on policy and legislations for epidemics prevention and control (Chanda et al., 2017). Pakistan is one of the Southeast Asian countries where dengue vector program is running successfully since 2011 by implementing three major measures. These three measures are reduction of *Aedes aegytpi* population by surveillance, community education, and law enforcement. The Epidemic Diseases Act, 1958 (West Pakistan Epidemic Diseases Act, 1958) was enacted to control the propagation of dengue vector and, consequently, dengue had been contained. Due to certain governance issues it is necessary to check out come of regulations (Manzoor et al., 2016). Vector-borne diseases (VBDs) are the main concern in urban areas. Malaria, dengue fever and naegleria spread due to improper environmental management. Climate change and urbanization are among the major causes for the spread of the vector-borne diseases. However, this is the challenge for Pakistan that has been moving toward decentralization the problems have surfaced at all levels regarding public health issues. The legislation and education can play major role to maximize general public participation (Caicedo et al., 2017). Internationally health regulations are made to main sustainability of environmental management to prevent and control epidemic diseases. Australia, Sri Lanka, Singapore and India have made regulations to control the dengue fever epidemics.

3. MATERIALS AND METHODS

Study Area

Lahore Cantonment is a garrison located in Lahore, Punjab, Pakistan. Although the cantonment is located within Lahore City District, it is an independent municipality under control of the Military Lands. Maximum numbers of dengue patients arise from the town. The population of Cantt Town is estimated at 838423. Cantt Town consists of following three major areas:

1. Lahore Cantonment Board (LCB)
2. Walton Cantonment Board (WCB)
3. Defense Housing Authority (DHA)

Data Base

Secondary data is collected for enforcement measures and primary data collected by interviews from the people.

Independent Variable

Enforcement: Firs Information Reports (FIRs), Notices

Dependent Variables

1. No. of Confirmed Patients
2. Outdoor Larva Found

Sampling Design

The study used quantitative, qualitative and descriptive methodology. A structured questionnaire developed and used to interview key stakeholders in Cantt Town of district Lahore selected for enforcement of dengue regulations from year 2012 to 2017.

Unit of Analysis

Cantt Town, District Lahore, Punjab, Pakistan.

Data Collection Method

Secondary data has been taken from District Lahore Cantonment Town Health Office and from field surveys.

Population Sample

Total 539 residents invited for study. After informing about study 376 showed willingness to participate in study.

Participants Description

Participants in sample surveys and related data collection exercises were given sufficient details on the research in question as to allow them to make an informed decision to participate or not in a research study.

- Total subjects n= 539
- Respond n=376
- Not Respond n=163

The data collection was completed through face-to-face interviews and telephonic interviews conducted by using questionnaire.

- Face-to-face interviews n=225
- Telephonic interviews n= 151

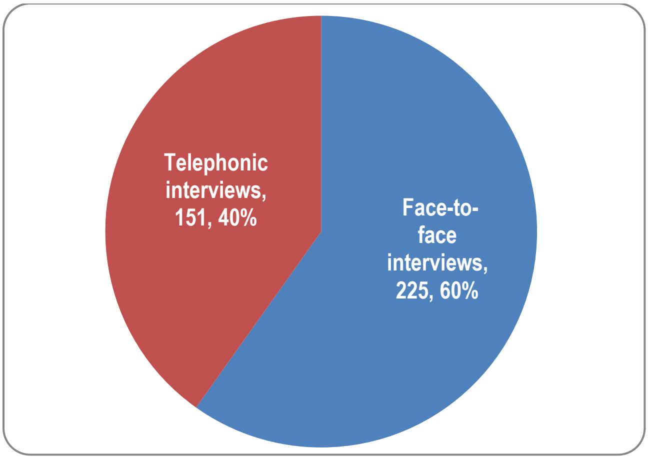

Fig. 1: Description of Interviews Participants: Number of Participants & Percentage of Participants

Data Evaluations & Statistical Analysis

In the study modified Ross-Macdonald model is used assuming that mosquitoes have constant activity for 6 years, four potential counter measures assessed: (1) FIRs, (2) Notices (3) Arrest (4) Premises seal. All collected information was introduced and records were double-checked. Statistical analysis was performed using and Statistical Package for Social Sciences 19.0 (SPSS, Inc., Chicago, IL, USA). Answers obtained from the questionnaire were recorded to obtain other categorical variables linked. Comparisons of percentage and distribution between socio-demographic groups have been done with normal distribution Skewness test. Linear regression models performed to explore Regulation factors on disease burden and outdoor larva prevalence.

Ethical Statement

Participants in sample surveys and related data collection exercises were given sufficient details on the research in question as to allow them to make an informed decision to participate or not in a research study. Participants in sample surveys and related data collection exercises agreed to participate in the research information to provide as part of the research allowed to putting the results of the research into the public domain.

4. RESULTS & DISCUSSION

Three major factors have been taken into study for impact of Legislation on disease and vector burden:

1. Impact of First Information Reports (FIRs) on Disease and Vector Burden
2. Impact of Notice issued on Disease and Vector Burden
3. Integrated approached and perception and behavior modification (n=376 participated in past faced dengue regulation procedure).

Table 1
Enforcement, Larva and Patients Data

Sr.	Year	Patient	Larva	Notice	FIR	Arrest	Seal
1	2017	7	219	545	144	23	9
2	2016	274	178	685	96	6	3
3	2015	4	143	317	55	2	1
4	2014	1	135	374	118	12	10
5	2013	10	105	474	87	26	13
6	2012	8	94	632	39	17	10

Table 1 gives the enforcement data, disease and vector burden details from the year 2012 to 2017.

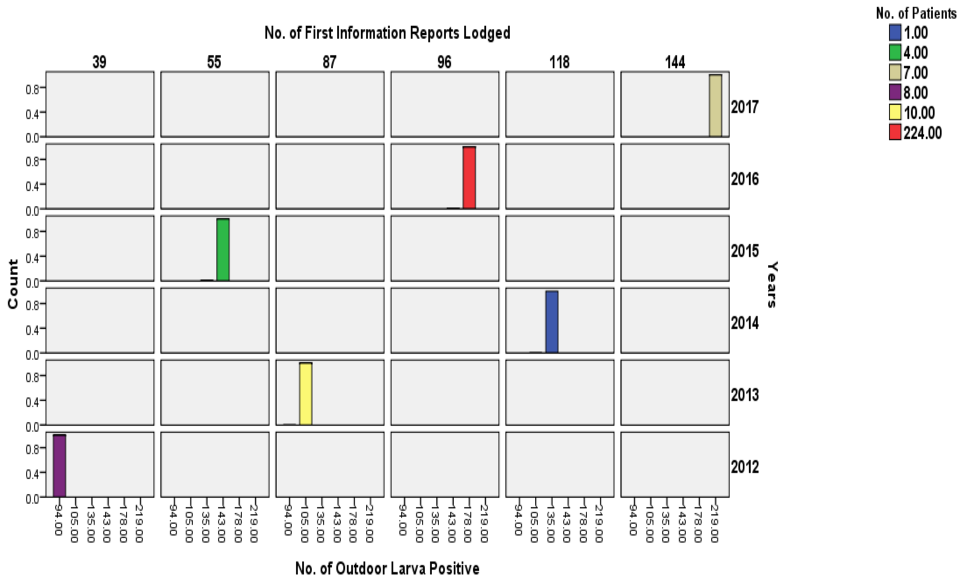

Fig. 2: Representation of Impact of FIRs on Disease and Vector Burden

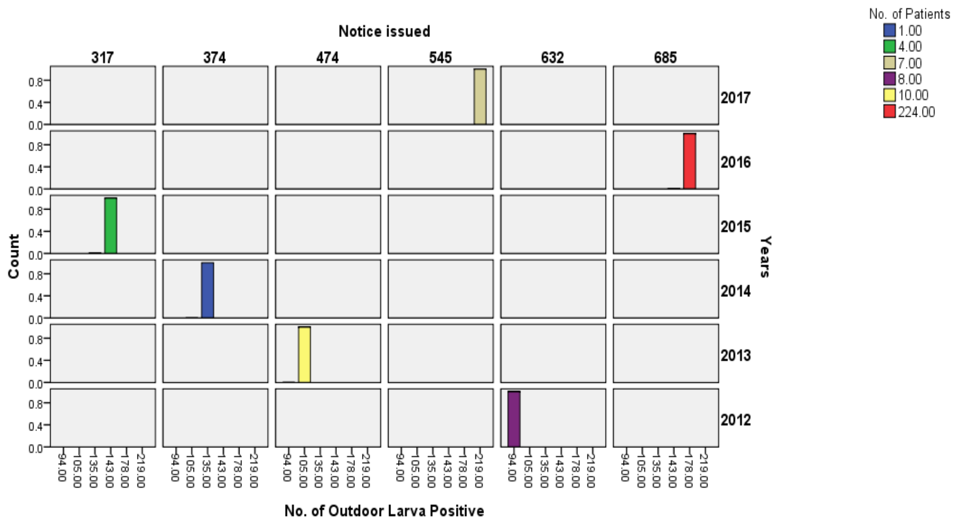

Fig. 3: Representation of Impact of Notices on Disease and Vector Burden

Linear regression models are drawn for the data analysis of enforcement variables.

Regression model of Impact of First Information Reports (FIRs) on Disease Burden

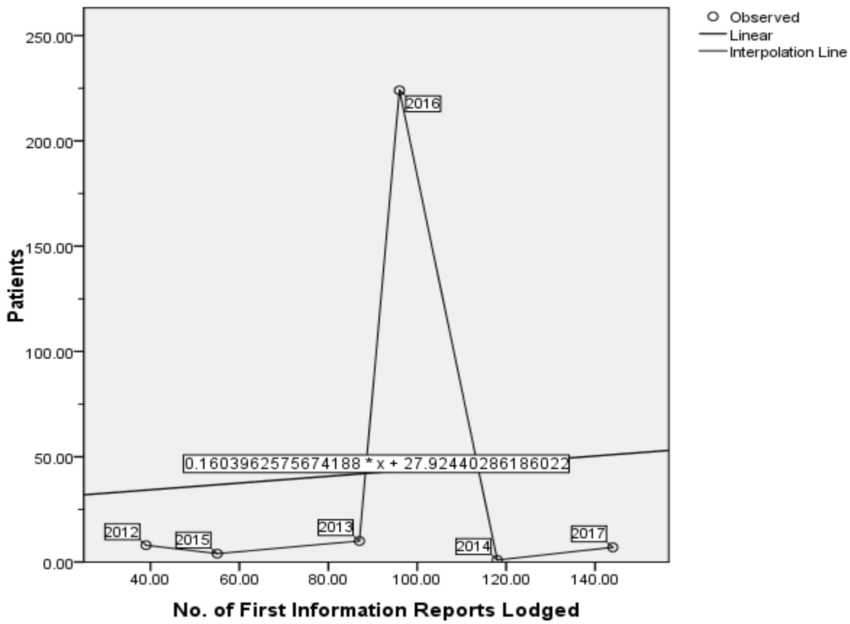

Fig. 4: Regression model of Impact of First Information Reports on Disease burden

Graph shows that dengue fever patients have been in control for the year 2012 to 2017 except the peak in 2016. Peak in 2016 is due to the non-implementation of the dengue regulations in Defense Housing Authority (DHA) area. In 2017 maximum numbers of FIRs have been done and also number of patients decreased. It showed a significant relationship between the enforcement and the disease burden. The disease burden increased due to less number of FIRs but it decreased when FIRs were increased.

Regression Model of Impact of Notice Issued on Disease Burden

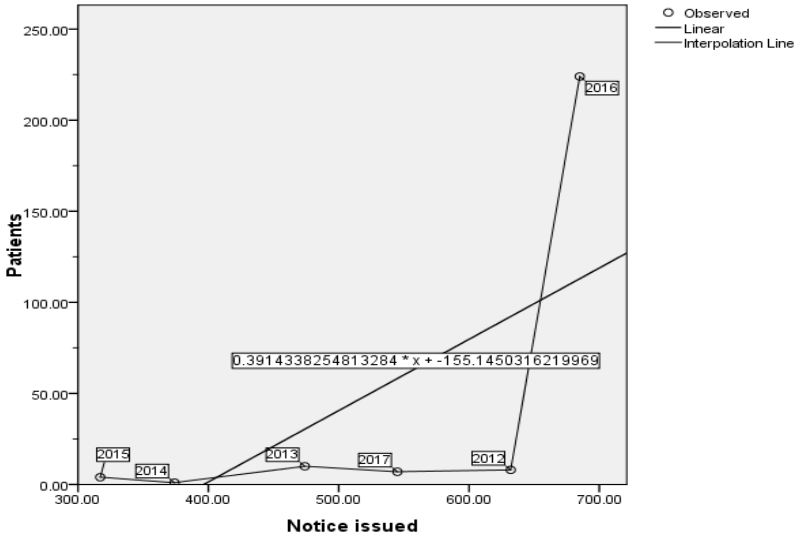

Fig 5: Description of Regression model of Impact of Notice issued on Disease burden

Regression Model of Impact of Firs on Outdoor Larva

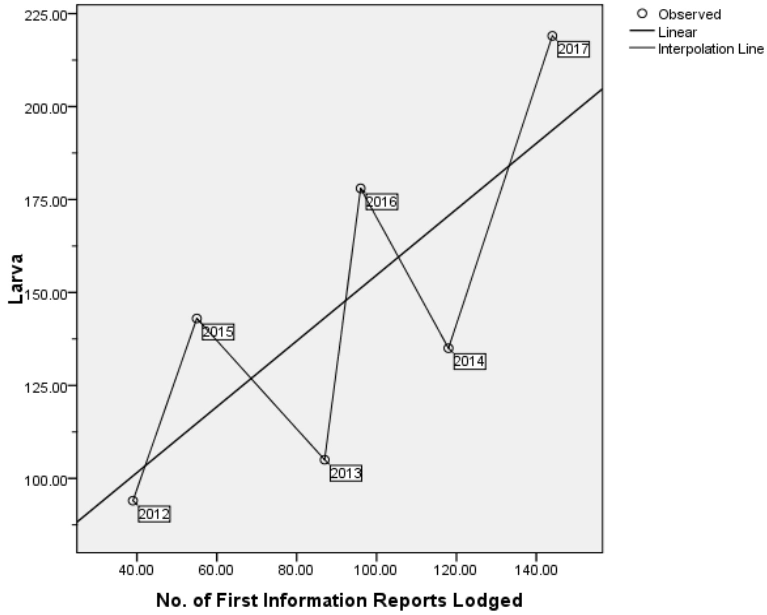

Fig. 6: Description of Regression model of Impact of FIRs on Vector burden

Regression Model of Impact of Notice Issued on Outdoor Larva

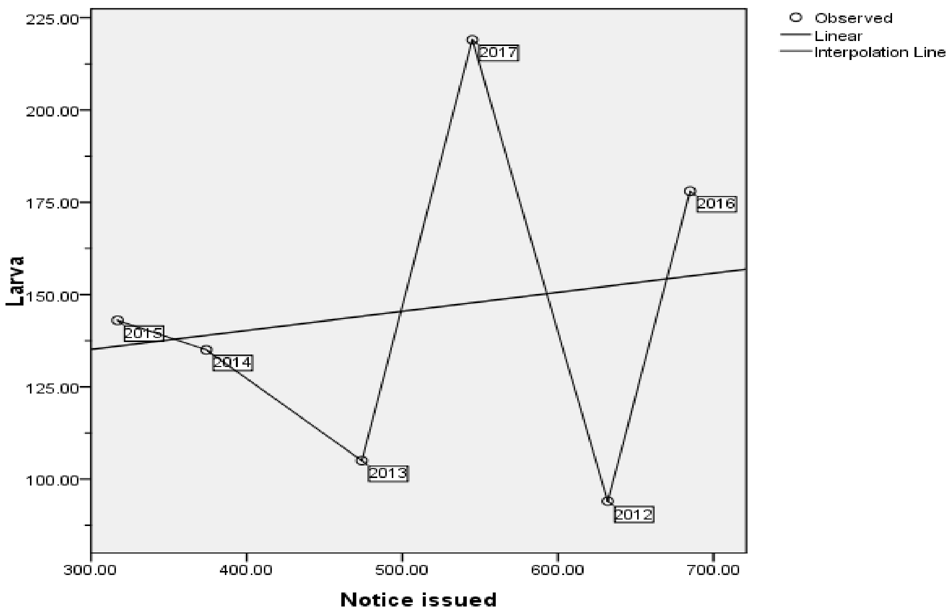

Fig. 7: Regression model of Impact of Notice issued on Outdoor Larva

Results of data of 2012, 2016 and 2017 showed highest number notice issued trend and 2015, 2014 and 2013 showed lowest number of notice issued trend. The years 2014, 2015 and 2017 have lowest disease burden trends and similarly 2016 >2013>2012 has highest disease burden trends. The legal framework indicates that the approach to the dengue epidemic should be an intersectoral response. The linear regression model curve indicates that a dramatic increase in the number and frequency of outbreaks followed in 2016 due to unattended surveillance and legal actions in past years. This result of study engages with the aforementioned discussion from a legislative perspective and underscores a strong relationship between health legislations, public law and the underlying systemic problems impacting the dengue epidemic promoting factors.

The purpose of legislation is to ensure compliance with advices/messages to speed up behavioral changes. However, it alone is not the solution. Nevertheless, legislation has its place in dengue control for the recalcitrant or those who pay lip service only, especially after repeated reminders and efforts to educate them.

Impact of Legislation Framework on behavioral modification Intervention for dengue prevention and Control

Table 2
Socio-demographic Characteristics

Economic Status	Frequency	Percent
Low (<500 Rs/day)	56	14.9
Medium (<1000 Rs/day)	105	27.9
High (>1000 Rs/day)	215	57.2
Total	376	100
Less<35	185	49.2
Less<50	163	43.4
>50	28	7.4
Total	376	100
Private business Owners	295	78.5
Government, Semi Government Employee	81	21.5
Total	376	100
Illiterate	72	19.1
Primary	184	48.9
Metric	61	16.2
Graduate	25	6.6
Post graduate	34	9
Total	**376**	**100**

Table 3
Vector Control Measure on basis of Age Groups

Vector Control Measure	Age Groups (Frequencies)		
	Less<35	Less<50	>50
Fumigation	25	32	7
Liquid Spray	97	72	14
Breeding Container Elimination	56	49	5
Others	7	8	2

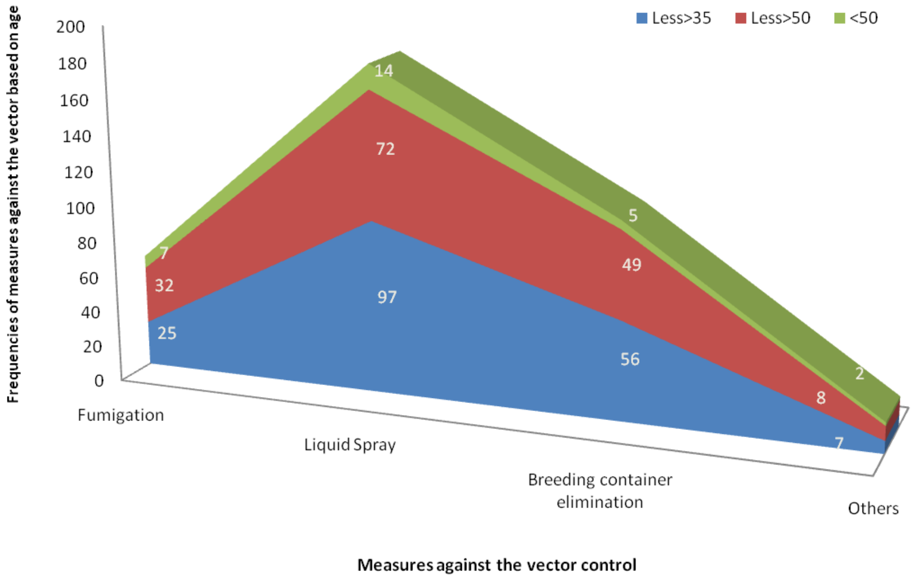

Fig8: Vector Control Measure on basis of Age Groups

Table 4
Vector Control Measure on basis of Educational Status

Vector Control Measure	Educational Status (Frequencies)				
	Illiterate	Primary	Matric	Graduate	Post Graduate
Fumigation	13	44	19	2	3
Liquid Spray	51	112	34	6	5
Breeding Container Elimination	7	21	5	17	26
Others	1	7	3	1	0

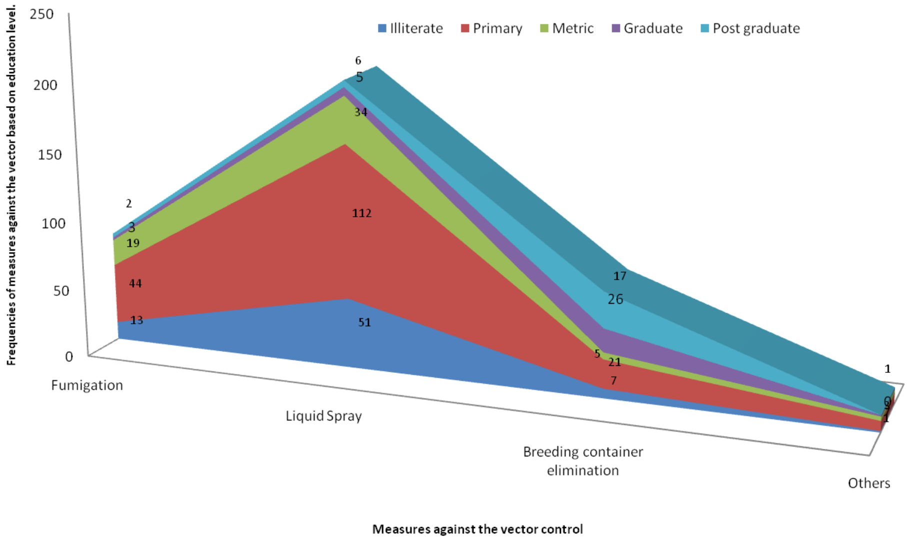

Fig. 9: Vector Control Measure on basis of Educational Status

In total 376 subjects measures against the vector control follow the trend Breeding container elimination, 49.5% > Liquid Spray46 %,> Fumigation, 2.9%, > others, 1.6 in other activities includes the Neem tree leave and seed boiled sprayed. Breeding container elimination showed an eco-sustainable aspect and environmental friendly. Monitoring for potential breeding site found following trends Weekly surveillance 37.8%> Twice Weekly surveillance 32.4%> Monthly surveillance 8.8%>After 2 Weeks surveillance 7.2 >Daily surveillance, 9.3%>Others 4.5 in other category as their premises opened at certain period of time related to businesses and services. Action applied to Control Mosquitoes found following trends Remove the Container 75.3%>Cover the Container16%>Physically Manipulate Container 8.8%.Container removal and Cover the Container showed behavior modification by legislation.

A comprehensive approach to achieving and sustaining behavioral impact recognizes that individual behavior change does not result from improved knowledge alone and cannot be promoted in isolation from the broader social context in which it occurs. Behavioral modification framework ensures team takes one of the most critical step in planning social mobilization and communication: to identify the few key factors that most influence the target behavior for particular audience (Parks &Lloyd, 2004).

5. CONCLUSIONS

Lowest Disease Burden due to Enforcement Implementation of Dengue Regulations showed significant role in control of the epidemic. Following major objectives have been concluded by results of the study:

- Years 2014, 2015 and 2017 have lowest disease burden trends with lowest number of confirmed dengue patients and 2016, 2013 and 2012 has highest disease burden trends as low implementation of regulations done
- Years 2017, 2016 and 2015 have highest vector burden and 2012, 2013, 2014 lowest vectors burden and showed highest number of premises seal trends. Warning models and implementations legal actions can allow health systems and vector control programs to respond more cost-effectively and efficiently.

6. RECOMMENDATIONS

To make Dengue Regulations fully effective following recommendations are proposed:

- The legislation is mostly applied to outdoor commercial sites and businesses for outdoor larvae positive sites, it would be more effective if the legislation is applied to indoor houses as well to control the disease completely and change people behavior.
- The Dengue Regulations have to be made known to public by advertisements, inclusion in public awareness seminars and workshops. Dengue Regulations can be sent to households in high risk areas to stop propagation of virus.
- There should be no private or public pressure in implementation of the regulations so that sustainable environmental management could be achieved for epidemic control.
- Complete implementation of all sections of the Dengue Regulations is required for sustainable environmental management and to prevent & control the disease.

REFERENCES

1. Caicedo-Torres, W., Montes-Grajales, D., Miranda-Castro, W., Fennix-Agudelo, M. and Agudelo-Herrera, N. (2017). Kernel-Based Machine Learning Models for the Prediction of Dengue and Chikungunya Morbidity in Colombia. In *Colombian Conference on Computing*, 472-484.
2. Chanda, E., Ameneshewa, B., Bagayoko, M., Govere, J.M. and Macdonald, M.B. (2017). Harnessing Integrated Vector Management for Enhanced Disease Prevention. *Trends in parasitology*, 33(1), 30-41.
3. Dengue Regulations (2016). Government of Punjab
4. Haque, M.A., Yamamoto, S.S., Malik, A.A. and Sauerborn, R. (2012). Households' perception of climate change and human health risks: a community perspective. *Environmental Health*, 11(1), 1.
5. Manzoor, R., Toru, S.K. and Ahmed, V. (2016). Legislative Gaps in Implementation of Health-related Millennium Development Goals: a case study from Pakistan. *Journal of Pak Med Assoc*. 66(6), 726-734.
6. Parks, W. and Lloyd, L. (2004). *Planning social mobilization and communication for dengue fever prevention and control: a step-by-step guide*. Book. WHO publication
7. Seufi, M.A. and Galal, H.F. (2010). Role of Culex and Anopheles mosquito species as potential vectors of rift valley fever virus in Sudan outbreak, 2007. *BMC Infectious Diseases*, 10(65), 1-8.

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ON BAYESIAN ANALYSIS OF MIXTURE OF FRECHET DISTRIBUTION UNDER TYPE-I CENSORING SCHEME

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ABSTRACT

In this paper, two component mixture model of Frechet distribution with known shape has been studied using Bayesian Analysis under type-I censoring scheme. Posterior distributions have been derived by using non-informative priors Asymmetric loss function has been used to obtain Bayes estimates. Loss functions are compared by using Monte Carlo simulation as well as real life example.

1. INTRODUCTION

Maurice Frechet (1878-1973), a French mathematician, introduced a distribution which was named after him as Frechet distribution. The distribution has also been known to others as Type II extreme value distribution. Its other applications include the estimation and forecasting of various phenomena associated with weather for instance the determination of wind velocity, flood, and probability of any famine or rainfall. A number of methods have been devised by Gumbel (1965) to estimate the parameters of Frechet distribution such as method of moments and methods of reciprocal moments, method of maximum likelihood, and some quick methods also. Feroze and Aslam (2012) has studied two component mixture model of Gumbel type-II distribution under informative prior using Bayesian analysis using type-I censoring scheme. Noor (2013) has studied the mixture model of inverse Weibull distribution using informative and non-informative priors under different loss functions. Feroze and Aslam (2014) has studied two-component mixture of Weibull distribution using doubly censored lifetime data. Sindhu et al. (2015) has studied mixture distribution of Kumaraswamy distribution using informative and non-informative prior. Different loss functions had been used to derive Bayes estimators and their cross ponding risks. Sindhu et al. (2016) has studied mixture density of Gompertz distribution using Bayesian analysis under different informative and non-informative priors using different loss functions.

2. POPULATION AND MODEL

Distribution function with characterization of convex combination of other specific probability distribution function. A mixture may be contain of a finite number of base elements, where usually a relatively small number of individual distributions are combined together, or an infinite number of base elements. Two component mixture model for Frechet distribution having two parameters β_1 and β_2 with mixing weight is

$$F(x) = \omega F_1(x) + (1-\omega) F_2(x) \quad 0 < \omega < 1, \tag{1}$$

where $F(x_i) = e^{-\left(\frac{\beta}{x}\right)^{\alpha}}$. Now eq. (1) can be written as

$$F(x) = \omega e^{-\left(\frac{\beta}{x}\right)^{\alpha}} + (1-\omega) e^{-\left(\frac{\beta}{x}\right)^{\alpha}} \tag{2}$$

Now, the mixture density will be

$$f(x) = \omega f_1(x) + (1-\omega) f_2(x) \tag{3}$$

The following model represents mixture Frechet distribution density with shape parameter $\alpha = 1$ is

$$f_i(x) = \beta_i e^{-\left(\frac{\beta_i}{x}\right)}, i = 1,2, \quad 0 < x < \infty \tag{4}$$

Now, Assume that a random sample of size n is selected for reliability of units i.e. $(x_1, x_2, ..., x_n)$.Let we assume r units in sample $[0, t_0]$ in which termination time is fixed which is t_0 and $n-r$ the remaining samples are survive for the rest when the termination time is over. Some situation is taken for two sub population model i.e. from n_i units have lifetime in the interval and $n_i - r_i$ have survived time in which $r = r_1 + r_2$ are uncensored units. Suppose x_{ij} be the failure time of j_{th} unit which is associated with i_{th} sub population i.e. $i = 1,2$, $j = 1,2,...,r_i$, $0 < x_{1j}, x_{2j} \le t_0$. The likelihood for above stated situation is

$$L(\beta_1, \beta_2, \omega) = \prod_{j=1}^{r_1} \omega\beta_1 \exp\left(-\frac{\beta_1}{x_{1j}}\right) \prod_{j=1}^{r_2} (1-\omega)\beta_2 \exp\left(-\frac{\beta_2}{x_{2j}}\right) \left[F(T)\right]^{n-r} \tag{5}$$

where $F(T) = 1 - F(T)$ and known as survival function and x=$[x_{1i}, x_{2i}]$

$$L(\beta_1, \beta_2, \omega) = \prod_{j=1}^{r_1} \left\{\omega\beta \exp\left(-\frac{\beta}{x_j}\right)\right\} \prod_{j=1}^{r_2} \left\{(1-\omega)\beta \exp\left(-\frac{\beta}{x_j}\right)\right\} \left[1 - \left\{\omega e^{-\frac{\beta_1}{t_0}} + (1-\omega) e^{-\frac{\beta_2}{t_0}}\right\}\right]^{n-r} \tag{6}$$

By simplifying above equation, we get

$$L(\beta_1, \beta_2, \omega) = \sum_{m=0}^{n-r} \sum_{l=0}^{m} \binom{n-r}{m} \binom{m}{l} (-1)^l \omega^{r_1+m-l} (1-\omega)^{r_2-m}$$
$$\exp\left\{-\beta_1 \left(\frac{1}{\sum_{i=1}^{r_1} x_i} + \frac{m-l}{t_0}\right)\right\} \exp\left\{-\beta_2 \left(\frac{1}{\sum_{i=1}^{r_2} x_i} + \frac{m}{t_0}\right)\right\}, \tag{7}$$

Prior and Posterior Distribution

The uniform prior is defined as

$$p(\beta_1,\beta_2,\omega)\propto 1,\quad \beta_1,\beta_2>0,0<\omega<1 \tag{8}$$

Then, the posterior distribution is

$$p(\beta_1,\beta_2,\omega)\propto \sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\\ \beta_1^{r_1}e^{-\beta_1 M_1}\beta_2^{r_2}e^{-\beta_2 M_2},\beta_1,\beta_2>0,0<\omega<1. \tag{9}$$

where $M_1=\sum_{j=1}^{r_1}\left(\frac{1}{x_{1i}}\right)+\frac{m-l}{t_0}$ and $M_2=\sum_{j=1}^{r_2}\left(\frac{1}{x_{2i}}\right)+\frac{m}{t_0}$.

The Jeffreys prior is defined as

$$p(\beta_1)\propto\frac{1}{\beta_1^2},\ p(\beta_2)\propto\frac{1}{\beta_2^2},\ p(\omega)\propto 1,\quad \beta_1,\beta_2>0,0<\omega<1 \tag{10}$$

Then, the posterior distribution is

$$p(\beta_1,\beta_2,\omega)\propto \sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\\ \beta_1^{r_1-2}e^{-\beta_1 O_1}\beta_2^{r_2-2}e^{-\beta_2 O_2},\beta_1,\beta_2>0,0<\omega<1 \tag{11}$$

where $O_1=\sum_{i=1}^{r_1}\left(\frac{1}{x_{1i}}\right)+\frac{m-l}{t_0}$ and $O_2=\sum_{i=1}^{r_2}\left(\frac{1}{x_{1i}}\right)+\frac{m}{t_0}$.

Bayes Estimators and Posterior Risks

Square error loss functions (SELF), precautionary loss function (PLF), simple precautionary loss function (SPLF) and weighted loss functions has been utilized for deriving Bayes estimators and posterior risks. They are presented as follow:

Bayes estimator and risks of β_1,β_2 and ω using uniform prior are

$$\beta_{1,SELF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^j B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1+2)}{M_1^{r_1+1}}\frac{\Gamma(r_2+1)}{M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^j B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1+1)}{M_1^{r_1+1}}\frac{\Gamma(r_2+1)}{M_2^{r_2+1}}} \tag{13}$$

$$V(\beta_{1,SELF})=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^j B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1+3)}{M_1^{r_1+3}}\frac{\Gamma(r_2+1)}{M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^j B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1+1)}{M_1^{r_1+1}}\frac{\Gamma(r_2+1)}{M_2^{r_2+1}}}-(\beta_{1,SELF})^2 \tag{14}$$

$$\beta_{2,SELF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^j B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1+1)}{M_1^{r_1+1}}\frac{\Gamma(r_2+2)}{M_2^{r_2+2}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^j B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1+1)}{M_1^{r_1+1}}\frac{\Gamma(r_2+1)}{M_2^{r_2+1}}} \tag{15}$$

$$V\left(\beta_{2,SELF}\right)=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{j}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+3\right)}{M_2^{r_2+3}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{j}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}}-\left(\beta_{2,SELF}\right)^2 \tag{16}$$

$$\omega_{SELF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+2,r_2-m+2\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}} \tag{17}$$

$$V\left(\omega_{SELF}\right)=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+3,r_2-m+3\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}}-\left(\omega_{SELF}\right)^2 \tag{18}$$

Bayes estimator and risks for β_1,β_2 and ω under PLF using uniform prior are

$$\beta_{1,PLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+3\right)}{M_1^{r_1+3}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}}} \tag{19}$$

$$V\left(\beta_{1,PLF}\right)=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+3\right)}{M_1^{r_1+3}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}}}-\beta_{1,PLF}\right) \tag{20}$$

$$\beta_{2,PLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+3\right)}{M_2^{r_2+3}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)}{M_1^{r_1+1}}\frac{\Gamma\left(r_2+1\right)}{M_2^{r_2+1}}}} \tag{21}$$

$$V\left(\beta_{2,PLF}\right)=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)\Gamma\left(r_2+3\right)}{M_1^{r_1+1}\,M_2^{r_2+3}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)\Gamma\left(r_2+1\right)}{M_1^{r_1+1}\,M_2^{r_2+1}}}}-\beta_{2,PLF}\right) \tag{22}$$

$$\omega_{PLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+3,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)\Gamma\left(r_2+1\right)}{M_1^{r_1+1}\,M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)\Gamma\left(r_2+1\right)}{M_1^{r_1+1}\,M_2^{r_2+1}}}} \tag{23}$$

$$V\left(\omega_{PLF}\right)=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+3,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)\Gamma\left(r_2+1\right)}{M_1^{r_1+1}\,M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+1\right)\Gamma\left(r_2+1\right)}{M_1^{r_1+1}\,M_2^{r_2+1}}}}-\omega_{PLF}\right) \tag{24}$$

Bayes estimator and risks for β_1,β_2 and ω under SPLF using uniform prior are

$$\beta_{1,SPLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1+2\right)\Gamma\left(r_2+1\right)}{M_1^{r_1+2}\,M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma\left(r_1\right)\Gamma\left(r_2+1\right)}{M_1^{r_1}\,M_2^{r_2+1}}}} \tag{25}$$

$$V\left(\beta_{1,SPLF}\right)=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1)\,\Gamma(r_2+1)}{M_1^{r_1}\;M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+1)}{M_1^{r_1+1}\;M_2^{r_2+1}}}}*\beta_{1,PLF}-1\right) \tag{26}$$

$$\beta_{2,SPLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+2)}{M_1^{r_1+1}\;M_2^{r_2+2}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2)}{M_1^{r_1+1}\;M_2^{r_2}}}} \tag{27}$$

$$V\left(\beta_{2,SPLF}\right)=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2)}{M_1^{r_1+1}\;M_2^{r_2}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+1)}{M_1^{r_1+1}\;M_2^{r_2+1}}}}*\beta_{2,PLF}-1\right) \tag{28}$$

$$\omega_{SPLF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}\,B\left(r_1+m-l,r_2-m+1\right)\frac{\Gamma(r_1+1)}{M_1^{r_1+1}}\frac{\Gamma(r_2+1)}{M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}\,B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)}{M_1^{r_1+1}}\frac{\Gamma(r_2+1)}{M_2^{r_2+1}}} \tag{29}$$

$$V\left(\omega_{SPLF}\right)=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+3,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+1)}{M_1^{r_1+1}\;M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+1)}{M_1^{r_1+1}\;M_2^{r_2+1}}}}*\omega_{PLF}-1\right) \tag{30}$$

Bayes estimator and risks for β_1,β_2 and ω under WLF using uniform prior are

$$\beta_{1,WLF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}\,B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)}{M_1^{r_1+1}}\frac{\Gamma(r_2+1)}{M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}\,B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1)}{M_1^{r_1}}\frac{\Gamma(r_2+1)}{M_2^{r_2+1}}} \tag{31}$$

$$V\left(\beta_{1,WLF}\right)=\beta_{1,SELF}-\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1)\,\Gamma(r_2+1)}{M_1^{r_1}\;M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+1)}{M_1^{r_1+1}\;M_2^{r_2+1}}} \tag{32}$$

$$\beta_{2,WLF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+1)}{M_1^{r_1+1}\;M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2)}{M_1^{r_1+1}\;M_2^{r_2}}} \tag{33}$$

$$V\left(\beta_{2,WLF}\right)=\beta_{2,SELF}-\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2)}{M_1^{r_1+1}\;M_2^{r_2}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+1)}{M_1^{r_1+1}\;M_2^{r_2+1}}} \tag{34}$$

$$\omega_{WLF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l+1,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+1)}{M_1^{r_1+1}\;M_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_1+m-l,r_2-m+1\right)\frac{\Gamma(r_1+1)\,\Gamma(r_2+1)}{M_1^{r_1+1}\;M_2^{r_2+1}}} \tag{35}$$

$$V\left(\omega_{WLF}\right)=\omega_{SELF}-\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}+1\right)\Gamma\left(r_{2}+1\right)}{M_{1}^{r_{1}+1}\;M_{2}^{r_{2}+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}+1\right)\Gamma\left(r_{2}+1\right)}{M_{1}^{r_{1}+1}\;M_{2}^{r_{2}+1}}} \tag{36}$$

Bayes estimator and risks for β_1,β_2 and ω under SELF using Jeffreys prior are

$$\beta_{1,SELF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}}\;O_{2}^{r_{2}-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}} \tag{37}$$

$$V\left(\beta_{1,SELF}\right)=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}+1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}+1}\;O_{2}^{r_{2}-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}}-\left(\beta_{1,SELF}\right)^{2} \tag{38}$$

$$\beta_{2,SELF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}} \tag{39}$$

$$V\left(\beta_{2,SELF}\right)=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}+1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}}-\left(\beta_{2,SELF}\right)^{2} \tag{40}$$

$$\omega_{SELF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+2,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}} \tag{41}$$

$$V\left(\omega_{SELF}\right)=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+3,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}}-\left(\omega_{SELF}\right)^{2} \tag{42}$$

Bayes estimator and risks for β_1,β_2 and ω under PLF using Jeffrey’s prior are

$$\beta_{1,PLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}+1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}+1}\;O_{2}^{r_{2}-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}}} \tag{43}$$

$$V\left(\beta_{1,PLF}\right)=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}+1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}+1}\;O_{2}^{r_{2}-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B\left(r_{1}+m-l+1,r_{2}-m+1\right)\frac{\Gamma\left(r_{1}-1\right)\Gamma\left(r_{2}-1\right)}{O_{1}^{r_{1}-1}\;O_{2}^{r_{2}-1}}}}-\beta_{1,PLF}\right) \tag{44}$$

$$\beta_{2,PLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2+1)}{O_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}} \tag{45}$$

$$V(\beta_{2,PLF})=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2+1)}{O_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}}-\beta_{2,SELF}\right) \tag{46}$$

$$\omega_{PLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+3,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}} \tag{47}$$

$$V(\omega_{PLF})=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+3,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}}-\beta_{1,SELF}\right) \tag{48}$$

Bayes estimator and risks for β_1,β_2 and ω under SPLF using Jeffrey's prior are

$$\beta_{1,SPLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1+1)}{O_1^{r_1+1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-2)}{O_2^{r_2-2}}}} \tag{49}$$

$$V(\beta_{1,SPLF})=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-2)}{O_1^{r_1-2}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}*\beta_{1,SELF}}-1\right) \tag{50}$$

$$\beta_{2,SPLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2+1)}{O_2^{r_2+1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-2)}{O_2^{r_2-2}}}} \tag{51}$$

$$V(\beta_{2,SPLF})=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-2)}{O_2^{r_2-2}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}*\beta_{2,SELF}}-1\right) \tag{52}$$

$$\omega_{SPLF}=\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+2,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}} \tag{53}$$

$$V(\omega_{SPLF})=2\left(\sqrt{\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^{l}B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}*\omega_{SELF}}-1\right) \tag{54}$$

Bayes estimator and risks for β_1,β_2 and ω under WLF using Jeffrey's prior are

$$\beta_{1,WLF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-2)}{O_1^{r_1-2}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}} \quad (55)$$

$$V(\beta_{1,WLF})=\beta_{1,SELF}-\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-2)}{O_1^{r_1-2}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}} \quad (56)$$

$$\beta_{2,WLF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-2)}{O_2^{r_2-2}}} \quad (57)$$

$$V(\beta_{2,WLF})=\beta_{2,SELF}-\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-2)}{O_2^{r_2-2}}} \quad (58)$$

$$\omega_{WLF}=\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}} \quad (59)$$

$$V(\omega_{WLF})=\omega_{SELF}-\frac{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l+1,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}}{\sum_{m=0}^{n-r}\sum_{l=0}^{m}\binom{n-r}{m}\binom{m}{l}(-1)^l B(r_1+m-l,r_2-m+1)\frac{\Gamma(r_1-1)}{O_1^{r_1-1}}\frac{\Gamma(r_2-1)}{O_2^{r_2-1}}} \quad (60)$$

3. SIMULATION STUDY

For comparing the performance of posterior distribution and loss functions, simulation study has been conducted. The $(\beta_1,\beta_2,\omega)\in\{(1.5,1.5),(0.30,0.20)\}$ for sample size of $n\in\{70,100,150,200\}$. To generate mixture data, probabilities mixing has been utilized. For each observation a random number u from $U(0,1)$. If $u<p$, the observation has been randomly taken from first subpopulation and if $u>p$, then the observation have been taken from sub population. The censoring rate in the respective sample has been 20%. The process has been repeated 10,000 times and average of results has been utilized to see the performance of Bayes estimators. The data has been obtained from Murthy and Jiang (1995). The data is of thyamic lymphoma (22 data points) and reticulum cell sarcoma (38 points). The sample characteristics of data is as follow $\sum_{i=1}^{r_1}x_{1j}=5349,\sum_{i=1}^{r_2}22433, r_1=17, r_2=37, r=54, n=60$ and $t=0.005$. The mixing weight of data is 0.367.

Table 1
Bayes Estimator and Risks when Using Uniform Prior $\beta_1, \beta_2 = 1.5$

n	$\beta_2 = 1.5, w = 0.30$				$\beta_1 = 1.5, w = 0.20$			
	SELF	PLF	SPLF	WLF	SELF	PLF	SPLF	WLF
70	1.5581 (0.1217)	1.5865 (0.0751)	1.5072 (0.0509)	1.4734 (0.0759)	1.5497 (0.1848)	1.6986 (0.1162)	1.5743 (0.0759)	1.5235 (0.1181)
100	1.5090 (0.0825)	1.5267 (0.0517)	1.4792 (0.0357)	1.4489 (0.0522)	1.5148 (0.1236)	1.6048 (0.0794)	1.5233 (0.0533)	1.4906 (0.0807)
150	1.4784 (0.0517)	1.4903 (0.0344)	1.4565 (0.0238)	1.4337 (0.0343)	1.4841 (0.0792)	1.5426 (0.0522)	1.4891 (0.0357)	1.4596 (0.0526)
200	1.4577 (0.0381)	1.4685 (0.0256)	1.4393 (0.0179)	1.4311 (0.0257)	1.4646 (0.0579)	1.5044 (0.0392)	1.4681 (0.0257)	1.4485 (0.0392)
n	$\beta_2 = 1.5, w = 0.30$				$\beta_2 = 1.5, w = 0.20$			
70	1.4226 (0.0443)	1.4410 (0.0305)	1.4109 (0.0219)	1.3968 (0.0307)	1.4028 (0.0377)	1.4322 (0.0266)	1.4072 (0.0191)	1.3766 (0.0264)
100	1.4020 (0.0302)	1.4219 (0.0213)	1.4023 (0.0153)	1.3886 (0.0213)	1.3993 (0.0261)	1.4125 (0.0186)	1.3958 (0.0134)	1.3759 (0.0185)
150	1.4039 (0.0201)	1.4056 (0.0141)	1.3924 (0.0102)	1.3848 (0.0142)	1.3845 (0.0171)	1.3993 (0.0124)	1.3845 (0.0090)	1.3749 (0.0123)
200	1.3872 (0.0148)	1.3970 (0.0106)	1.3867 (0.0077)	1.3810 (0.0106)	1.3823 (0.0116)	1.3910 (0.0101)	1.3808 (0.0068)	1.3715 (0.0092)
n	$w = 0.30$				$w = 0.20$			
70	0.3838 (0.0241)	0.3912 (.0087)	0.3888 (0.0034)	0.3781 (0.0091)	0.2013 (0.0137)	0.2973 (0.0099)	0.2921 (0.0029)	0.1998 (0.0092)
100	0.3827 (0.0170)	0.3892 (0.0062)	0.3814 (0.0024)	0.3805 (0.0064)	0.2017 (0.0096)	0.2942 (0.0070)	0.2903 (0.0021)	0.2008 (0.0079)
150	0.3820 (0.0113)	0.3865 (0.0042)	0.3833 (0.0016)	0.3803 (0.0042)	0.2027 (0.0063)	0.2911 (0.0047)	0.2887 (0.0014)	0.2020 (0.0056)
200	0.3828 (0.0052)	0.3860 (0.0031)	0.3837 (0.0012)	0.3808 (0.0032)	0.2028 (0.0041)	0.2896 (0.0037)	0.2620 (0.0010)	0.2025 (0.0036)

Table 2
Bayes Estimator and Risks when Using Jeffrey's Prior $\beta_1, \beta_2 = 1.5$

n	$\beta_1 = 1.5, w = 0.30$				$\beta_1 = 1.5, w = 0.20$			
	SELF	**PLF**	**SPLF**	**WLF**	**SELF**	**PLF**	**SPLF**	**WLF**
70	1.3903 (0.1102)	1.4315 (0.0747)	1.6568 (0.0567)	1.3186 (0.0758)	1.4099 (0.1807)	1.4516 (0.1149)	1.6852 (0.0897)	1.2816 (0.1176)
100	1.3996 (0.0757)	1.4243 (0.0518)	1.6469 (0.0384)	1.3468 (0.0523)	1.4075 (0.1196)	1.4417 (0.0791)	1.6694 (0.0598)	1.3216 (0.0803)
150	1.4054 (0.0496)	1.4199 (0.0343)	1.6492 (0.0251)	1.3667 (0.0344)	1.4087 (0.0768)	1.4329 (0.0521)	1.6608 (0.0384)	1.3550 (0.0526)
200	1.4025 (0.0366)	1.4171 (0.0256)	1.6491 (0.0186)	1.3673 (0.0257)	1.4149 (0.0567)	1.4310 (0.0386)	1.6613 (0.0281)	1.3700 (0.0391)
n	$\beta_2 = 1.5, w = 0.30$				$\beta_2 = 1.5, w = 0.20$			
70	1.4226 (0.0443)	1.3814 (0.0306)	1.3820 (0.0229)	1.3368 (0.0307)	1.4028 (0.0377)	1.3784 (0.0266)	1.3667 (0.0199)	1.3367 (0.0267)
100	1.4020 (0.0302)	1.3787 (0.0213)	1.3775 (0.0158)	1.3468 (0.0214)	1.3993 (0.0261)	1.3783 (0.0187)	1.3778 (0.0138)	1.3471 (0.0186)
150	1.4039 (0.0201)	1.3755 (0.0141)	1.3752 (0.0104)	1.3577 (0.0142)	1.3845 (0.0171)	1.3717 (0.0123)	1.3710 (0.0091)	1.3548 (0.0123)
200	1.3872 (0.0148)	1.3763 (0.0106)	1.3738 (0.0078)	1.3575 (0.0106)	1.3823 (0.0116)	1.3704 (0.0090)	1.3720 (0.0068)	1.3594 (0.0089)
n	$w = 0.30$				$w = 0.20$			
70	0.3864 (0.0205)	0.3912 (.0087)	0.3888 (0.0034)	0.3781 (0.0091)	0.2115 (0.0125)	0.2973 (0.0090)	0.2921 (0.0020)	0.1998 (0.0082)
100	0.3853 (0.0135)	0.3892 (0.0062)	0.3814 (0.0024)	0.3805 (0.0064)	0.2017 (0.0090)	0.2942 (0.0060)	0.2903 (0.0015)	0.2008 (0.0069)
150	0.3840 (0.0096)	0.3865 (0.0042)	0.3833 (0.0016)	0.3803 (0.0042)	0.2027 (0.0056)	0.2911 (0.0040)	0.2887 (0.0010)	0.2020 (0.0046)
200	0.3846 (0.0068)	0.3860 (0.0031)	0.3837 (0.0012)	0.3808 (0.0032)	0.2028 (0.0040)	0.2896 (0.0025)	0.2620 (0.0009)	0.2025 (0.0035)

Table 3
BEs and PRs using real life data

LF	SELF	PLF	SPLF	WLF
	β_1			
Uniform Prior	0.0044 (0.0561)	0.0046 (0.0002)	0.0045 1.126×10^{-6}	0.0043 (0.0002)
Jeffrey's Prior	0.0039 (0.0508)	0.0042 (0.0002)	0.0041 1.026×10^{-6}	0.0043 (0.0002)
Exponential Prior	0.0044 (0.0561)	0.0049 (0.0002)	0.0041 1.116×10^{-6}	0.0043 (0.0002)
Inverse Levy Prior	0.0042 (0.0594)	0.0042 (0.0002)	0.0042 1.126×10^{-6}	0.0040 (0.0002)
	β_2			
Uniform Prior	0.0016 (0.0270)	0.0017 (0.0000)	0.0017 7.67×10^{-8}	0.0016 (0.0000)
Jeffrey's Prior	0.0016 (0.0245)	0.0016 (0.0000)	0.0016 7.37×10^{-8}	0.0015 (0.0000)
Exponential Prior	0.0017 (0.0258)	0.0017 (0.0000)	0.0017 7.98×10^{-8}	0.0016 (0.0000)
Inverse Levy Prior	0.0016 (0.0278)	0.0016 (0.0000)	0.0017 8.126×10^{-8}	0.0017 (0.0000)
Uniform Prior	0.2343 (0.0048)	0.2348 (0.0010)	0.2337 (0.0002)	0.2332 (0.0011)
Jeffrey's Prior	0.2337 (0.0035)	0.2343 (0.0008)	0.2343 (0.0002)	0.2325 (0.0012)
Exponential Prior	0.2343 (0.0048)	0.2348 (0.0015)	0.2343 (0.0004)	0.2331 (0.0015)
Inverse Levy Prior	0.2341 (0.0049)	0.2334 (0.0018)	0.2345 (0.0010)	0.2329 (0.0018)

4. CONCLUSION

In this article, two component mixture model for unknown scale parameter of Frechet distribution with known shape has been studied using Bayesian analysis based on type-I censoring scheme. Bayesian analysis has been conducted by using informative and non-informative priors. Simulation study has been conducted to compare the performance of Bayes estimators and their corresponding risks. From the simulation study, we have concluded that posterior risks has been decreased by increasing sample size. SPLF is performing better as its posterior risks is minimum as compared to all others loss functions. Jeffrey's prior has minimum risks as compared to all other loss functions. Results of real life examples are same as the results of simulation study.

REFERENCES

1. Gumbel, E.J. (1965). A quick estimation of the parameters in Fréchet's distribution. *Revue de l'Institut International de Statistique*, 349-363.
2. Ali, S. and Aslam, M. (2013). Choice of Suitable Informative Prior for the Scale Parameter of Mixture of Laplace Distribution using Type-I Censoring Scheme under Different Loss Function. *Electronic Journal of Applied Statistical Analysis*, 6(1), 32-56.
3. Jiang, R. and Murthy, D. (1995). Modeling failure-data by mixture of 2 weibull distributions: a graphical approach. *IEEE Transactions on Reliability*, 44(3), 477-488.
4. Kazmi, S., Aslam, M. and Ali, S. (2012). On the Bayesian estimation for two component mixture of Maxwell distribution, assuming type I censored data. *International Journal of Applied Science & Technology,* 2(1), 197-218.
5. Feroze, N. and Aslam, M. (2013). Bayesian estimation of Two-Component Mixture of Gumbel Type II distribution under Informative Priors. *International Journal of Advanced Science and Technology*, 53, 11-30.
6. Feroze, N. and Aslam, M. (2013). On Bayesian estimation and predictions for two-component mixture of the gompertz distribution. *Journal of Modern Applied Statistical Methods,* 12(2), 269-292.
7. Sindhu, T.N., Aslam, M. and Hussain, Z. (2016). A simulation study of parameters for the censored shifted Gompertz mixture distribution: A Bayesian approach. *Journal of Statistics and Management Systems,* 19(3), 423-450.
8. Sindhu, T.N., Feroze, N., Aslam, M. and Shafiq, A. (2016). Bayesian Inference of Mixture of two Rayleigh Distributions: A New Look. *Punjab University Journal of Mathematics,* 48(2), 49-64.
9. Feroze, N. and Aslam, M. (2014). Bayesian analysis of doubly censored lifetime data using two component mixture of Weibull distribution. *Journal of the National Science Foundation of Sri Lanka,* 42(4), 325-334.
10. Frechet, M. (1927). Sur la loi de probabilite de lecart maximum. In *Annales de la societe Polonaise de Mathematique*, 6, 93-116. Bibliotheque des Sciences Humaines, Editions Gallimard.
11. Sindhu, T.N., Feroze, N. and Aslam, M. (2014). Bayesian estimation of the parameters of two-component mixture of Rayleigh distribution under Doubly Censoring. *Journal of Modern Applied Statistical Methods*, 13(2), 259-286.
12. Noor, F. and Aslam, M. (2013). Bayesian inference of the inverse weibull mixture distribution using Type-I censoring. *Journal of Applied Statistics,* 40(5), 1076-1089.
13. Sindhu, T.N., Feroze, N. and Aslam, M. (2015). Bayesian Analysis of Kumaraswamy Mixture Distribution under Different Loss Functions. *Journal of Statistics,* 22(1), 121-138.

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IMPACT OF MOBILE SERVICE ON STUDY AREA

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1. INTRODUCTION

Invention of the fixed telephone in the late 19th century in the United States changed the way that people interacted and communicate. This has been paralleled in early 21st century by the advent of mobile phone. The mobile phone was originally created for adults for business use (1).

This is extremely similar to the fixed telephone in the early 20th century, where telephone engineers explained that the telephone was made for the business world and not for social conversation (2). The growth of mobile phone technology is demonstrated by the fact that in 2002, the number of mobile phone users worldwide surpassed those of fixed phone users (03).

The mobile phone is a status symbol for young people. The features of the phone, the appearance and personalized accessories all attest to the phone's status (4).

The mobile phone had been in existence for about two decades before young people really adopted this technology. The reduction in the cost of the handsets, smaller in size and the introduction of the prepaid Phone card in 1990's contributed to the surprising rapid adoption rate by young people (5).

Various surveys worldwide have found high rates of mobile phone use amongst young people. In Norway in 1999, 80% of 13 to 20 years old owned a mobile phone, while in the United Kingdom more than 90 % young people have the mobile phone in use (6). Tokyo Japan the adoption rate is 100 % (03). In recent years number of adolescent owing a mobile phone has risen so dramatically that adolescents are now more likely to own and use a mobile phone than their parents and also adolescents upgrade their mobile phones (04).

Indeed, even the ownership of a mobile phone indicates that one is socially connected accessible and demand. It can also be seen as a symbol of independence from one's family. As (05) asserts "the introduction and adoption of the mobile telephone has led to various adjustments in a range of social institution, namely the adolescents the family and educational institution" as Ling (2001).

The paper explores both the positive and negative impact of the device on these educations. One of main reason use of mobile phone is functionality or "micro-coordination" of their social life.

However, along with positive impact there are negative aspects to the people's mobile phone use. This includes hiding behind the technology from emotionally distressing events. Such as ending relationships, ostracism of those without mobile phones and cyber bullying. some sociologist argue that as many people choose to text rather talk about awkward or emotionally difficult situations that this will impact on their capacity to interact with each other (03). Another negative aspect of people's mobile phone use is to bully other cyber bullying.

The main issue for teachers is the disruption to class room learning that can occur due to the disruptive nature of mobile phone calls and texting. The functionality of sms lets students send and receive messages unobtrusively (07).

HISTORY OF MOBILE PHONE

A mobile phone known as a cell phone in North America, is a portable telephone that can make and receive calls over a radio frequency linked while the user is moving with in a telephone service area. The radio frequency link establishes a connection to the switching system of a mobile phone operator, which provides access to the public switched telephone network. A handheld mobile radio telephone service was envisioned in the early stages of radio engineering. 1917, Finnish inventor Eric Tigerstedt field a patent for a pocket size folding telephone with a very thin carbon microphone.

First handheld mobile phone was demonstrated by John F. Mitchell and Martin Cooper of Motorola in 1973. In the first quarter of 2016, the top smartphone developers worldwide were Samsung, Apple and Huawei as of 2016 the largest were Samsung, Nokia and Alcatel.

GENERATION OF 1G, 2G, 3G, 4G

The first generation (1G) system starts 1983 and support for more simultaneous calls but still used analog cellular technology.

The second generation (2G) digital cellular technology was launched in Finland by Radio linja on GSM standard. After Ten years later in 2001, the third generation (3G) was launched in Japan. By 2009 it had become clear that at some point 3G, networks would be overwhelmed by the growth of band width, intensive applications, such as streaming.

The first two commercially available technologies billed as (4G), were the standard, offered in North America. The term (5G) is not officially used in official documents, but they are at this time seen as the under 4G umbrella.

TOP MOBILE/ CELL PHONE NETWORKS IN PAKISTAN

1. **Mobilink**
   Pakistan mobile communication limited (Mobilink) is one of the largest and old cellular network in Pakistan and since launch of its service in 1994, the company officers both postpaid and prepaid services to their customers.

2. **Telenor**
   Telenor is Pakistan's second largest cellular network. It is owned by Telenor ASA. The company provides both services postpaid and prepaid to their customers.

3. **Ufone**
   Ufone is third cellular company in Pakistan, owned by PTCL and Etisalat. Ufone is offering both prepaid and postpaid services to their customers.

4. **Zong**
   The old name for this service provider was Paktel. It was first ever company granted free license to carry out cellular phone service in Pakistan, setup cable and wireless, Paktel launched commercial service in 1990 (8), after completion of its acquisition by China mobile, Paktel was rebranded Zong Pakistan on April, 2008.

5. **Warid**
   Pakistan's fifth largest cellular service provider is Warid. It is owned by Warid Telecom Abu Dhabi Group and Singtel. Warid Telecom has been able to hold more than 12 million registered nationwide users, it offers both prepaid and postpaid services to customers.

A LOOK OF MOBILE PHONE TECHNOLOGY PROVIDES
MAJOR USES OF MOBILE PHONES

1- Voice
2- Short Messages Service (SMS)
3- Internet
4- Data transfer (Blue tooth)
5- Mobile instant Message (MIM)

and so many other services.

2. METHODOLOGY

A questionnaire was used to obtain information about the impact of mobile service on study area, in this connection interviewed from students of both sexes boys and girls. Information collected from 113 boys and also from 113 girls. Data collected on the primary source of information. The question asked from students based on their personal information.

e.g. currently residence of rural and urban wise, users feeling sight effects before and after use of mobile. Time period giving to their study, using packages of mobile service, under use of mobile sets of different manufacturing companies, types of mobile sets i.e. simple or touch screen sets, using networks services, impact of mobile service on study before and after using mobile phones.

The collected forms analysed and interpreterate and compiled the results according to the research article and finally prepared conclusion.

3. RESULTS

Table 1
Gender Wise Distribution of Mobile Users Currently Residence of Urban and Rural Areas Wise

Gender	Urban	Rural	No response	Total	Percent
Male	27 (23.89)	84 (74.34)	02	113	98.23
Female	74 (65.49)	31 (27.43)	08	113	92.92
Total	101	115	10	226	95.58
Total percentage	44.69	50.88	04.42	99.99	---------

Table 2
Gender Wise Distribution of Mobile Users using the Different Networks

Gender	U-Fone	Mobilink	Telenor	Zong	Warid	No reply	Total
Male	36	103	36	54	06	02	237 (2.08%)
Female	39	63	15	26	03	07	153 (1.29%)
Total	75	166	51	80	09	09	390
Total Percentage	19.23	42.56	13.08	20.51	2.31	2.31	100%

Table 3
Gender Wise Distribution of Mobile Users using Mobile Sets of Different Types

Gender	Touch Screen Set	Simple Set	Both Sets	No Response	Total
Male	55	30	25	03	113
Female	84	11	13	05	113
Total	139	41	38	08	226
Total percentage	61.50 %	18.14 %	16.14 %	03.54%	99.99

Table 4
Distribution of Mobile users having the Mobile Sets of Different Companies

Gender	Q- mobile	Nokia	Samsung	Others	No response	Total	Average%
Male	38	38	36	17	03	132	1.14
Female	44	18	47	11	05	125	1.06
Total	82	56	83	28	08	257	1.1
Total percentage	31.91	21.79	32.30	10.89	03.11	100 %	-------

Others: Mobile sets of other companies whose's names are not mentioned in this table i.e. LG-G3, OPP, I-PHONE, HUAWEI, MOTOROLA, EXPERIA-Z, GALAXY, VIGOTEL, INFINIX, APPLE, ETC.

Table 5
Gender Wise Distribution of Mobile Users uses the Net, SMS/ MSG and Call Packages

Gender	NET, CALLS, SMS/ MSG PACKAGES				
	Daily	Weekly	Monthly	No reply	Total
Male	24	38	39	12	113
Female	07	33	54	19	113
Total	31	71	93	31	226
Over all Percentage	13.72	31.42	41.15	13.72	100%

Table 6
Gender Wise Distribution of Mobile Users using Mobile Service in 24-Hours

Gender	One Hour	Two Hour	Three Hour	More than Three Hours	No Response	Total	Average time
Male	14	44	23	32	------	113	4.71
Female	28	38	18	26	03	113	4.58
Total	42	82	41	58	03	226	4.67
Over all percentage	18.58 %	36.28 %	18.14 %	25.66 %	01.33 %	99.99 %	24.67 %

Table 7
Gender Wise Distribution of Mobile Users giving time to their study in 24- hours(Excluded Class Attendance)

Gender	One Hour	Two Hour	Three Hour	More than Three Hours	No Response	Total	Average time
Male	17	27	41	27	01	113	4.67
Female	03	34	27	46	03	113	4.58
Total	20	61	68	73	04	226	4.63
Over all percentage	08.85%	26.99%	30.09%	32.30%	1.77	100%	24.56

Table 8
Gender Wise Distribution of Mobile Users Feeling Sight Effects after Using Mobile Service

Gender	Feeling Sight Effect	Not feeling Sight Effect	No Response	Total
Male	58	53	02	113
Female	60	49	04	113
Total	118	102	06	226
Over all percentage	52.21 %	45.13 %	02.65 %	99.99 %

Table 9
Distribution of Students having the Result of Examination before Using the Mobile Service

	Grade / Division			C.G.P.A			No Reply	Total
Gender	**A**	**B**	**C**	**2.09**	**3.09**	**4.0**		
Male	06	18	01	14	55	-----	19	113
Female	29	04	-------	03	36	----	41	113
Total	35	22	01	17	91	---	60	226
Over all percentage	15.49	09.73	0.44	07.52	40.27	----	26.55	99.99

Table 10
Distribution of Students having the Result of Examination after Using the Mobile Service

	Grade / Division			C.G.P.A			No Reply	Total
Gender	**A**	**B**	**C**	**2.09**	**3.09**	**4.0**		
Male	15	25	02	08	36	----	27	113
Female	06	02	---	04	45	---	56	113
Total	21	27	02	12	81	------	83	226
Percentage	09.29	11.95	0.88	05.31	35.84	------	36.73	100 %

Table No. 1 is concerned to see the mobile users of both sexes. It is found that mobile users belongs to rural areas are more users than urban areas interesting point is that female belongs urban areas are 65.49% and females of rural areas are only 27.43%, while boys are more users of mobile services which stands 74.34% and boys of urban areas are 23.89% only. On the other hand aggregate result stand that male students using more mobile service which is 98.23% and females 92.92%.

Table No. 2 shows that the mobile network service using students in which Mobilink network is more used by the students of both sexes which stand of 42.56% of total networks and averages networks sims having in use of students is more than two sims per student.

Table No. 3 is concerned to see the mobile sets in use of students, it is observed that 61.50% students having the touch screen mobile sets, simple sets stands 18.14% of total percentage while the students having both sets touch screen as well as simple sets are 16.81. This shows touch screen sets are more in use.

Table No. 4 is concerned to see the mobile sets using students of different companies, it is found that Samsung and Q-mobile are more using by the students which stands 32.30% and 31.91 % respectively, while mobiles of other companies like Galaxy, Apple, Huawei, Motorola etc. stands 10.89% and Nokia stands 21.79%.

Table No. 5 concerned to see the mobile packages uses by the students, in this table it is found that monthly and weekly packages of net, calls and SMS/MSG are more used by the students of different mobile networks, which stands 41.15 % and 31.42% while daily packages stands 13.72 %.

Table No. 6 and 7 is comparison of students giving time to mobile service and their study. It is found that students giving much time to their study which stands 30.09 to 32.30 % of total time within 24 hours which is three to more than three hours per 24 hours (excluded class attendance) while mobile users are giving time to more than three hours is which stands 36.28 % and 25.66% respectively.

Table No. 8, this table is concerned to see the sight effects of mobile users before and after using mobile service. It is found that 52.21 % students told that there is a sight effect of using mobile service, while 45.13 % told that there is no sight effect in using of mobile service, but it depends on user's trend to take care to their eyes during mobile using

Table No. 9 and 10, these two tables are comparison of before and after mobile service users, it is found that before using mobile service students having the good "A" grade and "4 GPA" which stands 15.49 and 40. 27 of total percentage but after using mobile service their percentage of Good grade "A" and CGPA is declined which stands 09.29% and 35.84% of total percentage. This shows that mobile service gives the bad impact on study and as well as effects on eye sight of students or other users.

4. COMMENTS AND CONCLUSION

In the present work we have presented a brief overview on mobile service as well as the use of its in different angles. In this connection a pilot study was conducted as the part of this research was aimed to see the impact of mobile service on study area.

A questionnaires was distributed among students of Shah Abdul Latif University, Khairpur, Sindh, Pakistan of different faculties to get the results. First hand, the questionnaires was divided into two genders male and female. By residence of Urban and Rural mobile users of both sexes mobile sets in use of different types and different types and different companies, calls and SMS packages using the mobile service within 24 hours and earning the time to their study. Effects of mobile service on eye sight, impact of mobile service on their study before and after using of mobile service.

It is observed that the male users of mobile service are more than the females. furthermore the in the case of mobile network, the Mobilink service is giving more service than the other networks i.e. Ufone, Telenor, Zong and Warid. The mobile sets using by students shows that touch screen is more in use than the other sets. It is observed that mobile sets of different companies used, by is on the first Samsung, second Q-mobile and third Nokia and mobile packages using by students for communication to others is monthly package.

The student using mobile service in 24-hours found more than two hours and students giving time to their study more than three hours. When checked the results of sight effects of mobile service, it is found that having the sight effects. Finally it is observed that it impact on their study after using mobile service having the more effect on their studies

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REFERENCES

1. Aoki, K. and Downes, E.J. (2003). An analysis of young people's use of and attitudes toward cell phones. *Telematics and Informatics*, 20(4), 349-364.
2. China mobile rebrands Paktel as Zong telecom paper.com
3. Flinchy, P. (1997). Perspectives for a sociology of the telephone. *The French Journal of Communication*, 5(2), 149-160.
4. Geser, H. (2004). Towards a sociological theory of the mobile phone. *E-Merging media: communication and the media economy of the future*, 235-260.
5. Ling, R. (2001). *Adolescent girls and young adult men: Two sub-cultures of the mobile telephone.* Kjelier, Telenor Research & Development (report r 34/2001).
6. Ling, R. (2003). Fashion and vulgarity in the adoption of the mobile telephone among teens in Norway. *Mediating the human body: Technology, communication and fashion*, 93-102. Mahway, NJ: Lawrence Erlbaum.
7. Netsafe. (2005). *The text generation: Mobile phones and New Zealand youth*: *A report of result from the internet Safety Group's survey of teenage mobile phone use.* Netsafe
8. Srivastava, L. (2005). Mobile phones and the evolution of social behaviour. *Behaviour & Information Technology*, 24(2), 111-129.
9. www.Capacity builders co.uk

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STATISTICAL INFERENCE OF SOCIAL FACTORS INFLUENCING STUDENTS ACADEMIC PERFORMANCE VIA BAYESIAN PARADIGM

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ABSTRACT

This research work focused on identifying the most important and influencing factors which affect student's academic performances overall. This study used methodology for inference of multi factors and rank over them. This research considers the Bayesian inference for the factors affecting the Students' academic performance of UOG. Bayes' estimators are computed which reflects the overall worth probabilities for each factors about social, psychological and biological life. The ranking is done and posterior analytical probabilities are computed for each of the six pairs of social life factor, ten possible pairs of psychological and also six pairs for Biological factor affecting the students' academic performance student for future single comparisons of each pair. Results for analysis are computed in C language and programs coding are developed for four and five parameters' inference. The model for ranking factors is also tested for appropriateness and it fits well to the observed sample data.

KEYWORDS

Paired comparison method; Bayesian Statistics, Posterior Means, Ranking, Posterior Predictive Probabilities Bradley-Terry Model, Non informative prior.

1. INTRODUCTION

In the globalization era, education becomes the first hand step for every human being. It plays widely role in individual factor of development and enables a person to make a person well-being of society and the great opportunities for better factor (Battle and Lewis, 2002). Geiser and Santeelices (2007) argue that Future admission and academic performance is the reflection of previous performance and points of admission. The Universities of Admission center (2006) reports that every third institution in Austria have found the best predictor of institution success is the selection based students overall academic achievements. In early of 1950s, Researcher adapted input-output model by Ludwing von Bertalanffy. According to Koontz and Weirich (1998) postulates that "an organized enterprise does not exist in a vacuum but is dependent on its external environment thus the enterprise receives inputs, transforms them and exports the output to the environment".

1.1 Social Factors

Social Scientists studied several prospective of Social Connection (Smith and Christakis 2008). Social relationship refers that exist surrounding an individual, in particular, such as the type and strength of social relationship.

Relationship refers the involvement of all level of formal and informal connection, Positive aspect of relationship, volunteer relation, and religious connection. Positive Connection as emotional support provided by significant others and strained effect is conflict and stress.

1.1.1 Socio-Economic Status

It is the most debating factor influence upon student achievements among professionals and researcher. Argument about this factor is from researcher is that socioeconomic status of a learner effect on their quality of educational performance. Adams (1996) also argued and say low income socioeconomic status of student negatively effect on the student performance because basic needs of student remain unfulfilled than it unable to make student perform better in academics.

Including that all aspects of connection like behavior, educational, and interaction. Research indicates that those Students have Low socio-economic status develop academic skills slowly as compared to those children who have higher SES (Morgans, Farkas, Hillemeier & Maczuga, 2009).

1.1.2 Social Networking Sites

The study shows that Social Networking sites became the most popular usage habit because of ease to access of technology now-a- days. These sites are known as the platform of getting knowledge and help them improve social connection and source of information (Coyle & Vaughn, 2008; Wang, Chen, & Liang, 2011).Some of the Most commonly using Networking sites (Facebook, twitter, Youtube, Instagram) in these days. The Statistical data shows that there are more than 500 Million people have account on Facebook more than 250 million visit Facebook at least one time each day(Boyd & Ellison, 2007).Williams &Merten (2009) and Raffterty (2009) shows that peoples who have different accounts on SNS because of some reasons like increase the circle of new friends and Sharing information. These peoples have highly addicted of SNS are called "Heavy User". Many researchers explain the Positive and negative effect of this addiction. Negative effects of Social Networking include Behavioral change, wasting of time; heavy adductors mostly spent their time on Facebook, instagram and other sites without any reasons, In return they got stress and sense of crises.

1.1.3 Healthy Environment

Providing the students healthy learning environment where they feel protect and comfortable by physically and mentally. Protected schools not just place with advanced security but this is a place help the student to develop confidence and communicate to teachers. Safe school protects as physically as also develop healthy environment which is helpful for learning skill; learn about fitness, nutrition and health of a Students.

Researchers and educators agreed that climate effect on student achievement. A report of National School Board association shows that positive school climate is critical variable influencing behavioral attitude, Attendance and Academic achievements. Most of the important in defining the environment of school is the quality of student connection with other students and faculty. These healthy environments encourage the student to pick new things quickly with confidence and enhance the learning skills.

1.1.4 Family Structure

No doubt, a lot of factors affecting on educational attainment of students so family factor influencing on performance as well as other have effects. Family background is the fundamental of student's social, psychological, moral advancement and socio-economic attainments. Ajila et al. (2009) shared their views that Home has great influence on child social, emotional, educational and moral advancements. This study purpose to find out the influence of family structure on students' performance during educational career.

2. BAYESIAN INFERENCE

2.1 Analysis of Social Factors

We use C++ language and also using the uniform prior distribution.

The Programming codes are design for the 4 objects which are given in above table of A1 Appendices.

2.1.1 The Bradley Terry Model for Paired Comparisons

Bradly-Terry (1952) developed the basic concept for paired comparison after Zermelo (1929) consideration the states, The Objects or treatments have merit η_i and η_j when we judge on some characteristic and may be represent by the continuous random variable with the following limit.

w_i,$(-\infty < w_i < +\infty)$,"

$$H(\eta_i - \eta_j) = \frac{1}{4} \int_{-(\ln\lambda_i - \ln\lambda_j)}^{\infty} \sec h^2 (w/2) dw \quad 3.11$$

where $i \neq j$ and $i=1,2\ldots m$

$$\omega_{ij} = H(\eta_i - \eta_j) = \frac{1}{4} \int_{-(\ln\lambda_i - \ln\lambda_j)}^{\infty} \sec h^2 (w/2) dw$$

This model shows the difference between two underlying variables $\left(\zeta_i - \zeta_j\right)$ and has logistic density with parameter ($\ln\lambda_i$-$ln\boldsymbol{\lambda}_j$) is called Bradley-Terry Model (1952) and formulated as

$$\omega_{ij}=\lambda_i/[\lambda_i+\lambda_j] \tag{2.0}$$

where ω_{ij} denotes the preference probability for object *i* when $\boldsymbol{\lambda}_I$ and $\boldsymbol{\lambda}_j$ are compared.

Appendix A (Table A3)

The table A3, ranking is done Here we follow the rule is that the factor among these have the highest probability, we give it the highest rank or rank 1 and the which one is lower than 1st rank we give it 2nd rank up to last one which have the lowest probability with lower rank. We have there 4 factors of social factor which are Socio-economic status, social networking, healthy environment and family structure. In Table A3 we collect the data from the students of university of Gujrat more preferred to Healthy environment as an affecting factor that has highest probability is 0.40552 so we give it the rank 1. The other factor which is preferred after healthy environment is Family structure which has probability of preference is 0.37405 so we gave it rank 2. The third factor which is social networking has the probability 0.15808 we gave it 3rd rank. At the end we have the factor Socio-economic status with lowest probability is 0.06235 and it has 4th rank. As we know that we gave the rank according to preference level. Healthy environment have the highest probability so it have strong position and the lowest factor is socio-economic status so it has week position. We also know that expected probabilities of preference have the sum is equal to 1 which is defined as $\sum\lambda_i$=1.

2.2.1 Posterior Predictive Probabilities for Factors of Social Factor Effecting Student Academic Performance

Posterior Predictive probabilities for four Factors effecting student social factor effect on academic performance is calculated for each pair of factors using the program C++. The predictive probabilities for **ith** and **jth** objects is given below for, I < j as;

$$P(ij) = \frac{1}{M} \int_{\lambda_1=0}^{1} \int_{\lambda_2=0}^{1-\lambda_1} \int_{\lambda_3=0}^{1-\lambda_1-\lambda_2} P(\underline{\lambda} \mid \underline{x}).\omega_{ij}\, d\lambda_3\, d\lambda_2\, d\lambda_1 \tag{2.1}$$

Here

$$P(\underline{\lambda} \mid x) = \frac{\prod_{i=1}^{4} (P(\underline{\lambda} \mid \underline{x})i \Big/ P(\lambda i + \lambda j)^{n_i}}{N\left(\prod_{i<j}^{4} (\lambda i/\lambda i + \lambda j + \lambda i/\lambda i + \lambda j)^{r_{ij}}\right)}$$ be the posterior distribution where as

$\omega_{ij}=\frac{\lambda_i}{\lambda_i+\lambda_j}$ be the preference probabilities of 'I' factor over 'j' factor. And here

$\lambda_4=1-\lambda_1-\lambda_2-\lambda_3$ is the constraint on the numerical integration.

Appendix A (Table A4)

In Table A4 we make the possible pairs of social factor to compare the results of preferred factor by students of UOG. From the pair of Socio-economic status and social networking sites, the first factor have the probability is 0.49206 i.e. students of university of Gujrat preferred socio economic status have the preference of this factor is 49.206% and other alternate factor which is social networking sites have the preference of probability is 50.794% total out of 100%. In second pairs of factor which is Socio-economic status and Healthy environment, and the preference probability of socio-economic status on healthy environment is 0.349939 which is 34.9939% of out of 100% and the other alternate of this pair is healthy environment gain 0.650061 probability is 60.001% out of 100% that has effect on student performance. In third of Pair of Socio-economic status and family structure, we get socio-status preferred by the students of UOG have the probability is 0.377153 i.e. students of university of Gujrat preferred the socio-economic status as compare to family structure is 37.7151% out of 100% and the family structured preferred this factor by the students that effects students' academic performance is 62.28% out of 100%. In the fourth one is possible pairs of factor we are going to talk is social networking and healthy environment, in this combination we have preference probability on social networking through the students of university of Gujrat is preferred among this combination is 0.357606 that effects on performance its mean students thoughts 35.606% of this factor out of 100% is effective factor influence on students' academic performance. The opposite pair of social networking is healthy environment that is 0.642394 or students preferred 642394% of this factor influence on performance. In the pair of 5th possible combination of factor that influence on student academic performance is social networking and Family structure, the preference factor of social networking as compare to family structure is chosen by the students has probability is 0.384836 and the probability of family structure is 0.615164 i.e. social networking is 38.4836% effects on the students' performance and family structure preferred more as compare to social networking have influence on performance is 61.5164% out of 100%. The last one pair of comparison of influence is between healthy environment and family structure and here we have probabilities are 0.529447 and 0.470553 respectively. Healthy environment is preferred by the students is 48.8% and family structured preferred on the basis of effects on students' academic performance and more preferred factor in this combination is healthy environment is 52.9447% influential on performance.

2.3 Appropriate of the Model

The null and alternative hypothesis for the model of fitness is given as below

H_0; The model is good fit for the given data

H_1; The model does not fit for the given data

We can calculate the expected values through the following formula

$$\widehat{x_{ij}} = r_{ij}(\omega_{ij}) \qquad \text{For all } i<j$$

Appendix A (Table A5)

With the significance of 5% and the test that follows the Chi-Square distribution as

$$\chi^2 = \sum_{i<j=1}^{m} \left\{ \frac{(x_{ij}-\widehat{x_{ij}})^2}{\widehat{x_{ij}}} + \frac{(x_{ji}-\widehat{x_{ji}})^2}{\widehat{x_{ji}}} \right\}$$

We follow the concept of Aslam (2002) about the degree of freedom in which he considered the choice of degree of freedom according to this formula d.f= m (m-2) =4 (4-2) =8.

In this table we have to use distinct terms that for different donation and here x_{ij} that indicates the observed number of preference of i^{th} factor compared with j^{th} factor. Similarly x_{ji} indicates the observed number of preference for j^{th} factor compared with i^{th} factor. The term $\widehat{x_{ij}}$ indicates the expected number of preference for i^{th} factor compared with jth factor and $\widehat{x_{ji}}$ indicates the expected number of preference by j^{th} factor the Chi-Square test statistic value or calculated value as

$$x_{cal}^2=9.8012455$$

And here we have the table value of chi-square is $x_{(0.05,8)}^2$=15.51

So the x_{cal}^2 doesn't not fall in the critical region that is

$$x_{cal}^2 \ngeq x_{(0.05,8)}^2$$

So we accept null hypothesis, this model is a good fitted.

3. CONCLUSIONS AND RECOMMENDATIONS

Social Factors

We concern about the most preferred factor chosen from students of university of Gujrat in social factor, so we rank the four factors according to highest probabilities as students more preferred among all factors is to the family structured and it have probability higher than of all other factor so we gave it 1st rank and then in 2nd number students gives preference healthy environment so we give it 2nd rank and this process will continued to the last factor. So we can conclude that students of university of Gujrat most preferred factor is Healthy environment and this factor have strong influence upon student academic performance, Family Structure have less effect then previous one on student performance and then the third factor which is social networking sites have lesser probability of previous so it have less effect on student performance then of above of 2nd factors. And at last the less effect on student performance among the entire factor is socio-economic status that influence on students' academic performance.

Recommendation

- In this study we have used paired comparison methodology is considered. It can be further generalized to multiple paired of comparison in experiments.
- We also used preference factors that influence on students' academic performance of university of Gujrat.
- Ranking of factor effecting students satisfaction among students of UOG that face different problems to face different kinds of issues.

REFERENCES

1. Akessa, G.M. and Dhufera, A.G. (2015). Factors That Influences Students Academic Performance: A Case of Rift Valley University, Jimma, Ethiopia. *Journal of Education and Practice*, 6(22), 55-63.
2. Brice, C. and Smith, A. (2001). The effects of caffeine on simulated driving, subjective alertness and sustained attention. *Human Psychopharmacology: Clinical and Experimental*, 16(7), 523-531.

3. Brice, C. and Smith, A. (2001). The effects of caffeine on simulated driving, subjective alertness and sustained attention. *Human Psychopharmacology: Clinical and Experimental*, 16(7), 523-531.
4. Childs, E. and De Wit, H. (2006). Subjective, behavioral, and physiological effects of acute caffeine in light, nondependent caffeine users. *Psychopharmacology*, 185(4), 514-523.
5. Tariq, W., Mehboob, M., Khan, M.A. and Ullah, F. (2012). The impact of social media and social networks on education and students of Pakistan. *International Journal of Computer Science Issues (IJCSI)*, 9(4), 407-411.
6. Farooq, M.S., Chaudhry, A.H., Shafiq, M. and Berhanu, G. (2011). Factors affecting students' quality of academic performance: a case of secondary school level. *Journal of Quality and Technology Management*, 7(2), 1-14.
7. Hasenfratz, M. and Bättig, K. (1992). No psychophysiological interactions between caffeine and stress? *Psychopharmacology*, 109(3), 283-290.
8. Hemmingsson, T. and Lundberg, I. (1998). Work control, work demands, and work social support in relation to alcoholism among young men. *Alcoholism: Clinical and Experimental Research*, 22(4), 921-927.
9. Higgins, A.Z. (2011). *Psychosocial Effects on Academic Performance* (Doctoral dissertation, Eastern Kentucky University).
10. Lara, D.R. (2010). Caffeine, mental health, and psychiatric disorders. *Journal of Alzheimer's Disease*, 20(1), 239-248.
11. Lorist, M.M., Snel, J., Kok, A. and Mulder, G. (1994). Influence of caffeine on selective attention in well-rested and fatigued subjects. *Psychophysiology*, 31(6), 525-534.
12. Martin, A.J. and Marsh, H.W. (2003). Fear of failure: Friend or foe. *Australian Psychologist*, 38(1), 31-38.
13. May, C.P., Hasher, L. and Foong, N. (2005). Implicit memory, age, and time of day: paradoxical priming effects. *Psychological Science*, 16(2), 96-100.
14. May, C.P., Hasher, L. and Stoltzfus, E.R. (1993). Optimal time of day and the magnitude of age differences in memory. *Psychological Science*, 4(5), 326-330.
15. Regina, E.G., Smith, G.M., Keiper, C.G. and McKelvey, R.K. (1974). Effects of caffeine on alertness in simulated automobile driving. *Journal of Applied Psychology*, 59(4), 483-493.
16. Ryan, L., Hatfield, C. and Hofstetter, M. (2002). Caffeine reduces time-of-day effects on memory performance in older adults. *Psychological Science*, 13(1), 68-71.
17. Shore, E.R. (1992). Drinking patterns and problems among women in paid employment. *Alcohol Research and Health*, 16(2), 160-164.
18. Smith, A., Sutherland, D. and Christopher, G. (2005). Effects of repeated doses of caffeine on mood and performance of alert and fatigued volunteers. *Journal of Psychopharmacology*, 19(6), 620-626.
19. Syed Tahir Hijazi, S.R. (2006). Factors Affecting Students' Performance. 01-10.
20. Walters, E.R. and Lesk, V.E. (2015). Time of day and caffeine influence some neuropsychological tests in the elderly. *Psychological assessment*, 27(1), 161-168.

APPENDIX A

Table A1

(Social Life)	Nij	Nji
SES,SN	27	23
SES,HE	9	41
SES,FS	18	32
SN,HE	24	26
SN,FS	20	30
HE,FS	24	26

Table A2

SES	SNS	HE	FS
0.06235	0.15808	0.40552	0.37405

Table A3
Ranking of factor Affecting Students' Academic Performance

Factors affecting students' academic performance	Expected probabilities (λ_i)	Rank
Socio-economic status	0.06235	(4)
Social networking	0.15808	(3)
Healthy environment	0.40552	(1)
Family structure	0.37405	(2)

Table A4
Posterior Predictive Probabilities for Factor Effecting Student Academic Performance

Pij	Estimates	Pji	Estimates(1-pij)
P(1,2)	0.49206	P(2,1)	0.50794
P(1,3)	0.349939	P(3,1)	0.650061
P(1,4)	0.377153	P(4,1)	0.622847
P(2,3)	0.357606	P(3,2)	0.642394
P(2,4)	0.384836	P(4.2)	0.615164
P(3,4)	0.529447	P(4,3)	0.470553

Table A5
(Observed and Expected Number of Preferences of Factor for hostel Students)

Xij	$\widehat{x_{ij}}$	Xji	$\widehat{x_{ji}}$
12	0.324662619	28	0.4149210170
9	0.819541346	41	.126007109
11	2.081734869	39	0.34700219
2	4.536073909	28	1.768254497
20	1.78316911	30	0.75359811
24	0.155211389	26	0.16826981

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CHILD LABOR IN PAKISTAN; BAYESIAN INFERENCE OF STRUCTURAL FACTORS VIA UNIFORM PRIOR

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ABSTRACT

This study focused the factors causing child labor in Pakistan after the consideration of Adiqa Kausar Kaini (2010) who considered the risk factors generating Child labor in Pakistan. This research is based on the Bayesian inference for the factors of child labor in Pakistan. Bayes' estimators against each factor are computed which reflects the overall worth probabilities for each factors. The ranking is done and posterior predictive probabilities are computed for each of the ten on pairs of Structural factors of child labor for future single comparisons of each pair. The ranking is done and posterior predictive probabilities are computed for each of the six on pairs of family factors of child labor for future single comparisons of each pair. The ranking is done and posterior predictive probabilities are computed for each of the six on pairs of individual factors of child labor for future single comparisons of each pair Another ranking is performed for the factors of child labor in Pakistan using the same Bayesian paradigm. Results for both analyses are computed in C language and programs coding are designed for seven parameters' inference. The model ensures good fit for data i.e., structural factors, family factors and individual factors of child labor of in Pakistan. The research aims to play a vital role in overcoming the national level problem of child labor by providing the preferences probabilities which reflects ranking of each of the factors.

KEY WORDS

Paired comparison method; Bayesian Statistics, Posterior Means, Ranking, Posterior Predictive Probabilities Bradley-Terry Model, Non informative prior.

1. INTRODUCTION

1.1 Child Labor

Child labor is a big problem all over the world which accounted for approximately 350 million children are child labor (Human Rights Watch, 2004). Child labor is a multifaceted problem in developing countries and forms of child labor vary according to the culture of the country, family, area of residence economic background and level of development (Halgad et al. 2014).

1.2 Child Labor in Pakistan

The children are used to render cheap labor in the factories, shops and business centers. They are also employed at the filling stations, bakeries, hotels and offices of different firms. In our country, there exists a law against child labor, but it does not operate practically. About one third of working children were literate, boys being more educated than girls and urban children were educated more than rural children (Kiani 2010).

Child labor is a common thing in Pakistan. It is generally and morally harsh for children and an invasion among the youngsters and their education, and powers them to leave of absence the school (ILO 2006).

1.3 Factors of Child Labor

For inference, we considered structural factors below;

1.3.1 Structural Factors

Structural factors including Economic development, Migration & urban poverty, societal norms, legal context and geographical area.

1.3.2 Economic Development

Pakistan's current population is a burden on Pakistan's economy. About 39 percent of Pakistan is living below the poverty line. "Poverty line means no access to their basic needs of the people" (Tesfey, 2003).

1.3.3 Migration & Urban Poverty

Movement from rural to urban areas is narrowly related to growing urban poverty and later, to child labor. In urban areas children are worked. They are also employed at the filling station, bakeries, hotels and offices of different firms. Many rural families migrate to urban areas in the rural push and urban pull factors. As a result, they often have children as vendors. These urban poverty, children of laborers in the food, accommodation and their children living in the street Workers on the street as there is lack of access to basic needs.

Many cities in developing countries have experienced rapid urbanization. This means that the urban population is increasing due to immigration and natural growth. Urban poverty is a multidimensional phenomenon. Urban poverty in developing countries faces many challenges in their daily lives. Much poor unemployment, housing shortages, due to the violence and unhealthy environment, living under great hardship (Osment, 2014).

1.3.4 Societal Norms

In every society, there is a set of rules that determine the responsibility of different behavioral standards within the household duties. This principle works are accepted by the community and the impact on their children and vice versa by parental attitudes. The different ways in which they can be enforced social norms or conventions (Calva, 2002).

1.3.5 Legal Context

Child labor is defined as 'work that deprives children of their childhood, their possible and their self-confidence, and that is injurious to physical and mental health.

1.3.6 Geographical Area

Children's work takes place in both rural and urban areas. In cities, children work as street vendors polishing shoes or selling water (Altuntaş, 2006). Majority of the child workers were found in Punjab. It was also found that children involved in work about 8 times in the rural areas which may be greater than in urban areas. About one third of working children were literate, boys being more educated than girls and urban children were educated more than rural children (Kiani 2010).

2. BAYESIAN ANALYSES

Bayesian analysis is performed for the data in Table 1 using complex integrating programming in C language and using the Uniform prior distribution. The programming codes are designed for 5 objects and given in the table of Appendix.

2.1 Structural Factors

Bradley-Terry Model

The Bradley-Terry model (1952) is used for the parameters of factors affecting terrorism in Pakistan. The probability of observed results in s^{th} recurrence of pair of parameters is given as follow.

$$\omega_{ij}=\frac{\theta_i}{\theta_i+\theta_j}$$

From Table 1 'Ed' denote the Economic development, 'Mu' denote the Migration & urban poverty, 'Sn' denote the societal norms, 'Lc' denote the Legal context, 'Ga' denotes the geographical area. 'Nij denotes the number of preferences of 1st factor, and Nji denotes the number of preferences for 2nd factor for each pair of factors respectively.

Appendix A (Table A2)

In this table Economic development has the probability 0.0778 i.e. faculty members of university of Gujarat preference of this factor are 7.778% out of 100%. Migration& urban poverty has the probability 0.3258 i.e. faculty members of university of Gujarat preference of this factor are 32.58% out of 100%. Societal norms has the probability 0.1289 i.e. faculty members of university of Gujarat preference of this factor are 12.89% out of 100%. Legal context has the probability 0.1289 i.e. faculty members of university of Gujarat preference of this factor are 12.89% out of 100%. Geographical area has the probability 0.33941 i.e. faculty members of university of Gujarat preference of this factor are 39.411% out of 100%. In structural factors geographical area has the greatest probability as compare to other factors.

Ranking of Structural Factors:

Appendix A (Table 3)

This table shows the ranking of the structural factors affecting child labor in Pakistan. The faculty of university of Gujarat more preferred the geographical area as an affecting factor that has the maximum probability of the preference that probability is 0.39411 so we give the first rank order of the Geographical area and the second rank goes to the migration & urban poverty the preferences probability is 0.32425 that probability is less than the se Geographical area so give the second rank and the further third, fourth, fifth. Order is given to the according societal norms, legal context, and economic development. In here ranks of third and fourth are same so we and her rank and divide by two so we get 3.5 its average rank and put in the place of third and fourth rank. We also know that has less probability of the preferences of the first one so we gave the less order or rank to the first one. In this table we can see the more preferred affecting factor of child labor is geographical area and have a strong position and that less preferred affecting factor of child labor is economic development most faculty members of university of Gujarat are not preferred that factor have a weak position. Expected probabilities of the preferences is a long term probabilities we know that the $\sum\theta_i$=1.

Posterior Predictive Probabilities for Structural Factors

Predictive probabilities for five structural factors of child labor are computed for each pair of factors using a program designed for five objects in C++. The predictive probability for **i**$^{\mathbf{th}}$ and **j**$^{\mathbf{th}}$ objects is given follow for, I < j as;

$p_{(ij)}$=

$$p_{(ij)}=\frac{1}{M}\int_{\vartheta_1=0}^{1}\int_{\vartheta_2=0}^{1-\vartheta_1}\int_{\vartheta_3=0}^{1-\vartheta_1-\vartheta_2}\int_{\vartheta_4=0}^{1-\vartheta_1-\vartheta_2-\vartheta_3} P(\vartheta/x).$$

$$\omega_i d\vartheta_4 d\vartheta_3 d\vartheta_2 d\vartheta_1$$

Here $p(\vartheta/x)=\dfrac{\prod_{i=1}^{5}(\theta i)^{n_i}}{M\left(\prod_{i<j}^{5}(\theta i+\theta j)^{r_{ij}}\right)}$ be the posterior distribution

$\omega_{ij}=\frac{\vartheta_i}{\vartheta_i+\vartheta_j}$ be the preference probabilities of 'I' factor over 'j' factor.

Here $\vartheta_5=1-\vartheta_1-\vartheta_2-\vartheta_3-\vartheta_4$ is the constraint on the numerical integration.

Posterior Predictive Probabilities for Structural Factors

Appendix A (Table 6)

From the above table we also see that in a single comparison the first factor is economic development have 0.364592 probabilities i.e. faculty members of university of Gujarat preference of this factor is 36.4592% out of 100%. Migration & urban poverty has the probability 0.635408 i.e. faculty members of university of Gujarat preference of this factor is 63.5408%. From this single pair of factors affecting the child labor we can see migration & has greater preference than the economic development. In second pair of economic development have 0.453647 probability i.e. faculty members of university of Gujarat preference of this factor is 45.3647% out of 100%. Societal norms has the probability 0.546353 an i.e. faculty members of university of Gujarat preference of this factor is 54.6353% and in this pair of Societal norms have greater preference than the economic development. From the third pair of factor economic development has 0.453647 probabilities i.e. faculty members of university of Gujarat preference of this factor is 45.3647% out of 100%. Legal context has the probability 0.546353 an i.e. faculty members of university of Gujarat preference of this factor is 54.6353% and in this pair of legal context have greater preference than the economic development. From the four pair of economic development has 0.353334 probability i.e. faculty members of university of Gujarat preference of this factor is 35.3334% out of 100%. Geographical area has the probability 0.646666 an i.e. faculty members of university of Gujarat preference of this factor is 64.666% and in this pair of f geographical area have greater preference than the economic development. From the five pair of migration & urban poverty has 0.59138 probability i.e. faculty members of university of Gujarat preference of this factor is 59.138% out of 100%. Societal norms has the probability 0.40862 i.e. faculty members of university of Gujarat preference of this factor is 40.862% and in this pair of migration & urban poverty have greater preference than the societal norms. From the next pair of factor migration & urban poverty has 0.59138 probability i.e. faculty member of university of Gujarat preference of this factor is 59.138% out of 100%. Legal context has the probability 0.40862 an i.e. faculty member of university of Gujarat preference of this factor is 40.862% and in this pair of migration & urban poverty have greater preference than the legal context. From the next pair of migration & urban poverty has 0.487765 probability i.e. faculty members of university of Gujarat preference of this factor is 48.7765% out of 100%. Geographical area has the probability 0.512235 an i.e. faculty members of university of Gujarat preference of this factor is 51.2235% and in this pair of geographical area have greater preference than the migration & urban poverty. From the next pair of factor societal norms has 0.5 probability i.e. faculty member of university of Gujarat preference of this factor is 50% out of 100%. Legal context has the probability 0.5 an i.e. faculty members of university of Gujarat preference of this factor 50% and in this pair of factors societal norms and legal context have same probability. From the next pair of societal norms has 0.396847 probability i.e. faculty member of university of Gujarat preference of this factor is 39.6847% out of 100%. Geographical area has the probability 0.603153 an i.e. faculty member of university of Gujarat preference of this factor 60.3153% and in this pair of geographical area have greater preference than the societal norms. From the next pair legal context has 0.396847 probability i.e. faculty of university of Gujarat preference of this factor is 39.6847% out of 100%. Geographical area has the probability 0.603153 an i.e. faculty member of university of Gujarat preference of this factor 60.3153% and in this pair of factors geographical area have greater preference than the legal context.

Appropriateness of Model for Structural Factors

The null and alternative hypotheses are as follow

H_0; The model is good fit of the data

H_1; The model does not fit the data

We calculate the expected frequencies by the following formula

$\widehat{x_{ij}}=r_{ij}(\omega_{ij})$

For all i<j

Appendix A (Table 7)

The level of significance is 5%

The test statistic follows the Chi-Square distribution as

$$\chi^2 = \sum_{i<j=1}^{m} \left\{ \frac{(x_{ij} - x_{ij})^2}{x_{ij}} + \frac{(x_{ji} - x_{ji})^2}{x_{ji}} \right\}$$

We follow the consideration by Aslam (2002) about the degree of freedom in which he considered the choice of degree of freedom by this formula d.f=m (m-2) =15

In Table 13 x_{ij} denoted the observed number of preference for i^{th} university compared with j^{th} university. Similarly x_{ji} denoted the observed number of preference for j^{th} university compared with i^{th} university. The term x_{ij} denotes the expected number of preference for i^{th} university compared with j^{th} university and x_{ji} denotes the expected number of preference for j^{th} university compared with i^{th} university.

We calculate the Chi-Square test statistic value as

χ^2_{cal} = 23.53062482

The table value of Chi- Square (0.05, 15) = 25.00

Since the χ^2_{cal} doesn't not fall in the critical region that is

$\chi^2_{cal} \ngeq \chi^2_{(0.05,15)}$

So as a conclusion we have no evidence to reject the null hypothesis. We conclude that the model good fits the data.

3. CONCLUSION

This study revealed the preferences of faculty members (experts) for the factors influencing child labor in Pakistan and the ranking with predictive inference factors affecting child labor. To know the preference of the faculty members is an important task for the child labor. For factors of affecting child labor ranks first. For the analysis of data we used paired comparison using Bayesian inferences via non-informative uniform prior. For this purpose C language is used for programming and results are generated for each program about estimation of parameters for posterior means and posterior predictive probabilities. The faculty members more preferred the structural factors of child labor factor are geographical area (rural, urban) that has the highest worth probability. and the second rank goes to the Migration &Urban poverty that probability is less than one so give the second rank and the future third, fourth, fifth order is given to according societal norms, legal context and economic development. The more preferred structural factor of the child labor in Pakistan is 'geographical area' most faculty members preferred that factor and have a strong influence on the child labor and that less preferred factor of child labor in Pakistan is 'economic development' most faculty members are not preferred that factor and have a weak influence on the child labor

The research aims to play a dynamic role in overcoming the national level problem of terrorism by providing the preferences probabilities which reflects ranking of each of the factors.

4. RECOMMENDATION

In this research paired comparisons methodology is considered. It can be further generalized to the multiple comparisons experimentations.

In our study we analyzed the preferences of factors affecting child labor in Pakistan for which the experts' opinions were collected from faculty. In future research may be conduct from experts the NGO's.

Ranking of factors of Child labor within Pakistan is done and can be attained for different countries facing child labor.

REFERENCES

1. Ali, K. and Hamid, A. (1999). Major determinants of female child labour in urban Multan (Punjab-Pakistan).
2. Bahar, O.S. (2014). An overview of the risk factors that contribute to child labor in Turkey: Implications for research, policy, and practice. *International Social Work,* 57(6), 688-697.
3. Basu, K. (2001). International labor standards and child labor *Globalization, Institutions and Social Cohesion,* 115-123. Springer.
4. Emerson, P.M. and Souza, A.P. (2008). Birth order, child labor, and school attendance in Brazil. *World Development,* 36(9), 1647-1664.
5. Haider, S.Z. and Qureshi, A. (2016). Are All Children Equal? Causative Factors of Child Labour in Selected Districts of South Punjab, Pakistan. *Journal of New Approaches in Educational Research,* 5(1), 3-10.
6. Hamid, S. and Ahmed, K. (1994). A Micro Analysis of Urban Child Labour: Some Determinants of Labour and its Conditions [with Comments]. *The Pakistan Development Review,* 33(4), 1249-1271.

7. Johansson, J. (2009). Causes of Child Labour: A case study in Babati town, Tanzania.
8. Kazmi, S.M.A. (2015). *How Does Socio-economic Factors Force Children Into Child Labour?: A Case Study of Sahiwal District, Punjab, Pakistan*. Sustainable Development Policy Institute.
9. Khalid, U. (2004). Socio Economic Conditions of Child Labourers in Pakistan: Evidence from the Labour Force Survey Umer Khalid & Lubna Shahnaz. *Lahore Journal of Economics*, 9(1), 86-105.
10. Kiani, A.K. (2010). A significant impact of child labor on Pakistan's Economy. *Globelics 2010, 8th international conference*, University of Malaya, Malaysia.
11. Koseleci, N. and Rosati, F.C. (2009). Child labour and the global financial crisis: an issues paper. *Understanding Children's Work Programme Working Paper*.
12. Labor, C. (2004). Christopher Udry Yale University udry@yale.edu.
13. López-Calva, L.F. (2002). A social stigma model of child labor. *Estudios Economicos*, 193-217.
14. Mridul, M. and Singh, M. (2008). *Factors affecting Child Labour in India*: Indian Institute of Management Ahmedabad, Research and Publication Department.
15. Nafees, A.A., Khan, K.S., Fatmi, Z. and Aslam, M. (2012). Situation analysis of child labour in Karachi, Pakistan: a qualitative study. *Journal of Pakistan Medical Association*, 62(10), 1075-1082.
16. Osment, L. (2014). Child labour; the effect on child, causes and remedies to the revolving menace.
17. Rashid, M.A.H., Farid, S. and Manzoor, M.M. (2015). Parents and Child Labour: A Testimony from Automobile Workshops in Islamabad. *Advances in Applied Sociology*, 5(8), 234-241.
18. Rena, P. (2009). The child labour in developing countries: a challenge to millennium development goals. *Indus Journal of Management & Social Sciences*, 3(1), 1-8.
19. Shunsuke, S. (2006). Parental Attitudes toward Children and Child Labor, Discussion Paper. HiHitotsubashi University Repository
20. Siddiqi, F. and Patrinos, H.A. (1995). *Child labor: Issues, causes and interventions*: World Bank Washington, DC.
21. Tesfay, N.K. (2003). *Child labour and Economic Growth* (Doctoral dissertation).
22. Tunca, G. (2009). *Gelişmekte olan ülkelerde çocuk çalıştırılması ve mücadele stratejileri.* DEÜ Sosyal Bilimleri Enstitüsü.
23. Zafar, T., Younes, I. and Malik, K.S. (2014) Socio Economic Conditions of Child Labor in Lahore District.

APPENDICES

Appendix A

Table 1

Pairs of Structural Factors	Nij	Nji
Ed,Mu	47	53
Ed,Sn	44	56
Ed,Lc	32	68
Ed,Ga	39	61
Mu,Sn	72	28
Mu,Lc	64	36
Mu,Ga	42	58
Sn,Lc	57	43
Sn,Ga	44	56
Lc,Ga	38	62

Table 2

Economic development	Migration &Urban poverty	Societal norms	Legal context	Geographical area
0.0778	0.3258	0.1289	0.1289	0.3391

Table 3

Structural Factors	Expected probabilities (ϑ_i)	Rank
Economic development	0.05379	(5)
Migration &Urban poverty	0.32425	(2)
Societal norms	0.13642	(3.5)
Legal context	0.13642	(3.5)
Geographical area	0.39411	(1)

Table 4
Ranking of Family Factors

Family Factors affecting Child labor	Expected probabilities (ϑ_i)	Rank
Household income	0.27514	(2)
Household size	0.19898	(3)
Parental education	0.34814	(1)
Parental beliefs	0.17773	(4)

Table 5
Ranking of Individual Factors

Individual Factors affecting Child labor	Expected probabilities (ϑ_i)	Rank
Age	0.38814	(1)
Gender	0.29782	(3)
Child birth order	0.31404	(2)

Table 6
Posterior Predictive Probabilities for Structural Factors

P(ij)	Estimate	P(ji)=1-P(ij)	Estimate
P(12)	0.364592	P(21)	0.635408
P(13)	0.453647	P(31)	0.546353
P(14)	0.453647	P(41)	0.453647
P(15)	0.353334	P(51)	0.646666
P(23)	0.59138	P(32)	0.40862
P(24)	0.59138	P(42)	0.40682
P(25)	0.487765	P(52)	0.512235
P(34)	0.5	P(43)	0.5
P(35)	0.396847	P(53)	0.603153
P(45)	0.396847	P(54)	0.603153

Table 7
Observed and Expected Preferences

X_{ij}	x_{ij}	X_{ji}	x_{ji}
70	58.02328	30	41.97672
43	44.1374	57	55.8626
50	60.7459	50	39.2541
46	36.722	54	63.278
55	52.825	45	47.175
75	66.195	25	33.805